3rd Polish Congress of Mechanics

21st International Conference on Computer Methods in Mechanics

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Tadeusz Burczyński
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Karol Winkelmann
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3rd Polish Congress of Mechanics

and

21st International Conference
on
Computer Methods in Mechanics

Short Papers

Vol. 2

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Preface

This book brings us a great honour and pleasure to present the papers selected for presentation at the PCM-CMM-2015 CONGRESS held on 8-11 September, 2015 in Gdańsk (Poland).

The PCM-CMM-2015 CONGRESS is a joint scientific event of the 3rd Polish Congress of Mechanics (PCM) and the 21st International Conference on Computer Methods in Mechanics (CMM).

The idea of a Polish Congress of Mechanics was firstly suggested in 2005 by the Polish Society of Theoretical and Applied Mechanics. The scope was intended to cover the whole range of problems of theoretical, experimental and computational mechanics as well as interdisciplinary issues, including industrial applications.

The 21st International Conference on Computer Methods in Mechanics continues the 44-year series of conferences dedicated to numerical methods and their applications to the mechanics-based problems. The meetings, organized biannually since 1973 provide a forum for presentation and discussion of new ideas referring to the theoretical background and practical applications of computational mechanics.

Both events – the 3rd Polish Congress of Mechanics (PCM) and the 21st Conference on Computer Methods in Mechanics (CMM) – are aimed at presenting current state-of-the-art research in the field of mechanics and providing a wide forum for discussion of new ideas on theoretical background, new technologies and computational methods in a vast domain of mechanics and related disciplines. We believe that the Congress becomes an event to trigger discussions, exchange of new ideas and valuable solutions in numerous aspects of mechanics. We hope that the book dedicated to academics, researchers, designers and engineers dealing with various problems of mechanics will take appropriate interest and meet a broad readers’ response.

Each paper submitted to PCM-CMM-2015 CONGRESS and printed in the book has been reviewed by members of the Scientific Committee and the International Advisory Board and refined by the Authors according to the referee comments. We are deeply indebted to all members the SC and IAB for their help in shaping the programme of the Conference and their important contribution to the publishing process of the book volumes. Most of the final texts have been additionally adjusted to technical requirements of the publisher, the English of some texts has been refined too. We would like to send our words of gratitude to our associates: Marek Skowronek, Marcin Kujawa, Anna Mleczek, Karol Daszkiewicz, Beata Zima, Aleksandra Mariak and Jacek Lachowicz for their assistance and help in bringing the volume to its final form.

Tadeusz Burczyński
Chairman of the Scientific Committee
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Mini-symposia
MS11

Mechanics of Complex Materials, Structures and Processes

organized by G. Mishuris and N. Movchan
Impact-induced fissuring of articular cartilage: an asymptotic modelling study

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Abstract

Articular cartilage is a soft connective biological tissue, which is prone to injuries under severe impact. In articular cartilage, fissuring is known to reflect mechanical damage of collagen matrix. The initiation of surface fissuring in cartilage layers subjected to impact loading is studied in the framework of the first-order asymptotic model for contact between thin incompressible elastic layers bonded to rigid substrates. The outcomes of the analysis are compared with results available in literature.

Keywords: blunt impact, articular cartilage, impact-induced fissuring, asymptotic modelling

1. Introduction

In recent years a substantial progress has been made in developing the impact testing methodologies for biological tissues and soft biomedical materials [17]. It was shown [2, 6] that linear and quasi-linear viscoelastic models are not capable of describing many sensitive features of observed impact phenomena for soft tissues [13, 14, 17]. The hypothesis that the damage accumulation during the impact of a biological tissue can be responsible of some peculiarities in the drop-weight impact experiments, which show increasing amounts of damage in the articular cartilage samples [13], was recently studied in [5]. In particular, the damage progression in high-strain rate and impact tests on articular cartilage was considered. A new type of kinetic damage evolution law is proposed and used to draw implications about the accumulated damage. Based on the developed damage model, a new fracture criterion is introduced.

In the study we consider the impact-induced fissuring of articular cartilage samples, while utilizing a simple asymptotic model for describing the contact pressure distribution. It is assumed that under impact loading, articular cartilage deforms like an incompressible material [15], and thus the contact problem can be considered using asymptotic solutions previously developed in a number of papers [1, 4, 8, 11].

2. Leading-order asymptotic model for the unilateral contact of thin bonded incompressible elastic layers

We consider two thin incompressible elastic layers of uniform thicknesses \( h_1 \) and \( h_2 \) ideally bonded to rigid substrates (see Fig. 1). Let \( \varphi(y) \) and \( \delta_0 \) denote the gap between the layer surfaces before deformation and the contact approach of the rigid substrates under the external normal load, respectively.

According to the asymptotic analysis [3, 4], the contact pressure density, \( p(y) \), which is assumed to be positive, should satisfy the differential equation

\[
-\left( \frac{h_1^3}{3G_1} + \frac{h_2^3}{3G_2} \right) \Delta_y p(y) = \delta_0 - \varphi(y), \quad y \in \omega. \tag{1}
\]

Here, \( \Delta_y = \partial^2 / \partial y_1^2 + \partial^2 / \partial y_2^2 \) is the Laplace operator, \( G_n \) is the out-of-plane shear modulus of the \( n \)-th layer \( (n = 1, 2) \), \( y = (y_1, y_2) \) are the in-plane Cartesian coordinates, and \( \omega \) is the contact area, which is not given a priori.

Figure 1: Contact of two thin incompressible elastic layers in the initial undeformed configuration

In the case of unilateral contact, the contact area should be determined in the process of solving Eqn (1) with respect to the following boundary conditions [8, 11]:

\[
p(y) = 0, \quad \frac{\partial p}{\partial n}(y) = 0, \quad y \in \Gamma. \tag{2}
\]

Here, \( \partial / \partial n \) is the normal derivative at the contour \( \Gamma \) of the domain \( \omega \).

Finally, denoting by \( F \) the total external load applied to the substrates, which produces the contact approach \( \delta_0 \), we write out the equilibrium equation

\[
\iint_\omega p(y) \, dy = F. \tag{3}
\]

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Equations (1)–(3) constitute the leading-order asymptotic model of the unilateral contact for thin bonded incompressible elastic layers.

3. Discussion and conclusion

An impact loading of the joints can be viewed as the action of extremely high non-physiological loads applied very rapidly (with the load rise-times of the order of milliseconds or shorter [7]), for instance, due to a car accident, sports injury, or a fall from a height. It is believed that severe impacts on articular cartilage layers can initiate post-traumatic arthritis [16, 18]. As in articular cartilage [12], fissuring is likely to reflect mechanical damage to collagen matrix.

The performed analysis supports the finding [9, 10] that shear stress can be used as a predictor for the initiation of surface fissuring in cartilage.

References

Regular vibration of the oscillator with two nonlinear serially connected springs – asymptotic approach

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Abstract

The analysis is aimed on dynamical regular behaviour of the nonlinear oscillator with two serially connected springs of cubic type nonlinearity. The dynamics of such a system is described by a set of differential - algebraic equations (DAEs). This system is solved by the asymptotic multiple scales method in time domain. The classical approach was appropriately modified to solve the governing DAEs. The analytical approximated solution was verified by numerical simulations and the permissible range of the nonlinearity parameter was estimated. The analytical form of the solution allows, among others, to formulate an explicit relationship between period and amplitude of the system periodic oscillations.

Keywords: nonlinear oscillator, free vibration, asymptotic methods, analytical approximate solution

1. Introduction

The linear simplification is sometimes too rough to describe the behavior of a physical object enough precisely. Therefore, models of nonlinear oscillators were widely considered in physics and engineering. Nonlinear oscillators with serially connected springs were investigated, mostly numerically, by many authors. Most papers concern a case, when one of the springs is linear and the second one is nonlinear \([1,2,3]\).

In the paper an oscillator with two nonlinear springs is analyzed using multiple time scale method (MMS). Telli and Kopmaz \([1]\) showed that the motion of a mass mounted by connected springs were investigated, mostly numerically, by many authors. Most papers concern a case, when one of the springs is linear and the second one is nonlinear \([1,2,3]\). The dynamics of such a system is described by a set of differential-algebraic equations. Similar situation occurs in our investigation. It implies a need to a modify the standard MMS.

2. Formulation of the problem

Let us consider a one-dimensional motion of a body of mass \(m\) attached by massless nonlinear springs to an immoveable support. The studied system is shown in the Fig. 1.

![Figure 1: Series connection of two nonlinear springs](image)

The restoring force of the spring with cubic nonlinearity is

\[ F_i = k_i \left( \Delta l_i^1 + \Lambda_i \Delta l_i^2 \right), \quad i = 1,2 \]

where \(\Delta l_i\) is the elongation of the \(i\)-th spring, \(k_i\) is the constant stiffness and \(\Lambda_i\) is the nonlinearity parameter.

Such type of nonlinear elasticity is widely discussed in papers concerning nonlinear dynamics \([1,4]\). When \(\Lambda_i > 0\) the characteristics of the spring is called "hard", while for \(\Lambda_i < 0\) the characteristics is called "soft". Further we consider only the case \(\Lambda_i > 0\).

Two equations describe motion of the system. One of them is the differential equation and results from the Newton second law. The second one is the algebraic equation describing equilibrium at the massless connection point S. The governing equations of the system are as follows:

\[ m \ddot{x}_1 + k_1(x_1 - x_2) + \Lambda_1(x_1^2 - x_2^2) = 0, \quad (2) \]

\[ k_2(x_1 + \Lambda_2 x_2^2) - k_1(x_2 - x_1)^2 = 0. \quad (3) \]

The initial conditions referring to equation (2) are

\[ x_1(0) = x_0, \dot{x}_1(0) = v_0. \quad (4) \]

Let us introduce the effective stiffness according to the linear part of the elastic spring force

\[ k_e = \frac{k_1 k_2}{k_1 + k_2} \quad (5) \]

If we define the characteristic length \(\delta = \frac{mg}{k_e}\) and characteristic time \(\tau = \sqrt{\frac{m}{k_e}}\), the dimensionless version of Eqs. (2)–(4) can be written as

\[ x_1^2(1 + \alpha x_1) = x_1^2(1 + \alpha x_1) = 0, \quad (6) \]

\[ x_2^2(1 + \alpha x_2) - x_2^2(1 + \alpha x_2) = 0. \quad (7) \]

*This work has been supported by the Polish National Science Centre, MAESTRO 2, No. 2012/04/A/ST8/00738 and by the grant 02/21/DSPB/3463.
where: \( x_i = X_i / \delta \), \( \alpha_i = \lambda, \delta^2 \) for \( i = 1, 2 \), \( \lambda = k_i / k_0 \), \( \delta = x_0 / \delta \), \( v_0 = V_0 / (\delta x_0) \). The dots over symbols denote derivatives with respect to the dimensionless time \( \tau = \alpha_0 t \).

Let us introduce the following two new functions
\[
\ddot{u} + \ddot{u} + \left(1 + \alpha \right) w \left(1 + \alpha \omega^2 \right) = 0, \tag{10}
\]
\[
\lambda w \left(1 + \alpha \omega^2 \right) - \left(1 + \alpha \omega^2 \right) = 0, \tag{11}
\]
\[
w(0) + u(0) = x_0, \dot{w}(0) + \dot{u}(0) = v_0. \tag{12}
\]

3. Method of solution

The problem (10)-(12) can be efficiently solved analytically using the asymptotic multiple scale method [4], although the approach requires some significant modification. The assumptions of smallness of the nonlinearity parameters are proposed in the form
\[
\alpha_i = \bar{\alpha}_i \varepsilon, \alpha_i = \bar{\alpha}_i \varepsilon, \tag{13}
\]
where \( \varepsilon \) is a small perturbation parameter.

Adopting two time scales in the analysis, the solution is investigated in the series form with respect to a small parameter
\[
w(t, \varepsilon) = \sum_{k=0}^{\infty} \varepsilon^k w_k(t, \tau), \tag{14}
\]
\[
u(t, \varepsilon) = \sum_{k=0}^{\infty} \varepsilon^k u_k(t, \tau), \tag{15}
\]
and the differential operator has the form
\[
\frac{d}{d\tau} = \frac{\partial}{\partial \tau} + \varepsilon \frac{\partial}{\partial \tau}, \tag{16}
\]
where \( \tau_0 = \tau \) and \( \tau_1 = \varepsilon \tau \) are the fast and slow time, respectively.

4. Results

Using the modified MSM, one may obtain the approximate solution for function \( w(t) \) in the analytical form
\[
w(t) = \bar{b}_0 \cos \left( \tau + \psi_0 + \frac{3b_0^2 \left(\bar{\alpha}_i + \bar{\alpha}_2 \right)}{2(1 + \lambda)} \right) +
\]
\[
\frac{b_0^3 \left(\bar{\alpha}_2 - 8 \bar{\alpha}_2 \lambda + 9 \bar{\alpha}_2 \lambda^2 \right)}{32(1 + \lambda)} \cos \left( \frac{9b_0^6 \left(\bar{\alpha}_i + \bar{\alpha}_2 \right)}{8(1 + \lambda)} + 3 \tau + 3\psi_0 \right). \tag{17}
\]

From solution (17) we can easily derive the period of the primary vibration
\[
T = \frac{16\pi \left(\bar{\alpha}_i + 1 \right)}{3 \bar{\alpha}_2 b_0^2 \lambda^2 + 3 \bar{\alpha}_i b_0^2 + 8(\lambda + 1)}. \tag{18}
\]

Expression (18) quantitatively describes dependence of the period with respect to amplitude, involved nonlinearities and the parameter \( \lambda \). For the case of linear springs the period would be equal \( 2\pi \), and when the springs are nonlinear, it differs from \( 2\pi \). For hard characteristics (\( \alpha_i > 0 \)), the period is shorter than \( 2\pi \). Generally, greater influence on the period comes from the nonlinearity of that spring, which has smaller linear stiffness coefficient \( k_i \).

The solution of the original system (6)-(8), i.e. functions \( x_1(t) \) and \( x_2(t) \), can be obtained from (17) using the definitions (9) and the algebraic equation (11). Time histories of the analytically estimated functions \( w(t) \), \( x_1(t) \) and \( x_2(t) \) are presented in Fig. 2. The calculations have were carried out for the following data: \( \lambda = 5 \), \( \alpha_i = 0.02 \), \( \alpha_i = 0.2 \), \( b_0 = 1 \), \( \psi_0 = 0 \). The numerical solution of \( w(t) \) is also presented in Fig. 2 in order to validate the analytical approach.

5. Conclusions

The analytical solution of the dynamic response of the oscillator with two serially connected nonlinear springs was obtained. A properly modified multiple scale method in time domain allows to solve efficiently the differential – algebraic system of equations, which describe motion of the oscillator. The range of parameter was estimated the error to be reasonably small.

The analytical solution allows to study directly the influence of the parameters on the dependence between period and amplitude of the free system vibration.

References

Numerical and experimental analysis of the screen operation in the parametric resonance conditions

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Abstract

In the paper the experimental and numerical studies of the screen operation in the parametric resonance condition are discussed. The investigations are conducted for laboratory parametric resonance screen. The measuring test is performed for four cases of tension force values. For each considered case the natural frequency of the plate and the parameter modulation frequency are determined. The numerical investigation is performed to determine the plate natural frequencies for different tension force of the plate. Numerical analysis of the parametric resonance phenomena is carried out too. It is shown that the highest sieve plate amplitude is obtained when the parameter modulation frequency is two times larger than natural frequency of the sieve plate.

Keywords: vibrating screen, parametric resonance, natural frequencies

1. Introduction

The problem of the parametric resonance in mechanical systems is studied by numerous researches and development centres [1,2,3,4]. This phenomenon is described by Hill and Mathieu and can be found as the fundamental theory related to the vibration of the slender structures and the parametric resonance phenomenon (the so-called Hill or Mathieu equation [3]). The analysis of parametric vibration in the beam system with constant transverse loading is presented by Osiński (1985). The numerical method, based on a finite element discretisation, is proposed to solve the parametric resonance problem of a shell structures by Başar et al. (1987). In some cases, the parametric resonance phenomenon, allows to achieve higher performance processing. A good example of confirmation, can be vibratory machines for screening and transport of aggregate (Bąk et al., 2013; Osiński, 1985). This paper continues the recent authors investigations concerning the vibration of screen systems [2]. The present paper deals with a numerical and experimental investigations of the laboratory screen operating in the parametric resonance condition. The aim of this investigations is to determine the natural frequencies of the sieve and the parameter modulation frequencies (the excitation frequencies) as a function of the sieve tension force.

2. Description of the laboratory screen

The laboratory parametric resonance screen (Fig. 1) is made in accordance to the project based on the GEPARD-2 screen. GEPARD-2 screen was formerly designed by V. Slepian from Loginov Partnership Mining Company in Kiev. The screen system consist of sieve which is mounted between two beams. Complete screening system is suspended on the frame by tension springs. The frame is composed of square profiles welded together. The excitation force is generated by two electrical vibrators.

3. Experimental methods

The experiment is performed for the plate without cut outs. The adjustment parameters which were applied during test are shown in Table 1. For each adjusted tension force the resonant plate vibrations can be done by changing of the rotational speed of electrical vibrators in the range of 2300÷3466 rpm. The eccentric masses are adjusted on 50% of nominal centrifugal force equal to 2972 N. By taking into account variable angular velocity of eccentric masses excitation force is calculated. Rotational speed data is measured by laser sensor and then converted to excitation frequency.

<table>
<thead>
<tr>
<th>Tension force, N</th>
<th>Excitation force, N</th>
<th>Excitation frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>1571 / 2151</td>
<td>49.4 and 57.8</td>
</tr>
<tr>
<td>4800</td>
<td>1818</td>
<td>53.13</td>
</tr>
<tr>
<td>5200</td>
<td>1913</td>
<td>54.5</td>
</tr>
<tr>
<td>6000</td>
<td>1987</td>
<td>55.55</td>
</tr>
</tbody>
</table>

*This work was supported by the European Research Agency - 7th FP PEOPLE PROGRAMME Marie Curie Industry-Academia Partnerships and Pathways, grant agreement No. 284544
Two PCB piezoelectric accelerometers with measurement range ±1000 g are used to determine the frequency and vibration level. Further signal processing is performed in MATLAB software by using Fast Fourier Transform functions (FFT) and Chebyshev filter.

4. Numerical modelling

The numerical model consider a simplified screen system. Only the plate with the rubber pads were modelled. The contact between plate and rubber pads is defined as bonded. To get a mass equal to the real screen system the density of rubber pads was increased. Screen system is fixed by the springs. In longitudinal direction additional preload was applied. For the plate and rubber pads the 3-dimensional 20-node brick elements were used. For each preload the modal analysis was carried out to obtain the natural frequencies of the system. Then the transient analysis was preformed. In this analysis the harmonic excitation force was applied in longitudinal direction to excite the parametric vibrations.

5. Results

The results of the plate vibration frequency are shown in Table 2. The first parametric vibration is observed when the plate natural frequency to excitation frequency ratio is nearly equal to 0.6 in all respected cases. The second parametric resonance is found when the rotational speed of vibrators increases. It is obtained for plate natural frequency to excitation frequency ratio equal to 0.5. This parametric resonance vibration was observed only for tension force equal to 4000 N because of the rotational speed limits of electrical vibrators.

Table 2: Plate resonant frequencies

<table>
<thead>
<tr>
<th>Tension force, N</th>
<th>Plate vibration frequency, Hz</th>
<th>Natural frequencies - numerical results, Hz</th>
<th>Plate resonant frq /excitation frq</th>
</tr>
</thead>
<tbody>
<tr>
<td>4000</td>
<td>29.1</td>
<td>24.925</td>
<td>0.589 and 0.503</td>
</tr>
<tr>
<td>4800</td>
<td>32.4</td>
<td>29.445</td>
<td>0.61</td>
</tr>
<tr>
<td>5200</td>
<td>33.5</td>
<td>31.992</td>
<td>0.615</td>
</tr>
<tr>
<td>6000</td>
<td>34.35</td>
<td>33.529</td>
<td>0.618</td>
</tr>
</tbody>
</table>

The first mode shape of plate vibration is observed in both excitation frequencies 49.4 Hz and 57.8 Hz. For second parametric vibration large plate deformation is visible caused by large vibration amplitude (Fig. 2).

Figure 2: Plate vibrations shape (maximum deformation) for tension force 4000 N and excitation frequency: 57.8 Hz

The plate natural frequencies obtained from the modal analysis are very close to the experimental results (Tab. 2). This confirms that prepared numerical model is correct. The stabilized parametric vibrations of the plate for the natural frequency to excitation frequency ratio equal to 0.6 and 0.5 were observed by a transient analysis. For the ratio equal to 0.6 the amplitude of 16 mm, when for ratio equal to 0.5 the vibration amplitude at the level of 26 mm is achieved (Fig. 3).

6. Conclusions

In the experimental investigation, natural frequency of the sieve plate and parameter modulation frequency are determined for four cases of a tensile force value. As it was expected the value of the sieve plate natural frequency and parameter modulation frequency grow parallel to the growing tension force.

The plate vibrations in parametric resonance conditions occurred for two resonant frequency to excitation frequency ratios - 0.6 and 0.5. The numerical analysis confirms the appearance of the parametric resonance for these conditions. The highest sieve plate amplitude is obtained when the parameter modulation frequency is two times larger than the natural frequency of the sieve plate.

References


Influence of temperature on the creep limit of microalloyed steel containing Nb, V and N

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Abstract

Microalloyed steel containing Nb, V and N, originally designed to be used for structures and equipment operating in the temperature range from –30 °C to about +50 °C, was tested for creep strength at temperatures ranging from 300 to 700 °C. Because of its relatively non-complex chemical composition, steel was expected to be influenced by microadditives such as Nb and V during creep testing. It was assumed that the creep limit would occur later than for non-alloy steels and that there would be additional effects related to the presence of carbides, nitrides and carbidenitrides formed by these elements. The creep limit curves obtained after 100, 1,000 and 10,000 hours of testing suggest that in certain temperature ranges the creep limit decreases more slowly than due to the others.

Keywords: microalloyed steel, microadditives, test temperature, creep limit

1. Introduction

A 10MnVNb6 steel containing Nb, V and a higher amount of nitrogen was patented in the 1970s [1]. Because of its chemical composition, it was classified as weldable microalloyed steel of increased strength. The steel is used in responsible structures, e.g. those for self-propelled truck cranes, Star truck chassis, pipelines, etc. Results of preliminary creep tests show that this steel is characterized by high creep strength at minimum temperatures of up to 500 °C [2]. Further investigations conducted at 700 °C focused on determining the yield point. The findings indicate that even at a temperature of about 700 °C, the yield point remains at a high level [3].

2. Material

The specimens used for the creep tests were cut from 8 mm thick plates. The plates were strengthened by controlled rolling followed by annealing at a temperature of 690 °C for 0.5 h to remove from the solid solution the rest of the microalloyed elements not bonded with the others during the metal working process. The chemical composition of the plate material is presented in Tab. 1.

Table 1. Chemical composition of 10MnVNb6 steel (8 mm thick plates)

<table>
<thead>
<tr>
<th>Element content, %</th>
<th>C</th>
<th>0.09</th>
<th>Al</th>
<th>0.031</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mn</td>
<td>1.49</td>
<td>N</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>Si</td>
<td>0.36</td>
<td>Cr</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>P</td>
<td>0.017</td>
<td>Mo</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>0.013</td>
<td>Ni</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>V</td>
<td>0.14</td>
<td>Ti</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>Nb</td>
<td>0.030</td>
<td>O</td>
<td>0.0030</td>
</tr>
</tbody>
</table>

The creep specimens were cut parallel to the rolling direction.

3. Method

Before the creep strength tests, the plate material was studied to determine its microstructure. The microscopic analysis was performed using a light microscope and a transmission electron microscope.

The creep tests were carried out for a set of specimens with a cylindrical cross-section in the laboratories of the Kielce University of Technology. The specimens were heated and held at a predetermined temperature. The heating was performed with an accuracy of up to ± 2°C. The elongated specimens were measured with an accuracy of up to 0.005 mm. The creep tests were conducted for a minimum of 10,000 h.

4. Results and discussion

From the microscopic analysis performed at different magnifications it is clear that the microstructure of the steel sampled from 8 mm thick plates consists of ferrite and pearlite, with the relative content of ferrite being 93-95 %, and the rest being pearlite (Fig. 1). The examinations conducted with the transmission electron microscope at high magnifications show that the ferrite matrix contains particles with varied dimensions ranging from several to more than a dozen nanometers (Fig. 2). The energy dispersive X-ray microanalysis reveals that the areas where the particles occur are rich in vanadium and that there is no niobium present or its content is below the detection limit of this method. At the grain boundaries there are numerous large precipitates varying in shape and size, with the size of up to several hundred nanometers (Fig. 2). The microanalysis of the composition shows that the phases are rich in niobium; vanadium is not present or its content is negligible.
The results of the creep tests were used to plot creep limit curves for a temperature of up to 700 °C and a creep time of 100, 1,000 and 10,000 h. The creep limit for the times considered changes. Its value decreases with an increase in temperature; however, the smallest drop is observed in the temperature ranges 450 – 500 °C and 550 – 650 °C (Fig. 3).

Considerable differences in creep behaviour were reported over the whole range of temperature of the creep tests. However, the creep limit curves look similar because under creep conditions, the mechanisms responsible for delaying the deformation processes are strongly related to the microstructure and chemical composition of the material. It was necessary to determine the ranges of temperature in which these phenomena would mostly affect the creep process. For this purpose, the difference quotient ΔR1/ΔT was used as it allowed us to study the increases in the creep limit with high accuracy (Fig. 4).

As can be seen from the curves in Fig. 4, there is a clear decrease in the creep limit curve from approx. 2 MPa/°C at 450 °C even to 0.8 MPa/°C at approx. 500 °C. In the range from about 500 °C to 550 °C the deformation processes are most likely to occur because steel is weakened. When the temperature is higher than 550 °C the material is more resistant to deformation. For R1/100/T and R1/10,000/T the resistance improves; it is high even at a temperature of 650 °C.

5. Conclusions

Deformation under creep conditions is a resultant of two opposite processes: weakening and strengthening of the material. An increase in the creep temperature causes the material to be more susceptible to deformation under stress. This results in a decrease in the value of the creep limit. The diagrams illustrate that in the case of steel containing vanadium and niobium and a higher amount of nitrogen, there are two ranges of temperature where the resistance to deformation rises under creep conditions. There are at least two causes of such behaviour: changes taking place in the microstructure and a change in the composition of the ferrite matrix due to the dissociation of the carbide, nitride and carbidenitride phases. The occurrence of two ranges of temperature where the resistance to creep is observed is probably a result of the presence of vanadium and niobium compounds characterized by different susceptibility to dissocation at elevated temperature.

References


The effect of technological parameters of shot peening on surface roughness of 51CrV4 steel

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Abstract

In the paper the effect of selected technological parameters of shot peening on surface roughness of 51CrV4 steel is presented. The experiments were conducted according to statistical Hartley’s plan PS/DS-P:Ha3. In the article the results of research carried out on stylus and optical profilometers were compared. The technological parameters are variable in the range: 1-3 min – shot peening time t, 1.5-2.5 mm – diameter of the balls db and 0.2-0.4 MPa – pressure p. For these parameters the cases of analysis of experiment reproducibility, impact significance and adequacy of equations was made. As an investigation result the adequate equations were obtained describing amplitude parameters such as: Rv, Rt, Rq, Rsk, Rku. The recommendation to use the stylus profilometer to measure surface roughness after shot peening was stated.

Keywords: Hartley’s plan, shot peening, 51CrV4 steel, roughness

1. Introduction

Increasing requirements for machine parts motivate the engineers to apply special treatments to increase the strength and durability of these parts as well as to use the sophisticated measurement techniques in their control [1]. The surface roughness has an important effect on machine part performance, therefore it is essential to choose an appropriate method of measurement. There are some techniques which can be employed such as the contact method with a mechanical stylus usage and the non-contact methods: interferometry, confocal microscopy, focus variation or electron microscopy.

Shot peening has a great impact on a machine part strength increase. This process is widely used to improve the fatigue strength of material by the creation of compressive residual stresses in surface layers of materials [5,6]. Fatigue strength of peened austenitic chromium-nickel steel demonstrates higher values compared to untreated one [3,4]. There are several technological parameters which influence the process and should be controlled. The statistical Hartley’s plan PS/DS-P:Ha3 is strongly recommended for the surface layer tests [2].

2. Methodology

The effect of selected technological parameters of shot peening on surface roughness of 51CrV4 steel was examined. The experiments were conducted according to statistical Hartley’s plan PS/DS-P:Ha3. All tests were conducted with threefold repetitions. The significance level α = 0.05 was adopted for calculations. The samples were peened according to Hartley’s plan. Three variable factors were selected:

- shot peening time t (1, 2, 3 min);
- ball diameter db (1.5, 2, 2.5 mm);
- pressure p (0.2, 0.3, 0.4 MPa).

The topography of specimen surface (Fig. 1) were examined using the stylus profilometer Surtronic 25 Taylor Hobson and optical profilometer Talyurf CCI Lite.

*This work was supported by the European Research Agency - FP7-PEOPLE-2011-IAPP - Marie Curie Industry - Academia Partnership and Pathways, grant agreement No. 284544.
### 3. Results and Discussion

Several adequate equations were obtained as results of the conducted experiments. According to the methodology of Hartley’s plan the significance of technological parameters was assessed.

For surface roughness parameters such as $R_v$, $Rsk$ and $Rku$ adequate mathematical models were obtained independently of the surface measurement method. Additionally, mathematical descriptions were found for $Rt$ and $Rq$ roughness parameters whose values were measured with the usage of a stylus profilometer. Roughness parameter $Rv$ representing maximum depth of valleys depends mainly on two technological parameters: the pressure and ball diameter. When the pressure was greater the bigger values of $Rv$ were achieved but with a larger ball diameter the depth values were smaller (Table 1 and Fig. 2). The time of the process duration has no significant effect on $Rv$ parameter except the interaction influence with the pressure and with the ball diameter.

![Figure 2: Averaged values of $Rv$ obtained with Surtronic (S) and Talysurf CCI Lite (T) instruments](image)

Simlar adequate models (1) and (2) with the same tendency were found comparing two different measurement methods.

$$Rv_S = 5.9814 - 0.7881t - 3.7254d + 18.156p + 1.2966td - 6.017tp \quad (1)$$

$$Rv_T = 7.934 - 1.5933t - 6.0176d + 27.071p + 2.0766td - 8.533tp \quad (2)$$

In comparison of the adequateness variance concerned $Rv$ parameter and taking into account the fact that more mathematical models were obtained from the results received from the stylus profilometer the mechanical contact method of a measurement is better for such applications. It is recommended to use the stylus profilometer to measure surface roughness after a shot peening. Height roughness parameters values increase considerably after realized process compared to initial values. The values measured by an optical instrument were usually higher than those from the stylus profilometer.

### 4. Conclusions

The pressure and the ball diameter have the significant effect on surface roughness. Substantial increase in height parameters values was obtained after a shot peening process in the whole range of applied technological parameters. The adequate models were obtained for surface roughness parameters $Rv$, $Rsk$, $Rku$, $Rt$ and $Rq$. The stylus profilometer is recommended to measure surface roughness after shot peening.

### References


Dynamic behaviour of three layer composite cantilever beam with viscoelastic core

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Abstract

The dynamic behaviour of a three layer composite structure with viscoelastic core is presented in the paper. Sandwich structure of A6060 aluminium alloy covers and Polyethylene terephthalate (PET) core is taken into consideration. To identify and model the viscoelastic properties of the structure the fractional rheological model is applied. For the purpose of the study a Zener rheological model and a four-parameter fractional rheological model are compared. In order to determine of the rheological parameters of the core material a Nelder-Mead method is applied. In the paper the impact of symmetry of the covers on dynamics of a composite structure is presented.

Keywords: composite, viscoelasticity, fractional model, complex shear modulus, loss factor

1. Introduction

Contemporary development of composite materials is strictly associated with the growing demand of various industries for lightweight and durable materials to replace steel and other metals. In many cases the viscoelastic properties of these materials are highly desirable. The example are aerospace and automotive industry, where reduction of structural vibrations by the application of surface damping is a frequent practice.

Many new laminates, invented at the end of the last century, caused growing interest in the field of dynamics of layered composites with viscoelastic cores. In the study by Cupiał and Nizioł [2] the equation of free vibration of the non-symmetrical three-layer plate with a viscoelastic middle layer was derived on the basis of a virtual work principle. However, as shown in later studies in this field (e.g. [3, 4, 5]), in order to describe the viscoelastic properties of laminated material it is necessary to introduce a more accurate tool than the widely used classical rheological models.

In order to describe the viscoelastic properties of laminated material instead of classical rheological models the fractional models are applied. The inaccuracy of the classical rheological models can be observed in the frequency domain, where the slope of the experimental amplitude curves is always smaller than that of the curves predicted by these models. The reason for this inaccuracy can be found in the stress-strain relationship defined in a time-domain by a linear differential equation of an integer order. Replacing the integer order of derivatives in the Zener rheological model with a fractional order, the four-parameter rheological model with fractional-order derivatives was introduced in the study by Pritz [6].

A great variety of composite materials and a trade secret hiding full results of the research carried out by the manufacturers, make it necessary to conduct fundamental research in this field. In the study the authors focus on dynamic behaviour of a three-layer composite structure with a viscoelastic core. The three-layer sandwich structure consisting of A6060 aluminium alloy covers and Polyethylene terephthalate (PET) core is taken into consideration. A method of loss factor determination for composite structures acts strongly on the identification results of the polyethylene core by means of the fractional rheological model. The investigation results show the advantages and disadvantages of various methods of the loss factor determination and their usefulness in the identification of rheological parameters of polyethylene. The impact of symmetry of the covers on dynamics of the composite structure is presented too.

2. Identification of modal parameters

A sandwich beam of the width \( b \) and the thickness \((2h_1+h_2)\) is considered. The top and bottom layers are made of the same elastic material and the core is of thin, soft polymeric material. In order to identify and model the composite structure the RKU equations are used [7]. The effect of bending of a viscoelastic core is neglected. Then the flexural rigidity of the composite cantilever beam is given by

\[
\left( EI \right) = \frac{E h_1^3}{6} + \frac{E h (h_1 + h_2)}{1 + 2 g^*},
\]

where:

\[
g^* = \frac{G_1^*}{h h_1^2}.
\]

and: \( E \) – Young’s modulus of the outer layer, \( G_1^* \) – complex shear modulus of the core, \( \xi_n \) the \( n \)-th eigenvalue of the cantilever beam.

In an experimental way due to a seismic excitation the flexural rigidity of the composite beam is determined. The measured resonant frequencies and loss factors of the composite beam and the response of outer beams taken separately lead to the modal flexural rigidity

\[
Z_k = \left( \frac{E I_1}{f_n} \right) = \left( 1 + \frac{2 \rho h_1}{\rho h_1} \right)^{\frac{1}{2}} \left( 1 + i \eta_n \right),
\]

where: \( f_n \) - the \( n \)-th resonant frequency of the composite beam, \( f_{0n} \) the \( n \)-th natural frequency of each individual outer beam, \( \eta_n \) - the modal loss factor.
In order to measure the modal loss factor the standard ASTM E 746-05 recommends the half-power bandwidth (HPB) method [1].

The HPB method is not the only method of determination the modal loss factor, described in the literature. The same standard applies the “n dB” bandwidth method. It is worth mentioning that the HPB method is the "n dB" bandwidth method for n = 3.01 dB.

On the basis of the measured resonant frequencies and the modal loss factors of the composite beam it is possible to perform the identification of the core. Comparing Equations (1) and (3) the shear parameter $g^*$, closely related with the complex modal shear modulus of the core can be determined

$$g^* = \frac{Z_0^2 - 2}{12 \left( 1 + \frac{h_2}{h_1} \right)^2 - 2Z_0^2 + 4} \tag{4}$$

According to the standard ASTM E 746-05 [1] the shear modulus and the loss factor of the material of the core are derived by

$$G_2 = \left[ A - B - 2(A - B)^2 - 2(A \eta)^2 \right] \left[ 1 - (1 - 2A + 2B)^2 + 4(A \eta)^2 \right] \tag{5}$$

$$\eta_2 = \frac{A \eta}{A - B - 2(A - B)^2 - 2(A \eta)^2} \tag{6}$$

$$\eta_2 = \frac{A \eta}{A - B - 2(A - B)^2 - 2(A \eta)^2} \tag{6}$$

where:

$$A = \frac{f_1}{f_n} \left( 2 + \frac{E h_1}{\rho h_1} \right)^2, B = \frac{1}{6 \left( 1 + \frac{h_2}{h_1} \right)^2} \tag{7}$$

$C_n$ - coefficient for the $n$ mode of clamped–free beam.

3. Experimental investigations

In order to characterise the flexural stiffness of sandwich composite with a viscoelastic core frequency domain investigations were carried out. A seismic excitation was generated by an electrodynamic shaker Ling Dynamic System V780. For the modification of shaker signals parameters the signal generator HMF2525 was used. The experimental configuration scheme is presented in Fig. 1.

![Test stand for investigation of the sandwich composite](image)

Figure 1: Test stand for investigation of the sandwich composite. 1 - generator HMF2525, 2 - Brüel & Kjær LDS HPA K amplifier, 3 - LDS V780 shaker, 4 - HSV-700 sensor head, 5 - HSV-800, 6 - HSV-2002 laser unit PC, 7 - Brüel & Kjær Pulse, 8 - oscilloscope, 9 - PC, 10 - specimen, 11 - fixture

During the tests the single base beams and composite beams were investigated. Six specimens of the beam were subjected to the pink noise input signal.

In order to determine the resonant plot the Fast Fourier Transformation (FFT) of the response signal was used. Afterwards the specimens were subjected to sinusoidal excitations in the vicinity of each resonant frequency. In this way the particular resonant frequencies were identified. Modal stiffness of the sandwich was determined according to Equation (3). The value of the modal loss factor was determined according to ndB methods. Standard deviation was calculated for the set of each mode.

4. Investigation results

A three-layer sandwich structure of A6060 aluminium alloy covers and Polyethylene terephthalate (PET) core is taken into account. For the description of dynamic properties of the core a material fractional rheological model is applied. For the purpose of the study the Zener rheological model and four-parameter fractional rheological model are compared. Identification of the composite structure is accomplished according to the RKU equations.

Investigations, by means of a seismic excitation result in modal stiffness and modal loss factor of the composite cantilever beam for particular resonant frequencies. In order to determine the modal loss factor three bandwidth frequency methods and the statistical error minimization fit method of the response of a single degree of freedom model system are taken into account. For the identification of the rheological parameters of the core material the Nelder-Mead method is used.

The results of investigations show that the loss factor values are similar according to the bandwidth frequency methods and the error minimization fit method of the response of a single degree of freedom model system. The method 1dB is characterized by the highest error, but its advantage is its usage in the case when 2dB and 3dB method cannot be applied. In the study the impact of asymmetry of the covers on dynamics of the composite structure is also presented.

References


Metamorphoses of resonance curves in systems of coupled oscillators

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Abstract

We study dynamics of two coupled periodically driven oscillators in a general case as well as in two simplified models. Periodic steady-state solutions of the system of these equations are determined within the Krylov-Bogoliubov-Mitropolsky approach. The amplitude profiles, A(\omega), B(\omega), which are given by two implicit equations, F(A, B; \omega) = 0, G(A, B; \omega) = 0, where \omega is frequency of the driving force, are computed. These two equations, each describing a surface, define a 3D curve - intersection of these surfaces. In the paper we analyse metamorphoses of amplitude profiles induced by changes of control parameters in three dynamical systems studied. It is shown that changes of dynamics occur in the vicinity of singular points of these curves.

Keywords: coupled oscillators, amplitude profiles, singular points

1. Introduction

We study dynamics of two coupled nonlinear oscillators, one of which is driven by an external periodic force. Equations of motion are:

\[\begin{align*}
    m \ddot{x} - V'(x) - R(x) + V_e(\dot{y}) + R_e(y) &= F(t) \\
    m_e \ddot{y} - V_e'(y) - R_e(y) &= 0
\end{align*}\]  

(1)

where \( x \equiv x_1 \) is position of primary mass \( m \), \( y \equiv x_2 - x_1 \) is relative position of another mass \( m_e \) attached to \( m \) and \( R \), \( V \) and \( R_e \), \( V_e \) are nonlinear elastic restoring force and nonlinear force of internal friction for masses \( m \), \( m_e \), respectively (we use convention \( x \equiv \frac{\dot{x}}{\omega} \), etc.). Dynamics of coupled, periodically driven oscillators, is very complicated. Dynamic vibration absorber is a typical mechanical model described by (1) (in this case \( m_e \) is usually much larger than \( m_e \) [1].

In the work we investigate a hierarchy of dynamical systems of form (1): we consider functions \( R \), \( V \), \( R_e \), \( V_e \) for which (a) system of equations can be reduced to one second-order effective equation of relative motion, (b) fourth-order equation for variable \( y \) can be separated off, (c) it is impossible to separate variables. We analyse approximate analytic solutions (amplitude profiles), obtained within the Krylov-Bogoliubov-Mitropolsky (KBM) method [2], using theory of algebraic curves. More exactly, singular points of amplitude profiles are computed. We demonstrate, that qualitative changes of dynamics, referred to as metamorphoses, induced by changes of control parameters, occur in neighbourhoods of singular points of amplitude profiles, see also [3, 4] and references therein.

2. Equations of motion

In what follows the function \( F(t) \) is assumed in form \( F(t) = f\cos(\omega t) \). When all the functions \( R, V, R_e, V_e \) are nonlinear, e.g.:

\[\begin{align*}
    R(x) &= -\alpha x - \gamma x^3, \quad R_e(y) = -\alpha_e y - \gamma_e y^3 \\
    V(x) &= -\nu \dot{x} + \beta \dot{x}^3, \quad V_e(y) = -\nu_e \dot{y} + \beta_e y^3
\end{align*}\]  

(2)

then we deal with general case of Eq. (1). For linear functions \( R, V \):

\[\begin{align*}
    R(x) &= -\alpha x, \quad R_e(y) = -\alpha_e y - \gamma_e y^3 \\
    V(x) &= -\nu \dot{x}, \quad V_e(y) = -\nu_e \dot{y} + \beta_e y^3
\end{align*}\]  

(3)

it is possible to separate off variable \( y \) to obtain the following exact equation for relative motion [4]:

\[\ddot{y} - V_e'(y) + c m_\omega^2 \ddot{\omega} \cos(\omega t) + c m_\omega^2 \ddot{\omega} \cos(\omega t) \]

(4)

where \( \ddot{\omega} = M \frac{d^2 \ddot{x}}{d \dot{x}^2} + \nu \ddot{x} + \alpha, \ddot{\omega} = (\nu \ddot{x} + \alpha) \frac{d^2 \ddot{x}}{d \dot{x}^2}, F = m_e \omega^2 f, \epsilon = m_e/M, \mu = m m_e/M \) and \( M = m + m_e \).

Finally, assuming \( m_e \ll m \), i.e. \( \epsilon \ll 1 \), we can reject the term proportional to \( \epsilon \) to obtain the approximate equation which can be integrated partly to yield the effective equation [3]:

\[\ddot{y} + \nu_e \dot{y} - \beta_e y^3 + \alpha_e y + \gamma e y^3 = F(t)
\]

(5)

where transient states were neglected.

3. Metamorphoses of the amplitude profiles

To find approximate solutions we insert \( y(t) = A \cos(\omega t + \varphi) + \varepsilon y_1 + \ldots, \) and \( x(t) = B \cos(\omega t + \psi) + \varepsilon x_1 + \ldots \) into Eq. (1). After appropriate steps of the KBM procedure [2] we get two implicit equations for amplitude profiles:

\[F(A, B; \omega; \Lambda) = 0, G(A, B; \omega; \Lambda) = 0\]

where \( \Lambda \) denotes parameters. Intersection of surfaces \( F, G \) is a curve and in singular points of the curve all three \( 2 \times 2 \) determinants of the matrix

\[\begin{pmatrix}
    F_A & F_B \\
    G_A & G_B
\end{pmatrix}
\]

(6)

*The corresponding author
are zero [5]. These conditions are used to compute singular points. Note that \( F_A' = \partial F / \partial A \), etc. We have shown in our previous papers that qualitative changes of dynamics (metamorphoses), induced by changes of control parameters, occur in neighbourhoods of singular points of amplitude profiles [3, 4].

4. Amplitude profiles and bifurcation diagrams

Applying KBM method to the effective equation (5) we obtain approximate formula

\[
y(t) = A \cos(\omega t + \varphi)
\]

where dependence of \( A \) on \( \omega \) is given by implicit equation

\[
F_1(A; \omega; \Lambda) = 0
\]

The form of function \( F_1 \) can be found in Ref. [3]. In Fig. 1 this implicit function is shown just after an isolated point is born.

![Figure 1: Amplitude profile \( F_1(A; \omega; \Lambda) = 0 \) with isolated point.](image1)

Then, applying the KBM method to Eq. (4) we obtain implicit equation

\[
F_2(A; \omega; \Lambda) = 0
\]

The form of \( F_2 \) has been described in [4]. In Fig. 2 we see that an isolated point has been just born.

![Figure 2: Amplitude profile \( F_2(A; \omega; \Lambda) = 0 \) with isolated point.](image2)

Figures 1, 2 are quite similar. Bifurcation diagrams show indeed that new branches of solutions have been born [3, 4].

Finally, applying the KBM method to the most general equation (1) we obtain

\[
y(t) = A \cos(\omega t + \varphi) \quad \text{and} \quad x(t) = B \cos(\omega t + \varphi)
\]

where dependence of amplitudes \( A, B \) on \( \omega \) are given by the set of two implicit equations,

\[
F(A, B; \omega; \Lambda) = 0, \quad G(A, B; \omega; \Lambda) = 0.
\]

The resonance surfaces are shown in Fig. 3 while the corresponding bifurcation diagram with a new branch of solution is shown in Fig. 4.

![Figure 3: Two resonance surfaces \( F(A, B; \omega; \Lambda) = 0 \) and \( G(A, B; \omega; \Lambda) = 0 \) with additional conical structure.](image3)

![Figure 4: Bifurcation diagram for Eq. (1), dependence of \( y \) on \( \omega \).](image4)

5. Closing remarks

We demonstrated, that computing singular points of two resonance surfaces,

\[
F(A, B; \omega; \Lambda) = 0, \quad G(A, B; \omega; \Lambda) = 0
\]

obtained by application of the KBM method to Eq. (1), we can detect birth of a new branch of a nonlinear resonance.

References

Structured waveguides: Floquet waves and polarisers in elongated systems

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Abstract

We present an analytical approach to the analysis of a challenging problem of controlling vibrations of elongated systems such as multi-span bridges or structured polarising interfaces. In this approach, the spectral analysis of these finite sophisticated elastic systems is aided by a more transparent and analytically treatable approximation based on the dispersion properties of Floquet waves existing in the corresponding periodic structures of infinite extent. In particular, for a flexural bridge supported by periodically distributed pillars, a quasi-periodic dynamic Green’s function is introduced as a building block of the analysis. Dispersion relations for Floquet waves are derived analytically and studied in detail. We give special attention to standing waves of zero group velocity, and also discuss localisation due to defects. We demonstrate that the waveguide model can be used to obtain the bounds for the eigenvalues of the finite-size elongated structure. Moreover, the positions of the boundaries of stop bands for Floquet waves identify the clustering regions for the eigenvalues of the finite structure. This approach opens a new way of optimal design of elongated systems by adding substructures that act as wave by-pass systems diverting vibrations away from the main deck of the bridge.

Keywords: multi-structure, vibration of elongated systems, Floquet waves, dispersion

1. Introduction

The modelling and implementation of cloaks and wave shields for acoustic and electromagnetic waves, as well as novel concepts of structured multi-scale media in the design of photonic crystals, have been developed to a significant extent, as outlined in the monograph [1].

Elastic waves are described by a vector problem, and while pressure and shear waves propagate with different speeds and have different polarisations, they are coupled via the boundary conditions. Standard cloaking geometrical transformations, which are highly successful in geometrical optics and acoustics [6], do not produce the same result in vector problems of elasticity [6]. Nevertheless, the idea of polarisers, shields and by-pass systems for elastic waves has proved to be successful, and several significant advances have been made over the recent years [2-6]. In the talk, we present a concept, based on the analysis of elastic Floquet waves in multi-scale elastic systems, which are linked to elongated elastic structures, such as bridges, tall buildings, or dynamic structured interphases separating different parts of an elastic solid [2, 3, 5]. The proposed approach, which is analytical in nature, complements the conventional eigenvalue finite element analysis applied to elongated structured solids by predicting clustering of eigenfrequencies as well as change of polarisation due to interaction of an incident waves with a structured interphase.

2. Polarisation of an elastic wave by a structured interphase

A surprisingly simple structure discussed in [2] provides an invaluable insight on the coupling mechanism between pressure and shear waves, which is also linked to an interesting phenomenon of transmission resonances.

Figure 1: Shear wave polariser, represented by a periodic multi-layer structured interphase, as in [2].

Resonant modes are associated with Floquet waves, supported by the structured interphase, and they also enable high transmission. Special, high-contrast structure of the interphase enables coupling between shear and pressure waves, which leads to change in polarisation. The inertia of the sub-structure of the interphase plays crucial role in controlling resonant transmission and shear polarisation. The talk will include numerical simulations and examples of resonant transmission due to resonances within the structured interphase.

3. Flexural structured waveguides versus eigenvalue analysis of elongated systems

A conventional method of spectral analysis of large-scale engineering structures is based on the finite element analysis produced with an aid of an industrial grade software.

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However, thousands of eigenvalues, as well as the corresponding eigensolutions, produced by a finite-element package for a large multi-scale elastic system are sometimes challenging to analyse due to the sheer volume of information. The Floquet analysis [3], developed for a structured periodic waveguide, delivers an additional tool that yields a clear understanding of clustering of the eigenvalues as well as the nature of the corresponding eigenmodes.

With the elegant asymptotic approximation [3], based on the use of quasi-periodic dynamic Green’s functions, the dispersion equation is derived in a closed form; the analysis of its solutions becomes straightforward and it will be demonstrated in the talk.

The lecture will also include a discussion of the connections between results concerned with the Floquet waves and the eigenvalue simulations for multi-scale elongated engineering systems.

The follow-up of these ideas is in the optimal design of wave by-pass systems, which are extremely helpful in the bridge design problems, where diversion of vibrations from the main deck of the bridge to one of the existing substructures may be beneficial, as discussed in the recent paper [5].

References


Active Cloaking of an Inclusion at Resonant Frequencies for Membrane and Elastic Flexural Waves

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Abstract

Active cloaking has become a widely used method in the past few years (Guevara Vasquez et al. Ref. [2], Guevara Vasquez et al. Ref. [3], O’Neill et al. Ref. [4]). Considering the dynamic response of a coated inclusion, it was shown by Farhat et al. Ref. [1], that (for small frequencies) a specifically designed coating provides the correct inertial contribution to reduce the magnitude of scattered fields. Here, we present a model of an arbitrary order active cloak for a coated inclusion against membrane or elastic flexural waves, considering small frequencies as well as the difficult problem of cloaking in a frequency range where the inclusion has strong resonances. We show how an appropriate choice of coating can reduce the sensitivity of the active source complex amplitudes when frequency is altered. Active cloaking is achieved by introducing control sources in the exterior of the coating and by choosing their complex amplitudes to eliminate propagating components of the scattered wave. The closed form analytical findings are accompanied by asymptotic estimates and numerical illustrations.

Keywords: Active cloaking, resonant regimes, membrane waves, elastic flexural waves, Kirchhoff plates, biharmonic operator.

1. Introduction

The popularity of active cloaking arises from the fact that there are many practical implementations due to the method flexibility and wide bandwidth capabilities.

In order to cloak an object, the algorithm employed by O’Neill et al. Ref. [4] uses the method of multipole expansions to produce effective cloaking. The procedure involves positioning a number of active control sources in the exterior of the object and choosing their complex amplitudes to eliminate selected multipole coefficients of the propagating components contributing to the scattered field. The active sources are represented by the Green function for the corresponding differential operator. The cloaking algorithm is fully analytic, and leads to a system of algebraic equations in order to find the necessary source amplitudes required for reconstructing the incident field.

For a given incident frequency, in non-resonant regimes the method used by O’Neill et al. Ref. [4] can deliver an approximate cloaking with a finite number of sources for a circular clamped void. At resonant regimes, we see a high rate of change in scattering. This in turn means that the complex amplitudes of the active sources undergo large changes in magnitude if the incident field frequency is altered by even a small amount.

Despite the wide bandwidth nature of active cloaking, the rapid changes in scattering from an inclusion in resonance regimes can be avoided by re-configuring the scatterer. We design a coating for the inclusion which delivers a flat reflectance as a function of frequency in a given frequency range. With this configuration, when the incident frequency is perturbed, the active source amplitudes do not undergo large changes in order to deliver effective cloaking. This coating design builds on that investigated by Farhat et al. [1] in that their coating was specifically designed to reduce scattering only at low frequencies. Our design will operate at high frequencies, and is applicable to a more general setting.

A coating for the inclusion can be chosen so that the average mass density of the inclusion and the coating together equals the mass density of the ambient medium. In the simpler case of membrane waves (Helmholtz equation) the monopole term associated with the scattered field then becomes zero. For flexural waves in Kirchhoff plates, this argument becomes invalid, except in the low frequency regime, which is dominated by the Helmholtz type waves. We present a method to overcome this deficiency and utilise the coating to control the frequency range of resonance regimes. We present the results of the algorithm used to calculate active source complex amplitudes for highly effective cloaking, which now covers high frequency regimes for coated inclusions.

![Figure 1: Total displacement field exterior to a coated inclusion at a resonant regime. (Membrane waves case.)](image-url)
2. Membrane Waves

We consider the case of membrane waves as a simple precursor to the more involved discussion of elastic flexural waves, due to the fact that solutions to the biharmonic equation are superpositions of solutions to the Helmholtz (propagating waves) and modified Helmholtz equations (evanescent waves). Figures 1 and 2 show that active cloaking remains effective in resonance regimes at high frequencies, as a result of the strategy outlined above.

3. Elastic Flexural Waves

Kirchhoff plates allow for a similar treatment of resonant regimes as the membrane wave case. However, the coated inclusion argument requires significant modification, as the mass-balance argument proves to be inadequate for the case of Kirchhoff plates. Additional features of elastic flexural waves scattered by an inclusion are related to the interaction between the Helmholtz and the modified Helmholtz waves.

Figures 3 and 4 show that cloaking can be achieved in resonant regimes, following a similar method to the membrane waves case.

References

Discontinuous Galerkin method for cracked solids with chemical compositions

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Abstract

A discontinuous Galerkin method is proposed to solution of cracked solids with chemical compositions. The combined effects of chemical compositions, interstitial concentration, stress and temperature fields as well as discontinuities in displacement field are considered. The discontinuous equations are formulated using the DG-derivative.

Keywords: discontinuous Galerkin method, cracks, chemical compositions

1. Introduction

In this paper we analyze cracks in solids taking into account combined effects of chemical compositions, interstitial concentration, stress and temperature [1,3,4]. The discontinuities in displacement field caused by cracks are considered with the above mentioned effects. The thermodynamical considerations of analyzed effects are described (see [4], for instance).

2. The energy equation

The internal energy $E$ of the system changes in time $t$ in domain $\Omega$ by the relation

$$\dot{E} + \text{div} (\text{div} S) = -\text{div}(E\text{div}U) + \text{div}\sigma + \text{div}R - \text{div}Q$$

(1)

where $\sigma$ is the stress tensor, $\varepsilon$ is the strain tensor, $Q$ is the heat flow vector and $R$ represents the volume rate density of the heat provided to $\Omega$ by possible external heat sources, $E_c$ is the internal chemical potential energy, $V$ is the material velocity and $U$ is the diffusion velocity.

Let $S_c$ be the entropy of chemical potential of the diffusing material. The total entropy $S$ reads

$$\frac{d}{dt}\int_{\Omega} S d\Omega = \int_{\Omega} \left( \dot{S} + \text{div}(SV) + \text{div}(S_c U) \right) d\Omega$$

(2)

Since the volume integral must be non-negative for any subsystem $\Omega$ we have

$$\dot{S} + \text{div}V + \text{div}(S_c U) + \text{div}\frac{Q}{\theta} + \frac{R}{\theta} \geq 0$$

(3)

where $\theta$ is the absolute temperature.

Multiplying Eq. (3) by $d\Omega$, the above equality becomes

$$\frac{d}{dt}\left( \text{div}S \right) + \text{div}(S_c U) + \text{div}\frac{Q}{\theta} + \frac{R}{\theta} \geq 0$$

(4)

which expresses the second law for the elementary material system $d\Omega$.

Let $\psi_c$ be the free enthalpy of the chemical potential defined by

$$\psi_c = E_c - \theta S_c$$

(5)

Now, let $\psi$ be the free volume energy defined by

$$\psi = E - \theta S$$

By (4) we have

$$\sigma \cdot V + \varepsilon \cdot U - \psi \text{div}V - S_c U \theta \text{grad} \theta + \text{div}(\psi \text{div}U) - \frac{Q}{\theta} \theta \text{grad} \theta \geq 0$$

(7)

and using the identity

$$\text{div}(\psi \text{div}U) = \text{U} \cdot \text{grad} \psi_c + \psi \text{div}U$$

(8)

we get

$$\sigma \cdot V + \varepsilon \cdot U - \psi \text{div}V - S_c U \theta \text{grad} \theta - \text{U} \cdot \text{grad} \psi_c + \psi \text{div}U$$

(9)

Equation (9) is the fundamental inequality extended to thermo-diffusion phenomena. The left-hand side of inequality (9) is the dissipation per unit initial volume $d\Omega$ and will be denoted by $\chi$.

3. Identification of dissipations

The second law requires the dissipation $\chi$ and the associated internal entropy production $\psi / \theta$ to be non-negative

$$\chi = \sum_{i} \chi_i \geq 0$$

(10)

where

$$\chi_i = \sigma \cdot V + \varepsilon \cdot U - \psi \text{div}V - S_c U \theta \text{grad} \theta - \text{U} \cdot \text{grad} \psi_c + \psi \text{div}U$$

(11)

is the intrinsic volume dissipation described in small deformation theory

$$\chi_2 = -\frac{Q}{\theta} \theta \text{grad} \theta$$

(12)

where $\chi_2$ is the thermal dissipation associated with heat conduction and $\chi_3$ is the dissipation associated with mass transfer.
If we define \( \chi_i = \chi_1 + \chi_2 \), then the energy equation (1) can be rewritten as:
\[
T[\dot{S} + \text{div}V + \text{div}(S,U)] = R - \text{div}Q + \chi_i
\]  
(14)

4. Equations of state

Let free energy be a function of variables \( \theta, \varepsilon_i, C, \kappa \), where \( \theta \) denote temperature, \( C \) concentration and \( \varepsilon_i \) strain components. These variables constitute a set of state variables which characterize the state of the system. The free energy volume density \( \psi \) will depend locally on the state variables, but not on their rates or on their spatial gradients. Letting \( \text{grad} \theta = 0 \) the non-negativeness of total dissipation \( \chi_i \) is derived independently of the non-negativeness of total dissipation \( \chi \)
\[
\chi_i = \sigma \dot{\varepsilon} + \psi \dot{C} - S \dot{\theta} - \dot{\psi} \geq 0
\]  
(15)

The above inequality results from the second law of thermodynamics.

The non-negativeness of the intrinsic dissipation (15) gives
\[
\left[\sigma - \frac{\partial \psi}{\partial \varepsilon_i}\right] \varepsilon_i + \left[\psi, - \frac{\partial \psi}{\partial \varepsilon_i}\right] \dot{\varepsilon}_i + \frac{\partial \psi}{\partial \kappa} \kappa + \left[\varepsilon_i - \frac{\partial \psi}{\partial \theta}\right] \dot{\theta} \geq 0
\]  
(16)

what follows
\[
S = - \frac{\partial \psi}{\partial \theta}, \sigma = \frac{\partial \psi}{\partial \varepsilon_i}, \varepsilon_i = - \frac{\partial \psi}{\partial C}
\]  
(17)

The above equations yield the symmetry relations
\[
\frac{\partial \sigma_{ij}}{\partial \varepsilon_j} = \frac{\partial \sigma_{ij}}{\partial \varepsilon_i}, \frac{\partial S_i}{\partial \varepsilon_i} = \frac{\partial \sigma_{ij}}{\partial \varepsilon_j}, \frac{\partial S_i}{\partial \varepsilon_j} = \frac{\partial \sigma_{ij}}{\partial \varepsilon_i}, \frac{\partial S_i}{\partial \kappa} = \frac{\partial \sigma_{ij}}{\partial \kappa}
\]  
(18)

Let the thermodynamic states of material be characterized by external variables \( \theta, \varepsilon_i, C \) and internal variables \( \kappa \). Assume
\[
\psi = \psi(\theta, \varepsilon_i, C, \kappa)
\]  
(19)

The state equations are
\[
S = - \frac{\partial \psi}{\partial \theta}, \sigma = \frac{\partial \psi}{\partial \varepsilon_i}, \varepsilon_i = - \frac{\partial \psi}{\partial C}, S_i = \frac{\partial \psi}{\partial \theta}
\]  
(20)

The above equations are based on the normality of external variables \( \theta, \varepsilon_i, C \) with regard to the whole set of state variables there are always actual evolutions for which each of these external variables varies independently of the other.

The expression of free energy \( \psi \) with respect to variables \( T, \varepsilon_i \) and \( C \) is
\[
\psi = \sigma_{ij} \varepsilon_i - S_i \theta + \frac{1}{2} \varepsilon_i C \varepsilon_i - BC \varepsilon_i + \frac{T \varepsilon_i^2}{2 \theta_i} - \theta \varepsilon_i^2 C - \frac{1}{2} \frac{M}{\theta_i} C^2 + U(\kappa)
\]  
(21)

where \( T = \theta - \theta_i, C \) is the elasticity tensor, \( A \) and \( B \) are the material properties tensors, \( U(\kappa) \) is the frozen energy and \( c_i \) is the volume heat capacity.

The state equations in elastic range read
\[
\sigma = \sigma_{ij} + C \varepsilon_i - BC - A \theta
\]
\[
S = S_i + CS_i + A \varepsilon_i + \frac{MC}{\theta_i} + \frac{T \varepsilon_i^2}{2 \theta_i}
\]  
(22)

5. Discontinuous Galerkin method

Let \( \mathcal{M} \) be a mesh defined for the domain of interest \( \Omega \). Let \( V_{in} \), \( V_{uc} \) be displacement approximation space, \( V_{ch} \), the concentration approximation space and \( V_{th} \), the temperatures approximation space. We define DG-derivative \( D_{DG}(\cdot) \) as introduced in [5] by
\[
D_{DG}(\cdot) = V_h(\cdot) + R(\cdot)
\]  
(23)

where \( V_h(\cdot) \) denotes differentiation within elements, i.e., it does not take into account the element discontinuities, \( R(\cdot) \) is the so-called lifting operator and \( \mathcal{U} \) is the jump operator.

Consider the following functional:
\[
I_h[u_h, \cdot] = \frac{1}{2} \int_\Omega D_{DG}(u_h)C_{DG}(u_h) d\Omega - \int_\Omega C_s B_{DG}(u_h) d\Omega +
\]
\[
\int_\Omega T_{DG}(u_h) d\Omega - \frac{1}{2} \int_\Omega \frac{c}{}\theta_i d\Omega - \int_\Omega \theta S_i C_{DG}(u_h) d\Omega +
\]
\[
- \frac{1}{2} \int_\Omega \frac{M}{\theta_i} C_{DG} d\Omega - \int_\Omega f u_h d\Omega - \int_\Omega t u_h d\Omega + W
\]  
(24)

where \( W \) is the stabilization term, \( u_h \) is the displacement filed, \( f \) are body forces and \( t \) surface tractions.

Note that \( D_{DG}(\cdot) \) operator in eq. (24) is used to displacement field only. If we consider discontinuous displacement problem then we can assume that
\[
W = \beta [\frac{\partial}{\partial n} R(\{u_h\}) R(\{u_h\})] d\Omega
\]  
(25)

The parameter \( \beta \) can be any positive real number. To find the solution we have to minimize the above functional.

References

Regenerative chatter stability as a design criterion in the design of rope threading lathe

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Abstract

The paper presents the design and modeling of mass-spring-damping structure of a CNC lathe capable of turning non-circular sections. A modern construction made of subassemblies is presented. Some construction elements are designed from scratch whereas others are selected from the offer of specialized suppliers. The lathe was constructed to meet requirements set to modern machine tools. The paper shows results of finite element modeling. These results are used to determine stability limit of the designed machine. Then results of theoretical modal analysis and stability analysis are used to modify the design.

Keywords: stability, stability lobe diagram, FE modeling, lathe

1. Introduction

For many years evaluation criteria of a machine tool construction have not changed significantly. Lately, modern design approach include usage of modular units. Also along with the development of high performance machine tools stability criterion becomes increasingly important [1]. This criterion consists in conducting stable (chatter free) machining at high productivity cutting parameters. Performing changes after building an actual machine may be very expensive, even impossible. Therefore a realistic mathematical model is a valuable tool to predict behaviour of a machine tool which helps to take corrective actions regarding its design. Such an approach leads to cost effective construction of the machine tool that meet set requirements. In the paper stability limit is used as a design criterion. The methodology is illustrated by a rope threading lathe design example.

2. Design of mass-damping-spring (MDS) system of rope threading

Cax systems have become standard tools used during design and numerical analysis processes. In the presented project geometry was modelled using CAD (SolidWorks), whereas MDS system analysis was carried out using finite element method (NeiNastran). The final analysis, i.e. stability analysis was performed in Matlab environment.

The MDS system system was created on the basis of simplified 3D geometrical representation of the lathe model. This system consists of all elements and units of the machine tool that are characterized by inertia, energy dissipation and spring properties. It includes load carrying system of the machine tool, guideway systems, spindle, feed drive systems, cutting tool fixture, cutting tool and the workpiece.

Application of standard subassemblies is a common approach: fixtures, rolling guideways, motors, spindle, tailstock are purchased from specialized suppliers. Frequently, machine tool manufacturers design load carrying systems, e.g. bed. At this stage of design, systems that do not participate in load transfer (e.g. machine tool shield) are not considered.
structure analysis. Thus, a relatively stiff workpiece (diameter 40mm, length 500mm) was assumed for structural analysis. The workpiece was fixed in a 3-jaw chuck and supported by live center of the tailstock.

![Figure 3: Discretized working space of the lathe](image)

2.2. FEM analysis

In order to perform stability analysis a set of frequency response functions (FRF) at discretized locations within working space have been computed (Fig. 3). Such an approach provides a global assessment of dynamic performance of the designed lathe [3] at various location of a machined shaft. Figure 4 and 5 show an illustration of the vibration mode at 237Hz and FRF at the tool-workpiece interface respectively.

![Figure 4: Vibration mode at 237 Hz](image)

![Figure 5: SUM FRF of the lathe in Y-axis direction](image)

3. Stability

Cutting process model and FRFs computed at selected points of the working space are used to generate stability lobe diagrams. This diagram is calculated according to Nyquist stability criterion [3]. It divides depth of cut-spindle speed plane into stable and unstable regions. Stability analysis gives also frequency at which regenerative chatter develops. This frequency corresponds to structural mode responsible for stability loss. Figure 6 presents a sample stability lobe diagram.

![Figure 6: Sample stability lobe diagram (given location at the working space)](image)

4. Model modification and numerical verification

A weak element is selected on the basis of stability analysis and animation of the mode responsible for the occurrence of chatter vibration. The weak element is subjected to redesign. The redesigned machine is evaluated using stability criterion, i.e. improvement of dynamic properties of the lathe is assessed by an increase in absolute limit depth of cut (bottom of the stability lobes).

![Figure 7: Stability diagram after lathe modification](image)

References


Dynamic and positioning analysis of the feed drive of the rope threading lathe

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Abstract

The paper sets out a rope threading method, defined by ISO 10208. This highly efficient solution allows to produce workpieces in a single pass of the tool. The main problem is to ensure high acceleration in the X axis changing with a high rate of frequency. Based on the analysis of thread geometry and pre-set machining operations of the lathe, the required kinematics of the tool path was determined. A servo drive model consisting of a mechanical part, an electric motor and a regulator was developed. The effect of spindle frequency of rotation on inertia forces, required torque of the electric motor and tool offset data affecting machining precision were analysed. Frequency response analysis of analysed signals was conducted. The initial one-dimensional model of the system is currently being extended to include the receptance of elements based on FEM models and more degrees of freedom. This should better reflect the reality.

Keywords: rope thread, rope threading, feed drive dynamic, CNC lathes, servomechanism

1. Introduction

The aim of the study was to develop a lathe for highly efficient rope threading of workpieces with non-circular cross-sections. The main workpiece used on the lathe is a rope thread, defined by ISO 10208:1991(E) Ref. [1].

In the past, rope thread was machined using copying lathes which required a pre-made master or template. Currently, copying lathes are being replaced by CNC tools which offer greater versatility and compatibility with CAM machining programs. The standard rope threading on CNC machine tools is conducted at a constant position of the tool bit in the X axis. It is possible owing to the synchronisation of spindle frequency of rotation with the tool path in the Z axis. For a single-start thread, the tool path is the same as the thread pitch for a single rotation of the spindle. The drawback of the process is that it requires many machine adjustment movements.

Rope threading may provide an interesting alternative. It requires, however, high dynamics of the tool feed system in the X axis. The problems of rope threading on a universal CNC machine tool were discussed by Jastrzębski Ref. [2]. Despite its drawbacks, the method allows to produce threads and other workpieces with non-circular cross-sections in a single pass of the tool. High efficiency of rope threading requires high speed and acceleration of the feed system in the X axis. These requirements must be met by the dynamic systems of a lathe and its efficient control systems.

The developed model provides data about the required torque of the electric motor at the pre-set spindle frequency of rotation. It can also determine the precision of machining operations. The model consists of a mechanical part, a PMSM motor and a regulator with pre-set setpoints. All these elements constitute a servo drive that controls the tool path along its pre-set geometry. The model also accounts for forces resulting from machining operations.

2. Kinematics

The thread can be understood as the workpiece with a cross-section other than circular. The cross-section is defined by a constant crest which turns in an axial displacement of the cross-sectional area. A full rotation is made at a length equal to the thread pitch b. During the machining, the tool moves in the X axis that compensates the difference between the cross-section and the circle. In the machining that involves movement in the Z axis, the workpiece rotates relative to the tool and the contour makes the virtual rotary motion that results from the tool moving in the Z axis. In a standard rope threading, the rotation angles of the movements are the same hence the position of the tool in the X axis is constant. In the proposed solution, the movement in the Z axis is significantly smaller which leads to differences in the rotation angles of the movements. Hence the need to move the tool in the X axis. To keep the difference at the lowest level, it is assumed that the thread direction is the same as that of the spindle rotation. The relative rotary movement resulting from the angle difference can be given by:

\[ \phi_{\text{total}} = \phi_{p0} - \phi_k \]
\[ \varphi_{po} = 2 \pi \frac{n}{60} \cdot t \]  
(2) \[ \varphi_{po} = \frac{b}{B} + 2 \pi \frac{n}{60} + t \]  
(3) 

where:
- \( \varphi_{po} \) – the rotation angle of the workpiece relative to the tool;
- \( \varphi_{e} \) – the virtual rotation angle of the crest resulting from the tool movement in the X axis;
- \( n \) – the spindle frequency of rotation [mm/rev];
- \( t \) – the time unit [s];
- \( f_{x} \) – the tool movement in the Z axis [mm/rev];
- \( b \) – the thread pitch [mm].

The relative rotational speed \( \varphi_{WS} \) is defined by the frequency of the tool feed in the X axis.

3. The mechanical part of the model

3.1. Basic model

The above model was extended with a dynamics model of the driven support with the tool and the tool. In this model, the point mass \( m \) is substituted with a multidimensional model. It enables the analysis of dynamic interactions between the control system, the drive and some selected points of the construction. The model is represented by the frequency response matrix.

Figure 2: Dynamic model of table feed in the X axis

\[ F_{p}(t) = M_{e} \cdot x + F \]  
(9) 

where:
- \( F \) – the vector of forces acting on the support (force from the drive unit, a component of machining force, movement resist forces);
- \( H \) – the receptance matrix;
- \( x \) – the displacement vector.

References

A multi-layer beam finite element for mixed-mode delamination in 2D beams

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Abstract

A 2D finite element formulation for a multi-layer beam with an arbitrary number of layers with interconnection allowing for mixed-mode delamination is presented. The layers are modelled as linear beams, while interface elements with embedded cohesive zone model are used for the interconnection. The only basic unknown functions of the system are two components of the displacement vector and one cross-sectional rotation per layer. A bi-linear constitutive law for a single delamination mode and a mixed-mode damage evolution law are used to model damage at the interconnection. Sharp snap-backs in the force-displacement diagram, which appear as an artefact of the numerical integration, are handled with the modified arc-length method in the solution procedure. The present model is successfully verified against commonly used models which use 2D plane-strain finite elements for the bulk material as well as against the analytical solutions on various numerical examples from the literature.

Keywords: multi-layer beam, mixed-mode delamination, damage, non-linear analysis, modified arc-length method

1. Introduction

Research and application of layered composite structures in many areas of engineering has been a topic of undiminished interest in the computational mechanics community over the last few decades. Due to an optimised performance of the components of composite structures they are economical and have a high load-bearing capacity. The mechanical behaviour of these structures largely depends on the type of connection between the layers, which is usually not completely rigid and allows for interlayer slip or/and uplift. Delamination between the components of composite structures is one of the most prevalent and severe failure modes in layered composite structures. In order to fill the gap between expensive computational procedures for delamination modelling in composite structures and a desire to have more effective and engineer-oriented design tools, in this work, a finite element formulation for a multi-layer beam with compliant interconnection is presented. Here, the processes of crack occurrence and propagation, damage-type material softening, and eventual delamination are modelled using Timoshenko’s beam-type finite elements to describe structural layers and interface elements with bi-directional stiffness [1] to describe interconnections between layers. Beam elements are more intuitive than solid elements, making use of a smaller number of degrees of freedom eventually reducing the overall computational effort, and can be used with very good accuracy for problems like double cantilever beam (DCB) and peel tests.

2. Problem description

An initially straight multi-layer beam of length \( L \) is considered. The layers are allowed to move with respect to one another depending on the properties of the interconnection is considered. The beam is composed of \( n \) layers and \( n − 1 \) interconnections. An arbitrary layer is denoted as \( i \), while an arbitrary interconnection, placed between layers \( i \) and \( i + 1 \), is denoted as \( \alpha \).

Kinematic equations for layers are defined according to Timoshenko’s beam theory, constitutive equations assume that the material of layers is linear-elastic, while the equilibrium equations are derived from the principle of virtual work (see [2] for details). Interface finite elements by Alfano and Crisfield [1] with embedded cohesive zone model (CZM) are adopted in the present multi-layer beam model. The interface is a zero-thickness layer with a non-linear constitutive law allowing for delamination in modes I and II including a mixed-mode delamination. Thus, depending on the conditions on the interface, the connection between layers can be linearly-elastic and after the softening of the interconnection material a complete damage may occur. Deformations of an interconnection \( \alpha \) are calculated from the relative displacements of adjacent layers \( i \) and \( i + 1 \), thus the interconnection does not introduce any new degrees of freedom. Basic unknown function for a multi-layer beam composed of \( n \) layers are two components of displacement \( (u_i(X_i) \text{ and } v_i(X_i)) \) and a cross-sectional rotation \( \theta_i(X_i) \) per layer which makes a total of \( 3 \times n \) unknown functions. Since the proposed formulation is non-linear due to the interconnection constitutive law, the solution is obtained numerically using finite element method.

3. Solution procedure

The total virtual work for the multi-layer beam analysed is composed by the virtual work of \( n \) layers and the virtual work of \( n − 1 \) interconnections. The nodal vector of residual forces is obtained from the condition that the total virtual work for the multi-layer beam must equal zero. Linearising the nodal vector of residual forces the nodal tangent stiffness matrix can be obtained. The global vector of residual forces, global tangent stiffness matrix and global vector of increments of the unknown functions are assembled using the standard finite-element assembly procedure, the system is solved using the Newton-Raphson procedure. Gauss quadrature with \( N = n − 1 \) integration points is used for the integration of the beam parts (layers) and Simpson’s rule with \( N + 1 \) integration points is used for the interconnection parts, where \( N \) is the number of nodes.

For each finite element and each interconnection the relative displacements are calculated. Then, at each integration point of the interconnection the current stage of delamination is determined (linear-elastic behaviour, softening, unloading and reload-
ing with a reduced stiffness or total damage). The total loss of adhesion at an integration point would lead to very sharp snapbacks in the load-displacement diagram. This behaviour cannot be captured neither with standard load- or displacement-control methods in the Newton-Raphson solution procedure, nor with the standard arc-length procedure. In the present work the modified arc-length method [1] is used.

4. Numerical Examples

The present model has been tested for mode I, mode II and mixed-mode delamination.

4.1. Mode I delamination

The double cantilever beam (DCB) test for mode I delamination from [1] is performed using the present formulation. An excellent agreement of the results obtained using the present model with analytical solution can be noticed (see Fig. 1). In [2] it is also shown that the present model, using significantly less degrees of freedom, gives results which basically coincide with the results from [1], obtained using 2D solid elements for the bulk material.

Figure 1: Results for the DCB test

4.2. Mode II delamination

The results of the present model for the end-notched flexure (ENF) specimen test again show excellent agreement with the analytical results (see Fig. 2). In [2] it is shown that these results are closer to the analytical solution using a smaller number of degrees of freedom than presented in [3] and obtained using 2D solid elements for the bulk material.

Figure 2: Results for the ENF specimen test

4.3. Mixed-mode delamination

The present model was successfully tested on different specimen for mixed-mode delamination [1, 3, 4] (see [2] for details), but here only results for a mixed-mode delamination specimen from [4] are presented (see Fig. 3). For three cases of material parameters (A, B, C) of the interconnection it was shown that the oscillations, typical for the numerical analysis of delamination problems, depend not only on the meshing and number of integration points, but also on the material parameters of the interconnection. Again, it was shown that multi-layer beam model, in comparison with the models using 2D elements for the bulk material, gives results of satisfactory accuracy using significantly smaller number of degrees of freedom.

Figure 3: Results for the mixed-mode delamination specimen

5. Conclusions

A multi-layer beam with interconnection allowing for delamination between layers was presented, where the bulk material is modelled using beam finite elements and the cohesive-zone model incorporated into the interface elements is used for the interconnection. Modelling the bulk material structure as beams, in comparison with the commonly used 2D plane-strain finite elements, gives comparable accuracy using a reduced total number of degrees of freedom. It was noticed that the mesh-dependent oscillations caused by the numerical integration are reduced with the mesh refinement or addition of the integration points.

References

Numerical and experimental prediction of the yield condition for porous materials

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Abstract

The porosity increases the plasticity of the elastic-plastic materials. Both microscopic and macroscopic plastic yields in porous materials are considered in this paper. In the microscopic approach the interactions between pores are examined in the sense of Mori-Tanaka scheme. The previous research has shown that in the macroscopic approach the plastic yield starts at the particular macroscopic strain and is independent of the porosity. Micromechanical and macroscopic considerations are verified by the elastic-plastic FEM analysis and by the experimental investigations as well.

Keywords: yield condition, porous material, Mori-Tanaka approach, micromechanics

1. Introduction

It is well known that the porosity increases the plasticity of elastic-plastic materials. This phenomenon has been investigated by a large number of researchers. In the porous materials the shape and orientation of pores influence the yield surface and cause the plastic anisotropy. The yield condition depends not only on the second invariant of the deviatoric stress but also on the first invariant of the stress tensor. It should be mentioned that the yield condition for porous material cannot be determined precisely. In the vicinity of the pores plastic strains occur initially at relatively small load magnitudes and form the so-called pockets. When the load increases these plastic pockets remain stable (they are locked next to pores) up to the state when the plastic zones grow rapidly connecting each other and finally causing the macroscopic yield. The construction of the macroscopic strain can be predicted by investigations made on the solid material (without pores).

2. Micromechanical approach to the yield condition for porous materials

The complete formulation for the yield condition for porous materials is presented in Ref. [3]. The following ratio of the intensity of the shear stress to the hydrostatic stress generated by remotely imposed displacements is introduced:

\[ \zeta = \frac{\sigma_{ij} \sigma_{ij}}{\text{ijkl} / 3} \]  

(1)

where \( \sigma_{ij} \) is the deviatoric stress. The macroscopic strain generated by the displacements imposed on the boundary on representative volume is expressed in terms of macroscopic stress as the sum:

\[ \varepsilon_{ij} = \left( S_{ijkl} + \sum_m H_{ijkl}^{(m)} \right) \sigma_{kl} \]  

(2)

where \( S_{ijkl} \) is the compliance tensor of the matrix and \( H_{ijkl}^{(m)} \) is the compliance tensor of m-th cavity. The interactions between pores are determined by Mori-Tanaka scheme (Ref. [2]). The macroscopic stress can be expressed in terms of applied strain as:

\[ \sigma_{ij} = \left( S_{ijkl} + \tilde{H}_{ijkl} \right)^{-1} \varepsilon_{ij} \]  

(3)

In Eqn (3) the explicit form of the inverse of the fourth rank tensor is required in order to analyze the effect of pores shapes and orientation. This non-trivial problem can be solved using the technique based on the representation of tensors in tensorial bases – Ref. [1].

The detailed equations of the plastic yield condition for porous materials are not presented here. In the simple case of overall isotropy the yield condition is reduced to the following form:

\[ 2 \sigma_{ij}^2 = A_1 (\sigma_{ij}^2) + A_2 \sigma_{ij} \]  

(4)

where \( \sigma_{ij} \) is the yield stress of the matrix material. For the porosity \( p \) in the case of spherical pores the coefficients \( A_1 \) and \( A_2 \) are:

\[ A_1 = \frac{p}{1-p} \frac{(1-v)(1-2v)}{4(1+v)^2} + \frac{p^2}{1-p} \frac{(1-v)^2}{(1-p)^2} \frac{7(1+v)}{32(1+v)^2} \]  

(5)

\[ A_2 = 1 + \frac{p}{1-p} \frac{15(1-v)}{4(7-5v)} + \frac{p^2}{1-p} \frac{225(1-v)^2}{4(7-5v)^2} \]  

(6)

The yield condition (4) can be expressed in terms of the \( \zeta \) parameter as follows:

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3. FEM analysis of the effective elastic constants and overall yield stress

In order to verify the results of micromechanical analysis the simple 2D tension test in modelled by the finite element method. The considered specimen includes circular pores of random diameters and locations. The lower edge of the body is fixed while kinematic load (displacement) is applied to the upper edge. This way assumed macroscopic strain is attained. Elastic-plastic bilinear material model is selected. Fig. 1 presents the distribution of the effective plastic strain for an early stage of loading when the plastic pockets appear in the neighbourhoods of pores as well as for the final state of loading when the whole cross-section of the specimen yields.

![Figure 1: Microscopic (left) and macroscopic (right) yield](image)

4. Experimental investigations of a porous sample

Experimental investigations are part of the presented research. The specimen shape is the rectangular sheet of 20x20 mm dimensions made of inconel AMS5599 material. The regular set of circular pores (Fig. 2) is made by application of laser beam technique. The tensile tests are completed on the testing machine for various overall strains up to the damage of the specimen.

![Figure 2: A specimen after plastic deformations](image)

Figure 3: Tension force [N] vs. strain [%]

5. Conclusions

Theoretical, numerical and experimental predictions of the yield stress are presented in this paper. In theoretical approach the typical methods for micromechanics are applied (Mori-Tanaka scheme). In numerical computations the elastic-plastic problem is solved by the finite element method. Finally, in experimental investigations several tests are made for 2D specimen. The reasonable conformity is achieved for all approaches (Fig. 3). FEM simulations have shown that plastic pockets which appear in the vicinity of pores remain stable for a wide range of loading and the overall response is almost linear up to the yield of the whole specimen cross-section. Experimental tests proved the presence of the overall plastic stress in materials with pores.

References

Influence of crack size on resonant frequency of compressor blade

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Abstract

The paper presents results of an experimental modal analysis of the compressor blade with the fatigue crack. In investigations the modern vibration system was used. The laser scanning vibrometer was utilized to measure the blade amplitude. In the first part of investigation, the blade was subjected to resonant vibrations. Transverse vibrations cause the stress oscillation and in consequence the fatigue of blade material. After a certain number of load cycles the first crack was detected. During the crack propagation process the fatigue test was interrupted in order to obtain the amplitude-frequency characteristic. As a result of the performed investigations, the influence of crack size on resonant frequency of the compressor blade was determined.

Keywords: compressor blade, turbine engine, fatigue crack, modal analysis

1. Introduction

Compressor blades belongs to the most important components of aero engines. They compress of air which is next mixed with fuel and used in combustion process. The blade of axial compressor during work is subjected to high rotational velocity and the large centrifugal force acts on the blade components. The operational speed of rotor of small turbine engines achieves 50000 RPM. Centrifugal forces cause that a large radial stress occurs in the blade.

Fracture problem of compressor blades were analyzed in the works [1,2,3,4,5]. In many publications the authors considered this problem from the point of view of the fatigue of materials. In this paper attention is focused on different research aspect. The objective of presented investigation is to determine the influence of crack size on resonant frequency of the compressor blade during crack propagation.

The work presents results of investigations concerned with modal analysis of compressor blades with fatigue cracks which are interesting both the research and practical point of view. A modal analysis of blades performed during engine main inspection can indicate which blade has a fatigue crack and should be replaced.

2. Experimental investigations

The crack propagation and modal analysis of the blade were performed with use of Unholtz-Dickie UDCO-TA-250 vibration system at Laboratory of Turbomachinery in Rzeszów University of Technology (Fig. 2). For control of amplitude

Figure 1: View of first stages of compressor

Figure 2: Compressor blade during vibration test

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of blade tip displacement the laser scanning vibrometer POLYTEC PSV-H400S were used. To measure the crack length (a dimension in Fig. 3) a non-destructive fluorescent penetrant inspection was utilized (Fig. 4). In all tests, the intensity of excitation 10 g was defined (where 1g equals 9.81 m/s²). During investigations only the first mode of transverse vibration was considered. The blade consists v-notch located on the attack edge, 3.0 mm above the blade lock. In first phase of investigations the blade was tested in resonance condition to first fatigue crack appearance (to crack length a = 2.5 mm). After that, the first amplitude-frequency characteristic presented in Fig. 5 was obtained (right plot, for a = 2.5 mm). The blade was next partially fractured.

During stops in crack propagation process (for crack length equals 4.5; 7.0; 10.0 and 16.5 mm) the modal analysis of blade with different crack size was performed. Results of this analysis are presented in Fig. 5 and in Table 1.

![Figure 3: Fracture of blade with crack length (a) definition](image3)

![Figure 4: View of fatigue crack in UV light (fluorescent NDE method)](image4)

![Figure 5: Amplitude-frequency characteristics for blade with crack length of: 2.5; 4.5; 7.0; 10.0 and 16.5 mm](image5)

3. Results and discussion

The results presented in Fig. 5 showed that the blade without defects has a resonant frequency $F_{rez}$ equal to 820 Hz. For blade with crack length 2.5 and 4.5 mm a small change of resonant frequency is observed (3 and 15 Hz). When the crack has a medium size (a = 10.0 mm) the difference of $F_{rez}$ between non-damaged blade and the blade with crack is 93 Hz (11%).

For the blade with crack size a = 16.5 mm a large decrease of resonant frequency is observed (289 Hz (35%)). Obtained results showed that proposed method can be useful in practice to detection of the fatigue crack in the compressor blades during the main inspection of engine.

<table>
<thead>
<tr>
<th>Crack length a [mm]</th>
<th>Resonant frequency $F_{rez}$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>820</td>
</tr>
<tr>
<td>2.5</td>
<td>817</td>
</tr>
<tr>
<td>4.5</td>
<td>805</td>
</tr>
<tr>
<td>7.0</td>
<td>779</td>
</tr>
<tr>
<td>10.0</td>
<td>727</td>
</tr>
<tr>
<td>16.5</td>
<td>531</td>
</tr>
</tbody>
</table>

Figure 5: Resonant frequency (first mode) in function of crack length

References

Modelling and Simulating Disperse Two-Phase Flows

organized by A. Soldati, J.P. Minier, B. Geurts and J. Pozorski
A stochastic approach for the deposition and resuspension of complex multilayered structures

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Abstract

The study aims at presenting a new stochastic Lagrangian approach for the simulation of particulate fouling (i.e. the accumulation of particles) for rough surfaces in turbulent flows. In particular, the study is focused on the later stages of fouling where multilayer deposition and resuspension occur. It is shown that the formation of complex structures on rough surfaces can be properly captured using a stochastic Lagrangian approach provided that the effects of particle-fluid, particle-wall and particle-particle interactions are properly accounted for. Besides, new models have been developed to account for deposit consolidation (i.e. modifications of the deposit morphology/structure with time due to small-scale chemical effects) as well as multilayer resuspension. Finally, the present work underlines the limitations in the current understanding of particulate fouling and outlines areas where further experimental and numerical analyses are required.

Keywords: fouling, stochastic approach, multilayer, deposition, resuspension

1. Introduction

Particulate fouling generally arises from the continuous deposition of small solid particles on initially clean surfaces. This accumulation generally affects the operating conditions and performance of various components in a wide range of fields (for instance, oxide particles in heat exchangers, soot particles in combustion engines, biological organisms in membrane filtration or medical fields). This process can lead to the formation of multilayered deposits with complex properties and geometries. From this quick overview, a distinction between four categories of fouling materials can be proposed [1]: particulate fouling (by solid particles), organic fouling (by organic species), biological fouling (accumulation of biological organisms) and precipitation fouling (crystallisation of salts).

The study is more specifically concerned with the simulation of fouling by solid inorganic particles, with a specific emphasis on colloidal particles (i.e. particles with sizes ranging from a few nanometres up to a few micrometres [2]). The main purpose of this study is to present a stochastic Lagrangian approach for the simulation of particulate fouling and, more specifically, for the simulation of the later stages of fouling (where multilayer deposition and resuspension occur).

2. A stochastic Lagrangian approach for particulate fouling

Particulate fouling is addressed as resulting from the coupling between four elementary phenomena: deposition, resuspension, agglomeration and clogging (i.e. multilayer deposition) [3, 4]. All four phenomena related to particle fouling are governed by two mechanisms: a transport step (particle-fluid interactions) and an attachment step (particle-surface-particle interactions). In the study, particle transport is described using a Lagrangian one-point pdf approach while particle attachment to the surface is described using the DLVO theory (with refinements to account for surface roughness as in [5]). The coupling between the transport step and the attachment step follows an energy-balance approach, where the kinetic energy $E_{\text{kin}}$ of a particle impacting the surface is compared to the energy barrier $E_{\text{barr}}$ encountered upon interacting with the fouled or clean surface: deposition occurs only if $E_{\text{kin}} > E_{\text{barr}}$.

2.1. Multilayer deposition

A stochastic model for multilayer deposition has been recently developed and is based on the following principle: upon approaching a fouled surface, the incoming particle interacts either with a clean area of the surface or with already deposited particles/clusters (see Fig. 1). The distinction between these two cases is based on geometrical considerations and on physical parameters (surface covered by already deposited particles $S_{\text{cont}}$). The number of deposited particles in contact with the particle $N_{\text{cont}}$ is generated randomly and the energy barrier is then evaluated considering particle-plate interactions and every particle-particle interactions.

![Figure 1: Sketch showing that incoming particles stick either to the clean surface (single particle deposition) or to existing particles/clusters. Reprinted from [3]. Copyright 2012 with permission from Elsevier.](image)

This model has been tested previously and shown to provide qualitative agreement with available experimental data [3]. Nevertheless, it should be noted that the main feature of the present model lies in the fact that the multilayer deposits are modelled with a set of cylinders growing on the surface, as depicted in Fig. 2. Therefore, no information on the detailed structure of the cluster is available since the model only provide the distribution of cluster height, radius on the surface and the number of...
particles within the cluster.

Figure 2: Clogging of a microchannel through sequential deposition of small particles (a,b,c) until the flow passage becomes blocked by a larger particle (d). Reprinted from [3]. Copyright 2012 with permission from Elsevier.

2.2. Consolidation effects

The present model was extended to account for deposit consolidation (i.e. modifications of the deposit morphology/structure with time due to small-scale chemical effects such as precipitation of oxides, Ostwald ripening, sintering and/or boiling-induced precipitation). Due to the mesoscopic level of description of the multilayer deposition model, we have retained a modelling approach where inter-particle forces are modified with time to account for the deposit consolidation.

As depicted in Fig. 3, the inter-particle force evolves continuously from the interparticle adhesion force $F_{adh,DLVO}^{p-p}$ to a consolidated force $F_{consol}^{p-p}$ with time (and height) of the deposit.

Figure 3: Sketches of the model retained for the deposit consolidation: inter-particle forces evolves from the interparticle adhesion force $F_{adh,DLVO}^{p-p}$ to a consolidated force $F_{consol}^{p-p}$ with time (and height) of the deposit.

The evolution of the inter-particle interaction potential energy at that point.

Figure 4: Monolayer resuspension model: a three-stage scenario. Reprinted from [6]. Copyright 2012 with permission from American Chemical Society

3. Conclusion

In the study we highlight the development of stochastic Lagrangian approaches for the simulation of multilayer deposition and resuspension including the effects of deposit consolidation. It is shown that satisfactory results can be obtained provided that the coupling between particle-fluid, particle-surface and particle-particle interactions is properly accounted for in the modelling approach. However, the current understanding of the later stages of particulate fouling is still limited and further experimental/numerical analyses are required.

References

A stochastic model for Lagrangian particle tracking in large-eddy simulation velocity fields

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Abstract

A Lagrangian stochastic model is proposed for tracking the inertial particles in fluid velocity fields obtained in large-eddy simulations; the model is formulated for the fluid velocity seen by the particles along their trajectory. The behaviour of the model is first investigated for tracer particles in a turbulent channel flow. It is checked that in this limit case similar statistics to those given by a fluid phase LES are obtained. Next, stochastic model is used for Lagrangian tracking of particles of different inertia. The results will be shown in the final presentation and will be compared to particle statistics and concentration obtained in DNS and in LES with no model for particle equations.

Keywords: inertial particles, Lagrangian tracking, large-eddy simulation, stochastic subgrid scale modeling

1. Introduction

The dispersion of small inertial particles in inhomogeneous turbulent flows is important in a number of industrial applications and environmental phenomena, such as, mixing, combustion, deposition, spray dynamics, pollutant dispersion or cloud dynamics. Direct Numerical Simulations (DNS) of turbulence coupled with Lagrangian Particle Tracking (LPT) demonstrated their capability to capture the mechanisms characterizing particle dynamics in turbulent flows. Due to the computational requirements of DNS, however, analysis of problems characterized by complex geometries and high Reynolds numbers demands alternative approaches; Large-Eddy Simulation (LES) is increasingly gaining popularity, especially for cases where the large flow scales control particle motion. LES is based on a filtering approach of the fluid phase governing equations; thus, only the filtered fluid velocity is available for particle tracking and particles are prevented from interacting with the small (unresolved) Sub-Grid Scales (SGS) of turbulence. This may strongly influence clustering of inertial particles and lead to significant underestimation of particle preferential concentration and deposition rates (see e.g. [1]). Hence, there is currently a general consensus about the need to model the effect of SGS turbulence on particle dynamics.

Different kinds of SGS models for particle motion equations were proposed in the literature, e.g. filtering inversion or approximate deconvolution [2], 3], fractal interpolation [2], stochastic modeling [4, 5] or mixed models [6]. Previous studies [7, 8, 9], focusing on the error purely due to filtering of the fluid velocity field seen by the particle along its trajectory, showed that this error is stochastic and may exhibit a non-Gaussian and intermittent nature.

The work is aimed at developing and appraising a new stochastic model for subgrid scales of large-eddy simulation of turbulent polydisperse two-phase flows. The model is based on the formalism for the filtered density function (FDF) approach in LES simulations. Contrary to the FDF used for turbulent reactive single-phase flows, the present formalism is based on Lagrangian quantities and, in particular, on the Lagrangian filtered mass density function (LFMDF) as the central concept [10].

A first example of Langenvin model constructed within the above formalism is proposed considering isotropic sub-grid fluctuations, but taking into account crossing-trajectory effects and paying attention to the consistency of the model with the fluid limit case.

2. Physical Problem, Numerical Methodology and Modeling

The physical problem considered in this study is the dispersion of inertial particles in turbulent channel flow. The reference geometry consists of two infinite flat parallel plates separated by a distance \(2h\). The origin of the coordinate system is located at the center of the channel with the \(x\), \(y\) and \(z\) axes pointing in the streamwise, spanwise, and wall-normal directions, respectively. Periodic boundary conditions are imposed on the fluid velocity field in the homogeneous directions (streamwise, \(x\), and spanwise, \(y\)), no-slip boundary conditions are imposed at the walls. The size of the computational domain is \(L_x \times L_y \times L_z = 4 \pi h \times 2 \pi h \times 2h\). The shear Reynolds number is \(Re_s = u_s h / \nu = 300\), where \(u_s = \sqrt{\tau_w / \rho}\) is the shear velocity based on the mean wall shear stress.

The flow solver is based on a Fourier-Chebyshev pseudospectral method to discretize the LES equations. The SGS models considered for the fluid phase are the classical and the dynamic Smagorinsky model.

The Lagrangian tracking is based on the following equation of motion:

\[
\begin{align*}
\frac{dx_p(t)}{dt} &= U_p(t) \\
\frac{dU_p(t)}{dt} &= \frac{1}{\tau_p} (U_s(t) - U_p(t))(1 + 0.15 Re_p^{0.687})
\end{align*}
\]

In these equations, \(U_s(t) = U(t, x_p(t))\) is the fluid velocity "seen", i.e. the fluid velocity sampled along the particle trajectory \(x_p(t)\), \(\tau_p\) is the particle relaxation times and \(Re_p\) the par-
particle Reynolds number. The fluid velocity seen is given by the following stochastic Lagrangian model:

\[
dU_{x,i}(t) = -\frac{1}{\rho_j} \frac{\partial p}{\partial x_i} dt + \sum_{j=1}^{3} (U_{p,j} - \bar{U}_j) \frac{\partial \bar{U}_i}{\partial x_j} dt - \frac{1}{T_{L,i}} (U_{x,i} - \bar{U}_i) dt + \sqrt{\varepsilon_r (C_0 b_i + \frac{2}{3} (b_i - 1))} dW_i(t)
\]

where the overbar denotes filtered quantities, output of LES of the fluid phase, \( T_{L,i} \) is a modified time scale obtained by multiplying the Lagrangian time scale, \( \varepsilon_r \) by the Csanady factor, \( b_i \) is the sub grid dissipation and \( b_i = T_{L,i} / T_{L,i} \). Finally \( dW_i(t) \) is a Wiener process.

Equations (1) and (2) are discretized through either a first or a second-order scheme, based on stochastic rules.

3. Results and Discussion

As a first test, behaviour is considered of the Lagrangian stochastic model for particles of zero inertia, i.e. fluid tracers. The LES of the fluid flow was carried out on a grid having a resolution of \( 32 \times 32 \times 33 \), which corresponds to a coarsening factor of 8 in each direction compared to the DNS resolution. Different SGS models have been used, namely no SGS model, the Smagorinsky and the dynamic models. The filtered fluid fields obtained from LES are compared with filtered velocities of zero inertia particles, in this limit case they should be nearly the same.

For particle tracking, 40 particles per cell initially randomly distributed was considered. It was checked that increasing the number of particles per cell does not significantly affect the results.

Figure 1 shows the mean streamwise velocity profiles computed from the stochastic model for tracer particles coupled with LES with different SGS models; they are compared with the corresponding LES fluid mean streamwise velocity profiles. It can be seen that, as expected, the stochastic model for tracer particles give the same mean velocity field as the one of the LES in which the particle are tracked.

![Figure 1: Mean streamwise velocity profiles computed from the stochastic model for tracer particles compared to those obtained for fluid in LES](image)

For the higher order statistics, Fig. 2 shows the streamwise velocity fluctuations on a plane at \( z^+ = 10 \) obtained in LES with no SGS model for the fluid part (top panel) and from the stochastic model for tracer particles (bottom panel). It can be seen that there is a good corelation and in particular streaky structures are clearly visible in both cases. The quantitative agreement is not perfect, as could be expected, since the stochastic model implicitly provides SGS terms, which have an impact on the higher order statistics. Hence, the filtered results of the stochastic model in the limit case of tracer particles can be considered as those of a LES of the fluid phase with a SGS model, which does not exactly correspond to any of the classical SGS models.

![Figure 2: Streamwise velocity fluctuations on a plane at \( z^+ = 10 \); LES (top panel) and stochastic model for tracer particles (bottom panel)](image)

The presented stochastic model was also used for tracking particles having different inertia in LES fluid velocity fields. The results will be shown in the final presentation.

References


Partial evaporation and total cut-off wet steam region on the shock wave

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Abstract

In the paper we focused on the phenomena of condensate re-vaporation in the shock wave zone. This process often occurs in the wet steam flow through de Laval nozzle. In the paper some original mechanistic model of droplet evaporation is involved, numerically implemented and compared with the IMP PAN experiment and experiment carried out by Dykas. The model of a single continuum with a special microstructure growing up during phase transitions, proposed by Bilicki and Badur, is taken to be a general non-equilibrium framework for description of condensation as well as vaporization. In our model, evaporation is governed not only by mass transport but also by internal structure energy based on balance of heat energy transported into a droplet. The novelty of our approach lies in modelling both the moment of initiation of a phase transition, as well as the moment of its reverse progress - called here re-vaporization of the condensate phase. The authors obtained two cases of re-evaporation of condensate droplets in the shock wave zone. The applied model include both partial evaporation and total cut-off wet steam region on the shock-wave.

Keywords: condensation, evaporation, shock wave, de Laval nozzle, Bilicki - Badur model

1. Introduction

Steam enters the LP turbine as superheated vapour and leaves saturated, with wetness forming as a result of several phase transition processes. The wetness mainly consists of large number of minute droplets which are initially nucleated within and generally carried by the flow [1,3]. The formation and evolution of these droplets has to lower the performance of the wet turbines stages and their effects on the efficiency are responsible for growth and evaporation of droplets:

\[ \frac{\partial r}{\partial t} + \nabla \cdot (\rho \nabla r) = \nabla \cdot (J_{\rho}r) + \rho S_r \]

where \( r \) is a critical Kelvin – Helmholtz radius of droplet, \( \rho \) - density of condensed phase, \( r_c \) - the critical Kelvin – Helmholtz radius of droplet, \( L \) - is a volumetric rate of nucleation, \( \alpha \) - volume fraction of condensate [1].

2. Evolution equation of dryness fraction

A wet steam model include governing equations which are based on balance of liquid-vapour mixture. For a consistent non-equilibrium condensation model a set of nine transport equations can be written in general form [1]:

\[ \frac{\partial (\rho \phi)}{\partial t} + \nabla \cdot (\rho \phi \mathbf{v}) = \nabla \cdot (\mathbf{J}_{\phi}) + \rho S_{\phi} \]

where \( \phi \) = \( I, \nu, e, k, \epsilon, \alpha, a \)

3. Conditions of experiments and numerical analysis

The IMP PAN experiment was carried out in a symmetric nozzle of rectangular cross-section. The nozzle dimensions and variation of cross section area along the nozzle axis are shown in Fig. 1. Modelling conditions were taken from description of experiment [3]. Inlet pressure was \( p_{inlet} = 2.26 \) bar, inlet temperature \( T_{inlet} = 502.15 \) K. The pressure at the point of intersection of the isentropic expansion line and saturation line was \( p_{sat} = 0.55 \) bar. These input conditions correspond to the V-th set of experimental conditions [3].

The Silesian experiment was carried out on two half arc de Laval nozzles of ,,rectangular” cross-section. The shape of two half arc de Laval nozzles is shown in Fig. 2. Inlet total pressure was \( p_{inlet} = 0.96 \) bar, inlet temperature \( T_{inlet} = 381 \) K. Back
pressure at the nozzle outlet was about $p_{outlet} = 0.4$ bar. These boundary conditions correspond to experimental conditions for case of D1 nozzle [2].

Figure 1: The shape and dimensions of the IMP PAN nozzle [3]

4. Results, discussion and conclusions

For a given de Laval nozzle (and two half arc de Laval nozzles) geometry the authors conducted calculations for wet steam model. Figure 3 shows a comparison of the results of the IMP PAN experiment with an employed model of wet steam for the V-th set of experimental conditions. Fig. 4 shows wetness for calculation of CFD simulation for this case. Results of our analysis (condensation and shock wave) is the same like in the experiment of Puzyrewski: For V-th set of experimental conditions, there were only very slight traces of mist ahead of the shock wave, disappearing in the shock. Due to the increase of steam conditions through the wave, there is a complete evaporation of the condensed water drops.

Figure 3: Static pressure versus nozzle length: IMP PAN experiment and calculations

Figure 5 shows a comparison of the static pressure distribution obtained from numerical simulation with Silesian experimental data (case of D1 nozzle). In Fig. 6 we can see photo of shock wave taken during experiment and the same shape and localization of shock for CFD simulation for the same flow conditions. The computed distribution of the wetness fraction (Fig. 6) shows partial evaporation of the liquid phase on the shock wave.

Figure 5: Static pressure versus nozzle length for D1 nozzle: Silesian experiment and calculations

Figure 6: Partial evaporation of condensate for D1 nozzle

Models of nine-equation non-equilibrium were tested and compared with the experimental data. This wet-steam model, describing both condensation and re-vaporization, applied to de Laval nozzle gives satisfactory results that agree well with experimental data (very well describes both partial and complete condensate evaporation in the shock wave zone). The presented data indicates correctness and utilization of employed model for condensing flows in more complex geometries.

References

Turbulent breakage of ductile aggregates

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Abstract

In this paper we study breakage rate statistics of small colloidal aggregates (modelled as sub-Kolmogorov massless particles) in non-homogeneous anisotropic turbulence. We focus specifically on ductile rupture, which is activated when the hydrodynamic fluid stress acting on the aggregate exceeds a critical value, \( \sigma > \sigma_c \), and is brought to completion when the energy absorbed by the aggregate meets the critical breakage value. We show that ductile rupture is associated to significant reductions of the breakage rate with respect to instantaneous rupture, which produces breakage as soon as the condition \( \sigma > \sigma_c \) is met. These discrepancies are due to the different energy values at play as well as to the statistical features of energy distribution in the anisotropic turbulence case examined.

Keywords: breakage rate, colloidal aggregates, turbulence, energy dissipation, ductile rupture

1. Introduction

Breakage rates of micro and nano aggregates in turbulent flow are of high relevance to a wide variety of applications [1, 2]. When aggregates are small with respect to the characteristic flow length scale and have density close to that of the fluid, breakage is caused by the hydrodynamic stresses exerted by the flow around the aggregate, which in turn induce internal stresses acting on the structure of the aggregate. If the response time of the aggregate to stress-induced deformations is small then breakage is instantaneous and the aggregate rupture can be referred to as brittle; otherwise breakage depends on the stress history and the aggregate rupture can be referred to as ductile. In either brittle or ductile rupture, the phenomenology of turbulent breakage is still not fully understood because the complexity of the flow field adds to the intricacy of the aggregate morphology in determining how the hydrodynamic forces redistribute and stresses accumulate over the structure of the aggregate. To provide a basic understanding of turbulent breakage, many investigations (see for instance [3, 4]) have focused on the influence that the hydrodynamic stresses have on the rate at which breakage occurs, neglecting the details of the aggregate inner structure. However, in the size range of interest for the present study (aggregates smaller than the Kolmogorov length), analyses were carried out considering brittle aggregates and instantaneous breakage [3, 4]. This assumption would be fully justified in highly viscous flows, such as dispersions in liquid polymers, where the stresses required to break the aggregate are very low [5]. But in low-viscosity systems the effective hydrodynamic stress required for breakage changes significantly depending on the nature of the flow and rupture can frequently be determined by the stress history [6].

The first attempt to assess the effects of flow inhomogeneity and anisotropy was recently put forward in the collaborative study of [3] where the breakage of small inertialess aggregates in different archetypal bounded flows (turbulent channel flow and developing boundary layer) was compared with those of homogeneous isotropic turbulence. This study showed that, regardless of the flow configuration, the breakage rate decreases when the critical stress required to break the aggregate increases. For small values of the critical stress ("weak" aggregates) the breakage rate develops a universal power-law scaling that appears to be independent of the flow configuration; whereas for high values of the critical stress ("strong" aggregates) large differences in the breakage rate arise among the different flows and no clear scaling is observed anymore. In the paper we examine a more realistic breakage process resulting from ductile rupture, focusing on turbulent channel flow. We assume that the breakage process is activated when the hydrodynamic stress acting on the aggregate, \( \sigma \), exceeds a critical value characteristic of a given type of aggregate: \( \sigma > \sigma_c \) (activation condition). As long as this condition is met the process continues, mimicking in the situation in which the aggregate is storing energy from the surrounding fluid. The process ends when the energy transferred from the fluid to the aggregate (deformation energy hereinafter): \( E = \int_0^t \epsilon(t| \sigma > \sigma_c |)dt \) , with \( \tau \) being the time spent by the aggregate in regions of the flow where \( \sigma > \sigma_c \) and \( \epsilon \) being the dissipation rate of fluid kinetic energy, exceeds the critical breakage value, which is also characteristic of the type of aggregate under investigation: \( E > E_c \) (breakage condition). Following [3] we assume \( \sigma \sim (\epsilon/\nu)^{1/2} \), where \( \mu \) (\( \nu \)) is the dynamic (kinetic) viscosity.

2. Physical Problem and Numerical Methodology

The physical problem considered in this study is the dispersion of tracer aggregates in turbulent channel flow. The reference geometry consists of two infinite flat parallel plates separated by a distance \( 2h \). The origin of the coordinate system is located at the center of the channel with the \( x, y \) and \( z \) axes pointing in the streamwise, spanwise, and wall-normal directions, respectively. Periodic boundary conditions are imposed on the fluid velocity field in the homogeneous directions (streamwise, \( x \), and spanwise, \( y \)), no-slip boundary conditions are imposed at the walls. The size of the computational domain is \( L_x \times L_y \times L_z = 4\pi h \times 2\pi h \times 2h \). The shear Reynolds number is \( Re_s = u_* h/\nu = 150 \), where \( u_* = \sqrt{\tau_w/\rho} \) is the shear velocity based on the mean wall shear stress. All variables discussed in this paper are expressed in wall units, obtained using \( u_* \) and \( \nu \). The flow solver is based on a Fourier-Galerkin pseudospectral method that solves for the full Navier-Stokes equations and thus yields the spatial derivatives required to calculate \( \epsilon \) along the aggregate trajectory with spectral accuracy. Lagrangian tracking is based on the following equation of motion: \( \mathbf{x}_p = \mathbf{u}_{ap} + \mathbf{u}_{ap} \), with \( \mathbf{x}_p \) the aggregate position and \( \mathbf{u}_{ap} \), the fluid velocity at \( \mathbf{x}_p \).
This equation is solved in time using a fourth-order Runge-Kutta scheme, whereas sixth-order Lagrangian polynomials are used to obtain the fluid velocity and the fluid velocity derivatives at the instantaneous aggregate position. Further details on the numerical methodology can be found in [7]. Breakage was measured by releasing $2 \cdot 10^7$ aggregates in two distinct regions of the channel: the wall region, $\Omega_W$, which comprises a fluid slab 10 wall unit thick where the viscous stress (representing the mean fluid shear) is maximum while the turbulent stress is close to zero; and the center-plane, $\Omega_C$, where all wall stress contributions drop to zero and turbulence is closer to homogeneous and isotropic.

3. Results and Discussion

Figure 1 shows the rates of ductile breakage obtained for the two release locations $\Omega_C$ and $\Omega_W$, highlighting the effect of increasing the critical deformation energy for different values of the critical energy dissipation. Results refer to three different values of $E_{cr}$: $E_{cr} = 0.04, 0.4$ and $4.0$, representing a case of low, intermediate and high ductility for the present flow configuration. Focusing first on the aggregates released in the channel center, we find that the breakage rate of brittle aggregates (solid curve) generally decreases with increasing aggregate strength, in agreement with the intuitive idea that weak aggregates in wall-bounded flows are broken by turbulent fluctuations faster than strong aggregates [3]. For small $\epsilon_{cr}$, the breakage rate is known to exhibit a power-law behavior of the type $f(\epsilon_{cr}) \propto \epsilon_{cr}^\chi$, where $\chi$ is a flow-dependent scaling exponent: [3] have demonstrated that the value of $\chi$ for aggregates released in the channel center is very similar to that of aggregates released outside a developing boundary layer but slightly larger than that of aggregates released in homogeneous flows. In figure 1(a) the power-law scaling of $f(\epsilon_{cr})$ for brittle aggregates is observed when $\epsilon_{cr} < -3$ and the best fit is obtained for $\chi \approx 0.5$. When ductile aggregates are taken into account (dashed curves), breakage rates change dramatically, especially for weak aggregates with low $\epsilon_{cr}$ threshold. The values of $f(\epsilon_{cr}, E_{cr})$ decrease significantly with respect to the case of instantaneous breakage, already at low thresholds for the critical deformation energy (e.g. $E_{cr} = 0.04$). In addition, no clear power-law scaling is observed anymore and the breakage rate profiles tend to flatten as the aggregate “ductility” increases. As could be expected, the effect of ductile rupture on $f(\epsilon_{cr}, E_{cr})$ becomes less important for strong aggregates: These must be subjected to extremely violent fluid stresses, typical of the intermittent nature of small-scale turbulence, to activate the breakage process and thus can store the level of energy required to break almost impulsively. As a result, there is just a little increase of the exit time with respect to strong brittle aggregates.

Breakage rates depend qualitatively on the specific location chosen to release the aggregates at time $t_0$. In the center of the channel strong aggregates, no matter if subject to brittle or ductile rupture, are mainly broken by the rare extreme excursions of dissipation from the mean, which are caused by intermittency. Most of such aggregates must therefore reach the high-dissipation, high-shear regions of the flow near the channel walls to undergo breakage. To examine the influence of the release location on breakage rates, in figure 1(b) we also show the behavior of $f(\epsilon_{cr}, E_{cr})$ for aggregates released in the region $\Omega_W$. For brittle aggregates (solid curve), we observe that the power-law scaling at small values of $\epsilon_{cr}$ is followed by a flattening for intermediate values of the threshold. For the very large threshold values associated to the right end of the profile, a drop-off in the breakage rate is observed, representing the case of aggregates that are too strong to be broken by the mean shear alone: intense but rare turbulent fluctuations within the near wall region are required to overcome the cohesive force of these aggregates [3]. The inclusion of ductile rupture effects (dashed curves) produces again a clear decrease of the breakage rate, which vanishes for large values of $E_{cr}$. Compared to the results of figure 1(a), we observe that the decrease is now almost negligible for aggregates with low ductility and flattening of the profiles is only attained for very high threshold values of the deformation energy. We also note that error bars are generally smaller, indicating a lower variability of the statistics: this is due to the fact that aggregates are already placed in the high-shear regions of the flow where they preferentially break and hence sample a reduced portion of the domain compared to aggregates released in $\Omega_C$. In spite of these quantitative differences, however, the reduction of $f(\epsilon_{cr}, E_{cr})$ associated with ductile rupture is evident independently of the initial aggregate injection location.

![Figure 1: Break-up rate $f(\epsilon_{cr}, E_{cr})$ for ductile aggregates released in the center of the channel (top panel) or in the near-wall region (bottom panel). Error bars represent the standard deviation from the mean value, and were computed using the variance of the exit time, $\sigma^2 = \langle \tau^2 \rangle - \langle \tau \rangle^2$. Error bars provide an indication of the dispersion of breakage rates around the mean value.](image)

References


On the rotation of rigid fibers in turbulent channel flow

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Abstract

We investigate the effect of local shear and turbulence anisotropy on the rotation of elongated rigid fibers. To this aim, statistics of the fiber angular velocity, Ω, are extracted from direct numerical simulation of turbulence coupled with Lagrangian fiber tracking. We show that elongation is important for fibers with small inertia ($St \leq 5$ in our flow-fiber combination), and that, in the strong shear region near the wall, fiber anisotropy adds to flow anisotropy to induce strong deviations on fiber rotation with respect to spherical particles.

Keywords: Wall turbulence, rigid fibers, rotation statistics, direct numerical simulation, Lagrangian tracking

1. Introduction

A possible approach for modeling rotation of fibers in turbulence is to describe the time evolution of fiber angular velocity as a stochastic (random walk) process in rotation space [1]: In analogy with the translational motion of tracers and particles in Homogeneous Isotropic Turbulence (HIT), one can hypothesize that the Lagrangian time series of angular velocity are characterized by a Gaussian probability distribution with Markovian properties. Based on this hypothesis, fiber rotation can be described within the theory of diffusion as an Ornstein-Uhlenbeck (OU) process [2], which is completely characterized by a statistically-stationary Gaussian distribution and autocovariance which takes the specific form of a negative exponential. This modeling approach assumes that both translational diffusion and rotational diffusion are homogeneous and isotropic. This assumption was proven acceptable for unbounded flows where dispersion is indeed stationary and Gaussian [3, 4, 5], and may be partly extended also to wall-bounded shear flows [6, 7], where Lagrangian velocity autocorrelations exhibit exponential decay if particles are not sampling near-wall regions of high turbulence anisotropy. All these studies, however, are concerned with spherical particles. Much less effort has been devoted to exploring the applicability of standard diffusion laws to non-spherical particles, as in the case of fibers: Previous modeling attempts focus on HIT [1, 8, 9], and thus neglect effects due to turbulence anisotropy. Recent studies [9, 10], however, show that orientation of elongated fibers is correlated with local velocity gradients and that the strength of this correlation is heavily influenced by fiber shape. These findings suggest that anisotropy of fibers adds to anisotropy of turbulence to deviate fiber behavior (in particular, rotation rates) from that predicted by stationary Gaussian dispersion models. In the work we examine the effect of local shear and flow anisotropy on the rotational dynamics of fibers with different elongation and inertia. This analysis is performed to assess the extent of diffusion laws application can be applied to describe fiber rotational dispersion in wall-bounded turbulence. More specifically, we are interested in assessing the possibility of modeling rotation as an OU process depending both on flow parameters (shear, anisotropy) and on fiber parameters (inertia, elongation).

2. Physical Problem and Numerical Methodology

The reference flow configuration is Poiseuille flow of incompressible, isothermal and Newtonian fluid in a plane channel at friction Reynolds number $Re = u_\infty h/\nu = 150$, with $u_\infty$ the friction velocity, $\nu$ is fluid viscosity and $h$ is the channel half-height. We performed pseudo-spectral DNS, imposing periodic boundary conditions in the streamwise ($x$) and spanwise ($y$) directions and no-slip conditions at the walls. Time integration a 2nd-order Adams-Bashforth scheme for the non-linear terms and an implicit Crank-Nicolson scheme for the viscous terms. The channel size is 1885 × 942 × 300 wall units (identified with the superscript “+” and obtained using $x_+ = x / \delta$ and $z_+ = z / \delta$, where $\delta = 128 \times 128 \times 129$ nodes. Lagrangian fiber dynamics related to [11]. The translational equation of motion of an individual fiber is given by the linear momentum equation $du_p/dt = F/m$, where $u_p$ is fiber velocity, $F = \mu K(u_{wp} - u_p)$, with $\mu$ the fluid dynamic viscosity, $K$ the resistance tensor and $u_{wp}$ the fluid velocity at fiber position, is the total hydrodynamic drag force acting on the fiber (strictly valid for an ellipsoid under creeping flow conditions) and $m = 3/2 \pi a^3 \lambda \rho_p$ is fiber mass with $a$ the the semi-minor axis, $\lambda$ the aspect ratio of the ellipsoid, and $\rho_p$ is fiber density. The resistance tensor $K$ is expressed in the Eulerian frame of reference, $x = (x, y, z)$. Two other Cartesian coordinate systems, both with origin at the fiber center of mass, are used to describe fiber motion: a Lagrangian frame of reference, $x'$, and a co-moving frame of reference, $x''$, with axes parallel to the inertial frame. The rotational motion of the fiber is governed by the following equation: $d(\Omega \cdot \Omega')/dt + \Omega \times (\Omega \times \Omega') = N'$, where $\Omega$ is fiber angular velocity, $I$ is the moment of inertia tensor and $N$ the torque acting on the fiber. The equations of fiber motion are solved using a mixed explicit/implicit differencing procedure [12]. The total tracking time in wall units is $t^+ = 3500$, with time step size equal to that of the fluid: 0.03 in wall units. The main simulation parameters are $a_+$ and the fiber response time [13]:

$$St = \frac{2(a^+)^2 \rho_p}{9 \rho} \lambda \ln(\lambda + \sqrt{\lambda^2 - 1}) / \sqrt{\lambda^2 - 1}.$$  \hspace{1cm} (1)

In this study, we have selected: $a^+ = 0.36$, $\lambda = 1$ (spherical particles), 3, 10, 50, and $St = 1, 5, 30, 100$. To ensure converged statistics, swarms of 200,000 fibers are tracked for each particle category, assuming dilute flow and one-way coupling.
3. Results and Discussion

Rotation statistics are computed for three distinct sub-regions of the channel: viscous sublayer \((0 < z^+ < 5\) with \(z^+\) the distance from the wall), buffer layer \((5 < z^+ < 50)\), and log layer \((50 < z^+ < Re_c)\). These sub-regions exhibit significantly different shear stress and dissipative flow scales, and may thus produce different fiber rotational dynamics.

3.1. Angular velocity statistics

The non-dimensional ensemble-averaged fiber angular velocity is shown in Fig. 1. Only the spanwise component \((\Omega_z)\) is shown since \((\Omega_x)\) and \((\Omega_y)\) are always zero. Profiles highlight the effect of fiber elongation on \((\Omega_z)\) for a specific value of \(St\). Results for tracer particles \((St = 0)\), are also shown (thick solid line). Regardless of inertia and shape, values of \((\Omega_z)\) are much higher near the wall, where accumulation of the dispersed phase is known to occur [11], than outside of the buffer layer \((z^+ > 50)\), where elongation does not seem to alter the expectation value of fiber rotational dynamics significantly. Elongation is important in determining the near-wall behavior of \((\Omega_z)\), which decreases as fiber length increases. This implies that fibers spin slower than spherical particles, a consequence of fiber alignment in the longitudinal \(x-z\) plane [11]. We also find that shape effects are evident for all values of the aspect ratio \(\lambda\) at small Stokes numbers \((St = 5)\) whereas longer fibers \((\lambda \geq 10\) in the present case) are required to induce significant reduction of \((\Omega_z)\) at large Stokes numbers (not shown).

![Figure 1: Fiber spanwise angular velocity, \((\Omega_z)\), inside the buffer layer \((z^+ < 50)\) for fibers with \(St = 5\)](image)

3.2. Statistics of rotational turbulent diffusivity

Rotational turbulent diffusivity, normalized by the angular velocity variance to allow direct comparison among different fiber families, is computed as: \(\Gamma_{\Omega_z}(\tau) = \int_0^\infty R_{\Omega_z,\Omega_z}(t) dt\), where \(R_{\Omega_z,\Omega_z}(\tau)\) is the autocorrelation coefficient of fiber angular velocity, computed in the inertial frame. Our aim is to verify if and under which conditions (degree of shear and turbulence anisotropy, fiber inertia and fiber elongation) the Lagrangian autocorrelation can be approximated as a decaying exponential: \(R_{\Omega_z,\Omega_z}(\tau) = \exp(-\tau/T_L^{\Omega_z})\), where \(T_L^{\Omega_z} = \int_0^\infty R_{\Omega_z,\Omega_z}(\tau)d\tau\) is the Lagrangian integral timescale of the fiber angular velocity. This quantity provides the simplest measure of the time span over which fiber orientation is self-correlated. This implies an exponential increase of the rotational diffusivity according to the relation: \(\Gamma_{\Omega_z}(\tau) = T_L^{\Omega_z}(1 - e^{-\tau/T_L^{\Omega_z}})\).

When the exponential term drops to zero for \(\tau >> T_L^{\Omega_z}\), \(\Gamma_{\Omega_z}(\tau)\) tends towards a constant value (known as the Fickian asymptote), which is equal to the Lagrangian integral timescale: \(\Gamma_{\Omega_z} = T_L^{\Omega_z}\). In Fig. 2 we compare the time evolution of \(\Gamma_{\Omega_z}\) computed from our DNS+LPT database with the corresponding prediction. Statistical convergence of the numerical results was obtained by ensemble averaging over at least \(10^4\) trajectories. The agreement in the log region (Fig. 2a) is excellent, even at short times. At longer times, \(\Gamma_{\Omega_z}(\tau)\) curves recover the Fickian asymptote, confirming that fiber rotation in the channel center is homogeneous and angular velocities have a Gaussian distribution that can be modeled as an OU process. Spherical particles exhibit the highest rotational diffusivity, which decreases monotonically up to \(St = 30\) and then increases for larger values of \(St\). As expected, deviations become larger in the viscous sublayer (Fig. 2b) where theoretical predictions generally fail, with the exception of short fibers with \(L_+^*/T_K \sim O(1)\) and with small inertia. Fiber rotation is strongly non-homogeneous and it may be speculated that angular velocities have a non-Gaussian distribution due to strong shear, high turbulence anisotropy and velocity gradients that are not \(\delta\)-correlated [10].

![Figure 2: Streamwise rotational diffusivity, \(\Gamma_{\Omega_z}(\tau)\), in the log layer (a) and in the viscous sublayer (b)](image)

References

CFD simulation of two-phase flow in the bearing chamber

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Abstract

The aim of the work is to investigate the structure of a two-phase flow in a bearing chamber. For this purpose the commercial code ANSYS FLUENT was used. Two approaches to two-phase flow modeling were taken into account: Euler-Euler and Euler-Lagrange. In the second case for modeling the oil film at the chamber walls the Eulerian Wall Film Model integrated with Disperse Phase Model was applied. The numerical results were compared with the experimental data available in the literature. The results are in a good qualitative agreement with the experiment. Based on the local wall film thickness, a local heat transfer coefficient was calculated.

Keywords: CFD analysis, two-phase flow, disperse particle flow

1. Introduction

In the case of combustion engines, hot exhaust gas flow requires applying the additional cooling system for the machine elements. A good solution used in practice is to apply a cooling liquid flow to the hot surfaces. Mixing the cooling liquid and hot gases results in a complex two-phase flow, which on the other hand intensifies cooling process of rotating machine elements. This study concentrates on the modeling of two-phase air-oil flow in the engine oil sub-system, and especially on calculation of thin wall liquid film. The exhaust gases flow outside the bearing chamber of the gas turbine. Inside the cylindrical chamber the shaft is rotating with high velocity. Both fluids (air and oil) flow into the chamber from the opposite surfaces. The oil creates a thin layer on the outside wall. Thickness of this layer determines the heat transfer coefficient and the heat flux between hot walls and fluid.

2. Two-phase flow modeling

The study concerns the numerical simulation of two-phase flow and heat transfer phenomena in the bearing chamber. For this purpose commercial code Ansys Fluent was used.

2.1. Euler-Euler approach

In the first step of calculations two-dimensional cylindrical closed chamber was considered. The calculation domain contains the air-oil mixture. For two-phase flow computations the mixture model was used. It is an Euler-Euler approach. The continuity, momentum and energy equations for the whole mixture are solved. The model uses mass-average velocity for the mixture and volume-average mixture viscosity. The volume fraction of oil phase is obtained from continuity equation. In Fig. 1 numerical results from 2D analysis are presented. As can be seen, in the chamber there is some asymmetry in the distribution of oil phase, which is caused by the rotation of the shaft. These results do not reproduce the real two-phase flow in the bearing chamber. In this approach, instead of separate two-phase flow with oil film at the external wall, the mixture of two substances was obtained.

2.2. Euler-Lagrange approach

In the next step of calculations the 3D computations were performed. In this case the Euler-Lagrange approach was applied. The Discrete Phase Model (DPM) and the Eulerian Wall Film Model (EWFM) were chosen. The Euler approach was used for air and liquid film flow. The DPM characterizes the oil droplets which are transported between oil inlet and oil wall film. EWFM is actually a specific boundary condition for modeling liquid droplets that hit the wall and form the thin liquid films. This model is dedicated to the internal combustion engines. In order to simulate the physical phenomena occurring during contact of droplets with the walls, EWFM uses Lagrangian approach for non-stationary molecules (DPM particles). This assumption leads to reproduce the very fast transient phenomena occurring at the contact surface. Interaction between liquid particle and the wall is a very important phenomenon in a film forming process. Small droplets collide with the surface, and then due to the splashing on the surface, a thin liquid film is formed.
In this approach the liquid film is treated as a two-dimensional phenomenon (limited by the height of the film and by the wall surface). A film is formed during the droplet collision with the wall. After the collision, liquid particle may be retained on the surface - the collision energy is small and droplet remains almost spherical. The droplet can be reflected from the surface and leaves the film surface practically untouched by changing only its own speed. Droplet that hits the wall with the relatively medium energy, can flow over the wall forming a uniform stream. Liquid particle can also be separated into the smaller droplets. Some of them can be attached to the liquid film, or reflected from the film. For the film conservation equations of mass, momentum and energy are solved. The movement of the liquid film is always treated as transient phenomena.

Film model is integrated with the DPM by the source terms included in the balance equations. Film is formed from the droplets defined in the DPM model. Droplets as a discrete phase can not only be broken into smaller particles, they may also increase its volume due to separation of the film from the wall. The appearance of shear stress high enough to cause the removal of large particles from the film surface, affects the particle volume too. The mass leaving the liquid layer in the form of separated droplets is taken into account by the source terms. Streams of the dispersed phase particles or individual particles hitting the film surface are absorbed by the liquid layer, and then their mass and momentum are added to the appropriate conservation equations in the form of source terms. If it comes to the separation of the film, from its surface mass is removed. It is defined by the negative mass source term and associated with momentum decrease. Separation of the droplets from the liquid film can also occur due to the high difference in speed between the gas phase and the liquid film. In this approach the volume fraction for each phase with continuity and momentum equations are solved.

This research was aimed at an analysis of the impact of operating conditions like shaft speed and mass flow rate of oil on the flow structure - especially on film oil thickness - which affect the value of the heat transfer coefficient at the bearing chamber walls.

In Fig. 2 numerical results from 3D analysis are presented. As observed, there is some asymmetry in the distribution of oil phase in the chamber, which is caused by the rotation of the shaft.

Figure 2: Instantaneous oil film thickness distribution for shaft speed of 15 000 rpm, \( m_{oil} = 0.01 \text{ kg/s} \).

The results of numerical simulations of oil thickness were compared with the experimental data from the laboratory in Karlsruhe Institute of Technology [1] (Fig. 3). These results are in a good qualitative agreement with experiment.

![Figure 3: Oil film thickness for shaft speed of 15 000 rpm - comparison with experiment: oil1=0.01 kg/s, oil2=0.0267kg/s, line1 15 mm from the air inlet wall, line2 - centre of the chamber, line3 - 15 mm from the oil inlet wall](image)

3. Heat transfer coefficient

Local heat transfer coefficient at the bearing chamber walls, presented in Fig. 4, were calculated as a function of Nusselt, Reynolds and Prandtl numbers [2]:

\[
Nu_L = 0.332 Re_L^{0.5} Pr_L^{0.33}.
\] (1)

![Figure 4: Local HTC for shaft speed of 15 000 rpm: oil1=0.01 kg/s, oil2=0.0267kg/s, line1 15 mm from the air inlet wall, line2 - centre of the chamber, line3 - 15 mm from the oil inlet wall](image)

4. Conclusions

The results of numerical calculations show the importance of appropriate model implementation to simulate the two-phase flow in the bearing chambers. The oil film thickness depends on the operating conditions of the bearing chamber and strongly affects the heat transfer.

References


MS13

Modelling and Simulation in Land Vehicles and Aircrafts

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Modelling and experimental investigation of the motion of a microbus passenger in the space between seat rows during a road accident

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Abstract

The dynamic loads acting on microbus passengers (both children and adults) during a road accident depend on many factors, which include vehicle construction, properties of the personal protection system used, and space available for safe movement of the passenger body during the accident. The major parts of the personal protection system are vehicle seat and seat belt. The effective functioning of such a system depends on appropriate selection and matching of the geometrical and spring-damping characteristics of the seat and the seat belt properties to the anthropometric features of the passenger involved. The threats are significantly affected by small distances between passenger’s head, torso, or legs and the rigid load-bearing structure of the preceding seat and by the resulting risk of impact of passenger’s body parts against the preceding seat structure. Within the work reported, the influence of seat elasticity characteristics and seat cushion angle on the motion of head, torso, and legs of test dummies (Hybrid III, P10) was analysed with regard to improvement of passengers’ safety during a frontal microbus impact against an obstacle

Keywords: bus passenger, individual safety, modelling, road safety

1. Introduction

For the vehicle passenger safety systems to be improved, the knowledge of the possible trajectories of passenger’s body during an accident must be acquired and the personal protection system must be adapted to the limited space available in the vehicle. The research works undertaken to date on the kinematics of motion of a human body were chiefly focused on the occupants of passenger cars [1,4,6]. In the work presented herein, this problem was explored for the case where an adult and a child (represented by a test dummy Hybrid III and P10, respectively) were simultaneously placed on the seats situated one after the other. The significant mass of an adult (a 50-centile male in this case) acting on the backrest of the preceding seat may cause severe deformation of that backrest and, in consequence, additional hazard to the occupant of the preceding seat (cf. Fig. 1) [3].

The objective of this work was to indicate the possibilities of improvement in passenger safety, based on an analysis of the motion of passenger’s torso, head, and legs in the space between seat rows during a frontal microbus impact against an obstacle. The topicality of this work results from the fact that short-distance transport services are provided with the use of increasing numbers of minibuses, where the personal protection systems are definitely inferior to those typical for passenger cars.

2. Model

The model employed (Fig. 1) represents a system chiefly consisting of bus floor, three seats, and two test dummies restrained in the seats with three-point seat belts. The major components of the seats (seat cushion and backrest) and of the dummies (head, neck, upper and lower parts of the torso, arm, forearm with hand, thigh, and shank with foot) were treated as rigid bodies connected by articulated joints. The model was developed taking into account the experience arising from the description of the dynamics of a human body as presented in [2].

Figure 1: General view of the model and head loads (one of the options)
Figure 1 includes the general view of the model as well as a representation of passenger’s head as an example of dummy’s elements with the system of loads acting on it; in this case, the system of loads consists of reactions acting in the joint (Mz and Rz), weight (m1g), and forces applied to the dummy’s element at the points of contact with other elements of the model. The model represents, inter alia, the following:

- process of deformation of the front part of the bus;
- flexibility of the joint between the seat and the bus floor;
- resistance moments at the articulated joints between elements of the seats and the dummies,
- seat belt reactions acting on the torso,
- contact forces between dummies elements and the seats and the floor.

The model geometry is defined by 33 generalized coordinates. For the motion of the system elements to be determined, a system of 66 differential equations was solved. The numerical computation was made by a HYBRID HYBv3 program, specially developed for this task and operating in the MATLAB environment. The model was assumed to move translationally with a known velocity \( \upsilon_0 \), treated as the velocity of impact against an obstacle at the initial instant \( t = 0 \). Starting from the initial instant, the bus floor is subjected to negative acceleration \( a(t) \), which is determined by the course of deformation of the front vehicle part.

3. Experimental tests and validation of results of the model tests

The primary selection of model parameters and characteristics was based on measurements of the characteristics of seats, test dummies, seat belt webbing, etc. The data thus collected were used as a starting point in the model parametrization process. The model validation process was based on the results of measurements carried out at a series of crash tests, where appropriately prepared seats and test dummies were used.

The validation calculations confirmed correct model response to an impact input defined by the microbus superstructure deformation process during a frontal vehicle collision with a rigid obstacle.

4. The results

The computer simulations provided a basis for an analysis of the influence of structural factors (seat elasticity characteristics and seat cushion angle) on the kinematics of dummies’ motion and on the risk of injury to microbus passengers during a road accident. At this work, the motion of dummy’s torso, head, and legs and the deformation of cervical spine were specially taken into account.

![Figure 3: Values of the biomechanical neck injury risk indicator \( N_i \) (calculated as described in [4]) for seat cushion angle values of 4°, 8°, 12°, and 16°; test dummy Hybrid III, \( \upsilon_{on} = 30 \text{ km/h} \)](image)

5. Recapitulation

The paper will include a full range of calculation results and constructional conclusions. The results of computation of dummies’ motion were interpreted with using the biomechanical properties of the human body, especially the relation between the dynamic impact loads and the risk of injuries, which was based, inter alia, on [5].

References


Fretting wear simulation in model studies

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Abstract

The article presents research methodology of fretting wear occurring in a wheel-axle connection of a railway wheel set with an automatic wheel track change by means of simulation of wear development conditions in model studies. In order to refer the results of the research to the actual object a condition of mechanical similarity must be fulfilled. In the article the authors propose to apply the similarity theory elements, whose mathematical tool is dimensional analysis.

Keywords: railway wheel set, fretting wear, model tests, dimensional analysis

1. Introduction

The notion of fretting comprises the effects of complex actions on the surfaces under pressure and loaded by a variable force of the elements. At present mechanical engineers agree that fretting is a phenomenon of a very complex wear mechanism, including: adhesive wear, surface fatigue, exfoliation, oxidation, irregularities apex wear and loosen products wear [2,3,6]. The researchers discuss, what is the result of taking one of these processes to trigger the fretting wear development. The studies of fretting wear were carried out mainly for matching concentrated or flat contacts and the proposed wear models also refer to them [1,6]. These studies were mainly carried out in the 50’s - 80’s of the past century. The present query of periodicals dealing with the above subject, especially Wear or International Tribology, shows that the fretting wear problem is mainly considered in the aspect of the influence of material selection or technological process on the initiation of wear in the tribological node. There are no publications on the studies of fretting wear development mechanism in actual nodes of machines or vehicles.

The authors of the paper made an attempt to explain the development of fretting wear in a wheel – axle connection of a railway wheel set with an automatic wheel track change. The initial exploitation of the set showed that after a low mileage (of about 1.5 thousand km) there occurred problems in the wheel track change. Observations of the surface of the axle wheel seat after dismantling the wheel set showed fretting damages in the area of contact with the wheel hub, causing locking of a wheel on the axle while changing its track.

2. Dimensional analysis

Dimensions of the actual wheel set imply great research problems. Therefore model studies were proposed on simple samples, simulating actual exploitation conditions, in agreement with Heinke’s experimental studies classification [4]. In the experimental studies carried out on models it is very important to retain mechanical similarity criteria which allow to transfer the results of these studies onto the actual objects [4]. Modelling only the basic parameters of the process on a test stand simplifies and facilitates the experiment, while reproduction of all the parameters is usually impossible, due to the complexity and range of the experiment. Difficulty in wear process modelling lies in the fact that adopting the same materials, contact conditions, loading and the like does not reproduce all wear phenomena encountered in practice. In these studies the theory of similarity, whose mathematical tool is dimensional analysis, may prove useful [5]. In this article elements of this theory were applied to determine mechanical similarity criteria accepted in the studies on the model of negative-allowance connection in reference to the actual object (a wheel - axle running fit connection of a wheel set with an automatic wheel track change).

Measures of fatigue strength of a wheel - axle running fit connection are the following:

- pressure on the wheel – axle contact surface $p$ [kg m⁻²]
- bending moment transferring the connection: $M_b$ [Nm]

Fatigue strength can be influenced by the following factors treated as independent variables:

- substitute modulus of elasticity: $E$ [kg m⁻²]
- connection length of frictional pair elements: $l$ [mm]
- axle wheel seat diameter: $D$ [mm]
- clearance gap $h$ [mm]
- bearing length of the effective roughness profile coefficient: $t_p$ [%]
- load frequency: $n$ [Hz]
- axle wheel seat surface roughness: $R_{a1}$ [μm]
- surface roughness of the wheel hub opening: $R_{a2}$ [μm]
- normal load: $P$ [N]

Normal contact stress can be presented as a function:

$p = F (t_p, P, h, R_{a1}, R_{a2}, l, D, E, n)$ (1)

In accordance with denotations, dimensional matrix of the quantities in dependencies (1) has the form:

Table 1: Dimensional matrix

<table>
<thead>
<tr>
<th>$p_0$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$p_4$</th>
<th>$p_5$</th>
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where: $L$ [m] - length, $M$ [kg] - mass, $T$ [s] - time
Function F takes its form on the basis of experimental studies. All the assumptions of matrix method application are fulfilled. Equation (1) is equivalent to the dimensionless equation (2) in agreement with Buckingham’s theorem which states that the number of dimensionless moduli is equal to the number of independent physical parameters reduced by the number of basic dimensions.

\[ \pi_0 = F(\pi_1, \ldots, \pi_i) \]  

where: \( \pi_i \ (i=0,1,\ldots,6) \) form a complete system of variable criteria numbers: \( p_0 \ldots n \).

From the above matrix the following dimensional system of equations results, which was solved for the unknown \( p_7, p_8 \) and \( p \):

\[
\begin{align*}
-p_0+p_2+p_3+p_4+p_5+p_6+p_7-p_8 &= 0 \\
-p_0+p_2+p_8 &= 0 \\
-2p_0-2p_2-2p_8-p_9 &= 0
\end{align*}
\]  

Having taken into consideration free variables \( p_i \ (i=0,1,\ldots,6) \) and a special sequence of numerical values we receive a solution matrix. Next after transformations, we get a system of criteria numbers e.g. in the dimensionless form:

\[
P = F(\frac{p_2}{D}, \frac{P}{D^2}, \frac{R_{hl}}{E}, \frac{R_{el}}{D})
\]  

From the comparison of the values of the actual object criteria numbers and from the model we get the answer about fulfillment of the mechanical similarity requirements. This allows to transform the results of experimental research from the model onto an actual object.

3. Research methodology

While choosing a sample modelling the connection sliding bush – the wheel set axle was to preserve dimensional similarity in the connection area. In order to achieve this a proportion of the connection length and the axle diameter and matching was assumed. Fatigue testing machine of MUJ type was used allowing to receive temporarily variable load together with bending the rotating sample thus simulating actual work conditions of the wheel set (Fig.1).

![Figure 1: A sample scheme and referring to it bending moment distribution](image)

Initially, a basic sample was tested (material of the roller and the bush steel- steel), the aim was to verify the model choice by gaining an image of wear similar to an actual object. Figure 2 presents a fretting wear image on the surface of a roller.

![Figure 2: Surface of the roller modelling the axle characteristic fretting wear, magnification up 5x](image)

Damages on the roller surface in the fatigue tests with respect to both the image and the place of damage, are similar to the damages on the axle surface. This proves the correct choice of the model and its work conditions. It also allows to carry out further tests on this simple model, comprising, among others,

References


Inverted joined-wing multidisciplinary optimization

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Abstract

The joined-wing system is an unconventional way to connect the lifting surfaces that is increasingly willing to use by designers in prototypes of new aircrafts. The article presents a way of multidisciplinary optimization of inverted joined-wing that can be useful during preliminary design. According to this methodology computer program is coded to test presented optimization algorithm results and performance.

Keywords: joined-wing, multidisciplinary optimisation

1. Introduction

The presented paper is part of a research program led by Polish Institute of Aviation dedicated to investigate inverted joined-wing configuration of an airplane. The project aim to collect wide range of information about this configuration, especially in the area of preliminary design and optimization, stability and control, manufacturing and technological process, verifying calculations in the wind tunnel and in flight. This paper is scoped on multidisciplinary optimization and presents results achieved so far.

2. Joined-wing configuration

The inverted joined wing configuration is an unconventional proposal for future airplanes. It consists of two similar size lifting surfaces. First of them is attached to the forward section of fuselage and acts as a classical wing, second is attached to the aft end of fuselage instead of a horizontal stabilizer. Usually the forward lifting surface is located in front of centre of gravity, second is behind. Moreover both surfaces join each other or are connected by wing tip surfaces. All lifting surfaces create a closed circuit – box wing.

Figure 1: Joined-wing model in wind tunnel

This configuration was first proposed by Prandtl in 1924 (Ref. [3]). It has many possible advantages like low induced drag or lower weight due to higher stiffness of closed wing. Unfortunately to avail all of joined-wing advantages, the aircraft should be well optimized during design process. This is not a simple task due to the strong aerodynamic coupling and static indeterminacy. Joined-wing is not like classical configuration a cantilever structure, being rather a closed frame. Searching for optimal solution is highly constrained by specific for this configuration phenomenons like global buckling of wings-fuselage system. That kind of constraint is not as important for classic configuration. For joined-wing it cannot be omitted if the minimum weight is a target of design process. Aeroelasticity is one more phenomenon highly important for closed wing configuration that should be take into consideration.

3. Optimization

A numerous of constraints during design of a joined-wing aircraft enforces large number of iterations to obtain satisfactionary results for many design variables. It is obvious to prepare automatic algorithm that can manage whole design process taking into consideration all specific for joined-wing behaviors, that target is a global optimum for specified assumptions. Multifield optimization process is proposed for preliminary design of joined-wing. Modular algorithm based on automatic geometry generator, FEM solver and aerodynamic panel method is developed. Whole process is optimized to decrease huge computation cost to the minimum.

The optimization algorithm was developed for small electric, general aviation aircraft and was presented in detail in previous paper Ref. [1]. The general objective is to maximize the range of aircraft for assumed mission, as it is the major disadvantage of electric aircrafts. The payload and battery capacity are fixed. Very small reserve of electric energy and high consumption during climb and descent induced to calculate the total range as a sum of ranges during climb, cruise and descent. During all of the phases of flight aircraft has constant mass that is opposite to other engine types. Only batteries discharges during mission.

Global, local geometry and structural parameters are selected as design variables. The main optimization loop is aerodynamic loop. For each aerodynamic step structure optimization is performed (simple iteration method) so that the geometries generated in following aerodynamic steps are comparable in terms of objective function. During structure optimization strength is checked for few sizing load cases obtained from loads envelope. Only structural parameters are

*This work was supported by The National Centre for Research and Development under grant No. PBS/A6/14.
variable in this loop. In the aerodynamic loop objective function (the range) is optimized by changing only geometrical parameters.

![Simplified optimization algorithm](image)

Figure 2: Simplified optimization algorithm

4. Aerodynamic calculations

In order to minimize flow computation time, panel method implemented in PanAir program is used (Ref. [2]). As a considered solver can obtain a solution of potential flow only, a simple estimation of friction drag is performed using engineering equations Eqn (1,2).

\[
C_{F_{\text{w}}} = \frac{1.328}{\sqrt{Re_x}} \quad (1)
\]

\[
C_{F_{\text{w}}} = \frac{0.074}{Re_x^{0.2}} \quad (2)
\]

All aerodynamic calculations are performed for balanced aircraft.

![Aerodynamic model](image)

Figure 3: Aerodynamic model

5. Structural calculations

Strength calculations are conducted by a FEM program Calculix (Ref. [4]). Simplified shell model of all structural parts of aircraft are prepared. Structure is divided into many regions with the same design variable like shell thickness or beam cross section area.

![Graphical representation of structural variables](image)

Figure 4: Graphical representation of structural variables

Stress distribution estimation comes from static linear analyses. Finally static strength and local stability of structure are checked.

References


Dynamic stability analysis of the inverted joined wing scaled demonstrator

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Abstract

The aim of the paper is to present dynamic stability analysis process and its results for inverted joined wing aircraft configuration executed in the scope of MOSUPS project. At the beginning, aeroplane aerodynamic characteristics are shown, obtained from wind tunnel tests carried out in the Institute of Aviation in Warsaw. Next, some additional calculations are presented, performed in order to complete existing aerodynamic data and to prepare appropriate input data set to be implemented into dynamic stability analysis software. This data set used in further analysis is compared with characteristics received from fluid mechanics computations. In the main part of the paper, general assumptions, method and results of dynamic analysis with the use of SDSA software is shown. Both longitudinal and lateral typical stability modes will be investigated and presented. The final part of the paper focuses on results discussion and determination of general conclusions about stability qualities of inverted joined wing aeroplane configuration.

Keywords: aeroplane stability, flight dynamics, computational fluid dynamics

1. Introduction

Various non-planar wing aircraft concepts have been investigated since 1920s [11,12]. Most of the studies suggested potential benefits that could be gained from specific configurations [7,8,10]. One of the proposed concepts is joined wing aeroplane that offers lower values of drag in comparison to conventional aeroplane configuration, resulting in L/D ratio improvement and better performance in general. This paper purpose is to present a research on joined wing airplane with wings inverted as a counter-proposal to the foregoing concepts, i.e. with front wing located above the aft wing. This solution seems to offer even better L/D ratio for wider range of angles of attack [3,8,9].

2. Aerodynamic data used for stability analysis

As mentioned previously, primary aerodynamic characteristics used in come from wind tunnel tests. These were C2(α), C10(α) and C2(α) characteristics in symmetric flight and C2(α), C10(α), C10(α), C10(α), C2(α) characteristics in asymmetric flight for different values of sideslip angle in range from 0 to 25°. Additional calculations were made with the use of panel method (PANUKL software) [3] to evaluate aeroplane characteristics in unsteady flight for non-zero angular velocities ρ, q, r, which in this case were defined to 0.05 and 0.1 rad/s. This gave full six-element aerodynamic force coefficients vector as a function of angle of attack, rolling, pitching and yawing velocities respectively. All these aerodynamic characteristics obtained from wind tunnel and additional data calculated in PANUKL were compared with the data received from fluid mechanics model (ANSYS/FLUENT software was used) [13,14] to define whether both methods gives consistent and therefore veritable results.

3. Dynamic stability analysis

Aeroplane dynamic stability was investigated in SDSA software [4] based on the aerodynamic data already described and prepared in an appropriate format. It is usually considered separately in longitudinal and lateral-directional stability modes of motion to describe its dynamic stability properties. Basic dynamic stability analysis results for both longitudinal and lateral-directional motion are shown below. Out of different methods of evaluation dynamic stability [1,5,6], root locus plots were used as it gives brief insight into aeroplane properties and allow to easily derive further data, such as damping coefficients, oscillations period, etc.

3.1 Longitudinal stability modes

Basic longitudinal modes of motion are phugoid and short period motion that are shown in roots locus plot in Figure 2. It should be noted that every eigenvalue λ = ξ + iη presented below has its corresponding conjugate λ∗ = ξ − iη.

As can be seen from Fig. 1, both phugoid and short period oscillations are convergent since real parts of eigenvalues are negative in the full range of flight speeds, thus the aeroplane is dynamically stable. Moreover, high values of real parts of eigenvalues describing short period mean this oscillation is heavily damped, i.e. calculated time to half amplitude equals 0.27 s for low and decreases further to approximately 0.1 s for higher airspeeds, respectively.
3.2 Lateral-directional stability modes

Basic lateral-directional aeroplane dynamic stability properties have been presented in root locus plot in Fig. 2, which shows eigenvalues for dutch roll and spiral modes in function of airspeed. Again, one should note that every eigenvalue for dutch roll mode exist in pair with conjugate $\lambda^*$. The aeroplane is dynamically stable in spiral since real parts of all roots, with moderate magnitude at low airspeed and consequently decreasing for high airspeeds up to less than 0.06, that corresponds to 12 s time to half the amplitude. Dutch roll mode is found to be convergent as well with relatively high real and imaginary parts values, especially for higher airspeeds. It decreases monotonically until the airspeed of 19 m/s, when damping increases up to minimum airspeed. It is caused by aerodynamic characteristics deterioration at high angles of attack (calculated stall speed equals 17.3 m/s).

4. Conclusions

On the basis of the analysis performed and data presented above, it can be said that the aeroplane investigated herein possess positive stability qualities in terms of both longitudinal and lateral-dimensional basic modes of motion. That in consequence means the aeroplane is safe in operation and poses interesting object for further research. However, this configuration needs to be investigated far thoroughly to understand dependencies between its properties and layout to find its optimal configuration for future projects.

References

A comparative study of the performance of the 4-slice Transverse/Oblique Slicing method for analysis of 3D residual stress in prismatic bodies

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Abstract

The Transverse/Oblique Slicing (T/O-S) technique is a destructive sectioning scheme devised originally for experimental analysis of 3D residual stress in railroad rails, but applicable to a broader class of prismatic bodies as long as their stress states are independent of the longitudinal coordinates. The T/O-S technique is not new, its original two-slice version dates back two decades ago, when it was proposed, tested and successfully applied to analysis of actual experimental data for several rail samples scanned for the residual states by a neutron diffraction technique. However, as the recent years saw a comeback of interest in new experimental studies of 3D rail residual stress, a newer version of the scheme was proposed, which called for cutting of two additional rail/body slices to form an enhanced 4-slice sectioning scheme. In the paper discussed are the concept of the new technique, the 3D stress reconstruction analysis of performance of the new technique and at making direct comparisons to the older two-slice sectioning scheme.

Keywords: 3D rail residual stress, experimental analysis, destructive testing, hybrid methods

1. Introduction

Back in the 80s of the 20th century, a number of experimental approaches were developed to investigate the rail residual stress [1]. The outburst of an immediate interest in the residual fields was driven in the first place by development of new rail grades that were devised to follow the needs of growing speed and tonnage of the trains. In order to improve overall strength, hardness and wear resistance, the industry practice shifted from application of rather ductile but of limited resistance to wear ferritic steels to newer grades of austenitic or even pearlitic/bainitic microstructure [2]. The side effect of this (otherwise reasonable) strategy was a growing number of various rail defects, for which the residual stress was found to be one of the key factors that drive crack growth and rail failures [1].

Among the methods developed back then for experimental analysis of residual stress in rails, the destructive Battelle 3D technique [3] has been considered the best in terms of resolution and completeness of the 3D results. Its winning features were a good spatial resolution (obtained thanks to application of very small strain gauges of ca. 1.3mm single sensor length, applied on a thin transverse Yasojima-Machi slice of 6.3mm thickness), and a numerical procedure for compensation of the stresses lost during sectioning. The Yasojima-Machi slice was diced on a 4x4mm grid to release the in-plane residual stresses, however, because during the slice removal some stress components were lost (normal stress was totally lost, the in-plane stresses were subject to partial stress relief due to Poisson effect), to obtain the 3D stress fields, a second, ca. 0.5m long piece of rail, had to be cut into thin rods on a corresponding to YM slice dicing grid. Cutting many 4x4x500mm rods was not only very time consuming and expensive, but had to lead to many severe errors.

The T/O-S procedure [4] was a direct response to these problems: by replacement of the Meier sectioning with a second inclinded YM slice, the method offered a substantial enhancement in terms of speed, cost and data accuracy. Applied to actual engineering problems [5] the approach was positively verified. However, in case of application of neutron diffraction (ND) technique applied for scanning of the residual stress, there were some concerns about the real quality of the 2D data. The source of these concerns was the problem of determination of the so called stress free lattice parameter do, fundamental for the neutron diffraction technique. In case of the rails in the post-service conditions, with texture, plastic flow, perhaps phase changes, etc., it is really a tough problem how to determine it correctly. Another concern was the problem of the independence of the stress state of the axial coordinate; for the old Battelle technique, the Meier sectioning averaged some possible axial fluctuations of the stress state, but is no longer happening in the case of the T/O-S technique.

2. The idea of the 4-slice T/O-S scheme

Figure 1 presents the classic, 2-slice [4] and the new, 4-slice setups. The classic scheme is limited to grey slices denoted here as “1” and “2”, the 4-slice version adds the red slices “3” and “4”.

Figure 1: The classic and the new T/O-S trepanation schemes

These redundant for the 3D stress reconstruction procedure data sets were introduced here for three important reasons:
- the second transverse slice (“4”) make it possible to assess the level to which the stress state is invariant along the longitudinal coordinate;
- the second oblique slice (“3”) – for which horizontal stress component theoretically should be exactly the same like for the both transverse slices (“1”) and (“4”) and the other oblique slice (“2”) – gives an instant assessment of the procedure accuracy;
- all four slices composed together provide a far better base for 3D reconstruction routine as averaging helps to reduce experimental error.

It is worth mentioning that all four slices are cut from the same piece of rail, shorter than in case of the Battelle 3D procedure, which helps to preserve a better data consistency.

3. Solution strategies and tests of the 4-slice method

The simulation tests of the 3D residual stress reconstruction procedure for the 2-slice version were performed back in the 1994 and the approach had been reported as highly effective [4]. Based on the theoretical hybrid finite element method (HFEM) 3D solutions obtained for UIC-60 rail profile by the shake-down approach [6], the tests numerically simulated the sectioning procedure (by relieving the normal stress component and calculating its impact on the 2D in-plane components due to the Poisson effect), and obtained 2D fields were treated as the pseudo-experimental data for restoration of the original 3D fields. This approach was proved highly efficient, able to restore the totally lost axial stress component and compensate for in-plane stresses partial relief with accuracy of 1.3%. If one used moderately disturbed (+/- 25%) pseudo-experimental 2D data, the reconstruction error grew by a factor of 3 to ca. 4% - still acceptable for the purpose. However, in case of actual neutron diffraction data, one may expect errors of much higher level. E.g., in case of neutron diffraction technique [5] one may face error magnitudes in the range of hundreds of percent and this is why the new 4-slice technique was proposed and is investigated here.

The program of the simulation tests performed in the paper is similar to that published in [4]: the HFEM based [6] 3D theoretical solutions are subject to a total normal stress component relief (simulated slice cut-off procedure, performed independently for all four slices), then the 2D data sets are randomized and these four 2D data sets are fed into the 3D stress reconstruction procedure. However, this time the amplitudes of the random error were not set as an arbitrary value, but were expected to mimic the error amplitudes seen in neutron diffraction data. An example of such an error estimate is presented in Fig. 2, calculated with help of the physically based approximation method [7]. What is important to notice is the fact that the error amplitudes differ spatially between components (the shear stress component was omitted due to scarcity of place), so randomization of the stress components’ error amplitudes should follow this spatially varying pattern.

Two strategies of a combined four-slice data based 3D stress reconstruction were considered:
- averaging raw slice data for both transverse and oblique slices and then processing such two averaged raw data sets as if it were a classic 2-slice data;
- averaging results of two independent 3D stress reconstructions, based on two 2-slice data sets combinations (set 1: slices “1” and “2”, set 2: slices “1” and “3”).

Figure 2: Estimates of actual error amplitudes for (a) horizontal stress component, and (b) vertical stress component for an ND scanned rail sample

In both versions, the numerical procedure consist of two main routines: raw data smoothing and enhancement on 2D level (slice data preprocessing), and 3D stress reconstruction (rail level).

4. Results and conclusions

The results of the tests will be shown during the conference. It was proved that the 4-slice version of the T/O-Step trepanation scheme leads to a better error control. Due to combination of the redundant data sets, the final errors were found to be smaller than in case of the respective two-slice combinations. Because the spatial distributions and amplitudes of the randomised data error were derived from actual ND data sets, it makes the simulations performed more realistic and feasible for assessment of the method in the case of actual neutron diffraction data.

References

Physical mode basis design for the flow past a Delta wing

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Abstract

Model order reduction is a key vehicle for flow control and better understanding of the flow physics. Most of the reduction techniques, like Galerkin method, are based on the approximation of the solution by superposition of global modes. In the paper a method for computation of global, physical modes is demonstrated on the example of the low-Reynolds number, incompressible flow past a Delta wing. The modes are computed in frequency domain as responses of the flow to random or localised perturbation. The presented approach, in contrast to the empirical methods such as POD and DMD, does not require prior solution of unsteady governing equations.

Keywords: modal decomposition, model order reduction, delta wing

1. Introduction

Computational simulation of fluid flow is very time-consuming, regardless of the governing equation (Naver-Stokes, LES/DES, RANS) being solved. All these high-fidelity models require computational grids in order of thousands (2D) or millions (3D) of degrees of freedom to cover all important vortex scales, which is unacceptable in the case of real-time, feedback flow control. When solution time is crucial, low-fidelity, Reduced Order Models (ROMs) might be used instead of high-dimensional data [1]. The proper choice of modal basis from empirical, physical and mathematical modes is crucial for the quality and robustness of the model [2]. While empirical modes, like POD or DMD are commonly - and successfully - used in the modelling of limit cycle oscillations, the modelling of transitional flow, either natural or under external forcing resulting from flow control, is more challenging task. Physical modes, representing fixed-point dynamics, traditionally may be obtained from global stability analysis.

2. Governing equation

The incompressible fluid flow may be described by the Navier-Stokes equation in non-dimensionalized form:

\[ \dot{V}_{i} + V_{i,j}V_{j} + P_{i} - \frac{1}{Re} V_{i,jj} = 0 \]  
\( i,j = 1,2,3 \)  
(1)

Assumption, that the unsteady solution of (1) can be expressed as the sum of its steady solution and a fluctuation:

\[ V_{i} = \bar{V}_{i} + \dot{V}_{i} \]
\[ P = \bar{P} + \dot{P} \]
leads to the disturbance equation in the form:

\[ \dot{V}_{i} + V_{i,j} \bar{V}_{j} + \dot{V}_{i,j} + V_{j} \dot{V}_{i,j} + \dot{P}_{i} - \frac{1}{Re} \dot{V}_{i,jj} = 0 \]  
\( i,j = 1,2,3 \)  
(2)

If small value of disturbance is assumed, equation (2) might be linearized, leading to generalized eigenvalue problem:

\[ \lambda \dot{V}_{i} + \bar{V}_{i,j} \dot{V}_{j} + \dot{V}_{i,j} + \bar{P}_{i} - \frac{1}{Re} \dot{V}_{i,jj} = 0 \]  
\[ \lambda \in C \]  
(3)

where:

\[ \bar{V}_{i}(x,y,z,t) = \bar{V}_{i}(x,y) e^{j \omega t} \]
\[ \bar{P}(x,y,z,t) = \bar{P}(x,y) e^{j \omega t} \]

In the method presented in the paper, global physical modes are computed as a response of the system to the global or localized perturbation in frequency domain instead of very challenging and time-consuming solution of generalized eigenproblem (3):

\[ Ax_{Re} + \lambda_{Re} Bx_{Re} + \lambda_{Im} Bx_{Im} = 0 \]
\[ Ax_{Im} + \lambda_{Re} Bx_{Im} - \lambda_{Im} Bx_{Re} = 0 \]

The choice of \( \lambda_{Im} \) influences the frequency / wavenumbers of obtained structures, while \( \lambda_{Re} \) is responsible for growth ratio.

3. Test case description

The aforementioned approach has been demonstrated on the incompressible flow past a Delta wing with vertical angle \( \Lambda = 35^{\circ} \). Reynolds number is set to \( Re = 1000 \), and the angle of attack \( \alpha = 30^{\circ} \). The disturbance is computed around a steady solution, depicted in Fig. 1.

Figure 1: \( V_{3} \) velocity isosurfaces for steady flow past a delta wing at \( Re = 1000 \)

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4. Global modes for a flow past a Delta wing

The perturbation was set on the surface of the wing. Three different values of the imaginary part of $\lambda$ coefficient have been chosen, leading to the complex structures (modes) with three different wave numbers and Strouhal numbers. They are depicted in Figs. 2, 3, 4.

Figure 2: $V_y$ velocity isosurfaces for real and imaginary parts of global mode computed for $\lambda_{Im} = 0.75$.

Figure 3: $V_y$ velocity isosurfaces for real and imaginary parts of global mode computed for $\lambda_{Im} = 1.50$.

Figure 4: $V_y$ velocity isosurfaces for real and imaginary parts of global mode computed for $\lambda_{Im} = 2.50$.

5. Summary

The method of computation of global, control-oriented physical modes was demonstrated on the example of the flow past delta wing. In contrast to traditional global stability analysis, this method allows continuous selection of the desired structures, for a given values of frequency and growth rate, as well as the position of perturbation. As a result, modes for slightly varying operating conditions might be computed by a simple change of a few parameters in the simulation.

References


Loadings in rail vehicle due to bulk material

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Abstract

The paper concerns loadings appearing during transport of bulk and aggregate materials such as sand, gravel, crushed coal etc. Nowadays, designers use general recommendations and then they solve problems of stress analysis in a simplified way. Author derived and solved numerically equations similar to those taken from soil mechanics allowing for a better description of bulk material pressure on retaining walls of containers. Significant achievement was to solve the problem of the bulk material pressure on the container walls of a running vehicle, when vertical and lateral accelerations occur. After application of the method, numerous results were obtained, which were next used for sensitivity analysis of the pressure force for various factors. The topic was undertaken after the author’s experience with experimental and theoretical testing of freight wagons as well as crash tests made on samples of cabs.

Keywords: rail vehicles, dynamic loadings, bulk material - structure interactions

1. Introduction

The subject of loadings caused by transport of bulk and aggregate materials was undertaken after an author’s experience with design and testing of wagons [1,2]. Usually designers apply general guidelines [1] concerning statics and dynamics of a vehicle and then they solve the problem of the pressure distribution on the container walls in a simplified way. The author derived and solved numerically equations similar to the ones taken from soil mechanics [3] but for a running vehicle, when vertical and lateral accelerations occur. Presented here model of pressure distribution caused by bulk payload is needed for better description of vehicle dynamics and also in order to optimize the vehicle structure.

2. Static loadings caused by bulk material

In this paper a method is presented to determine the static and dynamic loads originating from the bulk cargo transported in wagons, whose containers have elongated and prismatic shape.

The key problem is to determine the real value of thrust force and a pressure acting on the walls of the tank. Averaged pressure acting on the horizontal floor of the tank (vertical component, \( p \)) can be calculated according to the formula

\[
p = \gamma \cdot h,
\]

where \( \gamma \) – is average specific gravity of the bulk material, \( h \) - height of the pile.

According to theory applicable to bulk materials presented e.g. in [3] unit pressure \( p \) acting on the side wall of the tank should be determined as

\[
p(\zeta) = \gamma K \cdot \zeta,
\]

where \( K \) – lateral pressure coefficient (acc. [1,4]), \( \zeta \) – vertical coordinate measured down from the top of the pile.

Using the formulas adopted from soil mechanics, e.g. [3], it should be noted that the bulk material pressure on the retaining wall depends on whether it is in active state - if a small movement of the wall outward from the granular material is expected, passive state - if is possible substantial movement of the wall towards the pile, or the wall is at rest. There is a relationship

\[
K_a < K_s < K_p,
\]

where \( K_a \) – active lateral pressure coefficient, \( K_s \) – lateral pressure coefficient at rest, \( K_p \) – passive lateral pressure coefficient.

In order to determine the coefficient \( K_s \) and \( K_p \) the well known Coulomb or Rankine formulas may be used. The latter include only unilateral embankment slope with angle \( \beta \), but they neglect friction between the wall and the backfill. They also assume that the wall is positioned vertically.

Consider the container, which is commonly used to transport bulk material, filled with the pile with two-sided embankment. Since the above mentioned models do not include important parameters of the embankment, the author attempts to determine the pressure coefficient \( K \) also for more general situations. The presented approach consists of examining the balance of forces acting on factions (wedges) of the bulk material, as shown in Fig. 1. The thrust force of the wall loaded by the bulk material is given by formula

\[
P = W \cdot \frac{\sin(\theta - \phi)}{\cos(\theta - \phi - \omega - \delta)} = \gamma A L \cdot \frac{\sin(\theta - \phi)}{\cos(\theta - \phi - \omega - \delta)},
\]

where \( W \) – is weight of the faction, \( A \) – area of the faction, \( L \) – length of the container box, \( \theta \) - angle of quasi-rupture lines behind the retaining wall, \( \phi \) – average angle of internal friction of bulk material, \( \omega \) - angle of inclination of the wall face, \( \delta \) – an angle of friction between the wall and the bulk material.

The formula (3) can be rewritten as the following function

\[
P(\theta, \zeta) = \frac{1}{2} \gamma L \cdot \zeta \cdot K_s[\theta(\zeta), \zeta],
\]

where

\[
K_s[\theta(\zeta), \zeta] = \frac{\cos(\alpha - \beta)}{\cos(\omega)} \cdot k(\theta) \cdot \left[ 1 - k(\theta, \zeta) \chi(\theta) \right],
\]

\[
k(\theta) = \frac{1}{2} \cdot \frac{\sin(\theta - \phi) \cos(\theta - \omega)}{\sin(\theta - \beta_1) \cos(\theta - \phi - \omega - \delta)} \cdot \frac{\cos(\theta - \omega)}{\sin(\theta + \beta_1)},
\]

\[
\chi(\theta) = \frac{\cos(\theta - \omega)}{\sin(\theta + \beta_1)},
\]

\[
\alpha = \frac{\pi}{2} - \phi - \delta,
\]

\[
\beta = \beta_1 + \delta,
\]

\[
\delta = \frac{\pi}{2} - \omega - \phi,
\]

\[
\beta_1 = \beta - \omega.
\]
\( \lambda(\theta, \zeta) = \left[ 1 - \frac{\hat{\zeta}(\theta)}{\zeta} \right]^2 \sin(\beta_1 + \beta_2) \frac{\cos(\alpha_0 - \beta_1)}{\cos(\alpha_0 - \beta_1)} \), with \( \hat{\zeta}(\theta) = b \beta \tan(\theta - \theta_0g) \),

where \( \beta_1, \beta_2 \) are the embankment slope angles (Fig. 1), \( b \) – lateral position of the top of the embankment relative to one of the wall, \( \theta \) - angle of quasi-rupture lines for maximal force \( P \). The angle \( \theta \) ought to be determined from condition \( P^*(\theta) = 0 \) for a fixed value of the variable \( \zeta \). This equation is suitable for obtaining a solution for \( \theta = \theta(\zeta) \) with the use of one of the numerical methods. Next, it can be used for determining the pressure function \( p(\zeta) = \frac{\cos(\alpha_0 \theta)}{L \frac{d\zeta}{d\alpha_0}} \), which leads to equation

\[
p(\zeta) = \gamma \frac{\cos(\alpha_0 - \beta_1)}{\cos(\alpha_0)} \frac{k(\theta)}{k(\theta)} \left[ 1 - \frac{\gamma(\theta) - \lambda(\theta, \zeta)}{\eta(\zeta)} \right],
\]

where \( \eta(\zeta) = \left[ 1 - \frac{\hat{\zeta}(\theta)}{\zeta} \right] \left[ H(\zeta, \theta) \right] (H \text{ – Heaviside function}).

A series of results was achieved after application of the method described above in order to get the sensitivity analysis on various factors. The distributions of the function \( p(\zeta) \) (where \( z = h - \zeta \)) for the cargo containers filled with the bulk material with different shapes of the embankment were investigated. The slope angles \( \beta_1, \beta_2 \) as well as \( b \) – lateral position of the top of the embankment relative to one of the two walls were main variables of the analysis. The side walls were arranged vertically or were inclined relatively to the vertical at different angles \( \alpha_0 \). It was assumed that the internal friction angle of the bulk material was \( \phi = 31^\circ \) (for crushed coal) and the coefficient of friction on the side walls was \( \mu = 0.3 \). Figure 1 shows an example of the pressure distribution with the quasi-rupture lines inclined at the angle \( \theta \) marked with dashed lines.

![Figure 1: Exemplary distribution of pressure \( p(\zeta) \) acting on vertical wall of container filled with bulk material with embankment slope angles \( \beta_1 = \beta_2 = 29.75^\circ \)](image_url)

3. Dynamic loadings caused by bulk material

A more difficult problem is to define the impact of the bulk material on the walls of the tank of the running vehicle, when dynamic loads occur. In such cases usually various kinds of simplifications must be made. In this section, a certain approach is proposed in order to analyze the pressure effects occurring in the retaining walls like during the earthquakes. Taking into account the vertical and lateral accelerations caused by seismic waves was proposed for example in [3].

Pressure of the bulk material on the walls of the typical freight vehicle may be considered as the active state because the walls tend to deform outside the container while being under the load. Therefore, the weight of the faction of the bulk material makes the friction force acting down.

Figure 2 presents the most frequent case of the symmetrical embankment of the bulk material \( \beta_1 = \beta_2 = \beta_0 \), which is subjected to a vertical acceleration \( a_z = (1+\alpha_z)g \) and a lateral one \( a_\theta = \alpha_yg \), where \( \alpha_z, \alpha_y \) – coefficient acc. [1]. Due to these accelerations, a force of inertia \( G_i = \sqrt{\alpha_z^2 + (1+\alpha_z)^2 \gamma g m_3} (m_3 \text{ – mass of the bulk material, [1]}) \) acts in place of the weight of the bulk material. It is inclined from the vertical direction with an angle \( \varepsilon = \arctg \frac{\alpha_z}{1+\alpha_z} \).

![Figure 2: Nonstationary container filled with bulk material loaded with accelerations \( a_\theta \) and \( a_z \), \( \varepsilon \) - rotation angle of container caused by rail cant](image_url)

Therefore, the walls of the container are loaded with the forces \( P_\varepsilon, s = 1, 2 \) determined by taking into account the changed direction of the vector of the force of inertia \( G_i \) with the angle \( \varepsilon \). Moreover, modified specific gravity of the bulk material should be changed from \( \gamma \) into \( \gamma_\varepsilon = \sqrt{\alpha_z^2 + (1+\alpha_z)^2} \), and \( \gamma_\varepsilon \). Therefore, the coefficients \( K_\varepsilon^{\beta_1} \) were calculated with the method presented in section 2 (Eqn (5)).

References


Application of structural materials of ultra-high strength in rail vehicles

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Abstract

The paper presents a study of feasibility of weight reduction of selected examples of rail vehicle bodies. The project dealt with an application of structural materials such as an ultra-high strength family of steel instead of high strength steel, e.g. S355. First, a comparison between different sorts of materials was made. The results demonstrate reasons to choosing UHS steels. Examples of rail vehicles such as a container platform, a coal carriage wagon and selected wagon assemblies were taken into account in order to investigate the possibilities of mass reduction. As a result some structural members became lighter, while maintaining adequate strength and stiffness. The tare weight reduction make the mass of the payload (with different values for different load cases) may be noticeably increased.

Keywords: rail vehicles, weight reduction, UHS steel

1. Introduction

The main goal of the paper and the project is a study of feasibility of reduction of weight of structural members made on selected examples of freight wagons. The design process of railway vehicles in Europe should be based on so called technical specifications of interoperability (TSI), European norm EN 12663:2010 [1] and other regulations (e.g. ERRI reports, UIC leaflets). For the realization of the project mainly the European Standard EN-12633-1,2 was used as a base, here. This European Standard specifies minimum structural requirements for freight wagon bodies and associated specific equipment such as: roof, side and end walls, doors, stanchions, fasteners and other attachments. It defines the loads which have to be sustained by vehicle bodies and specific equipment, given material data, identifies their use and presents principles and methods to be used for design validation by analysis and testing.

The authors’ work was focused on an application of not typical materials such as an ultra-high strength (UHS) family of steel instead of high strength steel (HS) e.g. S355. Unfortunately, the European Standard [1] does not provide data concerning the UHS materials. The structural specification for them had to be part of the project, though it would be based on the general principles presented in the norm.

The application of structural materials of higher strength in rail vehicles gives the following advantages
- reducing of tare mass of the vehicle
- an increase of the payload
- an increase of strength and service life of structural members
- reducing the cost of the transport of goods
- a decrease of the manufacturing costs (welding, energy) as well as the maintenance costs
- assuring the safety and reliability of the vehicle.

Disadvantages of the products made of such materials are as follows
- increased costs of purchasing of the raw material
- increased manufacturing efforts – because of greater bending strength, lower cutting speeds etc.
- relatively lower flexural stiffness of the structural elements
- lower ductility of the material
- a risk of loss of stability of the structure
- a deterioration of some parameters of the empty vehicle.

2. Material study

In order to realize consequences of using different kinds of materials as the structural members of the rail vehicle bodies let us study Table 1 containing the basic data (Ref. [1,2,3]). Two parameters can be useful for a comparison: \( R_{ul} / \rho \) - a ratio of the ultimate strength to mass density and \( \sigma_{lim} \) – a permissible stress value. The ratio \( R_{ul} / \rho \) achieves the greatest value for aluminum and for the UHS, whereas the highest permissible stress \( \sigma_{lim} \) concerns the UHS and is equal (560÷1120 [MPa])/S. These results, as well as rather high cost of aluminum, are enough in order to encourage to choosing the UHS material.

Table 1: Structural materials – comparison; 7xxx – series of aluminum alloys of high strength (acc. International Alloy Designation System), MS – mild, HS – high strength, UHS – ultra high strength steel, S – safety factor

<table>
<thead>
<tr>
<th>Material</th>
<th>7xxx</th>
<th>MS</th>
<th>HS</th>
<th>UHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ), kg/dm³</td>
<td>2.8÷2.95</td>
<td>7.85</td>
<td>7.85</td>
<td>7.85</td>
</tr>
<tr>
<td>( E ), GPa</td>
<td>69.5÷72.5</td>
<td>210</td>
<td>210</td>
<td>210</td>
</tr>
<tr>
<td>( R_{ul} ), MPa</td>
<td>485÷575</td>
<td>200</td>
<td>350÷500</td>
<td>700÷1400</td>
</tr>
<tr>
<td>( R_{ul}/\rho )</td>
<td>-0.82÷0.97</td>
<td>-0.7</td>
<td>-0.72</td>
<td>-0.63÷0.96</td>
</tr>
<tr>
<td>( A )</td>
<td>&lt;10%</td>
<td>19÷26%</td>
<td>20÷22%</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>( R_{ul}/\rho )</td>
<td>164÷205</td>
<td>25</td>
<td>45÷63</td>
<td>89÷178</td>
</tr>
<tr>
<td>( \sigma_{lim} )</td>
<td>0.8R_{ul}/S</td>
<td>R_{ul}/S</td>
<td>R_{ul}/S</td>
<td>0.8R_{ul}/S</td>
</tr>
</tbody>
</table>

However, an important difficulty in using the UHS steel is to gain values of the limit stresses for different notch cases occurring in a structure, which are necessary to prove its fatigue strength. According to [1] five types of notch cases A, B, C, D or E are defined, where A is a parent metal or a machined butt weld. According to [1], a permissible range of dynamic stress
2σlim for the notch type A may be estimated from the material yield strength $R_y$ (or $R_{e-0.2}$) as 0.46$R_y$. The data for other cases of notches for the steel are not available presently.

### 3. Feasibility of weight reduction of freight wagons

A few specific examples of rail vehicles were taken into account in order to investigate the possibilities of the weight reduction of their bodies. Following freight vehicles such as a container platform Sggns (see its underframe in Fig. 1), a coal carriage Eanos (Fig. 2) as well as selected connections of components used in such vehicles. A frame of a freight bogie was also a good subject of a comparative analysis between the conventional and the new material.

A rationale for taking this task is the fact that the weight reduction has already been applied, to a limited extent, in other areas, e.g., in the truck industry.

A major difficulty in the analysis of the weight reduction of the rail vehicles is a large number of load cases, which the bodies of the passenger and the freight wagons as well as the bogie frames must be subjected to [1,4,5]. Some of these loadings are not defined accurately and their values are often given with an excess. Such case occurs when the load is acting on the walls and on their reinforcements in the vehicles carrying the bulk material.

The first task was to improve the material of the main members of the underframe of the container-wagon, having a large base length $L_b$, Fig. 1. The steel S355 was changed onto the UHS ($R_e = 1400$ MPa). Besides the strength conditions, the rigidity of the underframe had to be controlled by use of the limitation of its vertical deflection $|u|_{\text{max}} \leq 0.003L_b$.

As a result of the application of the ultra-high-strength steel (UHS) the side sills became lighter (the walls of the bars were thinner), while maintaining adequate strength for the vertical load. The tare weight was reduced by about 22%, so that the mass of the payload (in different values for different load cases) was noticeably increased. It was found that the material savings were possible while maintaining the stiffness condition, as shown in Table. 2. Regarding the fatigue strength of the investigated structural members, there are some problems with connections between the longitudinal and transverse beams.

![Figure 1: Underframe of freight wagon Sggns with long base $L_b$ for transportation of containers of length 20', 30', 40'](Image)

Figure 2 presents a body of the most frequent case of the freight wagon, e.g., a high sided open wagon for the transportation of bulk goods (coal carriage). There are some typical load cases given in [1], but mostly important is a static and dynamic impact of the bulk material on the walls of the tank of the vehicle. In the last case, certain simplifications usually are made or more accurate methods of stress calculations are to be applied. As the effect of the project concerning the weight reduction of the high sided open wagon, were savings of the mass presented in the Table 3.

![Figure 2: Body of freight wagon Eanos, which may be modified for weight reduction; deformations due to bulk material](Image)

### Table 3: Savings gained by changing structural material: S355J2G3 $\rightarrow$ UHS ($R_e = 850$ MPa), $t$ - thickness

<table>
<thead>
<tr>
<th>Change</th>
<th>Mass savings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper side walls, $t$</td>
<td>4 $\rightarrow$ 3 mm</td>
</tr>
<tr>
<td>Lower side walls, $t$</td>
<td>6 $\rightarrow$ 5 mm</td>
</tr>
<tr>
<td>Floor, $t$</td>
<td>6 $\rightarrow$ 5 mm</td>
</tr>
<tr>
<td>Underframe</td>
<td>UPN-380 $\rightarrow$ 350</td>
</tr>
<tr>
<td>Body without bogies Total savings</td>
<td>1.17 kg</td>
</tr>
</tbody>
</table>

Finally, a conclusion can be made, that the freight wagons would have decreased their weight through the modifications of their lower parts (underframes) as well as their upper parts (containers). The containers of different types of the wagons for the transportation of the granular or aggregate goods are particularly suitable for the task of gaining quite significant mass reduction by the improvements of the material.

### References


Wind tunnel tests of the development and demise of Vortex Ring State of the helicopter rotor

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Abstract

The Vortex Ring State (VRS) was the reason of several crashes of helicopters. This state of flight appears when a helicopter descents vertically (or with slight forward speed), and descent speed is nearly equal to induced velocity of the helicopter. However, this state may occur during hovering or slow horizontal forward flight, if a strong vertical gust appears. In the VRS a toroidal vortex is created around the rotor disc, which significantly reduces the thrust force generated by the helicopter’s rotor. The process of development and demise of the VRS phenomenon was investigated in a wind tunnel using 6-component strain-gage balance and an Particle Image Velocimetry (PIV) system. The investigated object was a remotely controlled (RC) helicopter with a rotor diameter of 0.71 m.

Keywords: aerodynamics of helicopter, VRS, Particle Image Velocimetry, PIV, strain-gage balance

1. Introduction

The Vortex Ring State was investigated for decades because of its threat for helicopters and their crews. However, most of these investigations were focused on the visualization of the streamlines during flight tests or wind tunnel tests in steady conditions ([1,7]). Some papers concern the CFD simulations as well ([2,5]). A development of measurement techniques and devices used in wind tunnels allow performing the unsteady experimental tests of the VRS phenomenon, including measurement of the rotor thrust as a function of the rate of descent. Due to application of Particle Image Velocimetry (PIV) method there was also a possibility to measure the velocity field around the rotor disc and to obtain a shape, size and location of the vortex. These results may be used for a better understanding of the process of development of the VRS, which should improve the safety of helicopter flight.

2. Methodology of investigation

2.1. Object of investigation

The object of investigation of the VRS was a remotely controlled (RC) helicopter T-REX 450 PRO Super Combo, with rotor diameter of 0.71 m and weight of ~0.78 kg. The pitch angle of the blades can be changed during the test in range from 0° to 10°. The maximum rotational speed of the rotor is 2400 rpm and the maximum induced velocity of the main rotor in hover is approximately 6 m/s. The helicopter was mounted in the wind tunnel via 6-component strain-gage balance. The axis of the rotor was parallel to the axis of the wind tunnel (Fig. 1).

The investigation was performed in the T-1 wind tunnel in the Institute of Aviation in Warsaw. It is a closed-circuit, open test section wind tunnel (with the test section diameter of 1.50 m), powered by 55 kW electric motor and 4-bladed constant-speed fan. The maximum airflow velocity in this wind tunnel is over 40 m/s and minimum steady velocity of the airflow is about 11 m/s. The measurements were performed during increment or decrement velocity (i.e. starting or stopping the tunnel motor).

2.2. Strain-gage balance investigation

The strain-gage balance was used to measure aerodynamic loads acting on the helicopter, especially the rotor thrust $T$. The loads were calculated based on the voltages of Wheatstone strain-gage bridges, measured by a computer with the NI USB-6259 I/O card. In the postprocessing phase timings of the aerodynamic loads were filtered by a forward-backward Butterworth lowpass filter.

The airflow velocity was calculated from:

$$V = \sqrt{\frac{2q}{\rho}} = \sqrt{\frac{2(p_0 - p_s)}{\rho}}$$

where $\rho = 1.225$ kg/m$^3$ is air density, $q$ means a dynamic pressure of the airflow, $p_0$ means total pressure of the airflow and $p_s$ – its static pressure. Both total and static pressure were measured by two Druck pressure sensors, connected with a Prandtl tube. The electrical outputs of these sensors were measured by the measurement & control system of the T-1 wind tunnel [3].

The timings of the airflow velocity were approximated as a polynomial of degree 5. This approach simplified an analysis of results, presented as $T=T(V)$ plots.

2.3. Particle Image Velocimetry investigation

The PIV is a modern technique to measure and visualize the flow velocity field. In order to make the measurement, seed particles (the DEHC oil in this case) must be atomized in the flow. Droplets of the seeding are illuminated by a ‘light sheet’
(i.e. a laser light, formed in a thin sheet by the lenses) and photographed by a camera. The diameter of a seeding droplet is few microns. At every measurement the camera grabs two frames. The time interval between frames in presented measurements was about $\Delta t=80 \mu s$. In the post-processing phase a software calculates the measured velocity field by obtaining displacements of the droplets (using the Adaptive Correlation scheme, with integration windows size of 64x64 pixels with 50% window overlap) and dividing them by the time interval $\Delta t$. The outlier vectors and missing data was removed in post-processing with the use of average and median filtering.

A scheme of the PIV test stand was presented in Fig. 2. More details can be found in [5].

3. Sample results

In Fig 3 a sample relation between the thrust of the rotor and the descent velocity was presented. The range of VRS appearance (theoretical and observed) was marked as well. In this case the thrust decreases from 12.5 N to 9.2 N, thus the difference caused by the VRS phenomenon is about 26%. The strongest decrement of the thrust appears when the descent rate equals 5.5 m/s.

The range of VRS appearance, marked in Fig. 3, is slightly narrower than the theoretical one and the curve $T=T(V)$ is smooth. It is caused by a relatively short time when the airflow velocity was appropriate for the VRS phenomenon (about 3 seconds). When this time was extended, the plot is more ragged – which means that the vortex around the rotor is more deployed and the flow becomes turbulent. It confirms conclusion written in [7].

![Fig. 2: Particle Image Velocimetry test stand](image)

![Fig. 4: Sample streamlines in the VRS](image)

4. Summary

Results of the investigation described in the paper illustrate a process of development and demise of the Vortex Ring State on the rotor of the helicopter. During the strain-gage balance investigation the loss of thrust caused by the VRS was obtained, as well as the rate of descent when the phenomenon appears. The PIV measurement allowed to present size and location of the vortex created around the rotor disc. All results may be applied to improve the safety of helicopter flight.

References


MS14

Multiscale Modelling of Materials and Structures

organized by T. Burczyński, W. Kuś, Ł. Madej and M. Pietrzyk
Multiscale evolutionary optimization of functionally graded materials

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Abstract

The paper is devoted to the multiobjective and multiscale optimization of composite structures made of functionally graded materials (FGMs). The aim of the optimization is the design of structures made of FGM by modifying the microstructure parameters in order to satisfy contradictory criteria at a macro scale. Numerical homogenization methods are used to perform a multiscale analysis. Finite element method is employed to perform the optimization procedure. Numerical example presenting the efficiency of the proposed attitude is attached.

Keywords: functionally graded materials, multiobjective optimization, evolutionary algorithm, multiscale modelling, numerical homogenization

1. Introduction

Composites are structural materials made of at least two different constituents. Typically, they consist of a continuous phase (matrix) and reinforcement. Due to their properties, especially the high stiffness-to-weight ratio, composites state an interesting alternative to traditional, usually isotropic, structural materials, like metals or their alloys.

The aim of the paper is to perform optimization of the structures made of a particular group of composites – functionally graded materials. As more than one criterion is considered during the optimization and the criteria are contradictory, the multiobjective optimization is performed. To avoid the premature convergence to local optima as well as problems with the calculation of the fitness functions gradient, an evolutionary algorithm is employed to perform the multiobjective optimization.

2. Functionally graded materials

Functionally graded materials (FGMs) are materials whose properties can be designed by a proper selection of constituent materials, the volume fraction of constituents and the shape and location of the reinforcement. An increasingly important group of composites is functionally graded materials. FGMs are characterized by the gradual variation in composition and structure along at least one specific direction [2]. The FGMs are readily used for applications related to severe thermal gradients and different conditions of its operation simultaneously reducing as well the maintenance of the product.

The performance of FGMs is not only a function of properties of relative amounts of constituents, but is also related to the optimal utilization of the constituent materials. The design of FGMs allows obtaining products well fit for the foreseeable conditions of its operation simultaneously as well as the material cost as the costs associated with the operation and maintenance of the product.

In order to obtain the structure made of FGMs best suited for a particular application, it is necessary to use optimization methods. Computational intelligence methods like evolutionary algorithms or artificial immune systems belong to global optimization methods and may be used as the optimization procedures in considered cases.

3. Numerical homogenization of heterogeneous materials

Some materials, like composites or porous materials are heterogeneous on a certain level of observation and their structure should be considered to model them with proper accuracy. The direct application of more than one scale (macro, meso, micro, nano) for the calculations performed by means of finite element method (FEM) or homogenization (homogenization) leads to extremely large equation systems. In order to overcome this problem, numerical homogenization methods may be used [3].

Assuming local or global periodicity of the considered structure, numerical homogenization allows determining the parameters of the equivalent, macroscopically homogeneous material. Typically, the numerical homogenization makes use of representative volume element (RVE) and consists in the determination of the constitutive relation between averaged field variables, like stresses and strains.

The RVE fully represents a part of the medium (assuming local periodicity) or the whole medium (global periodicity). The RVE is usually the smallest possible volume representing the entire medium (or its part). The conditions RVE has to satisfy are:

a) its size \( l_{RVE} \) is significantly larger than the microstructure characteristic dimensions \( l_{micro} \) and considerably smaller than the characteristic dimensions \( l_{macro} \) at the macro scale:
\[
l_{micro} \ll l_{RVE} \ll l_{macro}
\] (1)
b) Hill condition for the equality of the average energy density at the micro scale and the macroscopic energy density at the point of macrostructure corresponding to the RVE:
\[
\{\sigma_j, \epsilon_j\} = \{\sigma_i, \epsilon_i\}
\] (2)
where: \( \sigma_j \) and \( \epsilon_j \) – stress and strain tensors at the micro scale; \( \{\} \) – the averaged value of the field.

c) proper boundary conditions, e.g. periodic boundary conditions imposed on the opposite faces of RVE and strain boundary conditions taken from the higher scale.

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4. Formulation of the optimization problem

The aim of the optimization is to obtain the desired properties of considered structure made of FGM. The structure is divided into \(n\) sections of the same volume fraction of the reinforcement. The aim of the optimization is to simultaneously minimize two contradictory objectives:

a) the maximum deflection of the beam:

\[ \text{arg min} \{ f_1(x); \ x \in D \}; \ f_1(x) = \max(u_i) \]  \hspace{1cm} (3)

b) the total cost of the structure, represented by the total volume fraction of the reinforcement:

\[ \text{arg min} \{ f_2(x); \ x \in D \}; \ f_2(x) = \sum_{i=1}^{n} V'_i / V' \]  \hspace{1cm} (4)

where: \(x\) – a vector of the design variables, \(D\) – a set of admissible solutions being a subset of design space \(X\), \(u_i\) – the vertical component of displacement, \(V'_i\) – the reinforcement volume in the section \(i\) of FGM, \(V'\) – the volume of the section \(i\).

To solve the multiobjective optimization problem the multi-objective evolutionary algorithm (EA) based on NSGA-II algorithm and available in Matlab environment has been used [1]. MSC Patran/Nastran finite element method software has been employed to solve boundary-value problems in both scales.

5. Numerical example

A composite beam of dimensions \(b \times b \times l = 10 \times 10 \times 250\) mm and made of epoxy resin reinforced with longitudinally placed carbon fibres in form of FGM is optimized. The beam is divided into \(n=8\) sections of the same thickness \(b\) (Fig. 1a) and divided into 610 finite elements. The fibres in each section have the same diameter but fibre diameters may vary between the sections. Each section is represented by RVE or the same external dimensions \(a=10 \times 10\) \(\mu\)m. Reinforcement diameter values \(d\) state the design variables of the optimization.

A 3-dimensional RVE with centrally placed fibre (Fig. 1b) and divided into 648 finite elements has been used to calculate the equivalent material parameters for each section.

The averaged stresses obtained at the micro scale are transferred to the macro scale which allows calculating the equivalent material parameter values.

The parameters of the EA are: the number of chromosomes \(n_{ch} = 40\); the number of generations: \(n_g = 50\); the arithmetical crossover probability: \(p_{cx} = 0.8\); the uniform mutation probability: \(p_{mut} = 0.01\); the fibres diameter range \(d_i = 1\rightarrow 9\) \(\mu\)m.

The results of the multiobjective optimization in the Pareto frontier form are presented in Fig. 2. The values of the objective functions were normalised to the range 0÷10 and presented as dimensionless.

5. Numerical example

A composite beam of dimensions \(b \times b \times l = 10 \times 10 \times 250\) mm and made of epoxy resin reinforced with longitudinally placed carbon fibres in form of FGM is optimized. The beam is divided into \(n=8\) sections of the same thickness \(b\) (Fig. 1a) and divided into 610 finite elements. The fibres in each section have the same diameter but fibre diameters may vary between the sections. Each section is represented by RVE or the same external dimensions \(a=10 \times 10 \times 20\) \(\mu\)m. Reinforcement diameter values \(d\) state the design variables of the optimization.

The averaged stresses obtained at the micro scale are transferred to the macro scale which allows calculating the equivalent material parameter values.

The parameters of the EA are: the number of chromosomes \(n_{ch} = 40\); the number of generations: \(n_g = 50\); the arithmetical crossover probability: \(p_{cx} = 0.8\); the uniform mutation probability: \(p_{mut} = 0.01\); the fibres diameter range \(d_i = 1\rightarrow 9\) \(\mu\)m.

The results of the multiobjective optimization in the Pareto frontier form are presented in Fig. 2. The values of the objective functions were normalised to the range 0÷10 and presented as dimensionless.

Figure 2: Optimization results – Pareto frontier

The values of the design variables (fibre dimensions, from top to bottom sections) and the values of the objective functions for distinctive points A, B and C (Fig. 2) are collected in Tab. 1.

6. Final conclusions

The multiobjective and multiscale optimization of the beam made of FGM was performed by means of the evolutionary computations. The results presented in Fig. 2 and in Tab. 1 show that the considered objective functions are contradictory. Points A and B represent solutions with one of the criteria being dominating, which is proved by the obtained results. The values of design variables for the point B are high in the outer sections and much lower inside the beam, which results in obtaining a functionally I-beam structure. The choice of the one of the results at the Pareto frontier should state the next step and this is out of the scope of the present paper.

The proposed attitude allows the proper design of structures made of FGMs for certain purposes with a different number of different optimization criteria.

References

The use of a statistical representation of the microstructure in multiscale modelling of deformation of TRIP steels

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1 Abstract

The paper deals with modelling local phenomena occurring in TRIP steels during cold deformation. Process parameters at macro scale were calculated using the finite element (FE) method. Representative Volume Elements (RVEs) were attached to the selected Gauss integration points in the FE program. Initial microstructure in the RVE was generated using image analysis for various heat treatment parameters. First solution attempts with large number of grains involved very long computing times, therefore statistical methods were used to generate a Statistically Similar Representative Volume Element (SSRVE). This element contains simplified representation of phases in the microstructure, assuming periodic boundary conditions. The properties of phases were taken from the literature, where they were determined in plastometric tests. These properties were introduced to the SSRVE, and simulations of deformation of this element during process were performed for various drawing parameters and the deformation of retained austenite was predicted. Taking into account the relation between the amount of deformation and volume fraction of retained austenite, the distribution of the latter was determined. The results were validated by comparison with the experimental data.

Keywords: finite element modelling, FE2, TRIP steel, representative volume element, SSRVE

1. Introduction

Due to the nature of finite element (FE) simulations, coupled multiscale problems solved using a multilevel finite element method (FE2) require costly numerical procedures in both macro and micro scales. The cost of simulation is related to the number of finite elements used to discretize the volume. Representative Volume Elements (RVEs) contain often many grains and FE discretization requires using many elements to obtain reliable results. The problem of trade-off between accuracy and computing cost is still present in modern numerical simulations. This inspired scientists to search for methods of decreasing of computing time without losing the accuracy. Among various methods of microstructure simplification, Statistical Simplified Representative Volume Element (SSRVE) [1, 2] is one of the new and interesting approaches. The goal of this method is to provide statistically similar artificial microstructure, which contains only one or a few grains, but provides thereby the same rheological response in the macro scale. To make this possible, single SSRVE unit cell is considered with periodic boundary conditions (Figure 1).

2. SSRVE

The generation procedure of SSRVE element can be divided into few elementary algorithms. The preparation process for the SSRVE unit cell begins with typical image segmentation and reconstruction algorithms applied to the source metallographic image. The main goal of this part of the procedure is obtaining accurate representation of grains of each phase for shape analysis and discretization. After that, each inclusion is analyzed separately by the set of algorithms to determine shape coefficients, which are gathered into histograms to predict expected values. In addition, simulation of uniaxial stress and shear tests is performed to collect responses from the homogenization process. The obtained values are used as reference data inside the optimization procedure.

Figure 1: An example of two phase SSRVE Unit cell with periodical boundary conditions

2.1. Optimization procedure

SSRVE generation method is based on the multicriteria optimization function. The objective function is given by the equation composed of three internal elements responsible for identification of shape coefficients, statistical measures and rheological behaviour:

$$\Phi = \sum_{i=1}^{w_i} \sum_{l=1}^{k} \sum_{s=1}^{s} \sum_{p=1}^{l} w_i \sigma_i^p (\varepsilon_i)$$ (1)

$$\sigma_i^p = \frac{\sigma_{i,ref} - \sigma_{SSRVE}}{\sigma_{i,ref}}$$ (2)

$$\varphi_i = \frac{\varphi_{i,ref} - \varphi_{SSRVE}}{\varphi_{i,ref}}$$ (3)

where: $w_i$ - weights, $k$ – number of shape coefficients, $l$ – number of statistical measures, $s$ – number of rheological curves, $p$ – number of iterations in numerical simulations.

Equation (2) is the comparison of $i$-th shape coefficient, equation (3) is the comparison of statistical measures and
The implemented multi-iterative genetic algorithm is composed of the following steps:

a) Generation of initial population – random generation of $n$ specimens containing information about coordinates of control points and their weights.

b) Estimation of an objective function value – the procedure calculates values of function (1) on the basis of shapes of inclusions in succeeding SSRVEs. The shapes of these inclusions are described by NURBSs. A rheological model of the SSRVE is determined by processing of this element in virtual uniaxial compression in both directions, and shear deformation tests. The obtained stress-strain relations allow us to calculate the equivalent tensile stress, which describes the material rheology. The main statistical measure is lineal-path function, calculated directly from pixels of SSRVE.

c) Stop conditions – two fundamental conditions are implemented, i.e. number of iterations and mean square error between expected and actual objective functions.

d) Application of genetic operators – the following operators were implemented in the presented approach: crossing and mutation operators. A mutation operator changes the positions of control points regarding centre of gravity of the shape or by using random values of coordinates in the vector of translation. The crossing is responsible for exchange of random number of genes between the two specimens.

e) Generation of a new population – the specimens obtained after operations of crossing and mutation are included in the new population. In each subsequent iteration, every specimen is validated.

3. Applications

The SSRVE concept has already found application in multiscale simulation of wire drawing phenomena. As an example, the TRIP microstructure composed with retained austenite, bainite and ferrite phases was taken. 284587 elements were used for reliable discretization of the domain. Properties of phases were taken from the literature [3], where they were determined in plastometric tests. In the case of martensite, it was compression on nano samples. The properties of phases including flow stress were introduced into the SSRVE and simulations of deformation of this element during process were performed for various drawing parameters and deformation of the retained austenite was predicted. Consequently changes of the volume fraction of the retained austenite during the process could be calculated. The results were compared with the available experimental data [4, 5] and a good predictive capability of the multiscale model was confirmed.

A submodelling technique from Abaqus software was used to attach microstructure to selected places in the macro model. The time to calculate microstructure responses was estimated to be around 1 h, whilst, in contrast, the generated SSRVE with 5450 elements took only 20 minutes on a typical quad core processor. Comparison of RVE and SSRVE behaviour represented by the true stress is presented in Figure 2.

Figure 2: Comparison of compressive radial stresses obtained from SSRVE and RVE. Rheological behaviour

The tests proved high efficiency of SSRVE compared to typical RVE, with significant decrease of computational time necessary to obtain results, while predictive capabilities of both approaches were comparable.

4. Conclusions

The paper presents an approach to create of SSRVE and its application to wire drawing simulation. Obtained results show that SSRVE is a progressive alternative to the typical approach in multiscale computations. The SSRVE concept offers a significant reduction of computational time and resources in obtaining reliable results. A recent investigation showed that computing domain of SSRVE can be further reduced by replacement of the FE method by Isogeometric Analysis (IGA)

References


The FE modelling of residual stresses and void formation observed during the growth of semiconductor layers

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Abstract

In the current approach we consider interdiffusion, lattice distortions and chemical maps corresponding the growth of SiC/Si and InGaN/GaN layers. Dislocations and free surfaces are regarded as local regions for nucleation and annihilation of the vacancies transporting the mass between finite elements (FEs). As a result, the interface and FE mesh are convected with the crystal lattice drift. In the constitutive modelling applied [1] the lattice distortion and the Si and vacancy molar fractions are used as independent nodal variables. Due to the climbing down of misfit dislocations the plastic distortion tensor field is taken into account in the form of additional nodal variables. This tensor field is spanned on corner nodes of the second order Lagrangian finite elements [2]. The chemo-mechanical coupling is based on the use of Vegard’s law formulated alternatively in terms of Biot or Hencky strains. Due to the logarithmic strain applied in hyperelastic modelling, some transformation rule is considered for Vegard’s law. This rule allowed us to eliminate artificial residual stresses yielding from incompatibility of the fields of atom fraction and plastic distortions spanned by means of the same shape functions at the corner nodes.

Keywords: residual stresses, Kirkendall effect, diffusion, mass transport, semiconductor layers, crystal growth

1. Introduction

The interdiffusion of chemical components coupled with vacancy movement can cause the void formation driven by the chemical force and different mobilities the chemical components of crystal lattice. In the case of SiC layer growth in carbonisation process of Si wafer, the higher mobility of Si atoms compared to C can result the climbing down the SiC/Si interface and formation of voids in the substrate in some thermodynamic conditions, see Fig. 1.

A similar effect is observed in the case of the growth of In-rich InGaN layers on GaN substrate. The chemical force yielding from the spinodal decomposition of In-rich InGaN layer induces then the void formation on the border of metallic indium precipitations, see Fig. 2. In the talk we analyse the reasons of spurious stresses obtained during the FE calculation of stress equilibrium obtained by plastic relaxation of SiC layer grown on Si wafer.

Figure 1: Voids in a silicon substrate below the SiC film (black) formed in the growth process of SiC on Si wafer, see [3]

Figure 2: Voids formed in the In-reach InGaN layers. HTREM image courtesy of S. Kret

2. Constitutive model

In the presented approach, the thermodynamic potential function is the following molar density of free energy

$$\psi = \frac{1}{2} \hat{c} (\hat{\varepsilon} - \hat{\varepsilon}_{ch}) : \hat{\varepsilon} + (\hat{\varepsilon} - \hat{\varepsilon}_{ch}) + \psi_{ch}$$

(1)

where $\hat{c}$ is molar concentration, $\hat{\varepsilon}$ means the lattice strain referred to the perfect lattice of the reference material (substrate), cf. [1]. The chemo-thermal strain $\hat{\varepsilon}_{ch}$ and stiffness tensor $\hat{c}$ are governed by multilinear law

$$\hat{\varepsilon}_{ch} = \sum_{i=0}^{n-1} \hat{c}_i n_i + \hat{c}_T T$$

and

$$\hat{c} = \sum_{i=0}^{n-1} \hat{c}_i n_i + \hat{c}_T T$$

(2)
where $\varepsilon_i, \varepsilon_T, \bar{e}_i, \bar{e}_T$ are treated as material constants. By assumption, $\psi_{ch}$ is independent of strain and depends only on the remaining thermodynamic variables, i.e. on $n_1, \ldots, n_{m-1}$ and temperature $T$. In our case the elastic strain is identified with $\bar{e}_a \equiv \bar{e} - \bar{e}_{ch}$. The substitution of the energy function into balance equations gives the following formula for the driving forces governing the diffusion of the chemical components and vacancies

$$
\dot{f}_c = \tilde{a}_c \cdot \text{grad} \bar{\sigma} - \text{grad} \mu_{ch},
$$

where

$$
\bar{\sigma} = \tilde{c}_c : (\bar{e} - \bar{e}_{ch}) \quad \text{and} \quad \mu_{ch} = \frac{\partial \psi_{ch}}{\partial n_i}
$$

$\text{grad}$ means the gradient operator referred to the reference crystal lattice. The constitutive equations are considered alternatively for the Hencky and Biot strain measures

$$
\bar{e} = U - 1 \quad \text{or} \quad \bar{e} = \ln U
$$

where $U$ is the stretch tensor of crystal lattice.

3. Finite element approach

The nonlinear matrix equation used here for simulation of the crystal growth and silicon transport to SiC layer takes form

$$
\begin{bmatrix}
C_{Si} & -C_{Si} \\
C_{Si} & C_{Si}
\end{bmatrix}
\begin{bmatrix}
\hat{u} \\
\hat{n}_{Si}
\end{bmatrix}
+ \begin{pmatrix}
\hat{f}_u \\
\hat{f}_{Si} \\
\hat{f}_{vSi} \\
\hat{f}_{cSi}
\end{pmatrix} = \begin{pmatrix}
\hat{f}_u \\
\hat{f}_{Si} \\
\hat{f}_{vSi} \\
\hat{f}_{cSi}
\end{pmatrix}
$$

(6)

where the nodal variables $\hat{u}, \hat{n}_{Si}, \hat{n}_{vSi}, \hat{\beta}_{pl}$ denote the displacements, the molar fraction of silicon atoms and silicon vacancies as well as a few components of the plastic distortion tensor being indispensable for plastic relaxation of SiC layer. $C, P$ and $f$ denote the respective matrix and vectors obtained for nonlinear transient problems, cf. [4]. The distribution of Si fraction and the role of plastic distortions in relaxation of residual stresses is shown in Fig. 3-5.

4. Conclusions

Usually, the crystal growth processes are simulated from the viewpoint of the liquid deposited on the rigid substrate. In our approach, growth is considered from the viewpoint chemomechanical processes developed in the crystal lattice. The advantage of such approach is discussed.

In the talk we analyse the effect of different stress measures on the resultant residual and spurious stresses obtained in the finite element modelling. It was noted that the logarithmic strain leads to very high spurious stresses in comparison to the use of Biot strain measure. In the case of single transit finite elements situated at the interfacial zone, the mentioned approach allowed us to reduce spurious stresses in integration points from the level $10^3$ MPa to $10^{-2}$ MPa.

References

Optimal design of eigenfrequencies for a functionally graded piezoelectric plate by two-scale model and harmony search

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Abstract

The optimization problem of eigenfrequencies for the functionally graded piezoelectric plate is considered. The eigenvalue problem of the plate is modeled by the state-space approach. Two macroscale models of the plate are considered. The first case is related to the model of the functionally graded material where the physical properties are changed smoothly through the thickness of the plate. The mentioned model requires to determine the so-called transfer matrix by the Peano-Baker series. In the second case the discrete multi-layer model is considered. For the discrete model the transfer matrix of the state-space approach is given in a closed form by the exponential matrix function. To determine the effective properties of the FGM the micromechanical model is applied. The effective properties are determined by the Mori-Tanaka approach. The optimization problem of the plate eigenfrequencies is formulated across the micro- and macroscale, the harmony search method is applied to solve it. In the final version of the paper numerical examples and discussion will be presented.

Keywords: optimal design, functionally graded piezoelectric plate, harmony search, state-space approach, Mori-Tanaka model

1. Introduction

The multi-field composites, are also called the electronic composites detect many interesting properties, so they are widely used in the microsystem technology [5]. The physical properties of such composites are derived from the coupled field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity. Materials with a functionally graded microstructure (FGM) belong to the modern field phenomena, especially piezoelectricity.

2. Macroscale model

The macroscale model of the plate is based on the linear constitutive equations for the electroelastic FGM material [6]:

\[ \sigma_{ij} = c_{ijkl}(x) e_{kl} + \kappa_{ijkl}(x) (-E_i), \]
\[ D_i = e_{ijkl}(x) e_{ij} - \kappa_{ijkl}(x) (-E_i), \]

where \( \sigma_{ij} \) and \( \kappa_{ijkl} \) are the elastic stress and strain tensors, respectively; \( D_i \) and \( E_i \) denote the electric displacement and electric field vectors; \( e_{ijkl} \) and \( \kappa_{ijkl} \) are the elastic stiffness and the dielectric permeability tensors. The elastic field is coupled to the electric field through the piezoelectric moduli tensor \( e_{ijkl} \). The argument \( x \) denotes that the physical properties are functions of the spatial coordinates for the FGM material. For FGM plates the material tensors are usually dependent only on \( z \) coordinate measured along the thickness of the plate. In the present formulation, the electroelastic material is modeled as orthotropic, linear elastic and linear piezoelectric.

A state-space equation can be derived from the linear elastic kinematic equation, the equation of motion and the Gauss law as [1, 6]:

\[ \frac{\partial}{\partial z} V = \mathbf{M} V, \]

where \( V \) is the state vector consists of the mechanical displacement components, the electric potential, the elastic displacement in the thickness direction and the stress tensor components in the thickness direction; \( \mathbf{M} \) is the differential operator matrix depends on the in-plane derivative operators, the time derivative operators, the material constants and the spatial coordinates, especially \( z \) in the case of plates [6].

For the simply supported boundary conditions and the harmonic motion assumptions one can obtain the state equation expressed by the searched dimensionless amplitudes of the state vector components \( \hat{V} \), the investigated eigenfrequencies, the material constants and the coordinate \( z \) as shown in [1, 6]:

\[ \frac{\partial}{\partial z} \hat{V} = \hat{\mathbf{M}}(z) \hat{V}, \]

in the case of a simply supported plate and the harmonic motion assumption the operator \( \mathbf{M} \) has a form of the system matrix \( \hat{\mathbf{M}} \). The equation (3) has an analogous form to the state-space equation for the linear time-varying system known from the control theory [4].

The solution of the equation (3) can be written using the state transition matrix or the transfer matrix \( \mathbf{T} \) [4]. For the FGM material the transfer matrix is given by the Peano-Baker series [4, 6]:

\[ \mathbf{T}(z, 0) = \mathbf{I} + \sum_{s=1}^{\infty} \frac{1}{s!} \left[ \hat{\mathbf{M}}(s) \right] \hat{\mathbf{M}}(s) ds + \ldots, \]
where $I$ is the unit matrix, $z$ and $s_i$ are the spatial coordinates measured along the thickness of the plate. In this case the transfer matrix is often calculated numerically. For the exponential gradation of physical properties the transfer matrix can be expanded into a matrix polynomial form using the Cayley-Hamilton theorem [6].

For a discrete layer model the transfer matrix can be expressed in a closed form for each layer where the physical properties are constant [1]:

$$T(z_i, z_{i-1}) = \exp\left[\frac{M_i}{h}(z_i - z_{i-1})\right],$$

(5)

where $M_i$ is the system matrix for the $k$-th layer.

For the plate of a total thickness equal to $h$ the resultant system of equations has the following form [1,6]:

$$\nabla(h) = T(h,0) \nabla(0).$$

(6)

$\nabla(h)$ denotes the state vector at the upper surface of the plate and $\nabla(0)$ is the state vector at the lower surface of the plate. The total transfer matrix is denoted by $T(h,0)$. For a discrete layer model the total transfer matrix is calculated as a product of the transfer matrices for each layer [1].

The free vibration problem requires certain manipulations on the system (6) according to the prescribed boundary conditions, which gives the implicit transcendental equation for the eigenvalues of the plate [1,6].

3. Microscale model

The physical properties expressed by appropriate tensors in (1) are also strongly dependent on the microstructure of the material of the plate. To assure the properties of the FGM material gradation of the physical properties is obtained by changing the parameters of the microstructure [5,6], namely the volume fractions of the constituents, the morphology of the constituents, etc. To calculate the effective properties of the composite a generalized Mori-Tanaka approach is used. For the two-phase electroelastic composite the effective moduli tensor is given by [2]:

$$L^* = L_M + f_I (L_M - L_I) \cdot A^M_I,$$

(7)

where $f_I$ is the volume fraction of the inclusion; $L_M$ and $L_I$ denote the electroelastic moduli tensor of the matrix and the inclusion, respectively; $A^M_I$ is the strain concentration tensor for the inclusion in the Mori-Tanaka approach [2]:

$$A^M_I = \frac{1}{\frac{1}{A_I} - \frac{f_I}{1-f_I} L_I},$$

(8)

where $A^M_I$ is the dilute strain concentration tensor given by [2]:

$$A^M_I = \left[I + S : L\right]^{-1}. L_M^{-1} \left[I - L\right].$$

(9)

The electroelastic Eshelby tensor $S$ can be defined using the volume integral of the derivatives of the electroelastic Green tensor over the inhomogeneity domain [2]. The mentioned integral is calculated numerically using the Gauss quadrature method.

For given microstructural parameters, e.g. the range of the porosity through the thickness of the plate, relation between the effective properties and the microstructural parameters (also the spatial coordinate $z$) can be found using the Mori-Tanaka approach.

4. Optimization problem

The optimization problem for the eigenfrequencies $\omega_3$ of the plate can be formulated as the problem of the fundamental eigenfrequency maximization:

$$\max \min \omega_3(x), \quad x_{\text{max}} \geq x \geq x_{\text{min}},$$

(10)

where the design variables vector $x$ may depend on the micro- and macrostructure parameters. The upper and lower bounds are imposed on the design parameter vector $x$ as shown in (10). Another forms of the objective function are possible, e.g.:

$$\min \left[ \omega_3(x) - \omega_3^0 \right], \quad x_{\text{max}} \geq x \geq x_{\text{min}}.$$

(11)

The problem (11) is the design problem for a given eigenfrequency $\omega_3^0$. Due to complexity of the problem (10) and (11) the heuristic method, namely the harmony search method is applied.

5. Harmony search method

The harmony search method (HS) was developed on the basis of a improvisation process observed in music, especially in jazz and blues [3]. The main goal of improvised music is to search for the perfect state of harmony. The effort to search and to find the harmony in music is analogous to searching and to finding the optimum in an optimization process. The HS algorithm takes a few elements from the jazz improvisation process: usage of harmony memory, pitch adjusting and randomization [3]. The unknown optimum is the best harmony under investigation. The harmonies contain design variables and change during the optimization process. New harmonies are created from the harmony memory using pitch adjustment and randomization process. For the new harmonies the fitness (objective) function is evaluated and the better ones exchange the harmony memory. The main advantage of the HS method is the fact that this approach does not require any information about the gradient of the objective function and brings a height possibility to find the global optimum [3].

The numerical results and discussion will be presented in the full version of the paper.

References


Prediction of the polymer degradation: a molecular dynamics study

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Abstract

In the paper the results are presented of molecular dynamic simulations of degradation process of a polymer system. For the sake of this study a molecular model is developed for poly (lactic-co-glycolic acid) (PGLA) polymer and a set of molecular dynamics simulations is performed with constant temperature and volume conditions (NVT ensemble). In order to examine a hydrolytic type of degradation a model is located inside a simulation box filled with water. To include the reactive nature of degradation phenomenon we use ReaxFF force field. The results allow to determine the general trends for polymer dynamic viscosity and molecular mass and describe the state of bonds in a polymer internal structure. The obtained results provide a deeper insight into polymer degradation and may be used for a future prediction of polymeric material lifetime.

Keywords: MD simulations, reactive force field, hydrolytic degradation, dynamic viscosity loss, lifetime prediction

1. Introduction

Plastics are undoubtedly among the most useful materials currently used in various engineering applications. The possibility of manufacturing wide range of shapes, low density and many other advantages made plastics possible to replace other traditional materials, as metal or ceramics, providing all required needed durability parameters [1,14].

Unfortunately, each polymer is more or less susceptible to active chemical and biological environment or other external conditions i.e. high temperature, load or pressure, the change of material properties resulting from this factors is called degradation. Besides the factors connected with the environment there are also exist some associated with the polymer by itself. Therefore, all factors which may cause polymer degradation may be divided into the following sections [13]:

1) Physical factors – i.e. distribution of molecular mass, the percentage of the crystalline phase (material factors), mechanical load, light, ultrasounds, radiation (environmental factors),

2) Chemical factors – i.e. intramolecular forces of covalent bonds, cohesive energy, construction and spatial structure of the chain (material factors), the presence of water, acids, alkalines, oxygen molecules or other chemical compounds (environmental factors),

3) Biological factors – i.e. the presence of bacteria, fungi, and other prokaryotes (only environmental factors).

The abovementioned factors usually are not isolated cases but a combination – physical and biological agents typically occur in the presence of a chemical interaction. In addition, decomposition of polymeric material, during its use and storage, due to all influencing factors is called the material aging. The examples of polymers aging studies are presented in [5, 8–10, 15].

A general mechanism of polymer degradation is an example of a radical decomposition. The free radical is a molecule or atom with a nonzero spin value. This means that molecule has some unpaired (free) electrons make the molecule reactivity relatively high [11]. Since the start of radical decomposition depends on factors causing particular type of degradation we decided to investigate polymer decomposition induced by aquatic environment and elevated (comparing to standard conditions) temperature.

By the aim of Molecular Dynamics (MD) simulations we studied time changes for selected parameters directly acting on material durability and stability for a future prediction of its lifetime. Moreover, the use of polymer materials in industrial application requires an extensive understanding of their physicochemical and mechanical properties, especially variable in time, thus a deep insight into the polymer degradation is desired.

2. Polymer degradation study

A molecular model for poly (lactic-co-glycolic acid) (PGLA) is developed from the individual structure unit, through a single polymer chain, to a final amorphous structure. Subsequently, a developed model is subjected to geometric optimization in order to obtain the conformation located at the global energy minimum. The relaxed structure was placed inside cubic simulation box filled with water molecules.

The study of material degradation was performed by molecular dynamics simulation. In order to include the reactive nature of the studied issue a ReaxFF force field developed by A. van Duin [4] was applied. As the ReaxFF is based on atomic bond orders (instead explicit bond presence) thus makes it able to study chemical reactions occurring during material degradation and other aging mechanisms [2,3,6,7,12].

According to the objective of this paper, to study polymer properties under degradation a set of MD simulation assuming the NVT ensemble is performed and physicochemical material parameters affected by progressive degradation (dynamic viscosity, molecular mass, state of bonds and number of decomposition products) are concerned. The results for a degraded model are obtained for three different time cases and then compared. The calculated values are compared with the results for an initial (non-degraded) model in order to show decreasing/increasing time trends for the observed material properties. During simulation thermodynamics parameters are also monitored to make it sure that both constant temperature and volume conditions are satisfied.
3. Results and discussion

The results confirmed the adverse effect on the degradation of investigated material parameters. The highest rate was observed for polymer dynamic viscosity and for molecular mass averages. An internal material structure also changed. Qualitative assessment of these changes may be done by comparing two structures shown in Fig. 1. It is clearly visible that a degraded PLGA structure (lower image) is significantly less compact than in a non-degraded case (upper image). In addition, for the degraded case there occur short organic chains separated from the main part of degraded polymer.

Figure 1: The PLGA polymer amorphous structure before degradation (upper) and after degradation (lower).

Numerical estimation of degradation impact to a model geometry is based on the state of bonds. In particular we focused on the single order bonds as their breaking is the direct cause of an observed polymer decomposition.

Although the developed approach based of reactive force field includes only the processes occurring on the material surface, time variation for all investigated parameters follow the expectations. The obtained results allowed to formulate a general trend for investigated properties, but the exact numerical values should be verified by means of laboratory experiment.

Further research for polymer degradation studied by molecular dynamics methods will be focused on determination of mathematical model including the kinetic ratio of material decomposition for a more accurate prediction of polymer lifetime.

References

3D modeling and structural evaluation of ancient bozdogan (Valens) aqueduct in Istanbul

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Abstract

In the past, masonry aqueducts were used to convey water from one place to another to cross obstacles on the land. As a unique example, Bozdogan Aqueduct in Istanbul from the Roman time has survived to date despite many devastating earthquakes in the region over the course of time. To possibly address the true behaviour of the aqueduct, a detailed site investigation and damage diagnosis were conducted prior to developing a 3D computer based structural model for gravity and seismic analyses. Normal and shear stress distributions and locations of stress concentrations are demonstrated to determine the structurally weakest points. Other behavioral values such as modes of vibrations and maximum displacements are investigated. The analysis results confirmed the existing damages observed on the arched piers. Furthermore, computational modeling issues and expected behaviour of the aqueduct are discussed.

Keywords: historic masonry, structural analysis, damage, earthquake effect

1. Introduction

To convey water from one location to another, aqueducts being a kind of water supply systems, were used in the past to exceed obstacles such as valleys on the way. All around the World, there are many aqueduct structures as an invaluable cultural heritage. Aqueducts were mostly constructed as masonry (with brick or stone materials or brick-stone composite).

In the history, since several civilizations such as Ottomans, Roman Empire, and Byzantines ruled in the land of Turkey, a large number of heritage structures exist in the country especially in Istanbul. Bozdogan (Valens) Aqueduct is a unique example of this heritage that located at Fatih district of the city, and was constructed to water supply for Istanbul. The construction of the aqueduct was completed by Emperor Valens of Roman Empire in the 4th century. The structure was also used in Byzantine and Ottoman periods. Due to many factors, several repair, maintenance processes, and new additions were performed in various eras. A study on the structural behaviour of the aqueduct was carried out by Yorulmaz and Celik [1].

To determine structural and seismic behaviour of the aqueduct in the existing condition, a three-dimensional structural model is developed in SAP2000 structural analysis software and modal as well as response spectrum analyses are conducted for the aqueduct model [2]. Analytical results in terms of stress distribution, modes of vibration and comparison of damage patterns are discussed.

2. Geometric Properties and Damage Survey

The Aqueduct has a masonry structural system. The masonry unit is mainly stone and also brick parts exist that is mostly additions of different periods. The whole length is L=516.43m in plan, and the width varies in between b=3.82m to 5.99m. The structure accommodates 61 arched spans on ground story level (Fig. 1a). In the present study, spans are named as A1~A61 from northwest end to southeast end of the aqueduct.

Although the parts A1 to A47 are two-storey in its original construction, in the existing condition, the bottom level of the structure sinks gradually from span A35 to A1, where at the northwest end most of the ground story remained within the soil. The maximum heights are 13.14m and 22.02m for the one and two-story parts, respectively. Arch spans varies between l=2.40m–6.14m. During the reconnaissance visits, several damaged, decayed, and deteriorated structural parts were observed. The most important damages were observed on the arch piers of ground level in the tallest part of A35 to A47. This segment is the most vulnerable segment of the aqueduct due to its slenderness. Observed critical damages on piers are vertical and/or inclined shear cracks, large voids, and serious material and surface loss of masonry units (Fig. 1b). Other diagnostic damages observed commonly on several structural segments are surface contamination, deterioration, and vegetation etc.

Figure 1: a) Bozdogan Aqueduct, b) Pier Damage

3. 3D Modeling and Structural Analyses

Based on the damage patterns observed, only the part in between spans A41 to A52 of the aqueduct, which reflects the most critical two-story tall segment, transition section and a part of one-story segment, is included in the 3D model. Appropriate boundary conditions are defined. The model prepared in SAP2000 using 8088 solid elements (Fig. 2). In that model, the walls on two façades, arches and infills are considered separately. To determine the mechanical properties of stones and bricks, on-site sampling and laboratory tests were performed and average mechanical properties were obtained. The laboratory test report states that masonry units show good
quality, stiffness, and strength. Masonry is considered as a composite and unique material rather than micro or meso level modeling. The elastic modulus of masonry is considered as $E=5000\text{MPa}$ as per the test results of units and considering mortar. Elastic modulus of infill (between the faces of the wall) is assumed as $E=1000\text{MPa}$.

To investigate the structural and seismic behavior of the aqueduct, an evaluation according to the current Turkish Seismic Code is done [3]. Modal and response spectrum analysis are conducted in the model. The analyses are performed under linear elastic conditions. Two parameter sets (C1 and C2) are used in seismic analyses. C2 denotes a set of parameters that produce less seismic loading. In C1, the following is assumed: The Effective Ground Acceleration Coefficient of $A_0=0.4$, Seismic Load Reduction Factor of $R=2$, Building Importance Factor of $I=1.4$, Soil Class of Z3, and Spectrum Characteristic Periods of $T_a=0.15\text{sec}$, $T_b=0.60\text{sec}$. In C2, two parameters are different as $R=2.5$ and $I=1.0$. On top of the structure, the live load was assumed as $2\text{kN/m}^2$. Analyses are carried out under dead loads (G), live loads (Q), and seismic loads (EX and EY), where EX and EY represent the long and short directions, respectively. Load combinations of $G+Q\pm EX$ and $G+Q\pm EY$ are used.

### 4. Numerical results and evaluation

Structural and seismic responses of the aqueduct are investigated in terms of modes, stress distribution, damage patterns and displacements. The first natural period is $T=0.452\text{sec}$ and the ones for 2nd, 3rd, 4th, and 5th modes are 0.321, 0.204, 0.169, and 0.136sec. For the first two modes, structural vibration develops on higher segment and larger amplitudes are obtained along the short direction, where the structure is more slender. It seems that the structure is more vulnerable in this segment and out-of-plane behavior becomes important. To visualize stress distribution on the structure, a comparison of maximum stresses such as vertical stresses ($S_{33}$) and shear stresses ($S_{13}$) are given in Table 1. Stress values are significantly higher for EY earthquake direction.

Figure 3 shows normal ($S_{33}$) and shear stresses ($S_{13}$) distribution on the structure under gravity loads (G), $G+Q\pm EX$ and $G+Q\pm EY$ loading combinations. High values of $S_{33}$ vertical compression and tensile stresses localized on the ground level arched piers, which confirm observed damaged (See Figure 2). In C2, stress values decreases by $23\%$–$61\%$.

Shear stresses localizes mostly around the infilled transition zone from 1-story to 2-story part. According to the Code, shear strength of the structure may be calculated approximately as $\tau_{\text{crit}}=0.45\text{MPa}$, revealing that the obtained shear stresses exceed the limits for both C1 and C2 cases. However, these values are mostly at locations where the exceeded areas are less.

Maximum horizontal displacement values at the top reach $53.5\text{mm}$ and $30.6\text{mm}$ along the critical “y” direction leading to horizontal displacement ratios of $0.24\%$ and $0.14\%$ for C1 and C2, respectively.

### 5. Conclusions

Results of the analysis show that there is no considerable problem on the structure under dead/gravity loads. However, under seismic loading, short (transverse) direction EY is more critical as expected. The normal stresses concentrate around the piers in the considered highest parts of the aqueduct that also agree with the damage pattern observed. Beside, stress concentration near springing line of the arches are encountered. Transition part of the structure subjects to high shear stresses. Compared to C1, up to $61\%$ decrease in stresses is observed in C2. The tension stresses reach significant levels on piers on which some damage or local stability issues may be expected under a future significant seismic event.

### References


Selection of the proper work hardening law for non-linear deformation conditions

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Abstract

Metal forming processes under industrial conditions often involve complex loading sequences. To improve the prediction of some undesirable phenomena, such as backstress, physically-based models should be considered. This paper discusses selected problems connected with proper choice of work hardening law for cyclic deformation conditions. Advanced nonlinear hardening model and physically-based work hardening model that take into account strain-path changes were selected and compared in the light of their accuracy in terms of prediction of flow behaviour during plastic deformation processes with cyclic strain reversals.

Keywords: strain path change, work hardening models, cyclic deformation, inverse optimization

1. Introduction

The development of modern structural materials is closely linked with possibilities and limitations of prediction of the microstructure evolution that controls both the physical and mechanical properties. Deformation processes under industrial conditions in most cases are nonlinear [1]. One of the most important phenomena in such a case is Bauschinger effect that involves reduction of the yield stress as an effect of changed loading direction. With more understanding of the role of strain path change on microstructure evolution and deformation gradients in metals there is a need to either update existing or propose new models that will be strain path sensitive. The main aim of present work is to discuss modelling possibilities of cyclic deformation behaviour of microalloyed steels based on the existing materials models with respect to their strain path sensitivity.

2. Experiments procedure

In order to provide data for modelling part of the work cyclic forward/reverse torsion (F/R) and cyclic tension/compression (T/C) tests were carried out using cylindrical test specimens with basic chemical composition presented in the Tab. 1.

Table 1: Chemical composition of studied materials

<table>
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<tr>
<th>Experiment</th>
<th>C %</th>
<th>Si %</th>
<th>Mn %</th>
<th>Nb %</th>
<th>N %</th>
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<tr>
<td>T/C test</td>
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<td>0.161</td>
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<tr>
<td>F/R T</td>
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Cyclic T/C test was conducted at 800°C with forward/reverse strain of 0.06 per pass and strain rate of 0.05s⁻¹. Cyclic torsion test was deformed out of the room temperature in a cyclic manner applying F/R torsion with the strain of 0.25 per pass and the strain rate of 0.1s⁻¹.

3. Modelling

The key factor for robust computer modelling process is selection of an appropriate rheological model. During the selection of the proper hardening model during modelling of the cyclic deformation an important aspect is taking into account the Bauschinger effect [2]. Additionally, in the case of microalloyed steels the effects of interactions between recrystallization and precipitation kinetics as well as solid solution and precipitation strengthening are especially important problems. In this case, there is a need to use a more complex model, which includes not only isotropic part of the hardening, but also kinematic part.

In the study, three different hardening models were selected and studied with respect to their possibilities and accuracy in the modelling of nonlinear deformation processes.

3.1 Combined hardening model

The first selected model that was chosen is Chaboche model. [3] Its isotropic hardening part can be described by the following equation:

\[ \sigma^0 = \sigma_0 + Q(1 - \exp^{-b\epsilon}) \]  

where: \( \sigma_0 \) – yield stress; \( Q \) – maximum change in yield surface; \( b \) – rate of change of yield surface during plastic deformation; \( \epsilon \) – strain. EVolution law of the kinematic part of the Chaboche hardening model is described as a sum of the backstresses, which includes parameters that control the position of the stress for each backstress:

\[ \alpha = \frac{C_k}{\sigma^0}(\sigma - \alpha)\epsilon - \gamma_k \alpha \epsilon \]  

where: \( C_k \) – initial kinematic hardening modulus; \( \gamma_k \) – the rate at which the kinematic hardening modulus decreases with increasing plastic deformation; \( \sigma^0 \) – the size of the yield surface; \( \sigma \) – stress tensor; \( \alpha \) – backstress; \( \epsilon \) – plastic strain.

Inverse analysis tool was used for the determination of the model parameters. [4] In this method, an objective function has to be defined and used as a criterion to select an acceptable solution.

* Financial support of the Polish National Science Centre through the research project no. DEC-2012/05/D/ST8/02367 is gratefully acknowledged.
3.2 Dislocation density-based Rauch Gracio Barlat model (RGB)

General formula for the stress of the considered Rauch, Gracio, Barlat (RGB) model is based on the Kocks model and can be shown as follows [5]:

\[
\sigma = \sigma_0 + M[X + (1 - \alpha)(\tau - \tau_0)]
\]

where: \(\sigma_0\) is the initial stress, \(M\) is the Taylor factor, \(\tau_0\) is the stress related to lattice friction and solute contents. Concerning the backstresses, an internal variable \(X\) is introduced to describe the rapid changes in stress under reverse deformation:

\[
X = C(X_f' - X_f)
\]

Shear stress can be calculated from:

\[
\tau = \tau_0 + \alpha G b \sqrt{\rho_f + \rho_r}
\]

where: \(G\) is the shear modulus, \(b\) - the Burgers vector and \(\alpha\) - a factor that weights the dislocation interaction. Dislocation density parameter consists of two components that are related with forward and reverse loading of dislocation substructures:

\[
\frac{dp_f}{dt} = \frac{1}{\beta a} - f \rho_f
\]

\[
\frac{dp_r}{dt} = \frac{1}{\beta d} \rho_r
\]

where: \(b\) - Burgers vector, the athermal storage that depends on the mean free path \(A\) for mobile dislocations in Eqn 6a, and the thermally activated recovery term whose efficiency is given by the factor \(f\) that depends on the temperature and strain rate - Eqn 6b.

3.3 Mechanical Threshold Stress model (MTS)

The Mechanical Threshold Stress (MTS) model was developed to describe the post yielding behaviour of metals [6]. The scaling relationship between the flow stress and theoretical threshold values of materials can be described:

\[
\frac{\sigma}{\mu} = \frac{S_0}{\mu_0} + S_1(\varepsilon, A) \frac{S_c(\varepsilon, T)}{\mu_0}
\]

The temperature dependence of \(\mu\) is included in the scaling functions \(S_0, S_1\). The interaction kinetics for short-range and long-range obstacles is described:

\[
S_0(\varepsilon, T) = \left[1 - \frac{KT}{\mu_0^2} \ln \left( \frac{\mu_0}{\mu} \right) \right]^{1/q_0} \quad (8)
\]

\[
S_c(\varepsilon, T) = \left[1 - \frac{KT}{\mu_0^2} \ln \left( \frac{\mu_0}{\mu_0} \right) \right]^{1/q_0} 
\]

4. Results

Comparison of the differences between experimental and modelling results using all selected models for cyclic T/C test and F/R torsion tests is presented in Fig. 2. It can be seen that the best prediction was achieved using dislocation density-based work hardening model whereas the least accurate results were obtained using Chaboche model for both tests. The analytical solution based on dislocation density-based model gave the best prediction of the stress level - especially for higher strains upon strain reversal, therefore this model can be strongly recommended for the simulation of deformation behaviour of processes characterized by cyclic strain path changes.

5. Conclusions

The presented results show that a careful choice of the proper hardening model and utilization of the inverse approach for the model parameters identification is crucial when simulations of the strain path sensitive processes are considered. Application of the strain path sensitive rheological model gives the possibility to use it as a universal modelling tool that takes into account deformation phenomena occurring during multiple strain reversals.

References


Simulation of attractive motion of silica microparticles in aerosol under acoustic excitation

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Abstract

Numerical simulation of the attractive motion of dry silica microparticles in incompressible media under acoustic excitation is considered. The silica microparticles are spheres of the diameter are varying between 5 and 10 μm. A moving monochromatic sound wave initiates attraction forces between particles, resulting from the oscillatory motion of the media. The effect of orthokinetic and acoustic wake hydrodynamic interaction mechanisms, as well as gravity and buoyancy forces, is taken into account. The conventional time-driven Discrete Element Method (DEM) is applied for simulation purposes, and the developed in-house DEM code was modified for simulation purposes. Binary attraction of particles differently located with respect to the direction of the sound wave is considered numerically and agglomeration time is investigated. The results are compared with the analytical solutions, and the experimental data described in the literature. The simulation of a 2D polydispersed particle system is also demonstrated.

Keywords: DEM, silica microparticles, acoustic excitation, hydrodynamic interactions, orthokinetic effect, wake effect

1. Introduction

Silica dust is produced during various construction or mining works. Silica dust is formed by tiny particles which can be inhaled into the lungs and cause serious health problems. Silica particles released into the air can form aerosol clouds, systems of particles suspended in the air. Acoustic excitation is used in different devices to manipulate aerosol particles. The purpose of the work is to improve understanding of interparticle and fluid-particle interaction mechanisms under acoustic excitation.

An attractive behaviour of microparticles is observed under action of the acoustic sound wave. This behaviour is governed by various hydrodynamic particle-fluid and particle-particle interactions. It is stated [5], that the orthokinetic collisions and the hydrodynamic acoustic wake are dominant first-order effects causing the agglomeration of polydispersed particle systems. The earlier investigations on orthokinetic collisions were performed by Mednikov [7] and the important contribution was made by Dong et al. [3]. A theory describing the acoustic wake effect based on Oseen flow fields was first proposed by Pshenai-Severin [8]. Dianov et al. [2] extended the theory to include the interactions between particles of different size and derived an analytical solution. A detailed classification of particle forces acting in fluid can be found in the review paper of Deen et al. [1]. As concerns the acoustics induced forces, the paper of Li et al. [5] can be mentioned.

In the paper, attractive motion of silica dust microparticles under acoustic excitation is investigated using the discrete element method (DEM). Dynamic behaviour of the system is described using the Lagrangian approach. The motion of an individual particle is defined according to Newton’s laws of classical mechanics. Agglomeration patterns and the influence of particle arrangement on the approaching time is analysed.

2. Problem description

The silica dust microparticles are assumed rigid spheres with the diameters varying between 5 and 10 μm. Generally, the particle assembly is characterised by statistical distribution of the particle diameters, while characterisation of mechanical properties is restricted to their mass density ρp only. The particles are dispersed in the air, which is assumed a viscous incompressible medium. The medium is characterized by the density ρm, the dynamic and kinematic viscosities of the medium μm = umPm and um respectively.

External acoustic excitation presents a monochromatic sound wave propagating horizontally in plane Ox. It is described by sinusoidal acoustic velocity u0 (t) characterized by sound velocity amplitude US and frequency f. A travelling sound wave initiates the attractive forces between the particles, resulting from oscillatory fluid motion.

The modelled processes are considered in plane Ox on the scale of the inter-particle distance, while the attractive short-range forces are neglected.

3. A concept of DEM modelling

The conventional DEM approach was modified for simulation purposes. The translational motion of an arbitrary particle p with mass mp in time t is characterized by the position and velocity vectors of the particle mass centre xp(t) and up(t), respectively, with respect to the Newton’s second law.

The governing equation of the particle motion is written in a vector form as follows:

\[ m_p \frac{du_p(t)}{dt} = F_{dp} + F_{sp} + F_{bp} \]  (1)

The problem was solved numerically by time integration of the equations of motion (1), where \( F_{dp} \) denotes a drag force describing particle-fluid-particle interactions and comprising the

*Research of Lithuanian part was funded by the Project of Scientific Groups (Lithuanian Council of Science), contract Nr. MIP-072/2013.
Stokes force and the so-called Oseen’s correction, $F_{gr}$ is a gravity force and $F_{bp}$ is a buoyancy force. The conventional time-driven DEM approach [4] and the in-house developed DEM code were modified for simulation purposes. The detailed description of the DEM acoustic agglomeration model can be found in [6].

4. The numerical results

A series of numerical tests were performed to illustrate attractive motion of microparticles. The silica dust particles characterized by the density $\rho_d = 1500$ kg/m$^3$ were considered. The air medium at temperature 20º C is characterized by the density $\rho_a = 1.293$ kg/m$^3$ and the dynamic viscosity $\mu_a = 17.9 \times 10^{-6}$ m$^2$s. The generated monochromatic sound wave is characterized by the amplitude velocity $U_0 = 0.378$ m/s (SPL 135 dB) and the frequency $f = 24$ kHz.

The motion of particles from the position fixed at time instant $t_0 = 0$ is tracked by numerical integration of Eqn (1). Since the oscillation period of the sound is $T_s = 0.042 \times 10^{-3}$ s, a very small time step $\delta_t = 1.0 \times 10^{-7}$ s was used in simulation.

The possible contact between particles was checked at each time step during the simulation. If the contact was found, two particles were assumed to be merged into one bigger particle. The attractive motion during binary interaction of two particles with diameters equal to $d = 5$ µm and $d = 10$ µm, which were located at the initial interparticle distance of 2000 µm. The illustrations of the simulation results in terms of the trajectories of particles in plane $Oxy$ are presented in Fig. 1. The graphs demonstrate dependence of the attraction-driven approaching trajectories on sound direction, where the transformation of a straight line into the line of complicated shape can be clearly seen.

Figure 1: Trajectories of attractive motion of particles of different diameters: $d = 10$ µm (Particle 1) and $d = 5$ µm (Particle 2) without (left column) and with (right column) vertical forces for different direction of the sound

In the limit, when inclination line became perpendicular to sound direction, the attractive behaviour is transformed into repulsion. The line positions illustrate different contribution of the inclination angle (0º, 30º and 60º) between the initial interparticle line and the horizontal direction of the sound. This effect is illustrated by the increase in the agglomeration time. In particular, under the increase of the inclination angle from 0º to 60º, the agglomeration time increases from 0.039 to 0.163 seconds for interaction of equal 5 µm particles, and from 0.0199 up to 0.0817 seconds for interaction of equal 10 µm particles. The attraction behaviour of the particles of different size is more complicated (see Fig. 1.)

5. Concluding remarks

The results obtained in the present work show that DEM is a proper numerical analysis technique for the case of binary attractive behaviour of particles under acoustic excitation and for evaluation of the contribution of various parameters. It was also observed that binary interactions form the base of collective behaviour of multi-particle systems.

References

Plane anisotropy parameters identification based on Barlat’s model

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Abstract

The paper presents the method to determine the anisotropy parameters of a flat sheet, according to Hayer’s method, using anisotropic Barlat’s model. A Hayer’s method is based on the principle of a constant material volume. A numerical tensile test of three flat specimens was carried out. The first sample was measured according to the sheet rolling direction, the second at 45 degrees and a third perpendicular to the rolling direction. The Barlat’s model parameters were changed individually. Next width variation of the measurement section as a result of 20% elongation was measured, according to the standard. The paper also presents the results of a drawing process simulation, using designated anisotropy model parameters. In addition the differences in the amount of the resulting ears are compared on the periphery and their location relative to the rolling direction of the sheet have been compared.

Keywords: anisotropy, Barlat’s model, deep drawing, tensile test, numerical analysis, simulation

1. Introduction

Modelling of the drawing process usually applies an isotropic material. This assumption is true only in the case of shaping a high quality sheet metal for deep drawing without anisotropy phenomena. However, in most cases, is drawing sheet or deep drawing sheet are used characterized by different properties relative to the direction of rolling. Therefore a need arises, to determine the properties of the planar anisotropy of the drawing sheet.

In the paper, the method to determine the plane anisotropy parameters, based on the adopted Barlat’s model was presented [1,2]. Parameters can be also determined by linearization of the material function [3].

2. Numerical results of tensile test

In order to determine the Hayer’s anisotropy parameters, the flat tensile specimens model has been built. The total length of the samples was 250 mm. The measuring section was exactly 50 mm. A specimen has been numerically stretched in accordance with the x axis, compatible to the sheet rolling direction Fig. 1. A second sample was inclined at 45 degrees to the rolling direction stretched Fig. 2a and the last was perpendicular to the rolling direction Fig. 2b. The samples were stretched up to strain equal 0,2 of a measured section, which gives the final length 60 mm. An anisotropy value is determined by the Hayer’s formula (1). Depending on the direction of sampling, can be determined anisotropy for 0°, 45° i 90° relative to the rolling direction.

\[
r = \frac{\ln b_0}{\ln b - \ln b - l}
\]

(1)

Figure 1: Sample according to the rolling direction

![Sample according to the rolling direction](image1)

Figure 2: Samples a) at 45 degrees, b) perpendicular to the rolling direction

![Samples at 45 degrees and perpendicular to the rolling direction](image2)

In the Ansys / Ls-Dyna program anisotropic properties were implemented due to the Barlat’s model [1]. This model is based on the six anisotropy factors: a, b, c, f, g, h. The first three are the most important, relating to directions of the sheet, the last three are the tangents. The first stage of the research is to determine the impact of changes of individual factors on the anisotropy according to Hayer. The parameters in the Barlat’s model were individually changed, whereas the value 1 specifies isotropic material. The results of numerical calculations are presented in Tab. 1. The last line in Tab. 1 shows the normal anisotropy value, which, in the case of drawing process, should be as large as possible (2).

\[
F = \frac{1}{4} (r_0 + 2r_{45} + r_{90})
\]

(2)
Table 1: Numerical results

<p>| | | | | | | | | | | | | |</p>
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</table>

3. Numerical results of deep drawing process

Next, the drawing process analysis was performed using the Barlat’s model. The result for an isotropic material shown in Fig. 3 where all the parameters in the model have a value equal to 1. Figures 4, 5a and 5b show the drawpiece with adopted anisotropy parameter $a = 0.5$ in Barlat’s model (bolded column in the Tab. 1). The lowest value of the anisotropy has been for the $r_{90}=1,398$ obtained (Fig. 5b). Therefore, in this direction it was the largest recess achieved. The highest value obtained for $r_{45}=2,287$ gave rise to the so-called, ear in this direction.

![Figure 3: Drawpiece obtained with isotropic model](image)

![Figure 4: Drawpiece obtained with anisotropic model](image)

![Figure 5: Sample according to the rolling direction](image)

4. Conclusion

The paper presents a numerical method to determine the standard planar anisotropy parameters according to Hayer, using a numerical anisotropic model. The Barlat’s model parameters are correlated so it is impossible to provide a simple functional dependence between Hayer’s values. In the following research the inverse problem is planned to be conducted. That means, calculation of six Barlat’s model parameters: $a$, $b$, $c$, $f$, $g$ and $h$ based on the specified anisotropic parameters $r$, $r_{45}$ and $r_{90}$. This will allow, based on the results of the experiment, to develop an adequate model to simulate the anisotropic phenomena occurring in the deep drawing process.

References


Stochastic multiscale analysis of bioscaffolds

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Abstract

The paper is focused on application of stochastic finite element method to a multiscale analysis of bone scaffold. The numerical homogenization procedure with periodic boundary conditions is used in the paper. The parameters of the scaffold are random variables and the distribution of homogenized averaged material properties are computed in numerical example.

Keywords: multiscale modelling, numerical homogenization, stochastic, FEM

1. Introduction

The multiscale analysis of bioscaffold is shown in the paper. The stochastic finite element method is used in micro model and homogenized material properties for macro model are obtained. The micro model is created as a periodic structure with representative volume element (RVE). The geometrical parameters of the bioscaffolds RVE are random numbers and the resulting stress strain relation has uncertain coefficients. The results of the presented approach can be used in macro scale modelling of behaviour of implanted bioscaffold.

2. Stochastic multiscale modelling

The multiscale modelling takes into account two or more scales during analysis of a structure [6]. The numerical homogenization method is often used in multiscale modelling for structures with local periodicity (for example fibrous composites) [9]. The considered body is modeled numerically in micro and macro scale and other considered scales. The models in each scale are analyzed with use of numerical method like Finite Element Method (FEM) [10]. The transfer of stresses and strains between scales ensures proper solution of the multiscale problem for general case. The boundary value problem for lower scales should be solved six times for linear elastic problems, the homogenized, averaged values of stresses and strains allows to obtain constitutive parameters for upper level. The parameters of the micro model in real life problems have an uncertain nature. The dimensions of composite building blocks like fibers, pores can change for each produced structure and within structure, also the material properties may vary. One of the most important model of uncertainty is based on the theory of probability and stochastic processes [1,7]. The Stochastic FEM [4] can be used in stochastic multiscale analysis [3,8]. The Stochastic FEM based on Monte Carlo algorithm and deterministic FEM was used in the paper [5].

Figure 1: The bioscaffold model, a) front view with showed one dimension of the beam, b) models for different dimensions of the beams

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3. The bioscaffolds

Injuries and diseases of the human skeletal system are an important issue from both medical, biomechanical and economical point of view. Modern medicine and engineering helps to significantly reduce the time of treatment and in the case of very advanced degeneration of the tissues allows for implantation of artificial structures supporting or realizing their natural functions. A relatively new and emerging field of research is the tissue engineering, aiming at design and manufacturing of implants of the damaged tissues and organs. Research in the field of bone scaffolds [2] which are implanted in the place of surgically removed bone tissue is gaining increasing importance. Regarding the variability of bone structure between individuals and different bone regions, the bone scaffold should be a patient specific implant. The microstructure of the scaffold should mimic orthotropic material parameters of the healthy bone, but also the macro-scale shape should be also tailored to the irregular shape of the bone defect. Therefore, the Rapid Prototyping (RP) methods like Fused Deposition Modeling (FDM) are a good choice for manufacturing method of the scaffold structures built on the basis of the optimized CAD design. Micro-RP additive methods like 3D bio-printing allows to manufacture optimized, personalized bone scaffold structures from biodegradable biopolymers. The bioscaffolds are periodic structures and due to processing methods some uncertainty in dimensions and material properties are common.

4. Numerical example

The RVE of bioscaffold presented in Fig. 1 is considered. The front view of the bioscaffold with dimension of one of the beams is shown in Fig. 1a. The stochastic problem was solved using Monte Carlo method and deterministic FEM. The PCL function for generating geometry and hexahedral finite elements for MSC.Patran was created, two mesh examples generated by the procedure are shown in Fig. 1b. The dimensions of the beams in the scaffolds were treated as independent random variables with normal distribution. The mean value of dimensions were 0.3 and standard deviation were equal to 0.05. The PLGA material was used in the model with Young modulus equal to 2000MPa and Poisson ratio 0.33. The periodic boundary conditions were applied for the bioscaffold. The resulting coefficient of the stress-strain relation for main axis is equal to 1069MPa with standard deviation 146MPa.

5. Conclusions

The presented method can be used to determine stress-strain relation in multiscale analysis of bioscaffolds. The uncertainty of the geometrical dimensions of the scaffold beams were considered allowing more close to real life modeling of structure.

References

Modelling phase changes in thin metal film subjected to ultrafast laser heating using the two-temperature model

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Abstract

Thin metal film subjected to an ultrafast laser heating is considered. The thermal processes which take place in the domain considered are described by two-temperature hyperbolic model supplemented by appropriate boundary and initial conditions. The model presented takes into account the phase changes of material (solid-liquid and liquid-vapour) occurring at the constant temperature. At the stage of numerical computations the finite difference method with a staggered grid is used. In the final part the results of computations are shown.

Keywords: microscale heat transfer, two-temperature model, phase changes, finite difference method

1. Introduction

During the heating of thin metal films by a laser of high power and ultra-short pulse the phase transitions can occur, i.e. the melting and the evaporation. In order to analyze the process, a two-temperature hyperbolic model can be used. This model consists of equations describing the temporal and spatial evolution of the lattice and electrons temperatures, lattice and electron heat fluxes and also the isothermal solid-liquid (or liquid-vapour) phase change. In this paper the system of equations is solved using the explicit scheme of finite difference method with a staggered grid. In the final part the results concerning the heating of gold film are presented.

2. Governing equations

The one-dimensional two-temperature model describing the temporal and spatial evolution of the lattice and electrons temperatures (\(T_l\) and \(T_e\)) in the irradiated thin metal film together with the isothermal solid-liquid and liquid-vapour phase changes is of the form [1]

\[
C_e \frac{\partial T_e}{\partial t} = -\lambda_e \frac{\partial T_e}{\partial x} + Nk_B T_e \frac{\partial q_e}{\partial x} - G(T_e - T_l) + Q(x, t)
\]

(1)

where \(C_e\), \(C_l\) are the volumetric specific heats, \(G\) is the electron-phonon coupling factor, \(Q(x, t)\) is the source function associated with the irradiation, \(x\) is the spatial coordinate, \(t\) is time, \(q_e, q_l\) are the heat fluxes, \(\lambda_e, \lambda_l\) are the thermal conductivities, \(\tau_e, \tau_l\) is the relaxation time of free electrons in metals, \(\tau_f\) is the relaxation time in phonon collisions, \(Q_{ls}\) is the volumetric latent heat of fusion, \(T_{ml}\) is the melting temperature, \(t_{ev}\) and \(t_{me}\) are the times corresponding to the start and the end of the solid-liquid phase transformation, \(Q_{ev}\) is the volumetric latent heat of evaporation, \(T_b\) is the boiling temperature, \(R\) is the reflectivity of the irradiated surface and \(\delta = 4 \ln 2\) [1].

The mathematical model is supplemented by boundary conditions

\[
q_e(x, 0) = q_e(L, t) = q_e(0, t) = q_e(L, t) = 0
\]

(8)

and the initial ones

\[
t = 0: \quad T_e(x, 0) = T_f(x, 0) = T_p
\]

(9)

where \(T_p\) is the known constant temperature.

For high laser intensity the following formula describing the temperature-dependent volumetric specific of lattice is proposed in the form [2]:

\[
C_e = C_e(T_e) = \begin{cases} \frac{AT_e}{T_e - T_s}, & T_e < T_s \\ \frac{AT_e + Nk_s - AT_{e0}}{2T_e + Nk_s / \pi^2}, & T_s < T_e < 3T_s \\ \frac{3Nk_s}{2}, & T_e > 3T_s \end{cases}
\]

(10)

where \(T_s = T_F / \pi^2 \times \frac{2}{T_F - 3T_s}\) is the Fermi temperature, \(N\) is the density of electrons, \(k_s\) is the Boltzmann constant and \(A = \pi^2 Nk_b / (2T_F)\).
Electrons thermal conductivity and the coupling factor are dependent on the temperature of electrons and lattice [2], namely

$$\chi = \chi e^{-\Delta/2} + \chi_T e^{\Delta/2}$$

(11)

and

$$G = G(T) = G_0 e^{\Delta/2}$$

(12)

where $\chi, G_0, \Delta, B_i, B_f$ are the constants and $G_0$ is the coupling factor.

3. Method of solution

In order to solve the formulated problem the explicit scheme of the finite difference method presented in details in [3] is used. Here, only the modeling of isothermal solid–liquid phase change (c.f. equation (4)). In the differential grid the temperature nodes $i=0, 2, 4, ..., N$ and the heat flux nodes $j=1, 3, ..., N-1$ are distinguished. Let us consider $T_i = T(\Delta t)$ and $q_j = q_j/(\Delta t)$, where $h$ is the mesh size and $\Delta t$ is the time step, $f = 0, 1, 2, ..., F$. If at the node $x_i$ for time $t^f$ the lattice temperature $T_l$ is higher or equal to $T_m$ ($T_l \geq T_m$) then in the sub-domain $[x_i-1, x_i+1]$ the melting process starts. For this node the value

$$M_i = \left[ \frac{q_{i+1}-q_{i-1}}{2h} + G_i \left( T_{li} - T_{li}^f \right) \right] \Delta t$$

(13)

is calculated and it is assumed that $T_l^i = T_m$. For next transition $t^{i+1}$ the value $M_i^{i+1}$ is determined. If $M_i^{i+1} < Q_m$ then $T_l^{i+1} = T_m$ and the temperature field for transition $t^{i+1} \rightarrow t^{i+2}$ is found. The calculations are continued until

$$\sum_{k=f}^{K} M_k^i \geq Q_m$$

(14)

In a similar way the phase change liquid-vapor can be modeled.

4. Results of computations

The gold film of thickness $L = 100$ nm is considered. The initial temperature is equal to $T_p = 300$ K. The constants in equations (11), (12) are the following: $\chi = 353$ W/(mK), $\eta = 0.16$, $A_e = 1.2 \times 10^{15}$ 1/(Ks), $B_i = 1.23 \times 10^{11}$ 1/(Ks) and $G_0 = 2.2 \times 10^{-6}$ W/(mK) [2]. The Fermi temperature is equal to $T_F = 64200$ K and the density of electrons is equal to $N = 5.9 \times 10^{28}$ 1/m$^3$. The other parameters are as follows: thermal conductivity of lattice $\chi_T = 315$ W/(mK), volumetric specific heat of lattice $C_l = 2.5$ MJ/(m$^3$ K), electrons relaxation time $\tau_e = 0.04$ ps, phonons relaxation time $\tau_p = 0.8$ ps [1], reflectivity $R = 0.93$, optical penetration depth $\delta = 15.3$ nm. Melting temperature is equal to $T_m = 1336$ K and the latent heat of fusion $Q_m = 1229989$ MW/m$^2$, while the boiling temperature is equal to $T_b = 3127$ K and the latent heat of vaporization $Q_v = 32771.4$ MW/m$^2$. The problem is solved using a finite difference method [3] under the assumption that $\Delta t = 0.001$ ps and $h = 1$ nm.

In Figures 1 and 2 the electrons and lattice temperatures at the irradiated surface $x = 0$ for the laser intensity $I_0 = 4182$ W/m$^2$ (Fig. 1) and $I_0 = 40000$ W/m$^2$ (Fig. 2) are shown. In both cases the characteristic time of laser pulse is equal to $t_p = 100$ ps.

References


Description of packing and size effects in particulate composites by micromechanical averaging schemes and computational homogenization

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Abstract

Different approaches to model packing and size effects are studied to model overall properties of particulate composites of different morphological features of phase distribution. The micromechanical schemes originating in the composite sphere model and its extension by morphologically-based pattern approach are taken as a basis. Analytical predictions are compared with results of computational homogenization performed on the generated representative volume elements of prescribed statistical characteristics.

Keywords: micromechanics, morphologically representative pattern, computational homogenization, size and scale effect

1. Introduction

The micromechanical approaches employed in the present study originate from the concept of composite sphere model formulated by Hashin and its further modification to the generalized self-consistent (GSC) model by Christensen, cf. [1]. In the GSC model, called the three-phase model, a single composite sphere is considered that is embedded into the equivalent homogenized medium of effective properties (see Fig. 1a). The composite sphere is composed of a particle with an associated spherical matrix region. The ratio describing the size of inner sphere with respect to the outer sphere is constant and defined by the volume fraction of particle phase in the composite. The idea was expanded in [1] to tackle multiple coatings by formulating the n-phase model and in particular the four-phase model (see Fig. 1b).

Figure 1: Schematic of (a) generalized self-consistent (GSC) model (b) four phase model (4GSC)

The concept of a coated inclusion embedded in the homogenized medium was later employed in [2], within the morphologically representative pattern-based approach (MRP-based approach), to describe the packing effects and size effects in an elastic composite composed of a continuous matrix and dispersed spherical particles. In the simplest examples of 2-pattern approach:

- to account for a packing effect the first pattern is a GSC pattern where the coating thickness is specified by the mean distance between nearest-neighbor particles and the second pattern is a self-consistent type problem with an inclusion made of remaining pure matrix material,
- to account for a size effect the first pattern is modified towards the four-phase model i.e. an interphase with a thickness independent of the particle radius and with different properties than basic two phases is introduced.

The validity of these analytical schemes as concerns the predictions of the influence of packing and size of particles on effective properties are compared with results of computational homogenization performed on the generated representative volume elements of prescribed statistical characteristics.

2. Principles of MRP-based approach

In the MRP-based approach the representative volume $V$ with some morphological features is subdivided into the representative patterns $\chi$ with specified contributions $c_\chi$ to the overall volume $V$. Within each pattern one specifies the volume fraction $f_\chi$ of phase $\chi$ such that

$$f_\chi = \frac{V_\chi}{V}, \quad \sum_\chi c_\chi f_\chi = f_i,$$

where $f_i$ is the volume fraction of particles (inclusions) in the representative volume $V$.

When considering the linear problem (elasticity, viscosity) related to the subsequent patterns, the standard self-consistent procedure of micromechanics is followed. Each pattern is embedded in the infinite medium of homogenized properties to be found. For each pattern the concentration tensors $A_\chi$ are established that relate the auxiliary far-field strain $E_0$ with the average strain $\varepsilon_\chi$ in inclusion phase in the pattern $\chi$, that is

$$\varepsilon_\chi = A_\chi \cdot E_0.$$

In the analyzed cases the concentration tensors $A_\chi$ are isotropic fourth order tensors. The far field strain is not necessarily the overall average strain $E$ in the representative volume. The same

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far field strain is assumed for all patterns and it is derived from the condition
\[ \langle \varepsilon_i \rangle_V = E \varepsilon, \] (3)
where \( \varepsilon_i \) is the average strain in phase \( i \) in the representative volume \( V \) and \( \langle . \rangle_V \) is a volume averaging operation defined as \( 1/V \int f_i (. ) dV \), while \( \varepsilon_i \) is calculated as follows
\[ \varepsilon_i = \sum \phi_i \varepsilon_i^0, \quad \phi_i = \frac{V_i}{V}, \quad \sum \phi_i = 1. \] (4)

In the study there are three basic morphologies considered: the simple morphology of Eshelby problem, the morphology of the 3-phase model (Fig. 1a) and the morphology of the 4-phase model (Fig. 1b). Corresponding relations for \( \Delta \varepsilon_i \) for the considered morphologies are to be found in [1].

Let us demonstrate the predicted influence of packing on effective properties for elastic metal-ceramic composites. Results for two possible opposite cases are presented: ceramic inclusions dispersed in the metal matrix and reversely. Let us denote by \( \lambda \) the mean distance between nearest neighbor particles scaled by the particle diameter. Then in a 2-pattern approach the volume fraction of the GSC-type pattern can be calculated as
\[ c = f_i (1 + \lambda)^3, \] (5)
where \( f_i \) is the volume fraction of the inclusion phase. For specified volume fraction \( f_i \), the parameter \( \lambda \) may vary from 0 (nearest particles are in contact) to the value specified by the matrix phase fraction \( \lambda_{max} \). In the first limit case the MRP-based approach reduces to the classical self-consistent scheme and in the second case, since \( 1 - c = 0 \), only one pattern is considered, so the result of the generalized self-consistent scheme is recovered. In Fig. 2 the estimates of the overall Young modulus \( \bar{E} \) obtained for these two limit cases are presented as a function of ceramic (hard) phase content \( f_i \), being either inclusion (hard inclusion case) or matrix material (soft inclusion case). Obviously, following the known result for SC scheme, when \( \lambda = 0 \) specification of the inclusion phase does not alter the result. Note that in both dilute limits, when the content of the inclusion phase tends to zero, both curves (SC and GSC) are tangent to each other. For completeness the other classical bounds and estimates specified for the two-phase isotropic composite are also included in the figure.

Moreover, in Fig. 2 the MRP-based estimates of \( \bar{E} \) under assumption of constant value of \( \lambda \) are shown. Note that under such assumption there exists maximal volume fraction of an inclusion phase in a composite to be achieved (see Eqn (5)). For this value the corresponding curve sticks to the respective GSC limit curve. The smaller \( \lambda \) the closer MRP-based estimate is to the SC scheme prediction. Additionally, in Fig. 2 the MRP-based estimates of \( \bar{E} \) are shown for the regular cubic, bcc and fcc distributions of particles, or more precisely, for \( \lambda \) varying in the same way with inclusion volume fraction as for these ordered ways of particle distribution. It results with a constant \( c \) value, independent of \( f_i \), namely \( c^{soft} = \pi/6 \sim 0.52, c^{hard} = \sqrt{3}\pi/8 \sim 0.68, \) \( c^{fcc} = \sqrt{2}\pi/6 \sim 0.74 \). The maximum volume fractions of inclusion phase possible to obtain for such packing patterns are specified by conditions \( \lambda \rightarrow 0 \) and are equal to \( c \). Thus, for this limit values the MRP-based estimates stick to the SC curve. On the basis of the above analysis it can be concluded that other constant values of \( c \) assumed in MRP-based pattern would correspond to some isotropic distribution of particles within the representative volume with a radius specified by \( f_i \).

3. Computational homogenization

In order to verify micromechanical estimates presented in the previous section computational homogenization will be performed on representative volume elements with randomly distributed non-overlapping spherical inclusions [3]. Successive spheres are placed in the volume by random sequential addition, until a prescribed volume fraction of inclusions \( c \) is reached. To provide more control over the morphology, the spheres are positioned according to either a uniform or a cluster probability distribution over the volume. The width of clusters is adjustable and their spatial arrangement is itself random. The use of clusters allows to change the mean nearest-neighbour distance between inclusions while keeping \( c \) constant, as shown in Fig. 3.

References


Multiscale model of the proximal femur with implanted bone scaffold

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Abstract

The paper is focused on multiscale modelling of the proximal femur with implanted bone scaffold. The finite element method (FEM) macro-scale model of human proximal femur is built on the basis of computed tomography (CT) data. Micro-scale FEM model (RVE) of trabecular (cancellous) bone microstructure is built on the basis of micro-computed tomography (μCT) data. Effective anisotropic material parameters of trabecular bone are calculated using micro model with numerical homogenization algorithm and then are applied in the macro model. Homogenized orthotropic material parameters of the personalized bone scaffold structure are applied in the implantation area of the macro model and numerical analysis of proximal femur is conducted. Stain and stress state in the bone and implant microstructures are calculated using localization.

Keywords: multiscale modelling, numerical homogenization, bone scaffold, trabecular bone

1. Introduction

The task of trabecular bone multiscale modelling [8] is performed in the paper. In case of bone diseases like osteonecrosis of the femoral head, large areas of bone tissue must be removed through chisurgical intervention. Proximal femur is composed mainly from the trabecular bone. Trabecular bone can regenerate and heal, but in the case of large load-bearing areas of removed tissue, bone scaffolds [4] are implanted. Bone scaffold is a porous biodegradable structure implanted in a place of bone defect. Such an implant should provide mechanical support to bone and also should help to induce new bone tissue formation. The multiscale methods [7] are used to analyze bone-implant model and to calculate strain and stress fields at macro and micro scales of the structure.

2. Micro models

The micro model (RVE) of trabecular bone structure (Fig. 1) was built on the basis of μCT data [2] segmentation.

Figure 1: RVE model of trabecular bone structure

Cancellous bone is a locally periodic structure so periodic boundary conditions are applied in the RVE model. Effective anisotropic material parameters of bone tissue were calculated using numerical homogenization method and FEM [9]. The structure of personalized bone scaffold (Fig. 2), mimicking bone material parameters was found using evolutionary optimization methods [6].

Figure 2: RVE model of trabecular bone scaffold

Topology of the scaffold is also optimized in terms of stimulating new tissue formation (eg. appropriate porosity and pore size), because bone scaffold is a biodegradable temporary implant.

3. Macro model of proximal femur

The macro model of proximal femur was built on the basis of CT data [5]. Anisotropic material parameters of the micro models were applied in the macro model with respect to anatomical axis in femoral head (Fig. 3). The force resulting from the weight of upper part of human body during walking [1] was applied to femoral head in approximated contact area between pelvic bone acetabulum and the head of femur. Nodes at the lower part of model were fixed and numerical analysis was performed.
4. Results of multiscale analysis

The result of conducted numerical analysis are strain (Fig. 4a) and stress fields (Fig. 4b) in the macro model of the proximal femur.

Stress and strain concentrations can be observed in the femur neck area. Moreover, stress distribution coincides with the direction of anatomical axis in femoral head.

The strain states for the points inside the tissue and implant areas were applied to the micro models (localization). Stress and strain values in the microstructures were calculated with respect to force applied in macro model and macro strains in the proximal femur. The minimum principal strains distribution in the bone microstructure is shown in the Fig. 5.

5. Conclusions

Multiscale modelling methods allowed to analyze the structure of cancellous bone and bone scaffold at micro and macro scales. The calculated micro-strains are within the range of strains values reported for the human bone tissue [3].

References


A new approach to capacitance testing techniques - increasing dimension of the tests

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Abstract

Results of the original research aimed at increasing accuracy and applicability of capacitance testing techniques are presented. The essence of the innovation consists of scanning the test item by using an electric field of different topography called in this paper as a multidimensional scanning. This approach enables to design a comparatively simple and reliable capacitance testing hardware and software for independent measurement of the dielectric permittivity and distance between the capacitance sensor and the test object. The measurement procedure and validation experiment are carried out by a computer simulation.

Keywords: capacitance technique, computer simulation, non-destructive testing, sensors

1. Introduction

Testing of dielectric properties is important source of information for structure studies of different non-metallic objects and capacitance techniques are the most widespread instrument to carry out such tests [1]. Special importance these techniques have acquired for control of advanced composites and aerospace technologies.

The present research is devoted right to enhancement of the capacitance techniques and in this way to promote this instrumentation to qualitative higher level regarding applicability and accuracy of nondestructive tests. This goal has been achieved by providing tests at variable topographies of the scanning electric field and attaining a multi-parameter response in this way. The new qualities of this approach are confirmed by validation of the most demanded test assignments in terms of measurement accuracy and capabilities to minimize influence of the main disturbing factors. In the present research, these procedures have been carried out by computer simulation methods.

2. Research methods and procedures

Design of the multi-element sensors was performed using methods of electric field theory and mathematical physics, particularly potential theory. For computer simulation purposes, the actual design of the electrodes’ array is substituted by a schematic model, which comprises only one section in a series of definite number electrodes assuming that distribution of the field intensity in other sections of the array is equal. The proposal for multi parameter-tests is illustrated by a case study of test assignments widely demanded in practice of composite technologies. This case study relates to non-contact testing of dielectric permittivity and is implemented by the hardware (capacitance sensors) and software developed just for this study. Actually, this demonstration represents a two parameter test as the measurement process completes with quantitative estimates for both input parameters – the clearance \( h \) and dielectric permittivity \( \varepsilon \). However, other combinations of the input, for example clearance and thickness; thickness and dielectric permittivity would be applicable by the same principles.

The model for computer simulation of the case study is developed by methods of potential theory [2]. Following this theory, an equation for the potential of a single layer with the logarithmic kernel of the Euclidean plane with respect to the electric surface charge \( \sigma(x) \) should be compiled. Thus, adaptation of this theory for a series of flat electrodes in the two layer dielectric (Fig. 1) results to the system of integral equations in respect to potential \( V_i \) of these electrodes of a type [3]:

\[
\sum_{n=1}^{N} \left[ -\frac{1}{2} \int_{0}^{L} \sigma(b+z) \left( K_1(x,z) + \frac{\lambda}{2} L_1(x,z) + \frac{\lambda}{2} M_1(x,z) \right) \, dz \right] \varepsilon_1 = 2 \pi \varepsilon_0 K_1 \quad (1)
\]

where \( n \) – number of sections of the electrodes’ array, \( K_1(x,z), L_1(x,z) \) and \( M_1(x,z) \) dependences for the kernels observing presence of the two layer structure between the sensor and the test item, \( \lambda_1 \) and \( \lambda_2 \) take into account the two layer dielectric structure. This model enables to calculate capacitance of the electrode array as a function of the input parameters, particularly characteristics of properties and geometry of the test object.

3. Design of the multi-element capacitance sensor

The multi-element capacitance sensor comprises an array of flat or curved electrodes adapted to the surface of the test object, which is introduced in the electric field of the array (Fig. 1). By changing the field topography, it is possible to concentrate the field energy at different locations of the sensing area and thus to obtain feedback from the test item of special interest, for example, surface conditions, bulk properties, layered distribution, etc. An outline of the electric field lines corresponding to two combinations of potential over the electrodes are presented in Fig. 1. These potential distributions are denoted as the sensors S1 and S2. The output of the electrodes’ array in terms of capacitance may be considered as a two dimensional quantities, nevertheless the components are successive in time. The test object is represented by its real \( (\varepsilon_1) \) and imaginary \( (\varepsilon_2^*) \) component of dielectric permittivity.

4. Data processing of the two-dimensional capacitance response

Calibration for needs for this case study is performed by computer simulation means exploiting the model Eqn (1). This procedure is carried out by setting values for one variable, for example dielectric permittivity with certain interval through out
of the measurement range, while maintaining the second variable (the clearance) unchanged. The capacitance calculated by the model is assumed as the output of the sensors. The measurement process is simulated by the same mathematical model by computing the output for specific input accepted as the reference values.

The output of the multi-element capacitive sensor is a two argument function $C_1 = f_1(\varepsilon, h)$ and $C_2 = f_2(\varepsilon, h)$, where the arguments are dielectric permittivity ($\varepsilon$) and clearance ($h$), which represent input quantities of the measurement system. The output is submitted to data processing through out the whole processing chain. Actually, the output capacitances are mutually interdependent and therefore are not applicable for direct recovering of the input and needs some processing. This processing should to take into account that each capacitance is responsible for information about the specific locations of the test object (for example, the surface zone).

The essence of the data processing consists in transformation of the output by comparing it with the calibrated data file [3]. For example, subtraction of the output from the calibrated data, results to the derived functions possessing some important features.

![Schematic design of the multi-element capacitance sensor with variable potential distribution over the electrodes; field lines for S1 and S2 are symbolic.](image1)

![A fragment of zero transition functions for two sensors S1 and S2 at the region of zero crossing coordinates.](image2)

5. Validation of the case study

A summary of validation results of the algorithm for processing zero transition functions for the case study is presented in Table 1. Relative measurement errors of the recovered input are utilized as the validation criteria. The validation has been carried out for three test assignment representing the medium level of the input for data processing (M1), low level (M2) and high level (M3). As it follows from Table 1, three potential combinations over the electrodes have been applied.

<table>
<thead>
<tr>
<th>M</th>
<th>Sensors</th>
<th>Reference value</th>
<th>Recovered value</th>
<th>Relative error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\varepsilon$</td>
<td>$h$</td>
<td>$\delta \varepsilon$</td>
</tr>
<tr>
<td>M1</td>
<td>S1&amp;S2</td>
<td>3.362</td>
<td>0.334</td>
<td>3.360</td>
</tr>
<tr>
<td></td>
<td>S1&amp;S3</td>
<td>3.362</td>
<td>0.334</td>
<td>3.378</td>
</tr>
<tr>
<td></td>
<td>S2&amp;S3</td>
<td>3.362</td>
<td>0.334</td>
<td>3.365</td>
</tr>
<tr>
<td>M2</td>
<td>S1&amp;S2</td>
<td>3.537</td>
<td>0.531</td>
<td>3.540</td>
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<td>3.541</td>
</tr>
<tr>
<td></td>
<td>S2&amp;S3</td>
<td>3.537</td>
<td>0.531</td>
<td>3.540</td>
</tr>
<tr>
<td>M3</td>
<td>S1&amp;S2</td>
<td>3.917</td>
<td>0.127</td>
<td>3.925</td>
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<tr>
<td></td>
<td>S1&amp;S3</td>
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<td>0.127</td>
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<td>3.917</td>
<td>0.127</td>
<td>3.924</td>
</tr>
</tbody>
</table>

6. Conclusions

1. The validation results confirm that the measurement technology of variable electric field should be considered as a step forward for enhancement of applicability of the capacitance techniques.
2. The elaborated algorithms of data processing could be applicable for any other multi-parameter measurement data irrespective of instruments utilized for attainment of the information (modeling, physical experiment, tabulated data, etc.).

References

Application of the stress based finite element method to problems of isochoric plasticity

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Abstract

A stress based formulation of the finite element method is applied to equilibrium problems for an isotropic elastic–plastic material being nearly or fully incompressible. It is shown that such a formulation of the method is free of volumetric locking and provides more accurate results than the $u$–$p$ mixed approach applied usually as a remedy for the locking phenomenon. Three kinds of $C^1$ elements suited for the plane strain case are employed in the present formulation and the results obtained by use of them are compared with outcomes gained by the $u$–$p$ method.

Keywords: stress-based finite element method, volumetric locking, isochoric plasticity, $C^1$ elements, stress function

1. Introduction

It is well-known that the isochoric deformation is not embedded in the approximation of the deformation field by means of linear or quadratic interpolation function used commonly in the displacement-based finite element method (FEM). In an analysis of an isotropic material with Poisson’s ratio close or equal to 0.5, or having the property of plastic incompressibility, a phenomenon of volumetric locking is observed. There are some ways of alleviating the influence of locking like averaging (smoothing) the volumetric strain or the pressure or the use of the mixed formulation, e.g. the $u$–$p$ formulation where, beside the displacement field, the pressure field is a primary unknown, e.g. [1].

The stress based formulation of the finite element method allows to avoid the difficulties related to locking phenomenon. This formulation is based on the principle of complementary work. It is rather rarely used in the research and engineering (except in the case of torsion problems), however. The main reason seems to be a harder way of construction statically admissible stress fields comparing to the interpolation of kinematically admissible displacement fields in the displacement based FEM.

In the paper, the stress fields in the plain strain problem are constructed by means of the Airy stress function which is interpolated by the use of elements with shape functions of $C^1$ class. The approach is applied to an elastic–plastic material with the Huber–von Mises yield condition.

2. Problem statement

A quasi-static small deformation problem for an isotropic elastic–perfectly plastic body is analysed. Isochoric plastic strains are considered. No restriction for the value of Poisson’s ratio is assumed; it may be close to 0.5 or equal to this number.

Let it be defined the following set of plastically admissible stress ratios is assumed; it may be close to 0.5 or equal to this number.

\begin{equation}
\begin{aligned}
B = \{ \boldsymbol{\tau} : f(\boldsymbol{\tau}) = g(\boldsymbol{\tau}) - \sigma_Y \leq 0 \} \\
\end{aligned}
\end{equation}

where $\sigma_Y$ denotes the plastic limit while $g = (3/2) s_{ij} s_{ij})^{1/2}$ is the stress invariant where $s_{ij} = \tau_{ij} + p \delta_{ij}$, $p = -1/3 \tau_{ii}$. The constitutive relations can be written in the following form:

\begin{equation}
\begin{aligned}
\dot{\varepsilon}_{ij} &= \dot{\varepsilon}_{ij}^e + \dot{\varepsilon}_{ij}^p, \\
\dot{\varepsilon}_{ij}^e &= \frac{1}{E} ((1 + \nu) \sigma_{ij} - \nu \delta_{kk} \delta_{ij}) , \\
\dot{\varepsilon}_{ij}^p &= (\tau_{ij} - \sigma_{ij}) \leq 0 \ \forall \boldsymbol{\tau} \in B
\end{aligned}
\end{equation}

where $\varepsilon_{ij}^e$ and $\varepsilon_{ij}^p$ are the elastic and plastic parts of the strain tensor, respectively, $E$ is Young’s modulus and $\nu$ Poisson’s ratio.

Let us define the set of statically admissible fields of stresses

\begin{equation}
\begin{aligned}
Y = \{ \boldsymbol{\tau} \in [L^2(\Omega)]^9 : \tau_{ij} = \tau_{ji}, \tau_{ji,j} + b_i = 0 \ on \ \Omega, \\
\tau_{ji}, n_j = t_i \ on \ \Gamma_a \}
\end{aligned}
\end{equation}

where $\Omega$ denotes the region occupied by the analysed body, $n_i$ the vector outwardly normal to the boundary of region $\Omega$, $\Gamma_a$ the part of the body boundary where the Cauchy stress vector, $t_i$, is given, $b_i$ is the vector of volumetric forces and $L^2$ the space of square-integrable functions.

The following set of plastically admissible stress fields be defined:

\begin{equation}
\begin{aligned}
P = \{ \boldsymbol{\tau} \in [L^2(\Omega)]^9 : \tau_{ij} = \tau_{ji}, \boldsymbol{\tau}(x) \in B(x) \text{ a.e. on } \Omega \}
\end{aligned}
\end{equation}

The quasi-static evolution problem for an elastic–plastic body can be stated as follows: Find the stress field $\sigma_{ij} \in K$ such that the variational inequality holds

\begin{equation}
\int_{\Omega} C_{ijkl} \sigma_{kl} (\tau_{ij} - \sigma_{ij}) \, dx \geq \int_{\Gamma_a} U_i (\tau_{ij} - \sigma_{ij}) n_j \, ds \\
\forall \tau \in K
\end{equation}

where $K = Y \cap P$, $C_{ijkl}$ denotes the tensor of elastic compliances and $\Gamma_a$ the part of the body boundary where displacements $U_i$ are given.

3. Finite element solution

In the case of plane strain problem considered in the paper, in order to satisfy the equilibrium equations inside the region $\Omega$, the Airy stress function, $F$, is applied, $\sigma_{ij} = \epsilon_{ij} e_{kk} F_{,k} + \varphi \delta_{ij}$ where the Greek indices take values 1 and 2 and $\varphi$ is the potential of the volumetric forces, $b_{ik} = -\varphi, \epsilon_{ij}$. As the second derivatives of the stress function appear in the variational formulation (5), the function is interpolated by means of elements

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of class $C^1$. The Bogner–Fox–Schmit rectangular element with 16 degrees of freedom and the Hsieh–Clough–Tocher triangular macro-elements with 12 and 9 degrees of freedom are utilised in the present paper [3]. In order to fulfill the equilibrium conditions on boundary $Γ_e$, the Lagrange multiplier method is employed. As the Lagrange multipliers have the meaning of displacements in the present formulation of the finite element method, the displacements along boundary $Γ_e$ can be easily evaluated.

The problem is solved in an incremental way. The non-linear system of algebraic equations is solved by use of the modified Newton–Raphson algorithm for each load increment.

4. Numerical example

The plane strain state induced by a balanced loading is analysed for a body, the cross-section of which is shown in Figure 1. The following material properties are assumed for the body: Young’s modulus $E = 2 \times 10^{11}$ Pa, Poisson’s ratio $\nu = 0.4999$, plastic limit $σ_Y = 2.5 \times 10^8$ Pa.

Figure 1: Cross-section of the body, dimensions in centimetres

Computations were made setting the load increment $Δp = 1 \times 10^6$ Pa at the beginning of the plastic strain process with reducing it to the value $Δp = 0.2 \times 10^6$ Pa when the load has been approaching its limit value. Using a displacement interpretation of the Lagrange multipliers, an averaged deflection was computed for the element segment located near the symmetry axis indicated by $L$ in Figure 1. This allows to find the load–deflection paths shown in Figure 2 plotted for meshes with element size $h = 0.0125$ m. The results obtained by the equilibrium elements are compared to the results got by use of the 8-node quadrilateral element implemented in program ANSYS [2] and constructed in the mixed $u$–$p$ format. 0.4999 is the largest value of Poisson’s ratio acceptable in the ANSYS software.

Figure 2: Loading–deflection path

All the equilibrium elements: the Bogner–Fox–Schmit (BFS) element, the Hsieh–Clough–Tocher element with 12 degrees of freedom (HCT12) and the Hsieh–Clough–Tocher element with 9 degrees of freedom (HCT9) give similar results. An important feature from the engineering viewpoint of the stress based FEM is that it gives the lower bound for the limit load. As can be seen from Figure 2, the mixed method provides a higher value of the limit load. The difference is about 2%.

Colour maps representing the horizontal normal stress component obtained by the $u$–$p$ and the stress based approach are shown in Figure 3. Significantly smaller discontinuities of the field on the element edges are observed in the case of the stresses obtained by use of equilibrium elements comparing to the $u$–$p$ element.

Figure 3: Horizontal normal stress component [Pa] related to $p = 1 \times 10^6$ Pa obtained by mixed method (top), BFS equilibrium element (middle) and HCT12 equilibrium element (bottom)

5. Concluding remarks

The equilibrium model of the finite element method was applied to the isochoric strain problem of the elastic–plastic material. The lower bound of the limit load was determined. The proposed approach allows to find smoother and more accurate stress field than the mixed displacement–pressure formulation of FEM commonly applied in computational mechanics.

References


Effective conductivity of random 2D composites

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Abstract

Two-phase composites with non-overlapping inclusions randomly embedded in matrix are theoretically investigated. A new approach is applied to estimate the effective properties of random 2D composites. Firstly, deterministic boundary value problems are solved for all locations of inclusions, i.e., for all the events of the considered probabilistic space \( \mathcal{C} \) by the generalized method of Schwarz. Secondly, the effective properties are calculated in analytical form and averaged over \( \mathcal{C} \). The proposed method yields high order in concentration analytic formulae.

Keywords: random composite, RVE, effective properties, generalized method of Schwarz

The effective properties of random heterogeneous materials and methods of their computation are of considerable interest [1, 13, 14]. Randomness in such problems reveals through random tensor-functions locally describing the physical properties of medium. Despite the considerable progress made in the theory of disordered media, the main tool for studying such systems are numerical simulations. It is frequently asserted that it is impossible to get general analytical formulae for the effective properties except dilute composites when interactions among inclusions are neglected and except regular composites of a simple geometric structure. This opinion sustained by unlimited belief in numerics has to be questioned by the recent pure mathematical investigations devoted to explicit solution to the Riemann–Hilbert problem for a multiple connected domains [10] and by significant progress in symbolic computations [4], [5], [12]. In the talk we demonstrate that the theoretical results [10] can be effectively implemented in symbolic form that yields analytical formulae for random composites.

For conductivity problems governed by Laplace’s equation, the local conductivity tensor \( \lambda(x) \) can be considered a random function of spatial variable. We restrict to two-phase composites with non-overlapping inclusions when a collection of particles with fixed shapes and sizes is embedded in matrix. More precisely, consider a set of hard particles \( \{D_k, \ k = 1, 2, \ldots\} \) where each \( D_k \) has a fixed geometry. Let all the particles are randomly located in the space and each particle \( D_k \) occupies a domain \( D_k \) without deformations. Thus, the deterministic elements \( D_k \) are introduced independently but the joint set \( \{D_1, D_2, \ldots\} \) randomly. The diversity of random locations is expressed by joint probability distributions of the non-overlapping domains \( D_k \).

Various methods were applied to such random composites. The most known theoretical approach is based on the \( n \)-point correlation functions presented by Torquato [14]. This theory yields analytic formulae and bounds for the effective properties. The main shortage of this theory is computational difficulties to calculate the \( n \)-point correlation functions for large \( n \). Let a distribution of inclusions be fixed. The effective conductivity tensor \( \Lambda_n \) is uniquely determined through \( D_k \) and their physical properties. In order to estimate \( \Lambda_n \), various statistical approaches were applied to create the representative volume element (RVE). These approaches are based on the straightforward computations of the effective properties for various locations of inclusions and on the statistical investigations of the obtained numerical results. The main disadvantage of this physical theory is that the notation of the RVE is not correctly defined and the results have no precise meaning. This theory is rather a statistical analysis of the computational experiments and physical measures.

The main purpose of the talk is to work out constructive analytical-numerical methods to calculate the effective constants of composites with non-overlapping inclusions and to develop the corresponding RVE theory. We apply a new approach to estimate the effective conductivity of random 2D composites. First, deterministic boundary value problems are solved for all locations of inclusions, i.e., for all events in the considered probabilistic space \( \mathcal{C} \) by the generalized method of Schwarz (GMS) referred to decomposition methods. This method was proposed by Mikhlin as a generalization of the classical alternating method to multiply connected domains; convergence of the GMS was established in [7], [3].

After solution of the deterministic problem by the GMS the ensemble average for the effective conductivity (mathematical expectation) is calculated in accordance with the given distribution. This method is related to the classical method [14] based on the average probabilistic values involving the \( n \)-point correlation functions. In our approach following the GMS, we avoid computations of the correlation functions and compute their weighted moments. The effective properties are exactly expressed by means of these moments. Analytical formulae and numerical simulations show advantages of our approach in the case of non-overlapping inclusions. For instance, for circular inclusions, our method yields the effective conductivity for the uniform non-overlapping distribution

\[
\lambda_e = 0.8115 \left( \frac{x + 0.896}{x^2 + 0.3498x + 0.352} \right) \left( \frac{0.9069 - x}{x^2 + 0.4168} \right)^{4/3}
\]

Stochastic 2D problems are posed and solved in doubly periodic statement in the plane. Theoretically, doubly periodic problems constitute the special class of problems in the plane with an infinite number of inclusions [8]. However, the number of inclusions per a periodicity cell, \( N \), is arbitrary taken, and the final formulae for the effective tensor contains \( N \) in a symbolic form. Similar non-periodic statements can be used following [11]. Such an approach explains, for instance, the divergent sum (integral) arising in applications of self-consistent methods when the divergent sum does really diverge for some distributions. This implies that
the corresponding composites cannot be homogenised over the whole plane even if the concentration \( \varepsilon \) is properly defined. The homogenization theory (see [13] and works cited therein) justifies existence of the effective properties for statistically homogeneous random fields which constitute a subclass of heterogeneous fields discussed in [8], [14], [11]. The divergence and other similar effects do not arise for doubly periodic composites when homogenization is always valid. A periodicity cell can be considered a representative cell and vice versa. From a practical point of view, each sample is finite, hence, it can be considered as a representative cell with many inclusions. Application of the new RVE theory can reduce the number of inclusions per periodicity cell but not always significantly. Reduction to a small number of inclusions per cell can distort the effective properties of the original sample. It was noted in [2] that periodical arrangement of inclusions decreases the effective conductivity when conductivity of inclusions is greater than the conductivity of host. Moreover, the theory of elliptic functions can be applied to constructive symbolic computations for periodic problems that yields exact and approximate analytical formulae for the effective tensor.

The proposed method leads to the construction of rigorous RVE theory for non-overlapping inclusions [9]. The effective tensor can be written in the form of expansion in the moments of the correlation functions which can be considered as "basic elements" depending only on locations of inclusions. The RVE is defined as the minimum size periodicity cell corresponding to the set of basic elements calculated for the composite. A simple fast algorithm to determine the representative cell for a given composite is based on reduction of the number of inclusions per periodicity cell having the same basic elements as the given composite. It follows from simulations [4], [5] that for the uniform non-overlapping distribution of disks in order to reach high accuracy in the effective conductivity required to solve the corresponding boundary value problems for at least 64 inclusions per cell repeated at least 1500 times. These technical parameters essentially exceed possibilities of the traditional statistical approach to the RVE.

The goal of the talk is to show that the results obtained by the GMS [2], [7]-[12] can be considered a constructive approach to resolve some problems of the classic theory of random composites for non-overlapping inclusions. The GMS allows to overcome myths frequent in the theory of random composites:

1. It is asserted that only application of the correlation functions is the proper method to treat random composites. Hence, restrictions in this field are related to computational difficulties arising in numerical computations of the correlation functions. As it is noted above, we obviate the need for the correlation functions calculating the mathematical expectation of the effective tensor after a complete solution of the deterministic problems.

2. It is thought that the contrast parameter expansions are valid only for sufficiently small contrast parameters. However, it was first justified in [6] that such expansions for plane conductivity problems are valid for all possible values of the contrast parameters.

S. Self-consistent methods for random composites are considered a simple and constructive alternative to the method of correlation functions. However, applications of the GMS demonstrates essential limitations of these methods [11]. More precisely, the self-consistent methods do not capture interactions among inclusions. Even a solution to the \( n \)-particle problem in the infinite plane can be applied only to diluted distributions of the clusters consisting of \( n \) inclusions.

In some works, the homogenization theory is frequently oversimplified. For instance, the paper Representative elementary volume in en.wikipedia.org/wiki/, presents homogenisation of 2D composites as the replacement of the random locations by unit periodicity cells containing one inclusion per cell. Such a wrong treatment contradicts to the extreme principle of locations deduced in [2] for the shaking model. Shortly, the extreme principle of locations says that growth of the order in locations of inclusions decreases the effective conductivity of medium with highly conductive inclusions. In particular, the square array attains a local minimum and the hexagonal array the global minimum of the effective conductivity for a fixed concentration.

References


Multiscale analysis of metal matrix composite with ceramic reinforcement using embedded cell approach and Johnson-Holmquist constitutive model

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Abstract

The paper deals with an analysis of composite material reinforced with continuous ceramic fibres. The research is focused on an investigation of the material behaviour and damage under transverse loading. In this case a two-dimensional plane strain model is taken into consideration. In order to analyse the material at micro-level the embedded cell approach is applied. The displacement boundary conditions are prescribed and a boundary value problem is solved the finite element method using commercial LS-Dyna code. Fibres are modelled by Johnson-Holmquist material model while matrix is considered elastic-plastic. The properties of homogenous material that embeds the cell are determined in preliminary homogenization analysis taking into account simplified composite constituents material description. The effects of different loading conditions are investigated. Results of the analysis carried out on the micro-level are expressed at macro-level by stress-strain curves obtained by volume averaging of stress and strain fields inside the cell.

Keywords: Johnson-Holmquist model, ceramic reinforcement, embedded cell approach, metal matrix composite

1. Introduction

Modern metal matrix composites very often contain reinforcement made of ceramic material providing high strength and stiffness. Strength and damage evolution in continuous fibre reinforced composites depend on variety of factors such as fibre characteristics, matrix strength and work hardening, strength and nature of the interface between the phases [1]. In the case of a relatively weak interface fibre deboning and matrix shear mechanisms are predominant. When the interface is relatively strong coplanar failure, where single fibre fracture leads to precipitous fracture of adjacent fibres, is predominant [1]. During this research the strong interface is assumed and the fibre damage influence on the composite response is investigated. In order to simulate local phenomena like fibre damage a random model was created by applying the embedded cell approach [2,3]. The displacement boundary conditions are prescribed and a boundary value problem is solved by the finite element method in the framework of a commercial LS-Dyna software [4]. A two-dimensional model with the plane strain assumption is taken, so an influence of transverse loadings on the composite response is accounted. During this research aluminium alloy 6061-T6 matrix composite reinforced with Al₂O₃ fibres is analysed.

2. Microstructure modeling

Observations of the composite material microstructure reveal that fibres are usually randomly distributed through the matrix [3]. To study the random fibre arrangement and therefore non-periodic micro fields embedded the cell approach is taken into account. The embedding models typically contains a local heterogeneous region, in which the microstructure is well resolved geometrically, that is embedded in an outer region that serves mainly for transmitting far field loads [2]. This approach is particularly appropriate when a material damage is studied at the micro-level [3].

Figure 1 shows the geometry of the considered model. The cell contains 228 randomly distributed fibres that comprise 43.5% of the composite volume.

1.1 Cell with randomly distributed fibres

1.2 Embedding region-heterogeneous material

Figure 1: The considered embedded cell model

3. Constituents material modelling

Material properties of the embedding region are determined in preliminary homogenization of simple unit cell containing a single fibre. The applied homogenization procedure is based on finite element computations with periodic boundary conditions and averaging of stress and strain fields [5]. In this case fibres are described by elastic model while matrix is described by elastic-plastic model. At the cell level ceramic reinforcement material behaviour is described by Johnson-Holmquist constitutive model implemented in LS-Dyna code [4,6]. In this case the equivalent stress is given in terms of the damage parameter $D$ by:

$$\sigma^* = \sigma^*_f - D(\sigma^*_f - \sigma^*_m)$$

(1)
where
\[
\sigma_i^\prime = a (p^* + t^*)^n (1 + c \ln \dot{\varepsilon}^\prime)
\]
(2)
represents undamaged material behaviour and
\[
\sigma_i^\prime = b (p^*)^n (1 + c \ln \dot{\varepsilon})
\]
(3)
represents the damaged behaviour [4]. The superscript “*” indicates a normalised quantity. In particular the stresses \(\sigma\) are normalised by the equivalent stresses at the Hugoniot elastic limit, pressures \(p\) are normalised by the pressure at the Hugoniot elastic limit and the strain rates \(\dot{\varepsilon}\) are normalised by the reference strain rate. Parameters \(a, b, c, n, m\) are model constants and
\[
t = \frac{T}{p_{HEL}}
\]
(4)
where \(T\) is the maximum tensile strength and \(p_{HEL}\) is the pressure at the Hugoniot elastic limit [4]. The damage parameter \(D\) represents the accumulated damage by accounting increase in the plastic strain per computational cycle and the plastic strain to fracture defined as follows
\[
D = \sum \Delta \varepsilon^* \quad \varepsilon_i^* = d_i (p + t)^{d_i}
\]
(5)
(6)
where \(d_1, d_2\) are damage constants. As stated before initially the material response is considered to be elastic. For the undamaged material hydrostatic pressure is given by equation of state:
\[
p = K_1 \mu + K_2 \mu^2 + K_3 \mu^3
\]
(7)
where \(\mu = \frac{\rho}{\rho_0} - 1\) and \(K_2, K_3\) are equation of state constants [4].

Johnson-Holmquist constitutive model constants for Al₂O₃ are collected in Tab. 1 [6].

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
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<td>Density, kg/m³</td>
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<td>(K_3), GPa</td>
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</table>

### 4. Analytic results

The three different loading conditions were considered: compressive, tensile and shear. Figure 2 shows stress-strain curves at macro-level corresponding to different loading conditions obtained by volume averaging of stress and strain fields inside the cell. Figure 3 presents the way of particle damage at micro-level in the case of extension and shear.

In further analysis the presented methodology will be applied in investigation of the reinforcement volume fraction and arrangement on the material response. Another issues that will be raised are the estimation of strain rate dependency of considered composite and the consideration of matrix damage.

### References

The problem of computing costs of multiscale simulation and optimization of manufacturing of rails

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Abstract

The required operation properties of rails are obtained by controlled heat treatment of the head of the rail carried out after rolling. Design of cooling schedules should be supported by numerical simulation of the controlled cooling. Multiscale models of various complexity were developed by the Authors. In all solutions FE code was used to calculate temperatures during controlled cooling process. Micro scale models expand from simple phenomenological models describing microstructural parameters to advanced models based on the FE solution of the diffusion equation in micro scale. Computing times for these models differ noticeably, by few orders of magnitude. Therefore, the goal of the paper was searching for a compromise between the predictive capabilities of the models and computing times.

Multiscale simulations of the controlled cooling of rails were carried out with various models. The optimization of this process was performed with the objective function composed of required microstructural parameters. Computing times for various models were compared and confronted with the performance of these models.

Keywords: multiscale modelling, heat treatment of rails, pearlite, mechanical properties

1. Introduction

The constant development of rail transport is specifically connected with an increase in train speed, application of greater axle loads due to an increase in the weight of materials carried by rail transport, as well as linking of the railway networks with the tram infrastructure. Therefore, there is a continuous need for the improvement of the quality of rails, which is characterized by the increased wear resistance, fatigue strength and resistance to the occurrence of the contact-fatigue defects. These positive features can be obtained for pearlite structure after accelerated cooling with small distance between the cementite lamellae. Reduction of this distance results in an increase of pearlite strength/hardness and increase of plasticity of this phase [1,2].

Numerical models of heat treatment of rails allows to determine correlation between process parameters and microstructure and properties of rails and support of the design process.

2. Simulation of the rail head cooling

The main goal of the heat treatment of rails is to obtain a fine structure of pearlite. Increasing the cooling rate decreases interlamellar spacing, but it can also lead to an occurrence of unfavorable components of degenerated pearlite and bainite. For this reason, it becomes very important to conduct the heat treatment in a controlled manner.

Methods of the heat treatment are constantly improving. An example of such innovative solution is a method of cyclic cooling of the rail head in the aqueous solution of the polymer mixture, which was presented and investigated experimentally in [1] and was simulated in [2]. This method was used as a basis in the present work. The rail head was cooled according to schemas shown in Fig. 1.

Figure 1: Temperature changes at the point located 2 mm below the surface of the rail head for 2 variants of cooling

The process of accelerated cooling starts when the temperature of the rail head reaches about 820°C. Then the solution is supplied to the tank, which leads to gradual immersion of the rail head. Keeping the head immersed too long would lead to occurrence of the bainite in the microstructure. Therefore, the head is immersed cyclically into the polymer solution for short periods of time, after which the solution is removed from the tank and the rate at which heat is transferred from the head to the environment is substantially reduced. At this stage transfer of heat from the hotter head centre increases the temperature of the running surface. It is critical for the
running surface temperature not to increase above 570°C. To prevent this, the coolant level is raised again and accelerated cooling stage begins again. The whole process of cooling is repeated many times until the pearlite transformation is completed in the entire head.

FE model was used to calculate temperature in the rail head. Heat transfer equation was solved:
\[ \nabla k(T) \nabla T + Q(T) = c_p(T) \rho(T) \frac{\partial T}{\partial t} \quad (1) \]
where: \( T \) - temperature, \( k \) - thermal conductivity coefficient, \( t \) - time, \( c_p \) - specific heat, \( \rho \) - density, \( Q \) - heat of phase transformation.

Fourier boundary condition was applied on the entire outer surface:
\[ k(T) \frac{\partial T}{\partial n} = h(T)(T_0 - T) \quad (2) \]
where: \( T_0 \) – ambient temperature, \( h \) – heat exchange coefficient, \( n \) - unit vector normal to the surface.

3. Simulation of the austenite to pearlite phase transformation

The model of austenite to pearlite phase transformation is based on the assumption that carbon diffusivity is the main controlling parameter during this transformation [3]. Movement of carbon atoms is described by the second Fick law:
\[ \nabla \cdot (D \nabla c) = \frac{\partial c}{\partial t} \quad (3) \]
where: \( c \) – local carbon concentration, \( D \) – carbon diffusion coefficient.

Numerical solution assumes that the initial structure is pure austenite with carbon content of 0.71%. The first plate of cementite is created when the carbon concentration increases locally to 6.67%. This plate grows until the content of carbon in a distance greater than 3.5 times the thickness of the plate falls below \( c_{\beta} \), then two new cementite plates are nucleated. The value of \( c_{\beta} \) is determined by an inverse analysis.

During both the nucleation and the growth of plates the condition of the mass conservation must be complied. The initial and boundary conditions at the interface are:
\[ c(x, y, 0) = c_0, \quad c(\xi, t) = 0.02 \]
\[ \frac{\partial c}{\partial n}(x, y, t) = 0 \quad (4) \]
where: \( c_0 \) – carbon content in austenite, \( \xi \) - edge of cementite, \( n \) – unit vector normal to the surface, \( (x, y) \in \Gamma_1 \), \( \Gamma_2 \) – edges of the solution domain.

Numerical simulations of austenite to pearlite phase transformation were performed for rail steel 900A. Simulations in 2D domain were performed for three cooling rates: 0.1, 0.5, 0.7°C/s. Symmetry was assumed during occurrence of nucleation and growth of the plates. Sidewise growth of pearlite was analyzed, while front growth was not considered. Growth of pearlite colonies during cooling at 0.5°C/s is shown in Fig. 2. At lower undercooling thicker plates of cementite and greater interlamellar spacing were observed.

4. Multiscale approach

Temperature distribution calculated by the macro scale model was combined with simple closed form equations describing phase transformations and used to predict distribution of properties at the rail cross-section [4]. In the present work multiscale simulations of the controlled cooling of rails were performed with various models. Optimization task was formulated with the objective function composed of required microstructural parameters. Computing times for various models were compared and confronted with the performance of these models.

5. Summary

Model of austenite to pearlite phase transformation describes the mechanism of growth of cementite and ferrite plates during cooling of rail steels. Created model has a capability to determine changes of the carbon distribution and, in consequence, to predict the size of interlamellar spacing. Performed simulations confirmed qualitatively good accuracy of the model. Simulations showed reduced interlamellar spacing with increasing cooling rate. The computational costs of this model are very high.

References

Numerical model for brittle-ductile fracture analysis in DP steel

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Abstract

The subject of the work is development of a hybrid fracture model, which takes into account crack initiation and propagation at the microscale level in DP steels. At this stage of the research microscale numerical model based on the digital material representation concept was proposed. Models of brittle and ductile failure initiation and propagation during plastic deformation were simultaneously considered. Crack behaviour in the martensitic and ferritic phases is based on the XFEM and Johnson-Cook approaches, respectively. Comparison of the of simulations with and without fracture models is also presented within the paper.

Keywords: brittle fracture, ductile fracture, digital material representation

1. Introduction

Development of the numerical model based on the digital material representation (DMR) idea capable to take into account brittle and ductile failure mechanisms in Dual Phase steels was the main goal of this work. Recently in the literature simplified numerical models were published based on DMR idea that considered separately ductile and brittle failure models e.g. [1,2]. However, interactions between brittle and ductile failure play an important role and should not be neglected. Thus, authors decided to develop hybrid approach combining both brittle and ductile failure models and incorporate them into the DMR finite element solution.

2. Digital material representation

The concept of digital material representation was recently suggested and is dynamically evolving [3]. The main objective of the DMR is to create digital representation of microstructure with its features (i.e. grains, grain orientations, inclusions, cracks, different phases etc.) represented explicitly. Generation of material microstructure with specific geometrical features and properties is one of the most important algorithmic parts of systems based on the DMR. Such DMR is further used in numerical simulations of material processing. As a result, more accurate information on local material behaviour can be obtained. Details on the procedure used to generate DMR for DP steels can be found in [4]. The approach is composed of the three main stages responsible for image filtering (denoising), phases distinction and phases analysis. The last step is divided into the two separated algorithms dedicated to both ferrite and martensite phases processing. All these subsequently applied steps aim to detect the phases, phase boundaries, and, finally, to prepare the material microstructure in DMR format for numerical simulations. This image processing algorithm was applied to obtain a reliable digital material representation model for the present investigation as seen in Figure 1a. Separated ferrite grains (multicolour) and martensite islands (red) exactly reflect the real shape of features existing in the DP steel microstructure. Discretization with finite elements for the investigated image was performed with the DMRMesh software [5] as seen in Figure 1(b,c).

Figure 1: DMR of DP steel a) before discretization and b) after discretization c) magnification of mesh refinement along grain boundaries

3. Hybrid brittle-ductile fracture model

As mentioned, due to the fact that both fracture mechanisms play an important role when DP steel is considered a hybrid solution was proposed. This approach combines two different fracture models XFEM [6] and Johnson-Cook ductile criterion [2] at the same time. The model consists of three steps: brittle fracture model, data exchange module and ductile fracture model. After the first step, received information consists of specific locations of brittle fractures in the DP microstructure. These information are used to set an initial conditions for the subsequent ductile fracture model. The data transfer module was implemented by the designed script in Python language. The script directly modifies ABAQUS output (.odb) file by creating a new set of FE elements. New input file (.inp), contains microstructure morphology and elements sets with brittle failure needed for predicting ductile failure initiation. Elements that were grouped in the cracked set are then removed from the FE mesh and model is again ready for calculation focused on ductile failure. Schematic illustration of the developed hybrid failure model is shown in Figure 2.
4. Results

A simple tension test based on the DMR concept was performed to show functionality of the hybrid failure approach. The number of finite elements used for discretization was set to approx. 500000. A four node bilinear plane strain quadrilateral reduce integration and hourglass control finite elements (CPS4R) were chosen for the discretization purposes. Assumed level of discretization was selected after mesh sensitivity analysis performed during the research. The authors decided to use an implicit finite element solver to ensure good convergence of the solution. The results obtained after the deformation are shown in Figure 3. The maximum stress values are concentrated in the area where brittle cracks reached the martensite/ferrite interface. These zones were precursors of ductile fracture initiation in the ferrite phase.

As seen in Figs. 3 and 4 brittle cracks that occurred during the first stage of deformation led to ductile fracture initiation and subsequent propagation. As a result several fractures occurred across the investigated microstructure leading to slight softening in comparison to the prediction by the JC failure model.

5. Conclusions

A hybrid failure model gives opportunity to resolve brittle and ductile failure behaviour in DP steel. The developed model is mainly based on the finite element approach, with the incorporated digital material representation. In this form it can be easily applied for practical industrial research on fracture in dual phase steels. The user does not need any additional knowledge on advanced modelling techniques to use the proposed model.

References

The use of numerical studies in the assessment of mechanical characteristics of the highly deformable structures of the head

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Abstract

The head is one of the most complex and the most important parts of the human anatomy. It is interesting, from the point of view of mechanics, because of the multi-phase combination of the exposed highly deformable structures, with a predominance of those with nonlinear characteristics, characterized by different properties. During dynamic loading conditions that lead to the exceeding of the range of plasticity of the brain tissue, an instability of the tissue arrangement can be observed. Advanced methods for the detection of changes in the mechanical value of the multiphase medium are numerical methods, in particular, the finite element method (FEM) can be especially useful. Due to their research the authors constructed FE models of brain tissue fragments and conducted numerical studies taking into account the boundary conditions arising from violent overloads that result from combat operations. The results of the study are meant to contribute to the explanation of the phenomena of degradation of the heterogeneous structures within the head.

Keywords: nonlinear materials, multiphase systems, FEM, brain tissues

1. Introduction

The brain is among the most essential organs of the human body. Therefore, it is protected by a series of natural mechanisms against mechanical injury. The scalp, skull, meninges and appropriate pressure of the cerebrospinal fluid in the subarachnoid space contribute to a multiphase protective system possessing different mechanical characteristics. Brain tissue is highly vascularized, as it allows the communication of synapses by changing the energy density and deformation. Vascular and nerve structures of the brain are characterized by a high nonlinearity and demonstrate high deformability. Although the vascular system of the brain and nervous system is protected well, from the mechanical perspective, it is not adapted to dynamic load changes, e.g. on the battlefield or in the case of traffic accidents. Widely used in structural engineering numerical modelling, the FEM is also frequently used for the modelling of biological systems. However, biomechanical modelling of the head as a multiphase system affected by various impact forces is still rare [3]. Most of the work undertaken in the field of impact overload has been developed for the automotive and aerospace industries. Leading authors dealing with the modelling of the head structures include: Ruan [4], Zhang [6], Kleiven [2], Chafi [1].

However, mechanical studies of the destruction of bridging veins, meninges, or of the interruption of connections between neurons, as well as the grey and white matter density disturbances, increased blood and cerebrospinal fluid (CSF) pressure in the head resulting from combat are lacking. Currently, to assess the injury of the head resulting from an impact with a physical object, traumatic criteria, such as HIC, make it possible to better assess changes affecting the mechanical parameters (strain energy, stress) within the tissue.

In the work, the authors have undertaken a numerical study of the head fragments, based on experimental research. The numerical study was an attempt to analyse changes of parameters and mechanical characteristics of the head structures in rapid overload conditions that result from combat operations. We analysed, a system of protective structures: the skull, meninges, CSF, and the tissues responsible for other vital functions, namely, cerebral veins and neural structures.

2. Material and methods

Identification of the tissues of the head was based on the DICOM images derived from computed tomography (CT) with a very highly thickened mesh sampling. Next, using the Mimics software a simplified 3D geometry of the skull and the brain was formed. The model was imported into the program LS-DYNA, where an additional geometry of the structure was constructed, the different sections in the model were separated, and material properties and boundary conditions were assigned. The numerical model consisted of heterogeneous multiphase materials. The model includes the skull, meninges (dura mater, arachnoid mater, pia mater), falx, tentorium, cerebrospinal fluid (CSF), blood vessels, and nerve tissue. Mechanical properties of brain structures were obtained from experimental studies [5]. The selected fragments of the FE model are shown in Figs. 1 and 2.

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The boundary conditions were obtained from the real combat conditions in Afghanistan and battle range studies of explosive charge explosions of various sizes detonated from under the hull of vehicles. The values used in the simulation were read from sensors placed on the head and neck of the anthropometric Hybrid III dummy, which was located in an armoured transporter during each explosion.

3. Results

As the result of the conducted numerical analysis, critical values of strain, stress, strain energy density have been obtained. The following analysis showed high diversity in the properties of the different regions of the head. Figs. 3 and 4 show exemplary characteristics of the changes in kinetic energy over time for different brain structures, caused by the impact of the head with the roof of an armoured vehicle occurring at 3m/s.

4. Conclusion

The present work encompasses an analysis of the changes in mechanical parameters of the given brain structures that have been affected by heavy impact loading. In the numerical analysis, the boundary values of the protective structures, such as the skin, skull, and meninges, have been exceeded. Major changes in the mechanical parameters of the vessels and nerve tissue can initiate the mechanotransduction process and lead to the degradation of the brain structures. The mechanical loads initiates the dislocation of anatomical structures in the physiological areas and can cause damage to the lining of the vessels.

The described models explain the mechanism of trauma dealt to the different layers of the head and allow for the estimation of the degree of tissue destruction under a given impact load. The conducted numerical analysis, combined with widespread experimental studies can be of utmost significance in the assessment of stroke and impact loading effects among soldiers taking part in combat operations.

References

Numerical assessment of failure mechanisms due to transverse loading in unidirectional fiber-reinforced polymers

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Abstract

A method to determine the in–situ strength of fiber–reinforced laminas for three types of transverse loading including compression, tension and shear is presented. The framework of this method is the analysis of local stresses responsible for the coalescence of matrix cracks, carried out using a multi–fiber unit cell model and the finite element method. Random distribution of fibers, fiber–matrix decohesion and matrix plastic deformations are taken into account in micromechanical simulations. The present study also shows that the nonlinear hardening behavior of matrix reflects more realistically the influence of plastic deformations on the in–situ transverse strength of laminae than the perfectly plastic behavior of matrix. The prediction of in–situ transverse strength is verified against the experimental data for a cross - ply laminate subjected to uniaxial tension.

Keywords: computational micromechanics, matrix cracking, polymer–matrix composites, first ply failure load

1. Introduction

When a fiber–reinforced polymer–matrix composite lamina is subjected to transverse loading, it fails due to matrix cracking. The behavior of matrix cracking at the microscopic level is related to the appearance of fiber/matrix debonding, small cracks and plastic deformation within the matrix. The reduction of lamina stiffness due to matrix cracking can be determined at the fiber scale using computational micromechanics. The main objective of the paper is to present a simple procedure based on the use of the unit cell with random fiber arrangement and the finite element method to predict the limit load of the first lamina embedded in a laminate. Thus, an analysis of the hoop stresses that are responsible for the coalescence of the matrix cracks is carried out in the paper. To find the in–situ strength of lamina, the criterion of maximum hoop stress in matrix is used locally for the most loaded fiber.

2. Micromechanical model

Numerical simulations using a concept of the unit cell with random fiber arrangement are a current trend in computational micromechanics. The benefit of the use of such unit cells is that the effect of fiber array irregularities on transverse responses of composite can be accurately taken into account. In the paper the unit cell models of randomly distributed fiber composite are generated using Wongsto and Li’s algorithm [1]. Analyses were made on models that contained 39 fibers. The data required for the simulation study were taken from the world wide failure exercise WWFE [2] for a case of E–glass/MY750/HY917/DY063 lamina with the fiber volume content of 60%. Two–dimensional finite element meshes were built using ANSYS finite element code. Plane strain conditions were assumed. To ensure accurate displacement and stress field representation within each unit cell, sufficiently dense meshes comprising of approximately 45000 elements were used. Each fiber/matrix interface contained 100 contact elements equally spaced around the circumference.

Three types of transverse loading including compression, tension and shear are considered. For each loading type periodic boundary conditions [3] are imposed on the unit cell to reflect the repeatability of the microstructure and to ensure the compatibility of the displacement fields. By the assumption of periodicity, each displacement field $u_i$ may be decomposed in a part associated with the applied strain $\varepsilon_{ij}$ and a periodic one $u^p_{ij}(x_1, x_2) = \varepsilon_{ij}x_j + u^p(x_1, x_2)$. (1)

Figure 1: Influence of the ductility of matrix on the mechanical response of the unit cell model under transverse compression.

Although the extension of plastic strain zones in polymer matrices is inhibited by the nearest fibers, considerable plastic deformation may occur between the fibers. The epoxy matrix is therefore modeled within the framework of the finite deformations as an elasto–plastic solid which hardens isotropically. It is widely accepted that the deformation of polymeric materials is highly sensitive to the hydrostatic pressure. To address this requirement, the Drucker–Prager plasticity model [4], is used incorporating the linear dependence on the hydrostatic stress. In terms of the first invariant of stress...
$I_1$ and the second invariant of the deviatoric part of stress $J_2$, the yield function is given

$$
\ell = (\mu I_1 / 3)^{1/2} - k,
$$

(2)

where $\mu$ is the pressure sensitivity factor, $k$ is the flow stress of the material under pure shear. It is well known that polymer matrices cannot behave identically to unreinforced polymers. Thus, micromechanical models require in–situ properties of matrices that are different from bulk properties. In this paper the hardening curve of epoxy matrix is extracted from a micromechanical model subjected to in–plane shear loading. The non–linearity of epoxy matrix is identified such that the unit cell prediction matches the measured in–plane shear response reported by [2].

For the fiber/matrix interface failure, the cohesive zone model is employed, in which the constitutive equations of the interface refer to the normal $\sigma_n$ and tangential $\tau$ cohesive tractions to the normal $u_n$ and tangential $u_t$ opening displacement jumps and a scalar damage variable $d$, through [5]

$$
\sigma_n = k_n d(1-d), \quad \tau = k_t d(1-d),
$$

(3)

where $k_n = k_t =10^6$ GPa/m are initial contact stiffnesses in the normal and tangential directions, respectively. The variable $d$ represents the loss of stiffness, being a function of both opening displacement jumps. The variable $d$ takes values from 0 to 1. A cohesive layer consisted of interface elements with four nodes was introduced between the fibers and the matrix to reproduce the fiber–matrix debonding.

3. Selected results

Two models of matrix plasticity were applied in order to assess their utility for predicting the in–situ transverse strengths of lamina and to analyze the impact of the matrix ductility on the overall macroscopic responses of lamina under transverse loading. Fig. 1 compares the macroscopic stress–strain curves obtained from unit cell models with hardening and perfectly plastic matrix transverse compression. It can be seen that the application of the perfect plasticity theory in modeling transverse compression reduces the strength of lamina. Considering experimental responses it was found that the micromechanical models with $\sigma_y = 30$ MPa, $\tau_y = 60$ MPa and $G_t = 15$ J/m², $G_y = 30$ J/m² produce reasonable results. Fig. 2 shows the plastic shear bands obtained from the unit cell model with hardening matrix in the case of transverse compression at the post–critical stage of deformation. It is interesting to note that extensive plastic deformation of matrix takes place only in the vicinity of interfacial cracks. Furthermore, one clearly defined critical plane in this model oriented at angle $\omega_c = 54^\circ$, is consistent with theoretical predictions and test results. Fig. 3 shows the variation of the matrix hoop stress $\sigma_{\theta\theta}$ with increasing macroscopic strain in dependence on the polar angle $\theta$ for transverse compression. The evolution of the matrix hoop stress reveals that formation of the interfacial crack magnifies locally the tensile stress in matrix, as well as that the maximum of this stress locates at the interfacial crack tips. It is interesting to note that even for compressive loading, the tensile stress in the matrix appears [6].

![Figure 2: Contour plots of the effective plastic strain in hardening matrix in the unit cell model subjected to transverse compression at $\varepsilon_2 = 4 \%$.](image)

![Figure 3: Angular distribution of the matrix hoop stress $\sigma_{\theta\theta}$ at the interface between matrix and the most loaded fiber for transverse compression.](image)

References

Fractal behavior of the heat flux on the boundary of random composites

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Abstract

Stationary two-dimensional temperature fields in deterministic and random composites near the boundary are theoretically investigated by the method of functional equations. A simple structure of the obtained formulas allows to compute the heat flux for an arbitrary number of inclusions. It is stated that a large number of randomly packed inclusions drastically complicates the structure of the local temperature fields, irregular at a mesoscopic scale by comparing to regular composites.

Keywords: random composite, temperature fields, fractal behavior

1. Introduction and Method

The temperature and heat flux distributions in composites near the boundary are of considerable interest in various technological problems and theoretical investigations [7, 8, 14, 15]. As noted in [3] and the literature cited therein the problem of failure of composite materials cannot be solved correctly neglecting the analysis of the heat flux in the vicinity of walls of the composite solid. In particular, it was established that the length order of zones of rapid changes of the heat flux, i.e. zones of strong redistribution of the heat flux between fibres and matrix, matches the distance between fibres.

The paper focused on a theoretical investigation of the problem for stationary two-dimensional composites. More precisely, unidirectional fiber composites are considered with the boundary plane parallel to fibers. The number of fibers can be taken sufficiently large (~1000). The location of centers of inclusions can be arbitrary. A method of functional equations [10, 13] is applied to analytically describe the heat flux.

A large number of densely packed inclusions drastically complicates the local fields since the principle of superposition or numerical modifications cannot be applied. Practically, this requires refined mesh discretization between inclusions that becomes problematic for large number of inclusions. In the paper, approximate analytical formulas from [13] are extended to the boundary value problems. A simple structure of these formulas allows to compute the heat flux practically for an arbitrary number of inclusions. For simplicity, a two-component composite is considered made of a collection of non-overlapping, identical disks of radii \( r \) (fiber composites), embedded randomly in an otherwise uniform locally isotropic host. The conductivity of the host is normalized to unity. It is worth noting that the fractal behavior of the boundary flux takes place for the heat conduction when the process is governed by the simple 2D Laplace equation. General heat transfer models only amplify this effect.


The boundary flux for composites estimated in the previous works slightly differs from the flux computed for homogenous media. Frequently, a picture is presented with "randomly chosen" inclusions and fields computed for that picture. Usually, such a "random location" is chosen that the result is "smooth" and closely dense inclusions are excluded. However, such a case is too artificial and does not present real random fields. In the paper, all the locations of inclusions are chosen in accordance with the rigorously constructed algorithm ("True" simulations of random inclusions are important as well as a random number generator in the general theory of computations). A uniform non-overlapping distribution is simulated by the following procedure. First, a domain near the boundary is fixed, in the paper a rectangle \( P \). The first disk is randomly chosen by the uniform distribution in \( P \). The second disk is also randomly chosen. If it overlaps the first one, is deleted, so the next disk is randomly chosen. As a result we generate \( n \) disks satisfying a uniform non-overlapping distribution. Different distributions of inclusions do not impact the effective conductivity for low concentrations. Note that the local flux near the boundary can be drastically changed.

Figure 1: The broken bell-shaped line describes the prescribed temperature distribution at the half-plane boundary. Percolation chain (in black) of conductive disks: the accidental distribution of disks enabling a resurgent flow of heat.

As it was noted by Rychard Wojnar such an anomaly can be explained by the resurgence model [1, 2] presented in Fig.1. A percolation highly conducting chain can instantly transfer the heat from the initial point to its end point that produce inverse flux directed exterior to the composite.
2. Computer simulations

A computational example is given. Consider the boundary temperature distribution (broken line in Fig.1)

\[ f(x_1) = \frac{1}{(x_1 - x_0)^2 + h^2}, \quad -\infty < x_1 < \infty, \]

where \( b = x_0 + i h \), \( i = \sqrt{-1} \) and \( h > 0 \). Let \( z = x_1 + i x_2 \), the bar denote the complex conjugation. Let \( r \) be the radius of inclusions. Introduce the inversion \( z^*_{(m)} = \frac{r^2}{z - a_m} \) with respect to the circle \( |z - a_m| = r \).

The first order approximation is calculated by

\[ \psi^{(1)}(z) = \sum_{m=1}^{n} \frac{\psi^{(1)}_{m}(z_{(m)*)} - \psi^{(1)}_{m}(z_{(m)})}{(z - a_m)^2} \frac{1}{h} \frac{1}{(z - b)^2} + \]

\[ = \frac{i}{h} \left( \frac{1}{(z - b)^2} + r^2 \sum_{m \neq k} \frac{1}{(a_m - b)(z - a_m) + r^2} \right) + \]

\[ -r^2 \sum_{m=1}^{n} \frac{1}{(a_m - b)(z - a_m) + r^2} \]

in the disks \( |z - a_k| < r \) (\( k = 1, 2, \ldots, n \)).

Figure 2. The computed \( x_2 \)-component of the heat flux on the boundary for a regular composite for the given temperature distribution (1). The data refer to: a) composite of distance from the highest inclusions to the boundary equal 0.15 (broken line); b) the same composite with inclusions shifted up the distance from the highest inclusions to the boundary equal 0.05 (solid line).

\[ \psi^{(1)}(z) = \frac{\psi^{(1)}(z_{(m)*)} - \psi^{(1)}(z_{(m)})}{(z - a_m)^2} \frac{1}{h} \frac{1}{(z - b)^2} \]

References


Concurrent CAFE Model of Static Recrystallization during Multi-pass Hot Rolling

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Abstract

Development of a multiscale numerical model of hot plate rolling combining Finite Element (FE) and Cellular Automata (CA) approaches to simulate multi-pass rolling process to find interactions between strain state and microstructure evolution is the subject of the present paper. In order to describe the microstructure behaviour under deformation, a Digital Material Representation (DMR) concept is used during finite element simulation of rolling processes. As a result, detailed information regarding microstructure morphology as well as e.g. strain localization development along various micro scale features can be obtained. This is an input for the CA model of the static recrystallization occurring between subsequent stages of multi-pass hot rolling.

Keywords: static recrystallization, rolling, continuous annealing, cellular automata

1. Introduction

A multi-pass rolling process at elevated temperature can be realized using either continuous rolling mill or reverse rolling stand. In the latter case, material is fed forward and backwards through the set of the work rolls where it undergoes different direction straining compared to the case when it is fed forward through the consequent rolling stands in a continuous manner. This imposes different strain path what in turn, leads to different microstructure evolution and final properties between these two described cases. Therefore the strain path is a processing parameter to be taken into account when a complex history of deformation is considered. In steels, strain path changes affect microstructure evolution - especially during hot deformation in the austenitic range. They cause delays in the kinetics of recrystallization (both dynamic - DRX and static - SRX) [1], strain-induced precipitation process (SIP) [2] or phase transformations [3, 4]. The aim of current work is understanding of the progress of SRX in microalloyed austenite subjected to hot rolling process using reverse rolling mills. Numerical multiscale model of microstructure evolution during hot multi-pass rolling and subsequent cooling is proposed and tested in terms of its sensitivity for different strain paths. The idea is to develop a multiscale numerical model of hot rolling combining Finite Element (FE) and Cellular Automata (CA) approaches to model multi-pass rolling process and subsequent static recrystallization process upon post deformation cooling. To describe the microstructure behaviour under deformation, a Digital Material Representation (DMR) framework is used during finite element simulation of rolling processes [5]. As a result, detailed information regarding microstructure morphology as well as strain localization development along different micro scale features e.g. grain boundaries, etc. can be obtained. This is an input for the CA model of the static recrystallization occurring between subsequent stages of multi-pass hot rolling.

2. Multiscale Model

In the current study, multiscale model of two pass deformation followed by cooling was prepared in order to check its capability in term of strain path sensitivity. As presented in Figure 1, two passes of hot rolling (at 1100°C) with two different deformation routes were simulated in commercial finite element Abaqus Stadard software using submodelling approach combined with the DMR strategy [6].

Figure 1: Schematic representation of the developed multiscale model

As seen in Figure 1, DMR unit cells (1mm×1mm) with 7 grains were created and attached near the plate surface, where the effects resulting from strain reversal are the most significant. Material behaviour was described using elasto-
plastic model with combined isotropic-kinematic hardening rule [7]. Model parameters were specified using data from flow curves obtained during continuous cooling deformation test. Subsequently, static recrystallization during post deformation cooling was simulated using Cellular Automata approach [8]. During cooling, temperature dropped 20°C from deformation temperature during 20s. Input microstructure for the CA model was the deformed DMR from the micro scale FEM model.

### 3. CA Static Recrystallization Model

In the SRX CA model, a cell can be in one of two possible states: unrecrystallized or recrystallized, respectively. Each CA cell in the unrecrystallized state can become the nucleon of a recrystallized grain with the probability determined by:

\[ p = N S t, \]

where: \( S \) – volume in which nucleation can appear, \( t \) – time, \( N \) – coefficient computed by:

\[ N = M X \exp\left(-\frac{Q_n}{RT}\right), \]

where: \( Q_n \) – activation energy for nucleation, \( R \) – universal gas constant, \( T \) – temperature, \( M \) – coefficient.

After the CA cell becomes a nucleon of a new recrystallized grain, the grain growth process is simulated. This process is driven by stored energy or grain boundary curvature. The velocity of a moving grain boundary is calculated as:

\[ v = MP, \]

where: \( M \) – grain boundary mobility, \( P \) – net pressure on the grain boundary.

Finally, based on velocity, the cell coverage by the recrystallization front is calculated by:

\[ RX_{\text{fraction}} = RX_{\text{fraction}} - \sum_{j=1}^{n} \left( \frac{v_j f_{\text{step}}}{c_i} \right), \]

where: \( RX_{\text{fraction}} \) – the level of coverage of the i-th cell in the previous (i-1) time step, \( r_x \) – number of recrystallized neighbours (stored energy driving force) or number of neighbouring cells that belong to recrystallized grains (grain boundary curvature driving force), \( v_j \) – velocity of the recrystallization front or recrystallized grain boundary, \( f_{\text{step}} \) – length of time step, \( c_i \) – cellular automata cell size. When the recrystallized volume fraction reaches value of one, the cell changes its state to recrystallized. More details of the model can be found in earlier authors work [5, 8].

The proposed model of the initial stage of multipass deformation with two different deformation routes was used to investigate the strain path sensitivity during deformation and during post deformation cooling (with the same total strain).

The results obtained from the CA model after hot rolling under two deformation conditions is presented in Figure 2. Faster progress in SRX in forward/forward rolling, presented in Figure 2a, compared to forward/reverse rolling (Figure 2c) clearly indicates that applied modelling approach successfully captures effects of applied strain paths. These results are in line with the experimental study, where strain reversal also led to much slower kinetics of SRX in the reversed sample where there was much less potential nucleation sites and lower activation energy for SRX.

![Figure 2: Example of SRX simulation results: progress in SRX in the consecutive time steps during cooling after deformation with Forward/Forward -a), and Forward/Reverse -c) deformation schedules and corresponding accumulated energy values -b, -d) respectively](image)

The obtained material morphologies will be used in the future work to investigate material behaviour under subsequent rolling passes.

### References


Material model development for numerical simulation of the incremental forming process.

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Abstract

The main aim of the paper is to develop of a flow stress model of a commercially pure aluminium for subsequent numerical calculations of material deformation during an innovative incremental forming (IF) process. The IF is used to obtain light and durable integral elements with stiffening ribs dedicated for the aerospace industry. At first, a brief concept of novel incremental forging process is presented. Then, uniaxial compression plastometric tests are described. An inverse analysis is applied to transfer measured load-displacement data into stress-strain curves. Description of the main advantages of the inverse approach are presented within the paper. Finally, obtained flow stress data are applied to the numerical simulation of the incremental forging process.

Keywords: inverse analysis, incremental forming, numerical analysis

1. Introduction

Natural environment becomes more and more endangered by different pollutions. The rising necessity of its protection makes development of new materials as well as innovative manufacturing solutions a high request. Additionally, rigorous European Union regulations are introduced to limit the carbon dioxide and noise emission as well as electricity consumption during different stages of production. A worldwide environmental protection policy insists on limitation of factors that are dangerous for the natural environment, as observed in goals of the European Framework Program of Research and Innovation (2014-2020) – „Horizon 2020”. Most of the research is focused on reduction in emission of harmful means, produced by air transport. Such efforts were also visible in 7th Framework Program projects, such as: NINHA, X-NOISE EV, MARS or DAEDALOS.

Another possibility to meet the mentioned requirements is the weight reduction of commonly used conveyances (i.e. cars, trucks, airplanes, transport aircrafts). It is possible its way to reduce the amount of consumed fuel, in consequence to reduce carbon dioxide emission into the atmosphere. One solution is to replace several conjugate components by one integral element, made from a single part of the material. An integral element is lighter, more durable and less susceptible for cracking in vital locations during operation, due to joint avoidance. In order to obtain integral parts, manufacturing technologies based on machining, rolling, extrusion or casting were developed in recent years [1]. Regrettably, most of them detect huge costs, technical problems, small efficiency or limited applications [1].

Development of an innovative incremental forming process seems to be the best solution to obtain integral elements with required properties. A possible solution is an incremental bulk forming process to get manufacturing shapes impossible while using conventional forging methods. Overestimated loads recorded on the presses during conventional forging can be eliminated dividing the die into a series of small anvils to complete sequentially the entire deformation [2]. Such technology can be also successfully used to manufacture products from materials considered hardly deformable [2].

Incremental forming process, creates integral elements during a single process, without additional finishing and assembly an operations. Fig. 1 presents the novel technology with an additional die in the form of a roll with reciprocating movement that presses subsequent anvils into the material until the expected indentation depth is reached.

Figure 1: Concept of the incremental forming process [3]

Before this innovatory process can be applied into industrial conditions, a detailed knowledge on mechanisms that control deformation and microstructure evolution during mentioned complex conditions is required. Experimental studies are usually employed to provide such knowledge, but they are expensive and time consuming, especially when material behaviour at the microscale level is investigated. This problem can be solved applying less expensive numerical approaches, in order to support and broaden the experimental analysis. However, the quality of numerical results depends on material model used during numerical simulation. Thus, the objective of the paper is to develop a material flow stress model of the investigated aluminum alloy by means of the inverse analysis.

2. Inverse analysis

In order to obtain a material model in the form of stress-strain curves, on the basis of data from experimental tests, an inverse analysis method was used [4]. Such approach results in a flow stress model from force-displacement curves, obtained by e.g. uniaxial compression tests. The approach takes also into account influence of various disturbances related to deformation heating, friction, etc. Inverse analysis is based on three major steps: 1- performance of a series of plastometric tests, 2-
development of a direct problem model on the basis of the finite element method and 3- minimization of the objective function.

The first step is performed by a series of experimental uniaxial compression tests performed on the Gleeble 3800 thermo-mechanical simulator. Different deformation process parameters involving 3 strain rates (0.1, 1, 10 s⁻¹) and 3 temperatures (20, 50, 100 °C) were applied. The results in the form of load-displacement data are used as an input for the two subsequent steps: numerical simulation and optimization. Examples of forces recorded during mentioned tests are presented in Fig. 2.

A numerical model of the direct problem was developed within the in-house finite element code [3], and corresponds to the selected plastometric test (compression test) and process conditions. An optimization algorithm was based on a non-gradient technique, while the objective function was defined as a mean square root error between measured and recorded loads. Examples of results in the form of stress-strain curves are presented in Fig. 3. The material model was used to perform simulation of incremental forming of integral elements in the Abaqus software. The material data were implemented in a tabular form for different strain rates and temperatures. The developed numerical model of IF consists of 3 anvils only pressed by a moving roll into the material by 0.2 mm in each indentation, until the total indentation depth of 0.8 is reached. Thus, anvils are subsequently pressed into the sample from one side to the another and two times backwards t. The specimen and anvil dimensions are 50 × 20 mm and 5 × 10 mm, respectively (Fig. 4). The examples of obtained strain distributions are presented in Fig. 5.

In the further work a more detailed analysis of an impact of different process parameters (e.g. roll diameter, roll frequency, indentation depth, etc.) on the quality of obtained parts will be investigated. In order to improve numerical analysis, a multiscale model of proposed incremental forming process will be also developed, focused on material behaviour at the macro- and micro-scale level. The multiscale model will be based on a Digital Material Representation concept.

References


Geometric and mechanical Representative Elementary Volume for polydisperse granular materials

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Abstract

The 3D DEM simulations were conducted with sphere packings to determine minimal representative elementary volume for the parameters of interest. The geometrical REV for porosity and coordination number, and the mechanical REV for the elastic parameters and pressure ratio were estimated. The polydisperse systems with the same mean particle diameter $D_m$ and standard deviations of particle mean diameters ranging from 0 to 0.6$D_m$ were poured into a test chamber of rectangular cross-section and subjected to uniaxial compression. The thickness of the sample, related to mean particle diameter, ranged from 3$D_m$ to 25$D_m$. Experiments showed that the dimension of granular packing corresponding to 15 mean particle diameters was enough to minimize wall effects and to define average values. Particular systems of that size accurately represent a larger system, regardless of the degree of polydispersity of the sample. Thus the dimension not smaller than 15 times the particle mean diameter may be considered minimum REV thickness for packings comprising both, uniformly and non-uniformly sized spheres.

Keywords: REV size, polydisperse granular packing, Discrete Element Method

1. Introduction

Representative elementary volume (REV), is the smallest volume over which the macroscopic mechanical behavior of material can be defined from averaging. The concept of a representative elementary volume, introduced by Bachmat and Bear [1] and commonly applied in solid and granular mechanics, provides an effective means of developing macroscopic measures in the description of materials and is critical to understand and predict the behavior of effective properties of complex heterogeneous materials at different scales.

Two methods most frequently applied for determining REV size of sample for prediction of effective macroscale parameters may be stated. In the first method a minimum representative elementary volume of sample is assuring constant porosity; while the second method is based on macroscale parameters only.

The large-scale handling and processing of granular materials in many branches of industry requires improved insight into the complex nature of particular assemblies; however, due to the difficulties associated with the measurement and characterization of granular microstructure, the determination of REV size for granular packings has still remained an open issue. Therefore, this study deals with the minimum REV size for granular packings with various particle size distributions and influence of the sample size on the parameter of interest (porosity, elastic modulus and pressure ratio).

2. Model description

Discrete Element Method (DEM), based on a microstructural approach [2], with the non-linear viscoelastic Hertz-Mindlin contact model was applied to model granular packings. Three-dimensional simulations of uniaxial compression test were conducted using the EDEM software [3]. The uniaxial compression test is a common method to determine mechanical properties of granular materials which are of interest to technological process designers.

The polydisperse packings composed of spheres with the same mean particle diameter ($D_m = 7.3$ mm) and standard deviations of particle mean diameters (s.d.) ranging from 0 to 0.6$D_m$ were poured into a test chamber of rectangular cross-section (Fig. 1). The width and height of chamber were fixed at 0.12m, which corresponded to 16 particle mean diameters. The thickness of the sample ($T$), related to mean particle diameter, ranged from 3$D_m$ to 25$D_m$. The walls of chamber were frictional boundaries that did not deform under the applied load. The number of particles in the chamber varied from 350 to 7200, depending on the thickness of a specimen and standard deviation of a particle mean diameter. The input parameters for the spheres and the steel are listed in Table 1.

In the first stage of the simulation, the chamber was completely filled with randomly generated particles, which settled under gravity. As soon as equilibrium was reached, spheres were compressed through the top cover of the chamber that moved vertically downwards at a constant velocity of 3 m/min. The movement of top cover was continued until it exerted a pressure of 100 kPa on the uppermost spheres. Three replications were performed for each particle size distribution and sample size.

Figure 1: Initial configurations of the specimens with different sizes 5$D_m$ (a), 25$D_m$ (b)
Table 1: Simulation input parameters [e-Funda]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Particle</th>
<th>Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poisson’s ratio</td>
<td>0.26</td>
<td>0.3</td>
</tr>
<tr>
<td>Shear modulus (MPa)</td>
<td>560</td>
<td>2000000</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>1720</td>
<td>7800</td>
</tr>
<tr>
<td>Coefficient of restitution particle-particle</td>
<td>0.21</td>
<td>0.54</td>
</tr>
<tr>
<td>Coefficient of static friction particle-particle</td>
<td>0.4</td>
<td>0.29</td>
</tr>
<tr>
<td>Coefficient of rolling friction particle-particle</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

3. Results

3.1. Geometrical REV

Figure 2 presents evolution of initial porosity ($\phi$), defined as the ratio of the volume of voids in system to total volume of the particular system after deposition, in packings with various degrees of particle size heterogeneity with increasing thickness of packing. The mean values are shown; the error bars indicate ± one s.d. The decrease in $\phi$ value ranged from above 7% to 11% with the increase in system thickness from 3 to 15 particle mean diameters in monodisperse system and the one with standard deviation of particle mean diameter of $0.6D_m$, respectively. Slight differences between parameter values calculated for higher $T$ values were observed, which allowed $T=15D_m$ to be considered as a minimum REV size for porosity.

![Figure 2: Initial porosities of packings with various thickness and standard deviations of particle mean diameter: $0D_m$ (a) and $0.6D_m$ (b)](image)

3.2. Mechanical REV

The lateral-to-vertical pressure ratio ($k_y$), defined as the ratio of the horizontal pressure to vertical pressure, was calculated for the investigated samples. Figure 3 illustrates the evolution of the lateral-to-vertical pressure ratio in y direction (see Fig. 1) with increasing thickness of sample subjected to vertical load of 100 kPa. The $k_y$ values varied significantly with increasing sample size until its thickness exceeded 10 times mean particle diameter.

![Figure 3: Lateral-to-vertical pressure ratio in y direction for samples with various thickness and standard deviations of particle mean diameter: $0D_m$ (a) and $0.6D_m$ (b), when subjected to a vertical pressure of 100 kPa](image)

Figure 4 shows effective elastic modulus for monodisperse and polydisperse packings with various sizes subjected to vertical pressure of 100 kPa. Slight effect of the sample size on stiffness of granular system was observed, regardless of the degree of particle size heterogeneity. The differences between $E$ values for samples of different sizes lied within the range of scatter; however values of parameters calculated for the thinnest systems were affected by the highest errors. Therefore, $T=5D_m$ was considered as a minimum REV size for elastic parameters.

![Figure 4: Effective elastic modulus for samples with various thickness and standard deviations of particle mean diameter: $0D_m$ (a) and $0.6D_m$ (b), when subjected to a vertical pressure of 100 kPa](image)

4. Conclusions

The 3D DEM simulations were conducted with sphere packings to determine minimum representative elementary volume for porosity, elastic parameter and pressure ratio. Polydispersity of system was found to have no effect on the REV for the parameters of interest and geometric and mechanical REV for granular systems had different sizes. The results revealed that the dimension of granular packing corresponding to 15 mean particle diameters may be considered as a minimal REV size.

References


Numerical verification of two-scale approach for cancellous bone modelling

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Abstract

This document is focused on the numerical verification of two-scale modelling approach for two scale bone microstructure modelling. The study is a part of long-term project on bone remodelling which drives bone microstructure change based directly on trabeculae surface energy. The two-scale approach is based on a first-order computational homogenization technique. The set of test analysis variants will be presented. Both benchmark test and real cases are used to verify two-scale approach. The final verification was done based on calculation of the real bone structure (a piece of female Wistar rat bone) with the use of both one- and two-scale approaches.

Keywords: two-scale modelling, computational homogenization, bone microstructure modelling, numerical verification

1. Introduction

Bone tissue is a strongly heterogeneous and changing in time material. The structure and characteristics of two basic types of bone, cortical and cancellous, are different. In a macro-scale, both types of bone can be regarded as a continuum. However, in one order of magnitude smaller scale the cortical bone can be still considered continuum but the cancellous bone is visibly a non-continuous, heterogeneous material. From the mechanic viewpoint, it is a geometrically complex three-dimensional structure (see Fig. 3). Due to biomechanical process the shape and topology of the trabeculae are still changing. The orthotropic macroscopic properties of bone are changing through this process as well. On the other hand, the macroscopic behavior of the bone determines the internal strain state as a result of external loading. The internal strain state in the cancellous bone microstructure is a key to its remodeling.

The main goal of the work presented in this paper is to develop a robust method to model cancellous bone microstructure in the case of whole bone size FE simulation of bone remodeling phenomenon [4]. In this case numerical simulation needs a huge FE model which reflects a non-continuous structure of bone microstructure. The simplest way to reduce the size of FE model in such cases is to use homogenization. Unfortunately, the classical homogenization methods cannot be used directly here. The possible approach for this case should take into account the non-continuity of cancellous bone microstructure and enable the use average properties of cancellous bone in the analysis of the whole bone simultaneously. The method which passes these requirements is a two-scale computational homogenization [1].

Before the bone remodelling simulation the verification of the developed method has to be done. In the paper numerical verification based on calculation of the real bone structure (a piece of female Wistar rat bone) with the use of both one- and two-scale approaches is presented.

2. Two-scale approach fundamentals

The two-scale approach is based on the first-order computational homogenization technique, where the response at a macro-scale material point depends only on the first gradient of the displacement field. The basic scheme of the computational homogenization procedure is shown in Fig. 1.

![Figure 1: Computational homogenization procedure](image)

The macroscopic deformation gradient tensor $F_{M}$ is calculated for each integration point of the macro-scale FE model. Next, it is used to define the boundary conditions of the micro-scale model which corresponds to particular integration points. This micro-model defines the macrostructure boundary value problem and can be understood in the sense of the RVE. The macroscopic stress tensor $P_{M}$ is calculated based on the results from the micro-model. The stress-deformation relationship at the macro-scale level (macro-model element) is defined as a local macroscopic stiffness tangent matrix. This equivalent matrix is derived directly from the microstructural stiffness. We assume that micro-model is linear, the strains are small and the bone material is elastic. Taking into account the above assumptions, the tangent stiffness matrix can be easily calculated [2].

*The support of the Ministry of Science and Higher Education under grants R13 0020/06 and N518 328835 is highly acknowledged.
3. Models

In the presented approach the finite element method is used to realize computational homogenization procedure on macro- and micro-levels. Both macro- and micro-models are created in Abaqus code. In the preliminary stage of study the homogenization technique has been verified using a reference bone-like discontinuous microstructure (cube with spherical holes). Both the micro- and macro-scales are modelled as linear, homogeneous, isotropic elastic solids. Three load cases are checked: compression, shear and torsion.

4. Results

The results obtained for a two-scale modelling are close to results obtained with the use of one-scale approach. The relative errors of the global deformation are equal to 5.7% and 4.7% for compression and shear, respectively. They clearly correspond to the results of two-scale modelling verification presented in Table 1.

<table>
<thead>
<tr>
<th>Load case</th>
<th>Two-scale</th>
<th>One-scale</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>compression</td>
<td>5.5271e-3</td>
<td>5.2218e-3</td>
<td>5.7%</td>
</tr>
<tr>
<td>shear</td>
<td>2.6705e-2</td>
<td>2.5564e-2</td>
<td>4.7%</td>
</tr>
</tbody>
</table>

5. Conclusions

The obtained results show that proposed modelling approach can be used to simulate bone microstructure with satisfactory accuracy. The analysis of a piece of rat bone shows that presented two-scale approach makes it possible to reduce significantly the size of the problem from system of questions with more than 36 million unknowns to set of 2,774 systems of questions with no more than 27,000 unknowns each one. This technique opens a door to the effective simulations of bone remodelling on the level of particular trabeculae as a common practice in biomechanical studies.

References

Multiscale modelling of the acoustic waves in rigid porous and fibrous materials

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Abstract

This paper presents the multiscale approach to the problem of acoustic waves propagating in a fluid (air) inside rigid fibrous of porous materials with open porosity. The approach essentially consists of the finite element analyses of three relevant problems defined on the representative fluid domain of a porous medium, the averaging and up-scaling techniques applied to calculate some necessary parameters of the porous microstructure which are used to model the effective properties of a homogenized fluid equivalent to the porous medium, and finally, the solution of a relevant Helmholtz problem on the macro-scale level in order to estimate, e.g., the acoustic absorption of the porous medium. This approach is illustrated by two examples: experimentally validated analyses of a fibrous material made up of a copper wire based on two Representative Volume Elements, and an analysis of a foam with spherical pores using a randomly generated periodic representative cell.

Keywords: multiscale modelling, representative microstructures, porous materials, acoustic waves

1. Introduction

The absorption and propagation of acoustic waves in porous (or fibrous) media with sufficiently rigid frame (skeleton) and open porosity can be predicted by the so-called Johnson-Champoux-Allard-Pride-Lafarge (JCAPL) model [1], where a porous medium is represented on the macro-scale level by an effective equivalent fluid so that the classic Helmholtz equation of linear acoustics can be used. The effective speed of sound and density of the equivalent fluid are frequency-dependent, they are calculated from the JCAPL formulas which involve parameters of the actual fluid (air) which fills the pores and the so-called transport parameters which solely depend on the micro-scale geometry of the porous medium.

A multiscale modelling of the problem of acoustic wave propagation and dissipation in porous media consists of three stages: first, the transport parameters are calculated from a microstructure of porous medium, then, the JCAPL formulas are used to estimate the effective speed of sound and density of the equivalent fluid, which finally serve to solve a relevant Helmholtz problem on the macro-scale. The transport parameters are computed by solving three independent Boundary Value Problems (BVPs) on the macro-scale level of porous medium [2, 3], namely:

- the viscous incompressible flow (i.e., the Stokes flow) through porous medium with no-slip boundary conditions on the solid skeleton walls;
- the steady heat transfer with isothermal boundary conditions on the solid skeleton walls;
- the Laplace problem.

The BVPs are defined on the fluid domain of a Representative Volume Element (RVE) of porous medium; they are usually solved using the Finite Element Method (FEM). After the finite element analyses are carried out, the averaging over the fluid domain is applied to the solutions and the up-scaling formulas are used to determine the transport parameters.

2. Examples of multiscale modelling

2.1. Two RVEs for a fibrous material

Two fibrous samples were manufactured from a copper wire with diameter 0.5 mm: they were manually woven and fitted into an impedance tube with diameter 29 mm. The heights of samples are 30 mm and 60 mm, while the total length of wire used for the first sample was 10 m, and for the second one it was 20 m, so that both samples have the same porosity of 90%. The acoustic absorption coefficient was measured for both samples in the impedance tube: first, for each sample separately, and then, for two configurations of both sample, that is, with one sample on the top of the other, so that the total height of such fibrous layer was 90 mm (i.e., 30 mm + 60 mm or 60 mm + 30 mm).

![Figure 1: Simplified periodic representations of microstructure for the fibrous material of copper wire](image)

Two Representative Volume Elements, RVE-1 and RVE-2 (see Fig. 1), were constructed for the fibrous material. Both are periodic and contain only straight fibres, which entails the assumption that the copper fibres in fibrous samples are locally straight. The RVE-1 has uniformly-spaced fibres, while in the RVE-2 the fibres are grouped in a layer normal to the direction of the propagation of acoustic waves; the porosity is always 90%
and the diameter of all fibres is 0.5 mm. For both RVEs the microstructure-based calculations were effectuated to estimate the effective speed of sound and density, so that the sound absorption could be computed for layers 30 mm, 60 mm, and 90 mm thick. Figure 2 compares these results with the corresponding measurements from the impedance tube, in the frequency range from 500 Hz to 5 kHz. It appears that the results obtained for the RVE-2 are very similar to the experimental curves, which means that the simplifying assumption of straight fibres is admissible.

2.2. A foam with spherical pores

A periodic representative microstructure for a foam with spherical pores was randomly generated using an algorithm which simulates the dynamic mixing of spheres (see Fig. 3). In the algorithm, the spheres may to some small extent penetrate each other, and eventually, they become pores linked with windows. The procedure was carried out for a case with five total pores in a cubic RVE (each pore represented by a family of eight identical spheres), and it was accomplished when the designed porosity of 70% was reached (see Fig. 4).

The size of RVE was set so that the average diameter of pores was 0.33 mm. The micro-scale finite element analyses were carried out on the fluid domain of RVE in order to calculate the transport parameters, and then, to estimate the macro-scale effective celerity of acoustic waves propagating in such a foam (see Fig. 5).

3. Conclusions

Usually, more than a few pores or fibres in a periodic cell (RVE) are necessary to well represent real porous or fibrous materials, however, this makes the cell larger, and the size of a large RVE may at higher frequencies become comparable with the wavelengths, which may decrease the accuracy and reliability of estimations, because of a weak separation of scales.

References

 Numerical Modelling in Hydraulic Fracturing and Related Problems

organized by G. Mishuris and A. Linkov
On simulation and interpretation of seismicity accompanying hydraulic fractures

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Abstract

The objective of the work is to present a general approach to modeling of seismicity and its application to numerical simulation of seismicity induced by hydraulic fracturing (HF). The approach extends the method developed for mining problems to the HF problem taking into account the changes in the geometry and in the fluid pressure. Numerical implementation is performed on the basis of the hypersingular boundary element method. The net-pressure is prescribed in accordance with the approximate analytical solution accounting for the pumping rate, viscous properties of a fluid, elastic modules and in-situ stresses in rock formation. The approach is illustrated by results of numerical experiments. Being in agreement with the data of seismic monitoring, the results show the potential of the method developed for a better understanding and control of hydraulic treatments.

Keywords: seismicity, hydraulic fracture, boundary element method, numerical modelling

1. Introduction

Microseismic observations present an important means to make conclusions on the fracture growth and its influence on the state of embedding rock in real time (see, e.g. [4,6]). In view of high cost, quite a limited number of treatments are performed with registering seismic events. Even having seismic data, their interpretation is complicated by many uncertainties. Thus, it appears reasonable to complement and compare direct seismic observations with seismicity simulated numerically. This path of joining rock geomechanics with seismology gains growing recognition of scientists and engineers [7]. Below we present the general concepts of the numerical simulation of seismicity as regards to its application to hydraulic fractures (HF). The results of numerical experiments illustrate the potential of the method discussed.

2. Key components of seismicity simulation

The modern theory of numerical simulation of seismicity stems from the pioneering work by Salamon [5], who suggested it for mining applications. It has been further developed during the following decades. The key idea of the method consists in the analysis of potential sources of seismic events (crack-like flaws, pre-existing fracture network). The behavior of a source is defined by processes, which run in time and change stresses in rock and tractions on flaw surfaces. The theory includes general issues, which stay for wide classes of problems concerning with mining, hydraulic fracturing, geothermal processes and geotectonic activity. Specifically, the fundamental elements refer to modeling of a single seismic or aseismic event, to simulation of multiple events, to distinguishing the external and internal characteristic time, etc. On the other hand, the theory includes also flexible elements which change depending on a particular problem and on a particular computational implementation. Specifically, for HF problems, the changes in the net-pressure depend on such parameters as fluid viscosity and leak-off into formation. The comprehensive historical review and the theory of the numerical simulation with distinguishing fundamental and flexible elements may be found in the review [2]. We use the theory for the HF problem.

3. Numerical implementation

A numerical realization of the general method consists in complementing a basic conventional code, serving for stress evaluation (e.g. BEM, FEM, DEM, FDM, H-BEM), with a number of additional ‘seismic’ subroutines. The differences concern mostly with details of the subroutines defined by peculiarities of the basic code. In the considered problem, the basic code employed is the code of the hypersingular boundary element method (H-BEM). The density functions entering the starting hypersingular boundary equations represent the very physical quantities (displacement discontinuities and the tractions), which describe boundary and contact conditions on surfaces of both the hydraulic fracture and a flaw.

The deterministic input data for the basic code include the initial geometry, its changes caused by the fracture propagation, physical properties of rock, in-situ stresses and suggested in the paper [3] approximate analytical expression for the net pressure on successive time steps. The deterministic and statistical data used for seeding the initial flaws include prescribing the volume of possible event locations around the expected fracture, the number of seeded flaws, their locations, density, mechanical properties. These data are prescribed by the ‘seismic’ subroutine ‘FlawInput’. The seeded flaws, for which in situ tractions exceed the tensile or shear strength of a flaw, are excluded from further consideration. This initialization is done by the subroutine ‘ExclusInSitu’. Steps in time are performed by the subroutine ‘SeismTimeStep’. At the beginning of a step, the basic H-BEM code calculates tractions at the location of each active flaw. Then cycles are performed in (i) stages of events and (ii) chains of events. Tractions induced by flaws, at which events occur, are accounted for by the further subroutine ‘TracFlaw3D’. In time steps, a computer code saves data on each simulated event: its number; the number of the flaw in the

*The second and third authors appreciate the support of the TAMEP-PEOPLE-2013-IRSES program, grant number 610547.
global numeration; the time, at which the event is simulated; the
type of the event (seismic or aseismic) and its characteristics.
These are the output data, sorted and analysed by the subroutine
“OutputFlaw”.

4. Results of numerical simulation

We consider the hydraulic fracturing treatment at the depth
of 500 meters. The volume with seeded square flaws is taken as
a parallelepiped. The fracture height is constant, equal to 50 m.
After 10 ten-meter steps the final length of the fracture is 100 m
(for details, see [1]).

Both, number and magnitude of simulated events do not
change significantly on time steps. We observe notably fewer
and weaker events than in similar mining problems: there are no
simulated events with the magnitude exceeding -4. This agrees
with the data of observations [6]. The Gutenberg-Richter type
dependence for events simulated in 10 steps of HF propagation
is presented in Fig. 1 in comparison with the analogous depend-
ence for one step of mining. It can be seen that although the
forms of the curves are similar, there is remarkable difference in
magnitudes of the events. The difference is caused by a drastic
dissimilarity in the level and even in the signs of stresses, which
induce seismic events in these two problems.

5. Conclusions and further work

The conclusions from the study performed are as follows.
(i) The general features of the simulated events (magnitude,
scattering, distributions) agree with the recorded data.
(ii) The data on events occurred on time steps are more informa-
tive than that for the totality of the events. An analysis of these
data allows one to trace the front propagation and to conclude
on the final size of a hydraulic fracture.
(iii) The analysis shows the potential of coupling synthetic
seismicity and seismic monitoring. To use it in real time of HF
stimulation, there is a need to radically increase of the effic ien-
cy of the basic code. Its improvement is planned for further
work.

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Generalized Newtonian fluid flow and heat transfer in an internally finned tube

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Abstract

The paper deals with a problem of generalized Newtonian fluid flow and heat transfer in an internally finned tube. Both nonlinear governing equation of generalized Newtonian fluid flow and nonlinear energy equation are solved using the Picard iteration method. In the method nonlinear problem is transformed into a sequence of inhomogeneous problems. On each iteration step an approximate solution consists of two parts: the general solution and the particular solution. The particular solution is obtained using the radial basis functions and monomial interpolation and the general solution is obtained by means of the method of fundamental solutions.

Keywords: generalized Newtonian fluid, internally finned tube, method of fundamental solutions, radial basis functions

1. Introduction

Heat transfer and fluid flow in internally finned tubes is a very important problem from a practical point of view. In the literature on numerical analysis of such problems many different numerical methods were used: the finite element method [1], the finite difference method [5] or the finite volume method [4]. In the paper an alternative method is applied to solve the problem of heat and mass transfer in an internally finned tube. This method is called the method of fundamental solutions (MFS) and it is one of meshless methods.

In the modelling of many practical issues use of non-Newtonian fluid models is more appropriate, e.g. in food industry, chemical industry or biofluid flows. One of non-Newtonian fluid class is a generalized Newtonian fluid (GNF) class. In these models shear rate at any point is dependent only on the value of the shear stress. Another names for these kinds of non-Newtonian fluids in the literature are: time independent fluids, pure viscous fluids or inelastic fluids. The GNF are subdivided into three types of fluids: shear-thinning (or pseudoplastic) fluids, viscoplastic fluids and shear-thickening (or dilatant) fluids [2].

In the paper heat and mass transfer of the GNF in an internally finned tube is considered by means of the MFS and the radial basis functions (RBF).

2. Mathematical description of the problem

In this section geometry and governing equations for fluid flow and heat transfer of a generalized Newtonian (GNF) fluid are presented as follows.

2.1. Geometry of the problem

The geometry of the considered problem is depicted in Fig. 1. The repeating part of the region with characteristic dimensions is presented in Fig. 2.

2.2. Governing momentum equation

The problem of GNF flow in internally finned tube (in region $\Omega_1$) is described by a following dimensionless governing equation in the cylindrical coordinates ($r, \beta, \vartheta$):

$$\frac{\partial^2 W}{\partial r^2} + \frac{1}{r} \frac{\partial W}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W}{\partial \vartheta^2} = -1 \left(1 + \frac{\partial E}{\partial r} \frac{\partial W}{\partial r} + \frac{1}{r^2} \frac{\partial E}{\partial \vartheta} \frac{\partial W}{\partial \vartheta} \right)$$

where $W$ is a dimensionless velocity of the fluid, and $E$ is a dimensionless viscosity function. A form of the function is dependent on a specified model of the GNF.

Figure 1: Cross-section of an internally finned tube

Figure 2: The repeating part of the considered problem

The following boundary conditions are formulated for the fluid flow problem:

$$\frac{\partial W}{\partial r} = 0 \text{ on } AB \text{ and } EA,$$

$$W = 0 \text{ on } BC, CD \text{ and } DE.$$
The average velocity in region $\Omega_i$ is defined as:

$$W_i = \frac{\int_{\Omega_i} \mathbf{w} \, d\Omega_i}{\int_{\Omega_i} d\Omega_i}.$$  \hspace{1cm} (4)

A product of friction factor and Reynolds number for laminar flow can be calculated from the following formula:

$$f \text{Re} = \frac{2A'}{W_e P^*}.$$  \hspace{1cm} (5)

where $A'$ is dimensionless area of a cross-section and $P^*$ is a wetted perimeter.

2.3. Governing energy equation

The dimensionless governing energy equation for the fluid (in region $\Omega_1$) in the cylindrical coordinates $(r, \theta, z)$ takes the following form:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} = f \text{Re} W_e \frac{\Phi}{\Phi_{av}},$$  \hspace{1cm} (6)

where $\Phi$ is dimensionless temperature of the fluid and the average dimensionless temperature of the fluid is defined as:

$$\Phi_{av} = \frac{\int_{\Omega_1} \Phi \, d\Omega_1}{\int_{\Omega_1} d\Omega_1}.$$  \hspace{1cm} (7)

The dimensionless governing energy equation in the wall (in region $\Omega_2$) is represented by:

$$\frac{\partial^2 \Phi_w}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi_w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi_w}{\partial \theta^2} = 0,$$  \hspace{1cm} (8)

where $\Phi_w$ is temperature of the wall.

The boundary conditions for the temperature field are formulated as follows:

$$\frac{\partial \Phi}{\partial n} = 0 \text{ on } AB \text{ and } EA,$$  \hspace{1cm} (9)

$$\Phi = \Phi_{av} \text{ on } BC, CD \text{ and } DE,$$  \hspace{1cm} (10)

$$\frac{\partial \Phi}{\partial n} = K \frac{\partial \Phi}{\partial n} \text{ on } BC, CD \text{ and } DE,$$  \hspace{1cm} (11)

$$\frac{\partial \Phi}{\partial \theta} = 0 \text{ on } BG \text{ and } EF,$$  \hspace{1cm} (12)

$$\Phi_w = 0 \text{ on } GF,$$  \hspace{1cm} (13)

where $K$ is dimensionless thermal conductivity and $n$ is normal direction.

Nusselt number is defined as:

$$Nu = -\frac{2}{\Phi_{av}}.$$  \hspace{1cm} (14)

3. Proposed method of solution

In the proposed solution method the Picard iteration method is used to transform the nonlinear governing equations into a sequence of inhomogeneous problems. Then nonlinear Eqn (1) can be written in the following form:

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} = -\frac{1}{f} \left( \frac{1}{r^2} \left( \frac{\partial \psi}{\partial r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial \psi}{\partial \theta} \frac{\partial \psi}{\partial \theta} \right) \right),$$  \hspace{1cm} (15)

where $i$ is number of iteration. In the same way Eqn (6) can be written in the form:

$$\frac{\partial^2 \Psi}{\partial r^2} + 1 \frac{\partial \Psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2} = f \text{Re} W_e \frac{\Psi}{\Phi_{av}}.$$  \hspace{1cm} (16)

On each iteration step the solution consists of two parts: the general solution and the particular solution. In the presented numerical procedure firstly the right-hand side of Eqn (15) (or Eqn (16)) is interpolated using the RBF and monomials. Simultaneously the particular solution is known.

The general solution is obtained using the MFS. In the method the solution is assumed a linear combination of fundamental solutions which are functions of distance between the point and the source point. The source points are located on a pseudo-boundary which is similar to boundary of the considered region and it is located outside the region. Coefficients of the approximate solution are calculated using the boundary collocation technique by fulfilling boundary conditions [3].

The whole numerical algorithm is presented in Table 1.

Table 1: Numerical algorithm of the proposed method of solution

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 1</td>
<td>Calculate solution of fluid flow problem for Newtonian fluid using the MFS.</td>
</tr>
<tr>
<td>Step 2</td>
<td>Interpolate the right hand-side of Eq. (15) on the base of velocity value from previous iteration step using the RBF and monomials.</td>
</tr>
<tr>
<td>Step 3</td>
<td>Calculate the general solution using the MFS.</td>
</tr>
<tr>
<td>Step 4</td>
<td>Check condition for iteration process stopping.</td>
</tr>
<tr>
<td>Step 5</td>
<td>Calculate $W_e$ (using the trapezoidal rule) and the $f \text{Re}$.</td>
</tr>
<tr>
<td>Step 6</td>
<td>Assume that $\Phi/\Phi_{av} = 1$ and $i = 1$.</td>
</tr>
<tr>
<td>Step 7</td>
<td>Interpolate the right hand-side of Eq. (16) using the RBF.</td>
</tr>
<tr>
<td>Step 8</td>
<td>Calculate the general solution using the MFS.</td>
</tr>
<tr>
<td>Step 9</td>
<td>Calculate $\Phi_w$ (using the trapezoidal rule).</td>
</tr>
<tr>
<td>Step 10</td>
<td>Check condition for iteration process stopping.</td>
</tr>
<tr>
<td>Step 11</td>
<td>Calculate Nu.</td>
</tr>
</tbody>
</table>

References


Almost analytical evaluation of influence coefficients for ordinary and edge power-type boundary elements

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Abstract

The scope of the work is to present a method, which maximally employs opportunities for the analytical evaluation of influence coefficients required while solving 3D elasticity and potential problems by the boundary element method. We employ a boundary integral equations (singular and hypersingular) specially tailored for solving problems concerning piece-wise homogeneous regions (finite or infinite) with structural elements, with possible cracks, pores and inclusions. Ordinary and singular (with power-type asymptotics of densities) boundary elements are considered. Evaluation of their influence coefficients is reduced to arithmetic operations with a relatively small number of starting integrals. Specifically, for an ordinary element, the number of starting integrals is 3 only, and they are evaluated analytically. For a square-root singular element, the number of starting integrals is 4, and they are efficiently evaluated after transformation to the Carlson form. For an arbitrary exponent of the asymptotics, the number of starting integrals is at most 8, and being one-dimensional and non-singular, they are also efficiently evaluated numerically. Examples illustrate the accuracy and computational efficiency of the method developed.

Keywords: potential and elasticity problems, boundary integral equations, higher order approximations, ordinary and singular elements, influence coefficients

1. Introduction

There are many problems requiring accurate evaluation of local fields in strongly inhomogeneous media. Such are problems of engineering and material science concerning with stress concentration and stress intensity factors; simulation of crack nucleation and growth; tracing the propagation of the fracture front when modeling hydraulic fractures; evaluation of rock burst hazard in mines, etc. For more than thirty years, numerical modeling of these problems successfully employs 3D boundary element method (BEM), based on singular and hypersingular boundary integral equations (BIE). In fact, all the BIE, suggested for static 3D elasticity and potential problems, involve a limited number of standard integrals. Consequently, when discussing the key component of the BEM, which is the evaluation of the influence coefficients, it is reasonable to consider singular and hypersingular equations, which are general enough. Having in mind wide applications, we start from equations for inhomogeneous media with arbitrary interface conditions at the boundaries of structural blocks. The purpose of the work is to present a method for evaluation of the influence coefficients of higher-order accuracy in the most efficient way. We reduce the problem to arithmetic operations with a few starting integrals, and we show how the starting integrals may be efficiently evaluated. The method includes both ordinary and singular elements.

2. Problem formulation

Consider a domain consisting of isotropic elastic blocks, characterized by a shear module \( \mu \) and a Poisson’s ratio \( \nu \), containing multiple cracks and inclusions of arbitrary shapes. Denote \( S \) the total boundary of all blocks including surfaces of cracks, pores and inclusions in it. The boundary between adjacent blocks is treated as a single surface on which physical fields may experience discontinuities. Suppose firstly that the Poisson’s ratio \( \nu \) is the same for all the blocks. Then singular and hypersingular BIEs for piece-wise homogeneous media [1] are

\[
\begin{align*}
- \int_S U_S(x, \xi) \left( \mu^+ u^+ (\xi) - \mu^- u^- (\xi) \right) dS_\xi + \\
+ \int_S \mu U(x, \xi) \Delta t_n (\xi) dS_t = \frac{1}{2} \left( \mu^+ u^+ (x) + \mu^- u^- (x) \right) \\
\int_S \left( \frac{1}{2\mu^+} J^+_S (x, \xi) t^+_n (\xi) - \frac{1}{2\mu^-} J^-_S (x, \xi) t^-_n (\xi) \right) dS_\xi \\
- \int_S \frac{1}{2\mu} J_H (x, \xi) \Delta u (\xi) dS_\xi = \frac{1}{2} \left( \frac{1}{2\mu^+} t^+_n (x) + \frac{1}{2\mu^-} t^-_n (x) \right)
\end{align*}
\]

where \( x \in S \) is a field point, \( \Delta t_n = t^+_n - t^-_n \) is the traction discontinuity, \( \Delta u = u^+ - u^- \) is the displacement discontinuity. The normal \( n \) is fixed arbitrary on a contact of adjacent blocks, on cracks and inclusions. The index “plus” (“minus”) refers to the limiting value from the side with respect to which the normal \( n \) is outward (inward). The matrix \( U(x, \xi) \) of fundamental solutions is defined by the Kelvin’s solution. Matrices \( U_S(x, \xi) = [J_S(x, \xi)]^T, J_S = T_n(x) (U(x, \xi)) \) and \( J_H(x, \xi) = T_n(x) (J_S(x, \xi))^T \), where \( T_n(x) \) is traction operator. In fact, Eqs (1) and (2) include the typical integrals which appear in many

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other BIEs (direct and indirect) of the static elasticity and potential theory. The same integrals arise also when the Poisson’s ratio of blocks may be different.

3. Approximation of surface and density

We represent the total boundary $S$ as a set of planar trapezoidal boundary elements $S^h$ (in general case these elements may be four-vertex curvilinear boundary elements [2]). This type of boundary elements includes as particular cases commonly used parallelogram, rectangular, square and triangular elements.

Inspection of the integrals present in (1) and (2), shows that to evaluate any of them it is sufficient to consider the only integral:

$$\int_{S^h} \frac{f(y)}{R} \, dS_y, \quad (3)$$

and its first, second and third partial derivatives. In Eqn (3) $R$ is the distance between a field point and integration point. Integration is performed in local Cartesian coordinates of the element (Fig. 1).

![Figure 1: Trapezoidal element in local coordinates.](image)

The density $f(y)$ is a function, which corresponds to a component of the displacement or traction. Its quite general approximation is of the form:

$$f(y) = y^3 \sum_{i+j=0}^{m_p} c_{ij} y_2 y_3, \quad (4)$$

where $\alpha$ is the exponent accounting for the asymptotic behavior of the density near the edge $y_3 = 0$, $m_p$ is the degree of an approximating polynomial, $c_{ij}$ are coefficients of approximation. The exponent $\alpha = 1/2$ corresponds to problems of linear fracture mechanics; $\alpha = 2/3$ appears in problems of hydraulic fracturing, when a Newtonian fluid fractures impermeable rock; $\alpha = 5/8$ for hydraulic fracturing by a Newtonian fluid under strong influence of the fluid leak-off into rock formation; for non-Newtonian thinning fluids the exponent $\alpha$ may be arbitrary in the interval $(0, 1)$. Without loss of generality we may assume $0 \leq \alpha < 1$.

We distinguish the cases of (i) ordinary elements, for which the density is a smooth function of $y_3$ and consequently $\alpha = 0$, (ii) singular edge elements, for which $h_0 = 0$ while $\alpha \neq 0$ and (iii) singular ribbon elements for which $h_0 \neq 0$ and $\alpha \neq 0$. Singular edge elements serve to avoid singular edge elements with high aspect ratio ‘height-to-base’.

4. Evaluation of the main integral and its derivatives

As shown in [2], integral (3), with $f(y)$ defined by (4), and its derivatives are linear combinations of recurrently evaluated three types of integrals:

$$\int_0^h \frac{y^3 y^3}{R_\xi} \, dy_3, \quad J_1 = \int_0^h \frac{y^2 \, dy_3}{(y_3 - z_\xi)^2 R_\xi}, \quad K_1 = \int_0^h \frac{y^2 \, dy_3}{(y_3 - z_\xi)^2 R_\xi}, \quad 0 \leq \alpha < 1, \quad (5)$$

where $R_\xi = \sqrt{x_3^2 + (y_2 - b_\xi - a_\xi y_3)^2 + (x_3 - y_3)^2}$, with at most 16 starting values for $\xi = b, c$ and $s = 0, 2, 1$ when $\alpha = 0$ or $s = -1, 0$ when $0 < \alpha < 1$, $j = 1, 2, j = 1, 2, 3$ when $x_1 \neq 0$ or $j = 1, 2, \ldots, 4$ when $x_1 = 0$.

In cases of a smooth density ($\alpha = 0$) the number of starting integrals is 3 and they are evaluated analytically. For square-root asymptotics ($\alpha = 1/2$) the number of starting integrals is 4. They are elliptic and after transforming to the Carlson form, they are evaluated very efficiently by properly adjusted algorithms (see [2]). For an arbitrary exponent, the number of starting integrals is at most 8. They are also efficiently evaluated numerically.

5. Numerical results

The authors performed many numerical tests to study the accuracy, stability and time expense of the method suggested. They have confirmed the advantages of the approach developed. As an example, consider the 3D benchmark problem of a single planar crack of radius $a$ in an infinite isotropic linear elastic medium subjected to far-field uniform normal stresses $p$. The analytical solution for the SIF $K_1$ is $K_1^{\text{an}} = \sqrt{2\pi a}/\nu$. For the comparison purposes, the calculations are performed using (i) standard approach involving ordinary elements only (ApprC), and (ii) ordinary and singular edge element (ApprO). The results on the accuracy of evaluation the SIF $K_1^{\text{num}}$ for different meshes (the number of boundary elements $N = 18, 22, 1010, 3932$) are given in Table 1. It presents the relative error $\delta K_1^\text{num} = |K_1^{\text{num}} - K_1^{\text{an}}|/K_1^{\text{an}}$. As could be expected, the results show drastic increase of the accuracy when using singular edge elements. For fine meshes, the accuracy becomes an order better than in calculations with ordinary elements only.

<table>
<thead>
<tr>
<th>$N$</th>
<th>ApprC</th>
<th>ApprO</th>
<th>$N$</th>
<th>ApprC</th>
<th>ApprO</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>0.0388</td>
<td>0.1494</td>
<td>1010</td>
<td>0.0342</td>
<td>0.1193</td>
</tr>
<tr>
<td>220</td>
<td>0.0404</td>
<td>0.1870</td>
<td>3932</td>
<td>0.0135</td>
<td>0.1697</td>
</tr>
</tbody>
</table>

Obviously, for a square root edge element, the time expense is expected to be greater than for an ordinary element. However, the actual increase of the time is quite moderate. In the case of normal loading and a mesh of $N = 220$ (1010, 3932) elements, evaluation of the matrix of influence coefficients takes 0.62s (9.66s, 131.35s), while using edge elements with a square root approximation. For comparison, when merely ordinary elements, the time is 0.37s (7.53s, 115.85s). We see that the time expense for evaluation of the influence coefficients does not differ significantly; for fine meshes it is actually the same for calculations with and without singular elements. The time spent for solving the algebraic system is, in fact, independent on the approximation. It is on the level of 0.04s, (2.6s, 140s) for the meshes discussed. Therefore using singular edge elements developed notably increases the accuracy without increasing the computation cost.

We conclude that the method suggested provides an efficient tool for accurate evaluation of physical quantities near points of field concentration. In further work, it is to be combined with the fast multipole method to solve problems for strongly inhomogeneous media with many degrees of freedom.

References


Modified theory, universal asymptotic umbrella and efficient simulation of hydraulic fracturing

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Abstract

The paper presents a modified formulation of hydraulic fracture problem based on the revision of its theoretical rationale. The innovations include using (i) particle velocity instead of flux, (ii) fundamental speed equation to tracing the fracture propagation at each point of the front instead of a single equation of the global mass balance, and (iii) the system of universal asymptotic equations. It appears that the solution of the system (universal asymptotic umbrella) depends merely on the local propagation speed. The inversion of the solution in the speed provides the universal speed equation. It opens wide options for an efficient simulation of hydraulic fracturing by a well-established level set and fast marching methods, not available in conventional approaches. This delivers an efficient means for simulation of hydraulic fracturing in real time on laptops.

Keywords: hydraulic fracturing, revised theory, particle velocity, speed equation, asymptotic solution, numerical modeling, fast marching methods

1. Introduction

Hydraulic fracturing (HF) is extensively used in geological engineering applications such as stimulation of oil, gas and heat production, roof control in mines, waste disposal and remediation of contaminated soils. Its significance has further grown last years because it is a key element of gas production from low permeable shales.

In view of difficulties of direct observations, numerical modeling is of prime importance for HF designs and treatments (see, e.g. reviews in Ref. [1-3, 6-8, 10]). Despite numerous studies have been performed to obtain solutions, numerical simulation of HF remains a “formidable task” (ref. [2], p. 147). The work aims to summarize recent findings [5-8], which open wide options for a rapid improvement of HF modeling.

2. Speed equation. The particle velocity against the flux

The derivative of a volumetric integral with respect to a parameter \( t \) in the case, when the integrand \( \rho(x,t) \) represents a property of a medium, while the parameter is the time, is given by equation, called Reynolds transport theorem,

\[
\frac{d}{dt} \int_{V(t)} \rho(x,t) dV = \int_{V(t)} \frac{\partial \rho}{\partial t} dV + \int_{\partial V(t)} \rho \nu \cdot ds
\]

(1)

Herein \( \nu_n \) is the normal component of the particle velocity on the moving boundary \( S(t) \) of the volume. Note that Eqn (1) is essentially a mathematical result not involving conservation laws, motion equations and properties of a medium. By means of derivation, the normal component of the particle velocity \( \nu_n \) equals to the propagation speed \( v_p \) at a considered point \( x \), at the surface

\[
\lim_{\Delta x \to 0} \nu_n = v_p = \frac{dx}{dt}
\]

(2)

Equation (2) presents the fundamental speed equation (SE). Similar to Eqn (1), it does not involve conservation laws, dynamic equations or constitutive equations. For the first time, in hydraulic fracture studies, the speed equation (2) was distinguished in 1990 by Kemp [4], who used it solving the Nordgren’s [9] problem. Its significance has not been comprehended for twenty subsequent years. Recently it was re-discovered (Ref. [5]), and its crucial role has been clearly demonstrated for a proper tracing of fracture propagation, obtaining simple analytical solutions and developing efficient numerical techniques (Ref. [5-8]).

While applied to a flow of incompressible fluid in a narrow channel under the law of mass conservation, Eqn (1) yields the continuity equation

\[
\frac{\partial \omega}{\partial t} + \text{div}(\omega q) + q_t = 0
\]

(3)

where \( \omega \) is the width (opening) of the channel, \( q_t \) is leak-off of the fluid per unit area of the channel middle surface. Note that the continuity equation in its initial form (3) contains the particle velocity \( \nu \) as the primary physical quantity defining the movement of a continuum. Meanwhile, the conventional formulations (see, e.g. Ref. [1,2,10]) employ the secondary quantity, the flux \( q \), introduced by the definition as the product \( q = \nu w \). This strongly complicates application of the SE (2) to modeling HF, because, using \( q \), velocity is defined by the quotient \( \nu = \frac{q}{w} \). This leads to the uncertainty \( \frac{\partial v}{\partial t} \), neglecting a small lag and finding the velocity at the fracture front, both the flux and opening are zero. The flaw has hampered using the level set methods (LSM) and fast marching methods (FMM), where the SE is the starting concept of the entire theory given in Ref. [11]. The derivation of the motion equation of Poiseuille type for a fluid flow in a narrow channel involves the particle velocity as the primary quantity. Again the flux appears only by the mentioned definition as the product of the average particle velocity by the opening. Other equations of the HF problem, defining the dependence of the opening on the rigidity of the

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embedding solid and the fracture conditions, contain neither the velocity nor the flux. Consequently, the primary mathematical formulation of the HF problem, in contrast with the conventional formulation, does not involve the flux. The problem may be considered without mentioning this quantity. Then the well-established LSM and FMM of Ref. [8] become available for efficient numerical modeling of HF.

In further discussion the advantages of using the particle velocity are employed. The fluid is assumed viscous with the power-law dependence \( \sigma_{\tau} = M(2\dot{\varepsilon}_{\tau})^n \) of the shear stress \( \sigma_{\tau} \) on the shear strain rate \( \dot{\varepsilon}_{\tau} \). The leak-off term \( q_i \) is of a general form
\[
q_i = \frac{2C_i}{(t-t_s)^{\beta_i}},
\]
where \( C_i \) and \( \beta_i \) are constants, \( t_s \) is the time elapsed since the fluid front had reached a point. The embedding medium is assumed elastic with the Young modulus \( E \) and the Poisson’s ratio \( \nu \). The lag, usually very small, is neglected.

3. Universal asymptotic system. Universal asymptotic umbrella

The conditions of plane flow and plane strain occur near the front. This yields an asymptotic system, derived in Ref. [7],
\[
\left( \frac{w_{\xi}^{*}}{\mu'} \right)_{r=0}^{\xi} = \nu + \frac{2C_i}{(1-\beta_i)w} \left[ E' \nu^2 \sigma_{\tau} \right]_{r=0}^{\xi} = p(r)
\]
\[
\text{lim}_{r \to 0} \frac{w}{r^{n/11}} = \frac{4}{\pi} \frac{K_{lc}}{E} r^{n/11},
\]
where \( r \) is the distance from the front, \( p \) is the net-pressure,
\[
\mu' = 2 \frac{2n+1}{n} M, \quad E' = \frac{E}{1-\nu^2}, \quad K_{lc} \text{ is the critical value of the stress intensity factor.}
\]
The asymptotic system is universal, for it does not involve the opening and net pressure outside a vicinity of a considered point at the front. Its solution
\[
w = \phi(r, \nu_{*})
\]
on a half-axis \( r \geq 0 \) depends merely on the local propagation speed \( \nu_{*} \). Therefore the HF problem has the universal asymptotic umbrella (UAU), which completely defines the opening and net-pressure via the only characteristic of a particular hydraulic fracture, the local speed of the fracture propagation. This conclusion shows far reaching analytical and computational implications. On one hand, the revealed fact makes the HF problem ill-posed in the Hadamard sense while neglecting the lag and trying to solve it as a boundary value problem under a fixed position of the front. Initially, this general fact was disclosed in Ref. [5] for the particular case of the Nordgren problem. On the other hand, the properly employed UAU drastically facilitates numerical tracing of the fracture propagation.

4. Tracing fracture propagation by employing universal speed equation

Consider a quite general case when the UAU is found in a simple monomial form
\[
w = A_\nu(s) r^\alpha
\]
where \( 0 < \alpha < 1, \alpha = A_\nu(s) \) is the opening intensity factor. Expressions, defining \( \alpha \) and \( A_\nu(s) \) for a power-law fluid and an arbitrary propagation regime, are given in Ref. [7].

The power-type UAU (6) suggests application of the variable \( y = w^{1/\alpha} \) linear near the fracture front. Then simple analytical solutions of 1D self-similar problems become available (see, e.g. Ref. [6]). Besides, the UAU provides an easy evaluation of the lag between the fracture contour and the fluid front after obtaining an analytical or numerical solution neglecting the lag. These applications show the analytical advantages of using the revised formulation of the HF problem.

The computational gains are significant too. Solving Eqn (5) in \( \nu_{*} \), we get the universal speed equation
\[
v_{*} = \frac{dx_{\nu}}{dt} = A_{\nu} \left( \frac{w}{r^\alpha} \right)
\]
Hence after spatial discretization of a global system of equations, we add Eqn (6) at nodal points, closest to the fracture front, under the asymptotic umbrella. In this way, we arrive at a well-posed dynamic system. There are numerous options for solving it. In particular, methods of the Runge–Kutta type, employed for 1D problems (see, e.g. Ref. [8]), show high computational potential of such approaches. The conventional LSM and FMM are of immediate use, as well. This opens wide options for developing methods and codes for HF simulation in real time on conventional laptops.

References

Numerical simulation of hydraulic fracture: particle velocity based approach

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Abstract

The work presents a new effective mathematical formulation of the hydraulic fracture problem resulting in an universal numerical algorithm capable of tackling various HF models in the framework of a unified approach. The proposed numerical scheme is not limited to any particular elasticity model or crack propagation regime. Its basic assumptions are: i) proper choice of independent and dependent variables (with the direct usage of a new one - the reduced particle velocity), ii) tracing the fracture front by the use of the speed equation providing relationship between the crack propagation speed and the coefficients in the asymptotic expansion of the crack opening, iii) proper regularization techniques, iv) improved temporal approximation, v) modular algorithm architecture.

Keywords: hydraulic fracture, numerical modeling, fracture front tracing

Hydraulic fracture (HF) is a physical process of a hydraulically induced crack propagation in a brittle material. It can be found in nature, e.g. magma driven dykes or subglacial drainage of water. Moreover, it has many technological applications, e.g. exploitation of geothermal reservoirs or methane extraction from coal seams, but is now mainly associated with the stimulation of hydrocarbon reservoirs. Understanding and control of the process is also crucial for CO2 sequestration and storage of dangerous waste underground.

Mathematical modeling of the HF problem is extremely challenging task which requires accounting for a non-linear solid-fluid interaction. The main difficulties stem from: i) strong non-linearity of the system, ii) singularities occurring in the fracture front region (which depend on the employed HF model and propagation regime), iii) moving boundaries, iv) degeneration of the governing equations near the crack-tip, v) multiscaling, and others.

First HF models can be backdated to the 1940s and 1950s [1, 2, 3, 4, 5]. These studies together with the later works led to the formulation of the basic 1D models of hydraulic fractures: i) the PKN model [6, 7], ii) the KGD model [8, 9], iii) the radial or penny shaped model [10]. They were used up to 1990s in the design of hydrofracturing treatments. Later, the need for a more accurate modelling resulted in the formulation and practical utilization of more advanced models, like: pseudo 3D models (P3D) [11], the planar 3D models (PL3D) [12, 13] or attempts to full 3D models employing finite element method [14], boundary element method [15] and other numerical techniques [16, 17, 18].

Alongside with the development of various models, fundamental research has been carried out, it aimed at identifying the basic solution features related to the underlying physics of the process. In particular, the analysis of the near-tip behaviour of the solution demonstrated its multiscale character. It is now well understood that the coupling between non-linear, non-local and history dependent physical fields results in a complex solution structure, where relative impact of the mentioned processes depends on temporal and spatial scales. It has been proved that the global behaviour of a fluid driven fracture is controlled by the near-tip region, and this has consequences for the computational implementation. Furthermore, in the hydraulic fracture problem, the nature of the moving boundary results in degeneration of the governing equations at the crack tip, which makes tracing the fracture front an extremely difficult task [19, 28].

All these factors clearly indicate the challenge in understanding the solution structure and implementation of the knowledge into computational schemes. In the recent studies by [20] it has been shown that the algorithms which use the appropriate multiscale hydraulic fracture asymptote in the near tip region provide much better results than those which do not apply it. Moreover, when accounting for the proper tip asymptotics, good results can be obtained even for a coarse meshing. The analysis given in [21, 22, 23, 24] proves that proper mathematical formulation of the problem of hydraulic fracture facilitates the correct introduction of the basic asymptotic features of the solution to the numerical algorithm. This in turn results in an appreciable improvement of the accuracy and efficiency of computations.

Taking advantage of the latest advances in the area, the authors developed a new unified approach that yields universal numerical algorithm capable of tackling various HF models [25]. It enables one to account for all elasticity models, different fluid flow and fracture propagation regimes within a unified framework. The key algorithm points are: i) proper choice of independent and dependent variables (including a new one - the reduced particle velocity), ii) tracing the fracture front by use of

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the Stefan condition (speed equation) which can be integrated in a closed form to give an explicit relation between the crack propagation speed and the leading coefficients of the crack opening asymptotics, iii) proper regularization techniques, iv) improved numerical approximation of the temporal derivative of the solution, v) modular algorithm architecture.

The idea to use the particle velocity as a new dependent variable, instead of the usual fluid flow rate, was originally mentioned in [11, 21]. However, it was in [25] where the new variable in a more convenient form of the reduced particle velocity, was actually implemented. Moreover, the crack propagation speed is computed from the local relation based on the speed equation (Stefan condition). With respect to the HP numerical modeling, the condition was originally introduced in [26] but was later abandoned. Recently, it has been rediscovered in [27, 28].

The tip asymptotics is used in the numerical scheme together with appropriate regularization techniques. Some elements of the employed scheme have been presented in [24, 22, 23], where a broad discussion on the advantages of application of proper dependent variables and regularization techniques can be found. One of the key points of the developed universal algorithm is utilization of the explicit relation between the crack propagation speed and the leading terms of the crack opening asymptotic expansion. For the PKN model it was found and described in [24, 23], while for the KGD formulation respective results have been recently reported at a number of conferences.

The above developments lead to a robust and efficient numerical scheme. Its performance is demonstrated for both the PKN and the KGD models under different fracture propagation regimes. We show that the obtained numerical results are more accurate than others available in the literature. Some new semi-analytical solutions to the classical problems are proposed.

References

Numerical modeling of hydraulic fractures for non-Newtonian fluids

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Abstract

Recently a new unified approach has been proposed to build a universal algorithm allowing simulation of hydraulic fracture in its various configurations for Newtonian fluid. In the presentation an extension of this technique to the case of non-Newtonian fluids is presented. Although main ideas are preserved, several new techniques have been implemented in the code to tackle specific challenges associated with the flow rule. Numerical results demonstrating high effectiveness of the algorithm and comparison with existing benchmarks are presented.

Keywords: hydraulic fracturing, numerical modeling, non-Newtonian fluids

1. Introduction

Hydraulic fracture (HF) is a physical process of hydraulically induced crack propagating in a brittle material. It can be found in nature, e.g. magma driven dykes or subglacial drainage of water. Moreover, it has numerous technological applications, such as exploitation of geothermal reservoirs or methane extraction from coal seams, but is now mainly associated with the stimulation of hydrocarbon reservoirs [2]. Understanding and control of the process is also crucial in cases like CO2 sequestration and storage of dangerous waste underground.

Mathematical modeling of the underlying physical process is very challenging, due to complex interaction mechanisms between solid and fluid phases. Recently, a universal algorithm has been presented in [7] dealing with the case of Newtonian fluid being the driving force of the process. However, in many applications of HF non-Newtonian fluids propelling the crack propagation are used [5]. Moreover, the mass transport mechanisms within the fracture (proppant movement) can also be reduced to the problem of non-Newtonian fluid flow.

2. Numerical modeling

Alongside the development of modern stimulation techniques, the need for more efficient and accurate numerical modeling of the problem has emerged. Even in the simplest formulation, one needs to take into account interaction between solid and fluid phases. Main mathematical difficulties of HF stem from: i) strong non-linearity of the system, ii) singularities occurring in the fracture front region (which depend on the employed HF model and propagation regime), iii) moving boundaries, iv) degeneration of the governing equations near the crack-tip, v) multiscale, and others. Any robust numerical solver for hydraulic fractures needs to address these challenges.

Mathematical model consists of the following equations:

- Poiseuille equation describing fluid flow inside the fracture,
- continuity equation expressing mass conservation principle and taking into account possible leak-off phenomenon,
- elasticity equation defining deformation of the rock under applied hydraulic pressure.

The system is supplemented by the initial and boundary conditions, as well as the speed equation [4, 5].

In the paper we consider two classical models of HF:

- the PKN model [5],

Recently developed universal algorithm for numerical simulation of HF [7] allows one to consider various HF formulations and its different regimes within the common framework. It accommodates all the latest advances in the area [6, 5]:

- particle velocity as the main dependent variable,
- proper regularization techniques,
- improved approximation of temporal derivative,
- tracing the fracture front based on the Stefan condition (speed equation),
- adaptive meshing strategies.

All these techniques have been adopted when creating the universal solver for the case of non-Newtonian fluids.

3. Results

Below we present selected numerical results demonstrating the efficiency of the solver. Comparisons are done using analytical benchmarks developed specifically to verify the computation accuracy.

The relative errors of the computations for a number of shear-thinning fluids are presented in Figs 1 – 3, for the PKN model and the fluid-driven and toughness-driven regimes of the KGD model, respectively. All computations are based on \( N = 100 \) points of non-uniform spatial mesh.

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It can be easily seen that the accuracy of the numerical scheme is extremely high for all analyzed models and fluids. The solver is stable and allows to decrease the number of spatial points and still preserve a satisfactory accuracy level. In the presentation we will comment on the accuracy of other parameters (particle velocity, pressure, fluid flux) and compare our results with the ones available in the literature.

4. Summary

In the talk, we discuss additional challenges arising in the case of modeling HF for non-Newtonian fluids and present ways to overcome them. As a result, the numerical scheme from [7] is successfully extended to the case of shear-thinning fluids. We illustrate the performance of the algorithm using newly developed benchmarks and all available numerical and semi-analytical results from the literature.

Finally, we show that the modular structure of the computational scheme provides substantial flexibility which constitutes a universal tool for different variants of the problem.

References


On propagation of closely located hydraulic fractures

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Abstract

The work aims to study strong interaction of closely located, nearly parallel hydraulic fractures and the influence of their interaction on fracture propagation. Both computational and physical aspects of the problem are considered. It is shown that from the computational point of view, the system exhibiting distances between cracks small compared with their sizes becomes ill-conditioned. This leads to the deterioration of numerical results. The physical effect of strong interaction appears in notable decrease of crack opening and consequently even in greater decrease of conductivity. This leads to drastic growth of viscous resistance to fracture propagation, which results in the propagation of the individual fractures whose distance is large enough. Numerical experiments for 2D and 3D closely located parallel fractures are performed by employing the hypersingular boundary element method. The condition number of the matrix, the stress intensity factor at the tips and edges of cracks and the changes in effective resistance of a cluster of cracks, with various distances between them are computed. The results imply the possibility to replace a cluster of cracks by a single properly located crack, to avoid computational instability and to conclude on the physically consistent propagation of individual fractures.

Keywords: hydraulic fractures, cracks interaction, ill-conditioning, effective conductivity

1. Introduction

Hydraulic fracturing, used to recover gas from low permeable shale structures, generates a large number of fluid-driven cracks. Their growth and interaction with adjacent hydraulic fractures and with natural fractures strongly influences the efficiency of a hydrofracturing treatment. Thus it is desired to understand arising difficulties and to develop means to overcome them.

It is well-known that the decrement of distance between cracks makes interaction is stronger. When cracks are nearly parallel and the ratio of the distance to their size is small, the opening of a hydraulically driven fracture induces compressive stresses normal to the fracture surface. This decreases the opening of neighbouring, closely located fractures. As a consequence, some of the fractures may propagate in a lower rate, or even stop. This phenomenon, called the stress shadowing (see e.g.\cite{1, 2, 3, 6, 7, 10}), has a strong effect on the productivity of multiple fractures propagating from a wellbore. Its analysis tending to optimize a distance between successive treatments is of importance for the petroleum industry.

The work aims to study the interaction of 2D and 3D multiple, closely located, parallel cracks, including analysis of the stress shadowing effect. Numerical experiments are carried out using the hypersingular boundary element method, based on the complex (in 2D) and real (in 3D) variable hypersingular boundary integral equations, tailored to account for the asymptotic behaviour of fields in a close vicinity of singular points, such as tips and edges of cracks, multi-wedge points, etc. (see, e.g. \cite{4, 5, 9}). It is shown, that the decreasing distance between cracks the ratio \(d/L\) decreases. The medium is isotropic and elastic. Its Young’s modulus is \(E = 30\text{GPa}\), and Poisson’s ratio is \(\nu = 0.25\). Unit tensile stress normal to the crack surfaces is prescribed at infinity.

Calculations are performed by employing the hypersingular

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{clusters.png}
\caption{Clusters of parallel, straight cracks; a) \(N = 2\), b) \(N = 4\), c) \(N = 6\)}
\end{figure}
boundary element method with the second order approximation of the density functions (displacement discontinuities and tractions) and a square root asymptotics near the tips (edges) of cracks. High accuracy and stability of the method, while dealing with strongly inhomogeneous structures in 2D, have been confirmed in the paper [8]. The applicability of the method to solve similar problems in 3D is highlighted in [9].

Condition number. For a different number \( N \) of cracks and various distances \( d \), the opening (measured by calculated displacement discontinuities) of the cracks and corresponding values of the condition number, which is a measure of conditioning of a main matrix of a system, are calculated. Specifically, for 2D problem, when \( d/L = 0.05 \), the condition numbers are \( 6.1 \cdot 10^4, 1.7 \cdot 10^6, 2.2 \cdot 10^6, 2.3 \cdot 10^7 \) and \( 6.5 \cdot 10^7 \); for \( N = 2, 3, 4, 5, \) and \( 6 \), respectively. In 3D, for the same ratio \( d/L \), they are \( 3.3 \cdot 10^6, 3.8 \cdot 10^7, 1.2 \cdot 10^9, 1.4 \cdot 10^9 \) and \( 1.6 \cdot 10^9 \), respectively. Numerical tests show that the condition number rapidly grows with both decreasing ratio \( d/L \) and growing number \( N \) of cracks in a cluster.

Shadowing effect. Remarkably, when the distance between cracks decreases, the sum of the openings in the middle of the cracks \( \sum_{i=1}^{N} w_{c_i} \) in a cluster tends to the opening of a single crack \( w_S \). It appears, that due to the stress shadowing effect, the openings of the first and the \( N \)-th crack are approximately equal to \( w_{c_1} = w_{c_N} = w_S/2 \), while the openings of the intermediate cracks tend to zero, see Fig. 2. Similar conclusions follow from calculations for the SIFs at the tips (in 2D) and edges (in 3D) of the cracks. The values of the SIF at the internal cracks are much less than those at the external cracks, while their sum tends to the SIF at the tips (edges) of a single crack of the same size. This implies that under the conditions considered, the cluster may be replaced by a single crack located in the middle of a cluster as regards to its influence on stresses around the cluster.

Figure 2: Relative openings of the cluster of 5 square cracks visualized on its cross-section \((d/L = 0.25)\)

Resistance to flow. The flow \( q \) of a Newtonian fluid in a crack is prescribed by the Poiseuille equation \( q = -k \partial p/\partial x \), where \( k \) is the conductivity of a crack, \( p \) is the fluid net pressure, \( x \in (0, L) \). For a single crack, the conductivity is described by the relation \( k = k_\mu = w_S^2/(12 \mu) \), where \( \mu \) is dynamic viscosity. For a cluster of cracks it is \( k = k_{c,N} = \sum_{i=1}^{N} w_{c_1}^2/(12 \mu) \). The results of calculations show that the conductivity of the first and \( N \)-th crack tends to \( k_{c_1}/8 \), while conductivity of each of internal cracks tends to zero. The effective resistance of a cluster of cracks is inversely proportional to the total conductivity of a cluster \( k_{c,N} \); thus the internal cracks cannot propagate.

The results of numerical experiments are consistent with the field observations (see, e. g. the papers [1, 3]). The numerical data obtained provide a quantitative estimation of the observed effects important for proper design of hydraulic fracture treatments.

3. Conclusions

The conclusions of the research, employing the highly accurate and stable 2D and 3D hypersingular boundary element method, are summarized as follows. (i) The shadowing effect of nearly parallel cracks unavoidably leads to deterioration of numerical results when the distance between the cracks decreases and their number grows. (ii) A cluster of closely located nearly parallel cracks may be replaced by a single crack located in the middle of the cluster referring to its influence on the stresses around the cluster. (iii) The main influence of nearly parallel cracks consists in decreasing the fracture opening, resulting in a drastic growth of resistance to a fluid flow. (iv) Only individual hydraulic fractures, located sufficiently far one from another, may propagate.

References

Cavitation resistance of structural materials according to the fractional approach

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Abstract

The concept of fractional cavitation resistance was proposed by the author in the beginning of the previous decade as a response to continuous problems with incompatibility of cavitation resistance assessments based on tests conducted under different cavitation conditions. This contribution presents the background and the essence of the concept. The technique developed incorporates determining cavitation load at test facilities using erosion curves of reference materials. The considerations are illustrated with results of the analysis conducted with the use of data of the International Cavitation Erosion Test project and the recent laboratory tests conducted in the Szewalski Institute of Fluid-Flow Machinery (IMP PAN), Gdańsk, Poland. Insufficient accuracy in cavitation aggressiveness assessments based on direct cavitation pulse measurements at the reference test rig has been identified as the main reason for the lack of expected quality in fractional analysis results. An alternative approach based on analysing the pit size distribution at a soft surface (e.g. copper foil) in the initial period of cavitation impingement was proposed as a remedy.

Keywords: cavitation, erosion

1. Basic ideas

Erosion progress at cavitation flow confining walls depends both on cavitation load exerted at the walls and cavitation resistance of the wall material - usually meant as material capability to withstand the erosive action of cavitation. This obvious description is purely qualitative and clearly insufficient for the purposes of any practical mass loss prediction.

Cavitation load of a solid surface may be considered a stochastic process consisting in more or less random occurrence of a sequence of pressure pulses, characterised by their location, surface interaction area, amplitude and duration. A more detailed approach requires replacing the interaction area parameter by the pressure surface distribution around the interaction axis.

For the purpose of practical mass loss prediction it is required to develop quantitative relationships with cavitation load and material resistance described by a set of parameters.

The most obvious macroscopic relationship is that between energy absorbed by the impinged material and its volume loss as proposed in sixties by A.P.Thiruvengadam [1]. The absorbed energy is often considered to be proportional to the flux of cavity load and material resistance described by a set of parameters.

For the purpose of practical mass loss prediction it is required to develop quantitative relationships with cavitation load and material resistance described by a set of parameters.

The most obvious macroscopic relationship is that between energy absorbed by the impinged material and its volume loss as proposed in sixties by A.P.Thiruvengadam [1]. The absorbed energy is often considered to be proportional to the flux of energy delivered to the surface by pressure pulses with density estimated from the formula

\[ ME = \frac{1}{A} \frac{\tau}{2 \rho c} \sum_{i=1}^{n} F_i^2 = \sum_{i=1}^{n} ME_i \]

(1)

where \( A \) - pressure sensor active area, \( n \) - number of force pulses of amplitude \( F_i \) recorded in time unit at a unit surface area, \( \rho \) and \( c \) - liquid density and sound celerity, \( \tau \) - time constant often interpreted as average pulse duration.

As proportionality between the energy absorbed and delivered may be put in question and the low amplitude pulses are known to show negligible contribution to the erosion process, summation in Eqn 1 is often started from some threshold level [2].

Generally, material volume loss may not be assumed to follow proportionally to the exposure duration or energy delivered to the impinged surface. The classic S-shape volume loss curve is featured by several stages including those of erosion incubation, acceleration, deceleration and steady-state. Using several single-number parameters, such as nominal incubation time and maximum instantaneous or cumulative rate of erosion it is possible to describe the main characteristics of the curve. However, it is generally acknowledged that single number parameters are insufficient to describe properly material resistance to cavitation even under standardised test conditions.

Therefore, existing standards on cavitation resistance evaluation require using whole erosion curves as assessment basis while recommending the use of single number parameters as data helpful in comparative analysis.

2. Classic approach

The classic approach in assessment of material resistance to cavitation is based on testing a selected group of materials under specified cavitation conditions. The tests are followed by a comparative analysis of the resulting volume loss curves with the help of derived single-number parameters. Various analytical models of erosion progress were developed in order to get free of uncertainties due to human factor while plotting erosion curves based on experimental data.

In practice, cavitation erosion tests are conducted with a large variety of test facilities as developed throughout the previous century. The list of available cavitation facilities includes cavitation tunnels, rotating disks, vibratory rigs and cavitating jet cells. The last two types of test rigs are subject to the International Standards of the American Society for Testing and Materials (ASTM) [3,4]. While satisfactory consistency between results can be obtained by keeping to the standard specifications, lack of compatibility between test results obtained at different test facilities and/or under different cavitation conditions is a well-known problem. This observation was confirmed by results of the International Cavitation Erosion Test project co-ordinated some time ago by the IMP PAN team [5]. Common inversions in ordering of materials according to their erosion curves have shown that developing a technique essentially increasing compatibility between assessments based
on tests at different facilities and/or test conditions should be considered a top priority.

3. Fractional resistance concept

It is no doubt that the main reason for the mentioned incompatibilities is to be seen in qualitative differences between the cavitation loads, represented among others by the load vectors \( ME = [ME_i]_{i=1...N} \). The fractional resistance approach assumes material may show different response to each load fraction. Thus the approach is a generalisation of the threshold energy concept. With material performance under monofractional load characterised by a set \( R \) of numerical parameters (resistance vector), the volume loss after an exposure of duration \( t \) can be described by some analytic formula

\[
\Delta V = A U(R, Y) \tag{2}
\]

with \( A \) standing for the impinged surface area and \( U \) – mean depth of erosion function of the \( R \) resistance vector and the density of energy delivered \( Y = ME_i t \).

The volume loss due to a polyfractional load is considered a superposition of losses due to component loads. However, simple arithmetic adding is applicable solely in time intervals short enough to justify negligence of structural transformations and changes in mechanical properties due to cumulation of absorbed energy. Hence, the differential super-position law follows in form of the basic equation of polyfractional erosion process kinetics

\[
d(\Delta V) = \sum_{i=1}^{N} ME_i \frac{\partial U}{\partial \Theta}(R, Y)|_{\Theta(U(\Delta V/A))} \tag{3}
\]

with \( \Theta \) denoting a function reciprocal to \( U \) in respect to the delivered specific energy \( Y \). Derivation of this equation is shown schematically in Fig. 1.

![Figure 1: Differential superposition of erosion curves](image)

Following this approach material resistance to cavitation is described by the resistance matrix \( R \) constructed as a set of monofractional resistance vectors \( R \). The curve fitting technique based on erosion tests conducted under different cavitation conditions is used in order to derive the \( R \) matrix components. The same technique may be applied in order to derive the \( ME \) cavitation load vector basing on erosion tests of several reference materials with previously determined resistance matrices.

4. Experimental tests and cavitation intensity assessments

Based on extensive experimental tests at the IMP PAN cavitation tunnel with slot cavitator it appeared possible to determine the cavitation resistance matrix of selected reference materials and to use this data for determining the cavitation load vector at test rigs taking part in the ICET project [5]. At the same time the conducted study showed the analysis results to depend on the set of test conditions and some details of curve-fitting technique applied when determining the resistance matrices of the reference materials. After a deeper analysis – including studying the time course of recorded pressure pulse signals - it turned out that the main reason of problems was rather of physical and not computational nature. The cavitation load calculated by means of Eqn 1 in the area of intense erosion was clearly overestimated due to limited temporal and spatial resolution of the measurement system. A significant factor could be also signal distortion due to relatively low resonance frequency of the applied commercial pressure transducers (500 kHz). Including results from the areas of much lower load into the analysis suggested the low amplitude pulses could be more effective than these of a high amplitude which apparently was not true. Therefore, only results coming from high load intensity areas were accepted for the purpose of preliminary analysis. No steps were taken so far in order to recommend the technique for a widespread laboratory practice.

Fortunately, analysing signals coming from commercial pressure transducers is no more the only experimental technique available for determining the energy distribution of cavitation pulses. High time resolution may be achieved when using the PVDF sensors showing resonance frequencies of the GHz order of magnitude. An alternative approach based on analysis of cavitation pits having occurred in soft material (e.g. copper foil) in the initial period of exposure may appear even more attractive due to available results of computational studies on correlation between pit size and cavitation pulse energy [6].

5. Conclusion

The fractional analysis of cavitation resistance as outlined in this contribution is a technique potentially capable to overcome or at least to ease the problem of insufficient compatibility in erosion test results obtained under different test conditions. Unfortunately, at the current stage of development the technique cannot be considered mature enough to be recommended for widespread use in laboratory practice.

The main reason for observed problems seems to lie in inadequate determination of the cavitation pulse energy distribution. The approach based on the theoretically determined correlation between the erosion pit size and cavitation pulse energy seems a highly promising alternative at the moment.

References


Taylor-Couette flow with radial temperature gradient

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Abstract

The authors investigate the Taylor-Couette flow with heat transfer using Direct Numerical Simulations and Spectral Vanishing Viscosity methods (DNS/SVV). In the analysed flow cases the inner rotating cylinders are cooled and the stationary outer one is heated. The authors study the influence of the radial temperature gradient on the flow structure and on the statistical parameters. The Boussinesq approximation is used to take into account the buoyancy effects induced by the involved body forces. Additionally, for selected flow cases the influence of the periodic modulation (superimposed on the inner cylinder rotational speed) on the instability critical parameters and on flow structure is studied. The distributions of the local Nusselt numbers along disks and cylinders are presented as well as the axial and radial profiles of many structural parameters. The λ2 criterion was applied for numerical visualization.

Keywords: congress of mechanics, computational mechanics conference, instructions, formatting, camera-ready paper

1. Introduction

The Taylor-Couette flow (the swirling flow in the gap between two concentric cylinders and two disks, Figure 1) is one of paradigmatical systems in hydrodynamics, widely used for studying the primary instability, pattern formation, transitional flows and fully turbulent flows. This model flow is useful from numerical and experimental point of views, among others, because of the simplicity of its geometry.

The Taylor-Couette flow is widely used in industrial machines in the field of mechanics and chemistry, e.g., in ventilation installations, desalination tanks and waste water tanks, in cooling systems, in gas turbines, in axial compressors (rotating cavities) and in equipment for blood transfusion. Additionally, the Taylor-Couette flow is highly suitable for predicting various phenomena in geophysics and astrophysics.

The review of literature shows that one aspect of the Taylor-Couette flow, which has been insufficiently researched so far, is the laminar-turbulent transition and a fully turbulent flow in the varying temperature field which occurs in most industrial applications. An insight into laminar-turbulent transition in the Taylor-Couette flow (with temperature gradient) requires very detailed experimental and numerical investigations (both contribute to a better understanding of the transitional processes and, indirectly, increasing effectiveness of many designs). Additionally, the results obtained for geometrically simple Taylor-Couette flow can be extrapolated to far more complicated configurations typical of industrial devices. The paper can be also interesting for RANS modelers; such detailed numerical investigations can be useful to estimate the strengths and limitations of models which are presently used to predict transitional and turbulent flows with heat transfer in more complicated configurations.

In the paper the authors investigate the influence of radial temperature gradient on the Taylor-Couette flow structure. Such studies are less numerous and less advanced in comparison to the studies of the classical Taylor-Couette flow, very important from the applicational and fundamental points of view. The results are obtained for different aspect ratios $\Gamma = 2h / (R_2 - R_1) = 3.76, \ 20.0$, for curvature parameters $Rn = (R_1 + R_2) / (R_2 - R_1) = 2.2 - 10.0$, for the thermal Rossby number $B = \beta (T_2 - T_1) = 0.1$ (where $T_2$ is temperature of warmed walls and $T_1$ is temperature of cooled walls, $\beta$ is a thermal expansion coefficient), and for different Reynolds numbers $Re = R_1 (R_2 - R_1) \omega / \nu$.

The authors also investigate the influence of the harmonically modulated rotation of the inner cylinder on the Taylor-Couette flow instability. The angular velocity of the inner cylinder oscillates around some mean value $\bar{\Omega}$ with a given amplitude $\hat{\Omega}$, which is described as $\Omega(t) = \bar{\Omega} + \hat{\Omega} \cos(\omega t)$, where $\omega$ is the frequency of modulation.

Figure 1: Schematic picture of the Taylor-Couette flow
2. Numerical approach

The flow is described by the continuity, Navier-Stokes and energy equations. The equations are written in a cylindrical coordinate system \((R, \varphi, Z)\), with respect to a rotating frame of reference. Numerical investigations are performed using the Direct Numerical Simulations and Spectral Vanishing Viscosity methods (DNS/SVV). Numerical simulations are based on pseudospectral Chebyshev-Fourier-Galerkin collocation approximation. In the time approximation the authors use a second-order semi-implicit scheme, which combines an implicit treatment of the diffusive terms and an explicit Adams-Bashforth extrapolation for the non-linear convective terms. The correction/prediction method is used to maintain the incompressibility constraint, [1, 2]. To stabilize the discretization scheme for higher Reynolds number, the authors use the Spectral Vanishing Viscosity (SVV) method, [3].

3. Results

The authors observed different stages of the laminar-turbulent transition which appear with increasing Reynolds numbers: Taylor-Couette vortices - TVF, wave vortices - WVF, modulated vortices - MVF and a turbulent Taylor-Couette flow TTVF. An exemplary flow structure obtained in the cavity of aspect ratio \(\Gamma=1.47, Rm=2.2\) is presented in Figure 2 (\(\lambda_2\) criterion is used for visualization, the inner rotating cylinder is on the left side, the stationary cylinder on the right side, the upper disk is stationary and the bottom one rotates, \(Re=20000, B=0\)).

![Figure 1: Iso-lines of the \(\lambda_2\) criterion, meridian section of the cavity, \(Re=20000, \Gamma=1.47, Rm=2.2\)](image)

References


Optimization of Structural Topology

organized by T. Lewiński and B. Bochenek
Design and topology optimization of an aluminium alloy wheel

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Abstract

Competition in automotive industry has grown rapidly during the recent years. Through this perspective, manufacturers are developing low cost, high performance components for vehicles. For this purpose, various tools have been developed for structural analysis, based on finite elements, as well as multiobjective optimization techniques for structures and mechanical components. The paper presents an optimal design for an automotive aluminium alloy wheel, subjected to static loads. Topology optimization is carried out in order to define the formation of the wheel spokes.

Keywords: topology optimization, aluminium alloy wheel

1. Introduction

Wheels are critical parts of vehicles, being moving vehicle parts. In addition, in cooperation with the suspension system, they support the static and dynamic loads produced while driving and influence the dynamic characteristics of the vehicle. As a result, their design requires great attention in order to ensure safety.

Apart from safety, wheels meet given requirements in terms of style. As competition in automotive market grows, manufacturers are developing high styled, lightweight wheels in order to satisfy customers.

2. Wheel design

In this study we conduct optimal design for an aluminium alloy wheel. The final design will be created out of a simple, spokeless wheel, created with the CAD software Pro Engineer Wildfire 5.0. We choose a 17΄΄ wheel, shown in Fig. 1.

![Figure 1: CAD design before optimization](image)

3. Material properties

The Al-Si-Mg alloy is the main casting alloy used to produce wheels. In the present study, we choose the 6061-T6 aluminium alloy with the mechanical properties given in Table 1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density (kg/m³)</td>
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</tr>
<tr>
<td>Modulus of elasticity (GPa)</td>
<td>70</td>
</tr>
<tr>
<td>Poisson ratio</td>
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</tr>
<tr>
<td>Yield strength (MPa)</td>
<td>241</td>
</tr>
<tr>
<td>Ultimate strength (MPa)</td>
<td>300</td>
</tr>
</tbody>
</table>

4. Finite element analysis and optimization

Topology optimization is a mathematical approach that optimizes material layout within a given design space, for a given set of loads and boundary conditions. Using topology optimization, engineers can find the best concept design that meets the design requirements. Topology optimization is the modern approach to structural optimization, used for the design of both, microstructures and mechanisms [1,2,3]. Further information related to the presented application can be found in [4,5,6,7,8].

In this study, the CAE software Abaqus is used and its Topology Optimization Module. The classical settings of topology optimization as described in [1] was used.

4.1. Boundary conditions and loading

Two loads are applied, representing the cases of extreme driving conditions or collision with the curb of the road or a large obstacle, as shown in Figs. 2 and 3. The wheel is supported in the area of its hub.

![Figure 2: Lateral load of 7000 N](image)
4.2. Optimization settings

In order to define the optimal design, under the loading situation as above, topology optimization is conducted. The target of the optimization procedure is to reach a final design that minimizes stresses, with a volume at least 25% less than the initial design. The design space is defined in the area connecting the hub and the rim, where the spokes will be formed after optimization.

Formulation of the spokes is highly defined by the rotational symmetry that we choose to give to the optimal designed pattern. In this study, we choose a rotational symmetry of 36°, expecting as outcome a 10-spoke wheel.

4.3. Results

The optimization results give a good design approach to the final product, see Fig. 4. Not every solution is always applicable, and the decision of proceeding to production depends on the experience of the industrial engineer.

4.4. Optimal design verification

After the optimization is completed and the final design is defined, static analysis is essential, in order to ensure its suitability in terms of safety. Under the same boundary and loading conditions that the optimization was carried out, the static analysis confirms that the restriction of allowable stresses is not violated (Max. Von Mises Stress: 219 MPa) and the final result is highly recommended for production.

5. Conclusions

Topology optimization is a very powerful tool that makes structural optimization useful for industrial applications. Even with the simplest implementation the result is satisfactory, provided that several carefully chosen loading cases are used. Automatic transformation of the produced topology into CAD models for further usage needs further research.

References


Optimization of structural topology using unstructured Cellular Automata

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Abstract

In recent years the Cellular Automata concept has been successfully applied to structural topology optimization problems. The majority of results that have been obtained so far were based on regular lattices of cells. Practical engineering analysis and design require however using, in many cases, highly irregular meshes for complicated geometries and/or stress concentration regions. The aim of the paper is to extend the concept of Cellular Automata towards implementation of unstructured grid of cells related to non-regular mesh of finite elements. Introducing irregular lattice of cells allows to reduce number of design variables without loosing accuracy of results and without excessive increase of number of elements caused by using fine mesh for a whole structure. It is worth noting that the non-uniform density of finite elements can be, but not necessary is, directly related to design variables which are related to cells of Cellular Automaton. The implementation of non-uniform cells of Cellular Automaton requires a reformulation of standard local rules, for which the influence of neighborhood on current cell is independent of sizes of neighboring cells.

Keywords: topology optimization, Cellular Automata, unstructured mesh

1. Introduction

For a few decades topology optimization has been one of the most important aspects of structural design. Since the early paper by Bendsoe and Kikuchi [1] one can find in the literature numerous approaches to generating optimal topologies based both on optimality criteria and evolutionary methods. A general overview as well as a broad discussion on topology optimization concepts are provided by many survey papers e.g. Ref. [4]. At the same time hundreds of papers present numerous solutions including classic Michell examples as well as complicated spatial engineering structures, implementing specific methods ranging from gradient based approaches to evolutionary structural optimization, biologically inspired algorithms, material cloud method, spline based topology optimization and level set method. It is a permanently developing area and one of the most important issues stimulating this progress nowadays is implementation of efficient and versatile methods to generation of optimal topologies for engineering structural elements. In recent years the Cellular Automata paradigm has been successfully applied to topology optimization problems. In engineering implementation of Cellular Automaton the design domain is decomposed into a lattice of cells, and a particular cell together with cells to which it is connected form neighborhood. It is assumed that the interaction between cells takes place only within the neighborhood, and the states of cells are updated synchronously according to some local rules. The majority of structural topology optimization results that have been obtained so far were based on regular lattices of cells. Practical engineering analysis and design require however using, in many cases, highly irregular meshes for complicated geometries and/or stress concentration regions. The aim of the paper is to extend the concept of Cellular Automata towards implementation of unstructured grid of cells related to non-regular mesh of finite elements. Introducing irregular lattice of cells allows to reduce number of design variables without loosing accuracy of results and without excessive increase of number of elements caused by using fine mesh for a whole structure. It is worth noting that the non-uniform density of finite elements can be, but not necessary is, directly related to design variables which are related to cells of Cellular Automaton. The implementation of non-uniform cells of Cellular Automaton requires a reformulation of standard local rules, for which the influence of neighborhood on current cell is independent of sizes of neighboring cells. This paper proposes therefore new local update rules dedicated to unstructured Cellular Automata.

2. Unstructured Cellular Automata

The first application of CA to optimal structural design, and to topology optimization in particular, was proposed in mid nineties of the last century. In what followed, the last two decades have brought several papers implementing this approach. Most of to date applications of Cellular Automata in structural optimization are conventionally based on regularly spaced, structured meshes. On the other hand using unstructured computational meshes provides more flexibility for fitting complicated geometries and allows local mesh refinement. Some attempts to implement unstructured Cellular Automata have been already reported in the literature e.g. Ref. [3], but application to topology optimization is rather incidental (see Ref. [5]).

In the paper the concept of topology generator based on Cellular Automata rules is extended to unstructured meshes. Similar to structured (regular) Cellular Automata, several neighborhood schemes can be identified. The two most common ones are the von Neumann type and the Moore type. As can be seen in the Fig.1 in case of the von Neumann configuration only three immediate neighbors are taken into account. These neighbouring cells share common edges with the central cell. In the Moore type neighborhood any triangle that has common edges or common vertices with the central cell can be considered as a neighbor of the central triangle. It is worth noting that this type of neighborhood considers more neighbors around the central cell, but the number of neighbours can vary since it depends on the particular unstructured mesh arrangement.
3. The algorithm

The performance of Cellular Automata algorithms, reported in literature, is often based on heuristic local rules. Similarly, in the present paper the efficient heuristic algorithm, being extension of the one introduced by Bochenek and Tajs-Zielińska [2], has been implemented. The power law approach defining solid isotropic material with penalization (SIMP) with design variables being relative densities of a material has been utilized. The elastic modulus of each cell element is modelled as a function of relative density \( d_i \) using power law, according to Eqn (1). This power \( p \) penalizes intermediate densities and drives design to a solid/void structure.

\[
E_i = d_i^p E_0, \quad \text{d}_{\text{min}} \leq d_i \leq 1
\]  

(1)

The local update rule applied to design variables \( d_i \) associated with central cells is now constructed based on the information gathered from adjacent cells forming the Moore or von Neumann type neighborhood. It is set up as linear combination of design variables corrections with coefficients, the values of which are influenced by the states of the neighborhood surrounding each cell, as presented in Eqn (2):

\[
d_i^{(t+1)} = d_i^{(t)} + \delta d_i, \quad \delta d_i = (\alpha_0 + \sum_{k=1}^{N} \alpha_k) m = \tilde{\alpha} m
\]  

(2)

The compliance values calculated for central cell \( U_i \) and \( N \) neighboring cells \( U_{ik} \) are compared to a selected threshold value \( U^* \). The quantities \( A_i \) and \( A_{ik} \) stand for areas of central and neighboring cells, respectively. Based on relations Eqn (3) and Eqn (4) specially selected positive or negative coefficients \( C_{\alpha_0} \) for central cell and \( C_{\alpha_k} \) for surrounding cells are transferred to the design variable update.

\[
\alpha_0 = \begin{cases} 
-C_{\alpha_0} & \text{if } U_i \leq U^* \\
C_{\alpha_0} & \text{if } U_i > U^* 
\end{cases}
\]  

(3)

\[
\alpha_k = \begin{cases} 
-C_{\alpha_k} & \text{if } U_{ik} \frac{A_{ik}}{A_i} \leq U^* \\
C_{\alpha_k} & \text{if } U_{ik} \frac{A_{ik}}{A_i} > U^* 
\end{cases}
\]  

(4)

The move limit \( m \) implemented in the above algorithm controls the allowable changes of the design variable values. The numerical algorithm has been build in order to implement the above proposed design rule. As to the optimization procedure the sequential approach, has been adapted, meaning that for each iteration, the structural analysis performed for the optimized element is followed by the local updating process. Simultaneously a global volume constraint can be applied for specified volume fraction. If so the generated optimal topology preserves a specified volume fraction of a solid material.

4. Generation of optimal topologies

Selected examples of compliance-based topologies generated using the approach presented in this article are discussed in this section. The first one it is a rectangular Michell-type structure, clamped at the left edge and loaded by a vertical force applied at the bottom right corner. The irregular mesh that consists of triangular elements/cells has been applied. The more dense mesh surrounds right bottom corner of the rectangle, as shown in the Fig.2. The minimal compliance topologies have been generated for various numbers of cells, and the one found for 10594 cells is presented in the Fig.2. It is worth noting that in order to obtain equivalent topology using regular mesh the number of cells should increase to almost 30 thousand elements. That many elements are required to get almost the same compliance and maximal stress values for final topology. The second example regards optimization of machine part, a model of a wrench structure (see Ref. [5]), given in the Fig.3. The boundary of the circular hole on the right side is clamped, while loading is applied at the boundary of the hole situated on the left side. The irregular mesh with more dense areas surrounding holes has been applied. The lattice of almost 56 thousand cells have been used to obtain the topology presented in the Fig.3.

The subject is still under development but it seems that the approach demonstrates a significant potential of application to problems which cannot be adequately represented by regular grids. The use of unstructured meshes may be helpful while modelling a domain geometry, accurately specify design loads or supports and compute structure response.

References

New method of generating Strut and Tie models using truss topology optimization

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Abstract

The paper deals with the automatic generating of Strut and Tie models by means of truss topology optimization. A modification of the classic ground structure approach is developed. In order to rationalize the solution a penalty parameter, that represents cost of the nodes, is introduced. Stresses in concrete are controlled by an iterative model of the ground structure, hence the design code requirements are met. The linear programming problem is handled with the use of the adaptive ground structure by means of a solution of both primal and dual formulations of the optimization problem. Algorithms allowing generating the ground structure in arbitrary domains bounded by polygons assure versatility of the method.

Keywords: Strut and Tie models, reinforced concrete structures, truss topology optimization, Michell’s structures, ground structure.

1. Introduction

Strut and Tie (ST) models are used to approximate complex stress fields in reinforced concrete structures by trusses in employing bars in compression to map the stresses in the concrete, whilst the bars in tension represent the steel reinforcement. The ST models are incorporated in design codes, such as Eurocode 2 [1], as a tool for the ultimate limit state design of the reinforcement in two dimensional plate-like elements of the reinforced concrete structures. The bearing capacity of the truss is a lower bound of the capacity of the structure due to the limit load theorem known from plasticity. The codes, however, do not offer any indication on how such models should be constructed for arbitrary structures, hence leaving this problem for designers. The only restriction the codes set on the models is that directions of the bars must not differ significantly from the directions of principal stress in the corresponding solid, elastic body in order to satisfy the ductility requirements and to avoid excessive cracking.

Among several examples of ST models available in Eurocode 2 there is a model for a corner of the frame in which the height of the beam is more than twice the width of the column. In this model the directions of bars in tension do not coincide with directions of principal stresses in the corresponding solid body; moreover, the amount of steel required with the use of this model is alarmingly high. The example of the frame corner model involves creating ST models for nonstandard reinforced concrete structures. It implies a need for developing a versatile method allowing to automatically construct ST models for various reinforced concrete members. Literature review shows, that topology optimization can be used here, e.g. in paper [4] the authors obtain truss-resembling layouts by means of solving compliance minimization problem for 2D solid body using two materials. In the paper a topology optimization problem is formulated for a structure that is a truss by definition. The method used is a modification of the classic ground structure approach where a truss volume is minimized (see Sokół [3]).

2. Formulation of truss topology optimization problem

2.1. Definition of ground structures "T" and "C"

Let us consider a feasible domain bounded by arbitrary set of polygons. A regular mesh of points is stretched over a rectangle circumscribed about the domain, the points contained in the domain being called truss nodes. Each node is connected with each other by a line segment; the segments contained in the domain become bars of dense truss called initial ground structure. Versatile algorithms were developed by the authors to find if a given node or bar is contained in a feasible domain.

Two substructures "T" and "C" are extracted from the initial ground structure, the construction materials being assigned to steel and concrete respectively. Comparison to the initial one the ground structure "T" does not contain bars that are placed too close to the boundary of the domain due to concrete cover requirements. Construction of the structure "C" is more complex and will be discussed in section 3, at this point let us assume that the structure "C" coincides with the initial ground structure. The support conditions are implied, as well as the loads determined by vector \( P \), where \( s \) denotes the number of degrees of freedom of the initial ground structure. For each "T" and "C" trusses the following parameters are determined: number of bars \( n_t, n_c \), vectors of bar lengths \( L_{\{v\}}, L_{\{c\}} \) and geometric matrices \( B_{\{v\}}, B_{\{c\}} \). Vectors of cross section areas of bars \( A_{\{v\}}, A_{\{c\}} \) are design variables in the optimization process.

2.2. Modified problem of truss volume minimization. Primal and dual formulation

In order to effectively design the reinforcement of the structure with the use of ST models an optimization problem is stated as follows: find vectors \( A_v, A_c \) that minimize volume of the bars in tension and leave the ultimate limit state under the given load \( P \) not exceeded, namely under the stress constraints...
\((\sigma_c, \alpha)\) are yield stresses for steel and concrete according to the design codes). Solving such a problem a highly dense truss is formulated to approximate a Michell structure (see [2] and Fig. 1b), not applicable in practice.

Eliminating the design variables \(A_i, \phi_i\) in the first location, the following linear programming problem arises

\[
\tilde{V}_{\text{opt}} = \min \left\{ \tilde{V}_i = \frac{1}{\alpha_i} \tilde{L}_i \cdot T \right\} \quad \text{subject to} \quad B_i^j T - B_i^j C = P, \quad T, \quad C \geq 0
\]

which is a modified problem of truss volume minimization. The modification is introduced by the modified length vector \(\tilde{L}_i = L_{r_i} + p \) where \(i = 1, \ldots, n_r\); \(p\) is a penalty parameter representing the cost of the truss nodes, equal for all the bars in the truss "\(T\)". The larger the parameter \(p\), the less complex the solution, since efficiency of short bars decreases with its value.

The parameter \(p\) is set by the designer.

Regarding a problem \(\tilde{V}_{\text{opt}}\) in its primal formulation a dual problem arrives

\[
\tilde{V}_{\text{opt, dual}} = \max \left\{ \mathbf{P} \cdot \mathbf{v} \right\} \quad \text{subject to} \quad \frac{1}{\alpha_i} \tilde{L}_i \geq \Delta_i, \Delta_c \geq 0
\]

where: \(\mathbf{v}_{\mathbf{v}}\) is a vector of adjoint displacements of the truss nodes; \(\Delta_{[\mathbf{v}]} = B_i^j \mathbf{v}, \Delta_{[\mathbf{C}]} = B_i^j \mathbf{C}\) are vectors of adjoint elongations of bars of the ground structures "\(T\)", "\(C\)".

The following identity holds

\[
\tilde{V}_{\text{opt}} = \tilde{V}_{\text{opt, dual}}
\]

3. Other algorithms of the method

According to the design codes the stresses in bars at compression cannot exceed limit stresses for concrete. A bar of given compressive force exhibits a properly large cross section area, which is, on the other hand, bounded by geometry of the reinforced concrete structure. This limitation cannot be introduced directly in the problem \(\tilde{V}_{\text{opt}}\) as its constraint. The solution for the ground structure "\(C\)" being equal to the initial one contains, in general, bars in compression not fitting the feasible domain. This particular solution is the first iteration, namely \(\tilde{V}_{\text{opt}} = \tilde{V}_{\text{opt}}^{(1)}\). Elimination of the bars intersecting the boundary determines a new ground structure "\(C\)", used in the second iteration \(\tilde{V}_{\text{opt}}^{(2)}\). This procedure is continued until running the iteration \(i\), making all the compressed bars of solution \(\tilde{V}_{\text{opt}}^{(i)}\) cover the domain.

In order to make multimillion bars problems possible to solve each iteration \(\tilde{V}_{\text{opt}}^{(i)}\) is coped with using the adaptive ground structure. The algorithm starts with the use of sparse ground structure consisting of bars only at the angles of \(0^\circ, 45^\circ, 90^\circ\), yet generated for the full mesh of nodes. A problem \(\tilde{V}_{\text{opt}} = \tilde{V}_{\text{opt}}^{(i)}\) is solved both in primal and dual forms, the solution being a first sub-iteration of the iteration \(i\). Given the adjoint displacements \(\mathbf{v} = \mathbf{v}^{(i)}\) the elongations \(\Delta_i = \Delta_i^{(i)}, \Delta_c = \Delta_c^{(i)}\) of a full ground structure are computed, which allows to construct the ground structure in the second sub-iteration - the bars of new angles are added provided their elongations violate or equally satisfy the constraints of the dual problem. The procedure is continued until running the sub-iteration \(j\) where adjoint elongations \(\Delta_i^{(j)}, \Delta_c^{(j)}\) of all bars satisfy constraint, thus assuring that the solution of the problem \(\tilde{V}_j\) is the solution of the problem \(\tilde{V}_j\).

4. Generation of Strut and Tie model for the frame corner

The method described above was implemented in Wolfram Mathematica as a computer procedure. Using it the authors propose an ST model for a given frame corner. The optimization problem was solved for a ground structure of over 54 million bars and for a penalty parameter \(p = 0.2\) cm. In addition the solution is also obtained for a parameter \(p = 0\), thus arriving at a highly dense truss of minimum volume of bars in tension \(\tilde{V}_{\text{min}}\), which is an approximation of a Michell structure. Fig. 1 presents both solutions. The solution 1a) is far less complex than 1b), whereas the volume of bars in tension is only 1.5% higher.

References


Numerical study of slotted orifices shape influence on the downstream pressure distribution

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Abstract

One of the current problems is the measurement of wet gas stream, which is fundamental to the gas phase, while the liquid is present in the distributed form. The aim of the study is to analyse the impact of the slotted orifice geometry on the flow phenomenon. This orifice in various constructional variants is increasingly being used to measure the wet gas. The paper presents the results of numerical calculations of liquid flow through a standard and slotted orifices with an equal \( \beta \) ratio of 0.5. The conducted numerical calculations indicate that the standard orifice produces a larger pressure loss in comparison to the slotted orifices.

Keywords: orifice plate, slotted orifice, computational fluid dynamics, pressure distribution

1. Introduction

The flow measurements using a pipe orifices are widely used in industry due the simplicity of construction and high reliability. Many years of research led to the recognition of several types of normative documents containing guidelines for their design and methodology for calculating the fluid flow as a function of flow conditions \[1\]. Besides the standard orifice there are used special orifices, intended for measurements in non-standard flow conditions \[2\]. This group may include slotted orifice designed for wet gas measurement \[3,4\]. Appropriate design of the orifice allows to reduce the differential pressure pulsations associated with the presence of liquid in the stream of gas.

For many years, studies are being conducted in the field of exploration of new orifice construction and an initial assessment of their metrological properties supported using Computational Fluid Dynamics \[5\]. An example can be found in \[6\] where the effect of the velocity field deformation upstream of the orifice on its metrological properties were analysed.

2. Method of numerical research

The paper analyses the viscous flow of an incompressible fluid by a straight section of the pipeline in which the orifice is installed. Numerical calculations were done for a flow analysis through a standard orifice (Fig. 1a) and slotted orifices (Fig. 1b-1d). The considered orifices have the same area ratios, equal to \( \beta = 0.5 \), defined as:

\[ \beta = \frac{A_{\text{pipe}}}{A_{\text{slots}}} \] (1)

The motion of a turbulent incompressible fluid is described by the set of equations:

\[ \rho U \cdot V U = -\nabla p + \nabla \mu_{\text{eff}} \cdot (V U + V U^T) + \frac{2}{3} \rho \nabla k \] (2)

\[ \nabla \cdot U = 0 \] (3)

where \( U \) is the velocity vector, \( \rho \) – fluid density, \( k \) – turbulent kinetic energy, \( \varepsilon \) – turbulence energy dissipation rate, \( \mu_{\text{eff}} \) – effective viscosity, and \( \mu \) is turbulent viscosity:

\[ \mu_{\text{eff}} = \mu + \mu_t \] (4)

where \( \mu_t = \rho C_{\mu} \frac{k^2}{\varepsilon} \) (5)

\[ \rho N \cdot dU = \nabla \frac{\rho_e}{\sigma_e} \cdot \nabla \varepsilon + \frac{2}{k} \left( c_1 \mu_{\text{eff}} G - c_2 \rho \varepsilon \right) \] (6)

\[ \rho N \cdot k dU = \nabla \frac{\rho e}{\sigma_k} \cdot \nabla k + \mu_{\text{eff}} G - \rho \varepsilon \] (7)

where \( c_1, c_2 \) are empirical constants, \( \sigma_k \) i \( \sigma_e \) the Prandtl/Shmidt number of turbulent for \( k \) and \( \varepsilon \), component \( G \) is related to the generation of the kinetic energy \[5\]. The assumed constant values:

\[ \sigma_k = 1.00, \quad \sigma_e = 1.30, \quad c_1 = 1.44, \quad c_2 = 1.92, \quad c_2 = 0.09 \]

Figure 1: Orifices considered in the paper: a) standard orifice, b) slotted orifice 1, c) slotted orifice 2, d) slotted orifice 3
The system of equations was solved using the finite volume method. For numerical calculations of flow ANSYS Fluent 14 software was used. [7]

3. Results

The calculations were carried out for the value of the Reynolds number Re = 161240. Walls of the pipe were taken as smooth. Figure 2 shows the normalized pressure distribution along the pipe wall, while Fig. 3 illustrated the results of calculations relating to the velocity distribution.

Figure 2: Pressure distribution along the pipe wall

Figure 3: Velocity distribution in calculation area: a) standard orifice, b) slotted orifice

In case of slotted orifices, comparing to the standard orifice flowing stream is divided into several streams so that the fluid flows through the entire cross section of the pipeline, thereby reducing the big flow disturbance which cases standard orifice. Based on the obtained numerical results it can be concluded that, for the slotted orifices the length of flow stabilization is much shorter than in the standard orifice.

4. Conclusions

On the basis of the studies it can be concluded, that the shape of the slotted orifice substantially affects both: the differential pressure and a permanent pressure loss. The smallest permanent pressure loss occurs in the orifice with a large number of radial slots (Fig. 1c). In this case there is a division of the flow stream into multiple elementary streams, which causes a rapid loss of disturbances and large pressure recovery.

Division of the flowing stream into numerous elementary streams, however, reduces the differential pressure which reduces measurement accuracy. This requires the implementation of further studies to optimize the shape of the slotted orifice.

References

Biomimetic optimisation – differences and similarities in comparison to the SIMP method

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Abstract

This paper presents comparison of two methods of structural optimization. The first one is the SIMP method, common in topology optimization, the second is the biomimetic optimization, developed in Division of Virtual Engineering in Poznan University of Technology. These methods generated similar results in simple tests. However, some of the examined cases resulted in other geometries. One of them, the multiload case is discussed here. A type of a beam is regarded bent by alternating in time, perpendicular forces. This kind of example was repeatedly analyzed in the case of the SIMP method, therefore the new Biomimetic optimization method will be examined in particular attention.

Keywords: topological optimisation, parallel computing, biomimetics

1. Introduction

Topological Optimization plays an increasingly important role in the industry nowadays. Although, most of its customers are aerospace and automotive engineers, such industries as construction, medical engineering and furniture industries are involved as well. Undoubtedly the greatest success of topology optimization in recent years is the internal structure of the wing of Airbus A380. Another, perhaps comparable success was its use in the construction of the car’s body geometry in Mercedes SLS AMG. Both of these cases are a sign that in future we can expect more design requiring topology optimization. To meet expectations, the Division of Virtual Engineering in Poznan University of Technology have developed Biomimetic Optimization, a new approach to optimization of shape, size and topology in one procedure.

2. Biomimetic optimization

The base to develop a new type of structural optimization was the phenomenon of the adaptation of a trabecular bone. This effect, observed for the first time in 1892 by Julius Wolff [1] affects all the bones of vertebrates, however, the most visible is in the case of trabecular bone. This effect is based on the adaptation of the bone under the external force stimulation to avoid bone destruction. Any adaptation in trabecular bone is a very energy-intensive task for the body and any action in this matter must be conducted in an optimal way. For this reason, in case of closure of external stimulation for a long time organism loses unused cells. Due to continuity of the whole phenomenon, it takes place only on the surface of the tissue to avoid damage of the existing structure. The process of adaptation consist of resorption or absorption of cross-sectional area, and a topology change of existing trabeculae. For this reason, it can be regarded a specific example of optimizing the shape, size and topology.

2.1. The adaptation of a biomimetic optimization model into a computer system

Numerous studies of trabecular bone adaptation phenomenon lead to a mathematical explanation of the mechanism of absorption and resorption of tissue, which is based on Strain Energy Density [SED] on the surface of the bone. This equation is a base used to implement mathematical phenomenon of adaptation of the trabecular bone into ParallelCosmoProjector system [2]. This system, in its beginning dedicated to the aviation industry is able to perform a full optimization of the shape, size and topology based on CFD calculations and the forces generated by the pressure from the flow around optimized solid. The system at this stage has already archived some success, such as optimization of the Fowler flap of aircraft I-22 Iryda in cooperation between Division of Virtual Engineering and the Polish State Institute of Aviation in Warsaw.

2.2. The structure of the system

The presented system consists of a few stand-alone modules with unique functions. The most important are: ParallelCosmoProjector; Mesh generator based on 2D cross-section of the solid. TAU code (Deutches Zentrum fuer Luft und Raumfahrt) to CFD computing in case of FSI Biomimetical Optimization. Abaqus structural code to general computing and AE_Tools – procedure to deform generate mesh during optimization process. In the examples presented below no CFD parts were used. In figure 1 ParallelCosmoprojector loop were showed.

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2.3. SIMP topological optimization method

SIMP is one of the oldest and simplest methods of topological optimization. Its specify that SIMP require the input geometry not of the basis of solution or a simple connection of constraint and force, but the full area where the final optimization solution is available. This solution generates additional requirements for computing power. The main idea of SIMP method is to eliminate the least loaded elements in the mesh in each iteration until the planned percentage of volume is reached [3].

3. Comparison

3.1. Comparison of the 2D case

The study compared two geometries: The first is a simple stick loaded at the end by bending force and constrain on the opposite one. The fixed force acts continuously during all iterations. In case of the Biomimetic Optimization an additional constrain can be formed on each length on the surface joined to the constrained end of the stick. In this method the input is the geometry of a stick model. In the case, as the SIMP method was to achieve the same effect, it was necessary to create a mesh of the entire design space and set as input geometry. Then, one of its surfaces was chosen a constrain. Both of the optimization results are very similar.

3.2. Comparison of the 3D case

As shown in Figure 2, in 3D case the difference between these two methods is noticeable. In the input data not only the area of optimization changed, but also the method of force stimulation. Two perpendicular forces were applied to one end of the stick and affected alternately each of the two iterations. This case is different from the common tasks of Topological Optimization and prevents the development of solution from the point 3.1 but rotated of 45 degrees from the vertical. The results of both methods are clearly different. While the commonly accepted method SIMP formed a caisson structure with clearly closed section, The Biomimetic Optimization method results were clearly similar to trabecular bone without any larger surface of solid.

This solution seems to have a potential for technological use, especially in airplane industry where most of the structural frame consist only of simple beam similar to trabeculae.

4. Conclusions

Biomimetic Optimization for more complex tasks can give different results from the SIMP method. The reason of this difference is in mathematical approach in both methods. Biomimetical Optimization rejects large flat surfaces where, due to the torsional and bending forces, it is difficult to maintain the condition of constant SED on the surface. Flat surface in this kind of condition can be created only at very low energy thresholds, or in case of very broad range of the SED. This can be an advantage of this method especially in the airplane industry where mostly a large surface in structural frames are rejected due to its weight and difficulties in maintaining quality.

References

Stress based version of isotropic material design in two dimensions

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Abstract

The paper deals with the optimal distribution of the bulk and shear moduli (equivalently Young’s modulus and Poisson’s ratio) minimizing the compliance of an inhomogeneous isotropic elastic 2D body transmitting a given boundary loading to a given support. The isoperimetric condition is expressed by the integral of the trace of the Hooke tensor being a linear combination of both the moduli. The problem thus formulated is reduced to an auxiliary 2D problem of minimization of a certain stress functional over the stresses being statically admissible. The integral of the stress functional is a linear combination of the absolute value of the trace and of the norm of the deviator of the stress field. The auxiliary problem is solved numerically by introducing element-wise polynomial approximations of the components of the trial stress fields and imposing satisfaction of the variational equilibrium equations. The under-determinate system of these equations is solved numerically thus reducing the auxiliary problem to an unconstrained problem of nonlinear programming.

Keywords: free material design, non-homogeneous isotropic elasticity, compliance minimization

1. Introduction

In the FMD problem in its original setting by Bendsoe et al.[1] and in the further papers by M.P. Bendsoe, M. Kočvara, G. Leugering, J.M. Guedes, R.B. Haber, P. Pedersen, J.E.Taylor and other authors, see e.g [2] and further references in [3], all the moduli of the anisotropy are design variables. In the case of a single load condition the optimal material occurs to be singular, with only one nonzero load condition of the elastic moduli tensor, the remaining five eigenvalues being zero. One possible remedy to make the optimal Hooke tensor non-singular is to impose the body to multiple load conditions. In a 3D case at least 6 load conditions are necessary to make the optimal Hooke tensor non-singular, while in 2D three load cases suffice, see [3]. The other remedy is to impose certain symmetries on the Hooke tensor. The present paper introduces a priori the assumption of isotropy of the optimal layout; the design variables are the fields of bulk and shear moduli varying within the design domain. The optimization process is performed under the condition of the integral of the trace of the elastic moduli tensor (or Hooke tensor) being fixed, now expressed by both the bulk and shear moduli. This version of FMD will be called here Isotropic Material Design (IMD).

Most of the papers on FMD express the optimization problem with using displacement fields as behavioral unknowns, which needs keeping artificially the assumption of positive definiteness of the Hooke tensor during the computational process of optimization. The stress based version of FMD developed recently, see Ref. [3], circumvents this inconsistency by reducing the optimum design problem to an auxiliary minimization problem with the integrand being a norm of the stress field. To solve the latter problem we need implementation of a finite-dimensional approximation of statically admissible stress fields or stress functions. This feature clearly distinguishes the stress-based version of the FMD method from other, previously published works on the FMD subject and, at the same time, requires approximation of all components of the stress tensor. At present, available commercial systems of static analysis neither offer a stress-based version of finite element method nor packages of finding the solutions of linear systems of equations with rectangular matrices. To solve the optimization problem we have prepared our program based on the 4-nodal, isoparametric finite element C2D4 implemented in C++ for the stress-based IMD problem for plates loaded in plane. Numerical results obtained by the IMD method revealed surprising feature of the stiffest elastic structure made of the non-homogeneous isotropic material: almost always there appear subdomains in the optimal body that are characterized by almost extreme values of the Poisson’s ratio (often close to -1 or to 1, which is admissible in a 2D case). In particular, the optimal distribution of the isotropic material with negative values of the Poisson’s ratio points at the significance of using the materials of auxetic properties, Ref.[4].

2. Formulation of an optimal design and numerical example

Let us assume that a heterogeneous isotropic body is to be constructed within a given domain \( \Omega \) with \( \mathbb{R}^2 \) denoting its area, i.e. the two fields: \( k = k(x) \), \( \mu = \mu(x) \), \( x = (x_1,x_2) \in \Omega \subset \mathbb{R}^2 \) of bulk and shear moduli are to be placed into a domain \( \Omega \). Only one global isoperimetric condition is imposed on both of these fields, namely:

\[
\frac{1}{2} (2k + 4\mu) dx = \Lambda, \quad \Lambda = E_0 |\Omega|
\]

(1)

that could be interpreted e.g. as the cost of the design, the integrand (1) being viewed as a unit cost of the material and \( \Lambda \) being viewed as the entire cost of the design proportional to the given value of \( E_0 \) \( [N/m^2] \). We consider the following optimization problem: find the both fields \( k^* = k^*(x) \), \( \mu^* = \mu^*(x) \) minimizing the following functional:

\[
(k, \mu) \to \frac{1}{\Omega} \int_{\Omega}^{} u_1(k, \mu) \ dx,
\]

(2)

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where the displacement field \( \mathbf{u} = \mathbf{u}(k, \mu) \) is the solution of the equilibrium problem i.e.

\[
\forall \mathbf{v} \in V(\Omega) \int_{\Omega} H_{ikj} \sigma_{ij}(\mathbf{u}) \varepsilon_{ij}(\mathbf{v}) \, dx = \int_{\Gamma} t_i \mathbf{v}_i \, da .
\]  

(3)

In (2) and (3), \( \mathbf{v} = (v_i) \), \( \mathbf{t} = (t_i) \in \Gamma_\varepsilon \subset \partial \Omega \), \( \varepsilon = \left( \varepsilon_{ij} \right) \), \( \mathbf{H} = (H_{ikj}) \), \( (i, j, k, l = 1, 2) \) denote the fields of virtual displacements, boundary tractions, strains and Hooke tensor, respectively. The above problem can be simplified and reformulated resulting in the following computational scheme.

The first step is to solve the problem: minimize the stress-based functional

\[
\mathbf{t} \rightarrow \left[ \frac{\sqrt{2} \left\| \mathbf{t} \right\|}{2 + \sqrt{2} \left\| \text{dev} \mathbf{t} \right\|} \right]
\]  

(4)

over the virtual stress fields \( \mathbf{t} \) being statically admissible. Such trial fields form a linear affine set parameterized by the load applied. The integrand of the functional above has a linear growth, which implies that the solution can concentrate on certain curves. In general, the minimizer \( \mathbf{t}^* \) can be a Hausdorff measure, hence dx and a domain of integration is omitted in (4).

Having solved the problem (4) one can construct the solution \((k^*, \mu^*)\) of the original compliance minimization problem as below:

\[
k^*(x) = \frac{\left\| \mathbf{t}^* (x) \right\|}{2 \left( \sqrt{2} \left\| \mathbf{t}^* (x) \right\| / 2 + 2 \left\| \text{dev} \mathbf{t}^* \right\| \right)} \frac{\text{dev} \mathbf{t}^* (x)}{dx},
\]  

\[
\mu^*(x) = \frac{\left\| \mathbf{t}^* (x) \right\|}{2 \left( \sqrt{2} \left\| \mathbf{t}^* (x) \right\| / 2 + 2 \left\| \text{dev} \mathbf{t}^* \right\| \right)} \frac{\text{dev} \mathbf{t}^* (x)}{dx}
\]  

(5)

For illustration we consider a deep beam \( 1.0 \times 0.25 \) [m] with two clamped, vertical edges (clamped beam) – see Fig.1. The two vertical forces \( P = 1.0 \) [N] are applied to the middle points on each horizontal edge, respectively and modeled by the weight functions. The unity value of elastic modulus \( E_0 \) in (1): is assumed, i.e. \( E_0 = 1.0 \) [N/m²], because the optimal values of the design moduli and optimal compliances are proportional to \( E_0 \) or to its inverse.

![Figure 1: Clamped beam loaded with two vertical forces](image)

The images of the Young’s modulus \( E^* \) and Poisson’s ratio \( v^* \) in the optimal isotropic body are shown in Fig. 2 below.

![Figure 2: Oblique images of the Young’s modulus \( E^* \) and Poisson’s ratio \( v^* \).](image)

References


Selected problems of numerical analysis of Free Material Design

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Abstract

The paper deals with theoretical foundations and numerical issues of the implementation of stress-based version of the Free Material Design (FMD) method. The problem of finding stress trajectories in optimal anisotropic or optimal isotropic, three-dimensional, non-homogeneous elastic bodies is also briefly discussed. The discussion in the paper is limited only to the case of finding the optimal distribution of the bulk and shear moduli (equivalently Young’s modulus and Poisson’s ratio) minimizing the compliance of a non-homogeneous isotropic elastic 3D body transmitting a given boundary loading to a given support. The isoperimetric condition is expressed by the integral of the trace of the Hooke tensor being a linear combination of both the moduli. The problem thus formulated is reduced to an auxiliary 3D problem of minimization of a certain stress functional over the stresses being statically admissible. 

The problem is solved numerically by introducing element-wise polynomial approximations of the components of the trial stress fields and imposing fulfillment of the variational equilibrium equations. The under-determinate system of these equations is solved numerically thus reducing the auxiliary problem to an unconstrained problem of nonlinear programming.

Keywords: free material design, non-homogeneous isotropic elasticity, compliance minimization

1. Introduction

The present paper deals with optimum design of anisotropy of elastic structures with the framework of the Free Material Design (FMD) method in its stress-based version [1]. The target function is the total compliance, while the design cost is expressed by the integral of the trace of the Hooke tensor being the Euclidean norm. In the fully anisotropic version the FMD, for the single load case, reduces to solving the auxiliary problem

\[ Z = \inf \left\{ \int_{\Omega} \lambda \, d\mu \mid \lambda \in \Sigma(\Omega) \right\} \]  

(1)

see (50) in [1], in which the integrand, growing linearly is the Euclidean norm of a tensor-valued measure \( \lambda = \{ \lambda_{ij} \} \) standing for the virtual stress fields; \( \Sigma(\Omega) \) is the set of statically admissible stresses. Since the solution of (1) is expected to be a Hausdorff measure, the notation \( \mu \) and the domain of integration are omitted. A similar problem appears in the minimum compliance problem, see Eq. (2.12) in [2], corresponding to the case of \( \rho(z) \) being the Euclidean norm.

Then the integrand in (2.12) in [2] coincides with that in (1). Thus the theory of problem (2.12) given in [2] applies to (1). One should consider the virtual stress fields as \( R^3 \times R^3 \) valued measures on \( R^3 \), while the integral in (1) should be understood in the sense of convex functional on the space of measures. Paper [2] delivers a proof that the problem dual to (1) reads

\[ Z = \sup \left\{ \langle f, u \rangle \mid u \in Lip_{\rho,\rho}(\Omega, \Sigma) \right\} \]  

(2)

and it is well posed, provided that the load \( f \) is self-equilibrated; \( \Sigma \) is the support domain where \( u \) vanishes. Functions of \( Lip_{\rho,\rho}(\Omega, \Sigma) \) satisfy the inequality \( \|u\|_{\rho} \leq 1 \) in \( \Omega \), where \( \|u\|_{\rho} \)

is the symmetric part of \( Vu \); index \( \rho \) refers to the definition:

\[ \rho(Vu) = \left\| Vu \right\|_{\rho} \]  

The problem (1) can be expressed by

\[ Z = \min \left\{ \int_{\Omega} \|\lambda\|_{\rho} \, d\mu \mid \tau \in L^1(\Omega; R^3 \times R^3) \times \mu \right\} \]  

(3)

Here the minimizer \( \sigma \) is the function (not a measure). This function is integrable with respect to the (a priori unknown) scalar-valued measure \( \mu \); the relation \( \lambda = \sigma\mu \) expresses Radon-Nikodym theorem. The difficulty in (1) or (3) lies in that the measure \( \mu \) can be 1-, 2- or 3-dimensional Hausdorff measure. In the present paper we shall confines consideration to 3-dimensional measures, since by remeshing the domain we can approximate correctly elastic response of fibres and membranes (shells) to be formed within the optimal structure. The equilibrium equations in (1) or (3) are fulfilled in the weak sense. In the isotropic version (IMD) of FMD the problem (1) still holds, yet involves a different norm

\[ \|\lambda\|_{\rho,\rho} = \alpha \|\lambda\|_{\rho} + \beta \|\lambda\|_{\rho} \]  

\[ \alpha > 0 , \beta > 0 \]  

(4)

Consequently, the problem (2) is modified by new definition of \( \rho ; \rho(Vu) = \left\| Vu \right\|_{\rho,\rho} \) with \( \left\| Vu \right\|_{\rho} = \max_{\tau \in \Omega} \frac{\tau \cdot \mu}{\|\tau\|_{\rho,\rho}} \). The above changes do not affect the essence: both the problems (1) and (2) can be put in the form (3) with \( \tau \) being functions, not measures.

2. Formulation of an optimal design problem and case studies

Let us assume that a heterogeneous isotropic body is to be constructed within a given domain \( \Omega \) with \( [\Omega] \) denoting its area, i.e. the two fields: \( k = k(x) , \mu = \mu(x) , x = (x_1, x_2, x_3) \in \Omega \subset R^3 \) of bulk and shear moduli are to be placed into a domain \( \Omega \). Only
one global isoperimetric condition is imposed on both of these fields, namely:

\[
\int_{\Omega} \left( 3k + 10\mu \right) dx = \Lambda, \quad \Lambda = E_0 |\Omega| \tag{5}
\]

for a given \( E_0 \left[ N/ m^2 \right] \). We consider the following optimization problem: find the both fields \( k' = k'(x) \), \( \mu' = \mu'(x) \) minimizing the following functional:

\[
\int_{\Gamma_1} f_t \, u_t(k, \mu) \, da,
\]

where the displacement field \( u = u(k, \mu) \) is the solution of the elasticity problem i.e.

\[
\int_{\Omega} \int_{\Omega} \varepsilon_{ij} \sigma_{ij} \, dx = \int_{\Gamma_1} v_t \, u_t \, da. \tag{6}
\]

In (6) and (7), \( v = (v_t) \), \( t = (t_t) \in \Gamma_2 \subset \partial \Omega \), \( \varepsilon = (\varepsilon_{ij}) \) denote the fields of virtual displacements, boundary tractions, strain tensor, respectively. It can be shown that above problem can be reduced to the problem (1) with integrand (4) for \( \alpha = \sqrt{10} \), \( \beta = 5\sqrt{6} \). After dividing the design domain \( \Omega \) into finite elements and upon element-wise approximation of stresses, the solution of the problem (1) with the imposed isoperimetric condition (5) assumes the form

\[
k'(x) = \frac{\Lambda}{3} \left[ \begin{array}{c} \tau^* \varepsilon \left( x \right) \cr \alpha 3 \cosh 3 \cr \alpha 3 \cosh 3 \end{array} \right], \quad \mu'(x) = \Lambda \left[ \begin{array}{c} \frac{3}{2} \cosh 3 \cr \frac{3}{2} \cosh 3 \end{array} \right], \tag{8}
\]

where \( \tau^* \) is the statically admissible minimizer of the functional (1) with approximated integrand (4).

Construction of stress trajectories in the optimal, isotropic and non-homogeneous elastic body requires finding the solution \( [0, \infty) \ni s \rightarrow r(s) \in \mathbb{R}^3 \) of the system of three ordinary differential equations

\[
\frac{d}{ds} r(s) = \sigma_i [s, r(s)]
\]

where \( s \) denotes the natural parameter of the investigated curve \( r(s) \) and \( \sigma_i [s, r(s)] \in \mathbb{R}^3 \), \( (k = 1, 2 \text{ or } 3) \) is one of three eigenvectors of the stress tensor \( \sigma_i [s, r(s)] \) (sorted with respect to its eigenvalues \( \sigma_i [s, r(s)] \)). The heterogeneity of optimal body \( \Omega \) as well as the optimal Hooke tensor degeneracy in many subdomains of \( \Omega \) (a common property in topology optimization) severely hampers the implementation of well-known numerical methods, e.g. the use of any Runge-Kutta method.

The first example concerns a plate \( 1.0 \times 0.5 \times 0.25 \) [m] with two clamped, vertical sides – see Fig.1. Two vertical forces \( P = 1.0 \) [N] are applied to the middle points on each horizontal sides, respectively and are modeled by the weight functions. The unit value \( 2 \times 1.0 \) \[ \text{N/m} \] of elastic modulus in (5) was assumed.
Topology optimization of spatial continuum structures made of a non-homogeneous material of cubic symmetry

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Abstract

The paper deals with the minimum compliance problem of spatial structures made of a non-homogeneous elastic material of cubic symmetry. The elastic moduli and the trajectories of anisotropy directions are design variables. The isoperimetric condition fixes the value of design cost being an integral of the unit cost assumed as a linear combination of the three elastic moduli of cubic symmetry. The problem was reduced to the pair of mutually dual auxiliary problems similar to problems of the theory of materials with locking and the transshipping theory. Integrando of the auxiliary minimization problem grows linearly, which transforms the problem considered to the topology optimization problem to simultaneously obtain the shape of the structure and its material characteristics. In contrast to the Free Material Design leading to the optimal Hooke tensor with a single non-zero eigenvalue in a single load case, the optimal Hooke tensor of cubic symmetry gives either three or four non-zero eigenvalues.

Keywords: topology optimization, free material design, cubic symmetry

1. Introduction

The paper puts forward a topology optimization method aimed at constructing a stiffest continuum structure transmitting a given load to a given boundary. The problem is specified assuming that the designed structure is formed of a non-homogeneous elastic material of cubic symmetry at each point. All the fields which determine the cubic anisotropy within the whole body are design variables. The isoperimetric condition imposed is the design cost, expressed by the spatial integral of a linear combination of the eigenvalues of the elastic moduli. This expression encompasses the popular definition of cost, as the integral of the trace of the Hooke tensor.

Isotropic Material Design (FMD) to the case of materials of cubic symmetry. In its original formulation the FMD involves no restrictions on the anisotropy. The peculiar feature of the compliance minimizing FMD problems is a possible elimination of all design variables, leading to an auxiliary problem of the form:

\[
\min_{\Omega} \left\{ \int_{\Omega} \| \sigma \| \, dx \mid \tau \in \Sigma(\Omega) \right\}
\]

where \(f(v)\) represents the virtual work of the load on the displacement field \(v\); \(V(\Omega)\) being the space of kinematically admissible displacement fields; \(\varepsilon(v)\) is the strain tensor, defined as the symmetric part of the gradient of \(v\).

The problems \((P_1), (P_2)\) have been derived in papers by Czarnecki and Lewiński [4]. The mathematical structure of these mutually dual problems is similar to the Kantorovich-Rubinstein transshipping problem, while their equivalence can be proved in the manner the Theorem 3.3 in Bouchitté et al.[2] was proved. The inequality involved in \((P_2)\) can be written as the inclusion: \(\varepsilon(v(x)) \in B\) where \(B\) is a locking locus. Here \(B\) is the unit ball with respect to the Euclidean norm.

A natural modification of the FMD is a priori imposing certain material symmetries. The strongest assumption is isotropy; this modification was proposed by Czarnecki [3], is called the Isotropic Material Design (IMD). The only design variables are the bulk \(k(x)\) and shear \(\mu(x)\) moduli in each point \(x\) of the feasible domain. In the spatial setting the collection of the eigenvalues of the Hooke tensor is: \((3k, 2\mu, 2\mu, 2\mu)\) and the trace of the Hooke tensor equals \(3k + 10\mu\). Let the cost of the design be the integral of the trace of the Hooke tensor.

The composite materials and crystals are usually anisotropic. Thus it is useful to extend the FMD method to the class of design of lower symmetry. In the paper the material design will be confined to a cubic symmetry case. In each point of the structure the six parameters: the three elastic moduli and a triplet \((n,m,p)\) of mutually orthogonal unit vectors satisfying:

\[
||n|| = ||m|| = ||p|| = 1; \quad n \cdot m = 0, n \cdot p = 0, m \cdot p = 0
\]

are to be determined. The Hooke tensor of a material of cubic symmetry is represented by the celebrated Walpole formula [6]:

\[
C = aJ + bL + cM
\]

where \(a, b, c\) are elastic moduli while the fourth-rank tensors \(J, L, M\) are expressed as below

\[
J = \frac{1}{3} I \otimes I, \quad L = I - S, \quad M = S - J
\]

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The design cost is taken as the integral of the linear combination of the elastic moduli:

\[
J = \frac{1}{\Lambda} \left( \min_{\Omega} \left( \int_{\Omega} ||\tau|| \, dx \right) \right)^2
\]

(7)

where \(\alpha_i > 0\) are fixed. If \(\alpha_1 = 1\), \(\alpha_2 = 3\), \(\alpha_3 = 2\) then the unit cost is equal to \( \text{tr} C \). We consider the following problem of optimum design:

find the layout of the elastic moduli \(a, b, c\) and the orthogonal trajectories of the vector fields \((m, n, p)\) at each point of the feasible domain \(\Omega\), satisfying the isoperimetric condition (6), such that the structure made of this non-homogeneous material is characterized by the smallest total compliance among all structures designed in the same feasible domain, obeying the same isoperimetric condition and capable of transmitting the same load to the same boundary. The minimal compliance of the optimal structure equals

\[
J = \frac{1}{\Lambda} \left( \min_{\Omega} \left( \int_{\Omega} ||\tau|| \, dx \right) \right)^2
\]

where

\[
||\tau|| = \sqrt{\frac{\alpha_1}{3} ||\tau|| + \min(\sqrt{\alpha_2}, \sqrt{\alpha_3})||\text{dev} \tau||}
\]

(8)

The problem (8) has almost the same form as that occurring in the similar problem concerning non-homogeneous isotropy, see Czarnecki [3]. It can be proved that for each choice of the triplet \((m, n, p)\), satisfying (1) the following estimates hold

\[
\frac{1}{2} (\text{tr} \sigma)^2 \leq \tau (S \tau) \leq ||\tau||^2
\]

(9)

In case of \(\alpha_2 > \alpha_3\) the right inequality becomes sharp and the minimizer \((m^*, n^*, p^*)\) coincides with the principal directions of the stress tensor \(\tau\). In case of \(\alpha_2 < \alpha_3\) the left inequality becomes sharp and the minimizer \((m^*, n^*, p^*)\) can be found from the Norris condition (28), see Norris [5].

Case of \(\alpha_2 < \alpha_3\). The optimal moduli are expressed below

\[
a^*(x) = \frac{1}{\sqrt{\alpha_2}} \sqrt{\frac{1}{3} ||\text{tr} \sigma(x)||}, \quad b^*(x) = \frac{1}{\sqrt{\alpha_2}} ||\text{dev} \sigma(x)||, \quad c^*(x) = 0
\]

(10)

where the multiplier \(\lambda\) is defined as

\[
\sqrt{\lambda} = \frac{1}{\Lambda} \int_{\Omega} \left( \sqrt{\frac{\alpha_1}{3} ||\text{tr} \sigma|| + \sqrt{\alpha_2} ||\text{dev} \sigma||} \right) \, dx
\]

(11)

Case of \(\alpha_2 > \alpha_3\). The optimal moduli take the form

\[
a^*(x) = \frac{1}{\sqrt{\alpha_2}} \sqrt{\frac{1}{3} ||\text{tr} \sigma(x)||}, \quad b^*(x) = \frac{1}{\sqrt{\alpha_2}} ||\text{dev} \sigma(x)||, \quad c^*(x) = 0
\]

(12)

where multiplier \(\lambda\) is described below

\[
\sqrt{\lambda} = \frac{1}{\Lambda} \int_{\Omega} \left( \sqrt{\frac{\alpha_1}{3} ||\text{tr} \sigma|| + \sqrt{\alpha_2} ||\text{dev} \sigma||} \right) \, dx
\]

(13)

The case of \(\alpha_2 > \alpha_3\) is particularly important as encompassing the isoperimetric condition expressed in terms of the trace of the Hooke tensor. Then \(\alpha_1 = 1, \alpha_2 = 3, \alpha_3 = 2\) and the norm (8) assumes the form

\[
||\tau|| = \frac{\sqrt{3}}{3} ||\text{tr} \tau|| + \sqrt{2} ||\text{dev} \tau||
\]

(14)

The problem dual to (P) with the norm (14) assumes the form (P’)

\[
||\varepsilon(x)||^* \leq 1
\]

(15)

where \(\varepsilon \in \mathbb{E}^3\) and a new norm in \(\mathbb{E}^3\) is defined by

\[
||\varepsilon||^* = \sup_{\tau \neq 0} \frac{\tau \cdot \varepsilon}{||\tau||}
\]

(16)

This is a norm dual to (14). If the design cost is expressed by the trace of \(C\) then the locking locus is the form

\[
\max \left\{ \frac{\sqrt{3}}{3} ||\text{tr} \varepsilon||, \frac{\sqrt{2}}{2} ||\text{dev} \varepsilon|| \right\} \leq 1
\]

(17)

In the space of principal strains the locking locus assumes the shape of a cylindrical domain of the axis along the vector of \(e = (1, 1, 1)\). The length of the cylinder equals 2 while its radius equals \(2\sqrt{3}/3\).
On the optimization of hyper-stress fields

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Abstract

In [1, 2, 3], the authors considered optimal stress fields that equilibrate some given loading. Neglecting any constitutive properties of the material, the stress fields are instigated of the smallest possible norm. The analysis led to the concept of load capacity ratio, a number $C$ depending only on the geometry of the body such that a perfectly plastic body will not collapse plastically as long as the maximum magnitude of the applied load is not greater than the product of the load capacity ratio and the yield stress. This abstract describes an extension of that work to continuum mechanics of the order higher than one.

Keywords: high order continuum mechanics, hyper-stresses, optimization, worst case loading, load capacity ratio

1. The Mathematical Framework

Let $B : X \rightarrow Y$ be a surjective linear mapping from the normed vector space $X$ to the vector space $Y$. We consider the following optimization problem. For a given $y \in Y$, determine

$$\xi_y^{\text{opt}} = \inf \left\{ \| x \| : x \in B^{-1}(y) \right\}.$$  

This problem may be analyzed conveniently in the particular setting where for Banach spaces $W$ and $S$, and a norm preserving mapping $\varepsilon : W \rightarrow S$, so that $X = S^\ast$, $Y = W^\ast$ and $B = \varepsilon^\ast : S^\ast \rightarrow W^\ast$ are the respective dual objects. Thus, for some $f \in W^\ast$, the optimization problem considers

$$s_f^{\text{opt}} = \inf \left\{ \| \sigma \| : \varepsilon^\ast(\sigma) = f \right\}.$$  

Consider the linear functional $f \circ \varepsilon^{-1} : \text{Image}(\varepsilon) \rightarrow \mathbb{R}$. By definition,

$$\sup_{x \in \text{Image}(\varepsilon)} \frac{\| f \circ \varepsilon^{-1}(x) \|}{\| x \|} = \sup_{w \in W} \frac{\| f(w) \|}{\| w \|} = \| f \|.$$  

By the Hahn-Banach Theorem, there is an element $\sigma^{\text{hb}} \in S^\ast$ such that

$$\| \sigma^{\text{hb}} \| = \sup_{x \in \text{Image}(\varepsilon)} \frac{\| f \circ \varepsilon^{-1}(x) \|}{\| x \|} = \frac{\| f \circ \varepsilon^{-1}(\varepsilon^\ast(\sigma)) \|}{\| \varepsilon^\ast(\sigma) \|} = \| f \|,$$

and for all $w \in W$, $\sigma^{\text{hb}}(\varepsilon(w)) = f \circ \varepsilon^{-1}(\varepsilon(w))$, so $f = \varepsilon^\ast(\sigma^{\text{hb}})$. Since any solution $\sigma \in (\varepsilon^\ast)^{-1}\{ f \}$ satisfies $\sigma(\chi) = f(\varepsilon^{-1}(\chi))$ for all $\chi \in \text{Image}(\varepsilon)$, for such a generic solution $\sigma$ one has in general,

$$\| \sigma \| = \sup_{\chi \in \mathbb{R}^n} \sigma(\chi) \geq \sup_{\chi_0 \in \text{Image}(\varepsilon)} \frac{\sigma(\chi_0)}{\| \chi_0 \|} = \| f \|.$$  

We conclude therefore that $s_f^{\text{opt}} = \| f \|$ and that the optimum is attained for a Hahn-Banach extension $\sigma^{\text{hb}}$ of $f \circ \varepsilon^{-1}$.

2. The Application to Hyperstress Analysis

We now apply the foregoing framework to the optimization of the solutions of the equilibrium equations of high order continuum mechanics.

The principle of virtual work for $p$-th order continuum mechanics in $\mathbb{R}^n$ may be written as

$$F(w) = \sum_i F_i(w_i) = \sum_{i, \alpha, |\alpha| \leq k} \int_{\Omega} \sigma_{i,\alpha} w_i,\alpha d\lambda^n.$$  

Here, $\lambda^n$ is the Lebesgue measure in $\mathbb{R}^n$, $w$ is a virtual vector field over $\Omega$ and $w_i$ are its components, $F_i$ are its components, $\alpha = (\alpha_1, \ldots, \alpha_n)$ is a multi-index, $|\alpha| = \alpha_1 + \cdots + \alpha_n$, and

$$w_i,\alpha = \frac{\partial^{|\alpha|} w_i}{\partial x_1^{\alpha_1} \cdots \partial x_n^{\alpha_n}}.$$  

The fields $\sigma_{i,\alpha}$ are the hyper-stress fields for $k = 1$ retrieving standard continuum mechanics. For a given $F = (F_1, \ldots, F_n)$, the solution $(\sigma_{i,\alpha})$, that satisfies (1) for all $w$ in an appropriate function space is not unique. Without a constitutive relation, the equilibrium equation is not possible to be solved uniquely.

Among all solutions $(\sigma_{i,\alpha})$ of (1), the solutions are investigated to detect the least supremum of $\|\sigma_{i,\alpha}(x)\|$ over all $x, i, \alpha$. In other words, is minimized the maximum component of the hyper-stress field.

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For an open set $\Omega$ with a Lipschitz boundary, we consider the Banach space of vector fields
\[
W = \{ w : \Omega \to \mathbb{R}^n \mid w_{i\alpha} \in L^1(\Omega), \ 0 \leq |\alpha| \leq k \},
\] (3)
where $L^1(\Omega)$ is the space of Lebesgue integrable functions on $\Omega$. As a norm on $W$ we use
\[
\|w\| = \sum_{|\alpha| \leq k} \|w_{i\alpha}\| = \sum_{|\alpha| \leq k} \int_\Omega |w_{i\alpha}| \, d\lambda^n.
\] (4)

In other words, $W = W_1^1(\Omega; \mathbb{R}^n) = W_k^1(\Omega)^n$, where $W_k^1(\Omega)$ denotes the corresponding Sobolev space of functions and $W_k^1(\Omega; \mathbb{R}^n)$ is the corresponding Sobolev space of vector fields.

Consider
\[
J^k = \bigoplus_{0 \leq j \leq k} L^1_{\text{sym}}(\mathbb{R}^n; \mathbb{R}^n)
\] (5)
where $L^1_{\text{sym}}(\mathbb{R}^n; \mathbb{R}^n)$ is the space of completely symmetric $j$-multilinear mappings from $\mathbb{R}^n$ to $\mathbb{R}^n$, and let $\pi_j : J^k \to L^1_{\text{sym}}(\mathbb{R}^n; \mathbb{R}^n)$ be the natural projection. For $A \in L^1_{\text{sym}}(\mathbb{R}^n; \mathbb{R}^n)$ one may use $\tilde{A}_{\alpha j} = A_{i_1\ldots i_j\alpha}$, where $i_1, \ldots, i_j = 1, \ldots, n$, to denote the components of the corresponding tensor. Since $A$ is symmetric in all indices $i_1, \ldots, i_j$ we may represent $\tilde{A}_{\alpha j}$ by $\tilde{A}_{\alpha j} = A_{i_1\ldots i_n\alpha}$, where $\alpha$ is the number of times $i$ appears in the sequence $i_1, \ldots, i_j$. While the array $(A_{i\alpha})$ contains repeated terms, the collection $(\tilde{A}_{\alpha j})$ terms that are unique.

We define $L^1(\Omega, J^k)$ to be the collection of mappings $\chi : \Omega \to J^k$ such that the components of $\pi_j \circ \chi$ belong to $L^1(\Omega)$ for all $0 \leq j \leq k$. On $L^1(\Omega, J^k)$ we use the norm
\[
\|\chi\| = \sum_{|\alpha| \leq k} \int_{\Omega} |\chi_{i\alpha}| \, d\lambda^n.
\] (6)

The dual space of $L^1(\Omega, J^k)$ is identified with $L^\infty(\Omega, J^k)$ and we identify the field $\sigma \in L^\infty(\Omega, J^k)$ with the functional it induces by
\[
\chi \mapsto \int_{\Omega} \sum_{i,|\alpha| \leq k} \sigma_{i\alpha} \chi_{i\alpha} \, d\lambda^n.
\] (7)

The dual norm of $\sigma \in L^\infty(\Omega, J^k)$ is given by
\[
\|\sigma\| = \text{ess sup} \{ |\sigma_{i\alpha}(x)| : x \in \Omega, \ i = 1, \ldots, n, \ 0 \leq |\alpha| \leq p \}.
\] (8)

For $w \in W$ we may define $j_k(w) = (w, Dw, D^2w, \ldots, D^k w)$, where $D^k w$ is the $k$th weak derivative of the vector field. We observe that
\[
j_k : W \to L^1(\Omega, J^k)
\] (9)
is a linear, injective and norm preserving mapping.

With the notation introduced so far, the variational form of the equilibrium equation (1) may be written as
\[
F(w) = \sigma(j_k(w)).
\] (10)

Using the dual mapping of $j_k$, $\sigma(j_k(w)) = j_k^*(\sigma)(w)$ and the equilibrium equation may be written in the form
\[
F = j_k^*(\sigma).
\] (11)

In fact, the principle of virtual work, Equation (10) above, may be a representation theorem for a linear functional $F \in W_k^1(\Omega, \mathbb{R}^n)^*$. Since $j_k$ is norm-preserving, the Hahn-Banach theorem implies that there is an extension $\sigma$ of $F \circ j_k^{-1}$ to $W_k^1(\Omega, \mathbb{R}^n)$ for which (11) holds. As mentioned above, $\sigma$ may be represented as in (7)—proving the presence of hyper-stresses in this setting.

For a solution $\sigma$ of (11), $\|\sigma\|$ is an indication of the magnitudes of the hyper-stresses in the body. Actually, the norm in $L^\infty(\Omega, J^k)$ may be altered to assign different weights to the tensors of different order $j = 1, \ldots, k$. Thus we are concerned with the following optimization problem. Given $F$, find $s_{\text{opt}} \in \mathbb{R}^+$ such that
\[
s_{\text{opt}} = \inf \{ \|\sigma\| : F = j_k^*(\sigma) \}.
\] (12)

Using the analysis of the foregoing section, it is concluded that
\[
s_{\text{opt}} = \|F\|.
\] (13)

In addition, there is some optimal stress field $\tilde{\sigma}$ such $s_{\text{opt}} = \|\tilde{\sigma}\|$.

3. Load Capacity and Results for the Case $k = 1$

Consider the case where a body is subjected to a surface force $t$. Due to properties of Sobolev spaces, the trace mapping $\gamma : W \to L^1(\partial \Omega, \mathbb{R}^n)$ is a well defined continuous linear mapping so dual mapping exists $\gamma^* : L^\infty(\partial \Omega, \mathbb{R}^n) \to W^*$ mapping boundary loadings into generalized forces on the body.

Thus, may be considered the worst loading case for the optimal stress
\[
K = \sup_t \frac{s_{\text{opt}}(t)}{\|t\|} = \sup_t \frac{\|\gamma^*(t)\|}{\|t\|},
\] for all $t \in L^\infty(\partial \Omega, \mathbb{R}^n)$. However, it follows that
\[
K = \|\gamma^*\| = \|\gamma\|.
\]

Assume that we are given the admissible stress $Y_0 \in \mathbb{R}^+$. It follows from the construction above that as long as $\|t\| \leq Y_0/K$, $s_{\text{opt}}(t) = \|\gamma^*(t)\| \leq Y_0$ independently of the distribution of $t$.

This result is particularly applicable to perfectly plastic bodies of standard continuum mechanics, $k = 1$, where the optimal stress is attained in the limit, on the verge of plastic collapse, if the optimal stress is equal to the yield stress. We refer to $C = 1/K$ as the load capacity ratio of the body (see [1, 2]). With some adaptations required due to the fact that yield functions are usually semi-norms, it may be shown that the body will not collapse plastically as long as the maximum of the applied surface force is not larger than $Y_0/C$.

References


Optimization of auxetic structures using MMA algorithm

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Abstract

The document presents the optimization of topology of two components in composite material. The goal of the optimization is gaining of the least value of the negative Poisson’s ratio in the whole composite structure. When material has a negative Poisson’s ratio, it has also auxetic properties – wider during process of stretching, and narrower during compressing [2,3]. Computing and the whole optimization is made with the help of the MMA algorithm – Method of Moving Asymptotes.

Keywords: auxetics, optimization, MMA method, computational mechanics, negative Poisson’s ratio

1. Introduction

Composites are materials, which have at least two different components in their structure. The topology of this two or more components is various and can be modified and optimized. The optimization can lead to the occurence the auxetic properties.

Auxetics are materials with a negative Poisson’s ratio, so they have properties, that during process of stretching in one axis, are made wider dimension in axis perpendicular to one of them. Auxetics are remarkable, when the negative value of NPR (negative Poisson’s ratio) in structure is great [2,3].

In this paper is shown optimization of topology in composite to gain as big as possible NPR is computed using the MMA method [2,3].

2. MMA algorithm

2.1. MMA – fundamental equations

The MMA algorithm was invented in 1986 by the Norwegian scientist – Svanberg [1]. This method is development of CONLIN (convex linearization) method with the wider range of successfully solved computational problems. MMA uses the interventing variables $y_j(x_j)$ [1]:

$$y_j(x_j) = \frac{1}{x_j - L_j}, \text{ or }$$

$$y_j(x_j) = \frac{1}{U_j - x_j}, j = 1,...,n,$$

where $L_j$ and $U_j$ are called moving asymptotes. They are changes during the iterations but always satisfy the condition:

$$L_j < x_j < U_j$$

The MMA approximation of $g_i$, $i=0,...,l$ at the design $x^k$ reads:

$$y^M_k(x) = r^k_i + \sum_{j=1}^n \left( \frac{p^k_{i,j}}{U_j - x_j} + \frac{q^k_{i,j}}{x_j - L_j} \right),$$

where:

$$p^k_{i,j} = \left( (U^k_j - x^k_j) \right)^2 \frac{\partial g_i(x^k)}{\partial x_j}, \text{ if } \frac{\partial g_i(x^k)}{\partial x_j} > 0 \text{ and }$$

$$p^k_{i,j} = 0, \text{ otherwise}$$

$$q^k_{i,j} = \left( (x^k_j - U^k_j) \right)^2 \frac{\partial g_i(x^k)}{\partial x_j}, \text{ otherwise}$$

$$u^k_{i,j} = g_i(x^k) - \sum_{j=1}^n \left( \frac{p^k_{i,j}}{U_j - x_j} + \frac{q^k_{i,j}}{x_j - L_j} \right)$$

Thus $p^k_{i,j}$ is not zero, then $q^k_{i,j}$ is zero and vice versa.

2.2. Features of MMA

The MMA algorithm has some specific features [1]. The MMA approximation is a first order approximation, i.e.:

$$y^M_k(x) = g_i(x^k)$$

$$\frac{\partial y^M_k(x)}{\partial x_j} = \frac{\partial g_i(x^k)}{\partial x_j}$$

$g^M_k$ is an explicit, convex function and the approximation in MMA method is separable. The MMA approximation of (SO)n$f$ at iteration $k$ is written [1]:

$$y^M_k(x) = \min_{x} g^M_k(x)$$

$$y^M_k(x) \leq g^M_k(x), \text{ if } x^k(t), t = 1,...,l$$

$$x^k(t) \leq x_j \leq \beta^k_j, \text{ if } j = 1,...,l$$

where: $\alpha^k$ and $\beta^k$ are move limits to be defined below. This is convex, separable problem may be solved using Lagrangian duality.

Updating the asymptotes during the iteration is heuristic. In each iteration the design variables are made to satisfy the constraint [1]:

$$\alpha^k_j \leq x_j \leq \beta^k_j,$$

where the move limits $\alpha^k$ and $\beta^k$ are chosen as:

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\[
\alpha_j^k = \max(y_j^{\min}, L_j^k + \mu(x_j^k - L_j^k)) \\
\beta_j^k = \max(y_j^{\min}, U_j^k + \mu(U_j^k - x_j^k))
\]

(12)

(13)

where \(0 < \mu < 1\). It will always holds that:

\[
L_j^k < \alpha_j^k \leq x_j \leq \beta_j^k \leq U_j^k.
\]

(14)

3. Experiment

3.1. Boundary conditions

The model has a shape of the square with the area 1 m². At the beginning of the experiment only one material is in the square (blue on the Fig. 1 and Fig. 3) with Young’s modulus equal to \(10^7\) Pa (like rubber) and Poisson’s ratio is equal to 0,1. The second material (dark red in the Fig. 1 and Fig. 3) has Young’s modulus equal to \(10^{10}\) Pa (near steel) and Poisson’s ratio 0,33. From the top on the structure acts the force with the value 10 kN.

3.2. Computing

The analysis was made in COMSOL 5.0. Using the MMA algorithm computing leads to appear in the structure the second material with the specific shape – which makes all structure auxetic. One of the most important parameters in the computing process is the second material’s fraction of area (Afrac) in the whole structure. Optimizing algorithm makes the Poisson’s ratio as small as possible. For the purpose of computing parameters of a composite material were also used in the equations using SIMP (Solid Isotropic Material with Penalization).

3.3. Examples of the results

In the Fig. 1 is the structure with Afrac = 40%, the Poisson’s ratio of the composite structure is negative – the Fig. 3 – total displacement of the structure proves this fact.

When the second material fraction area is 20% (Fig. 3) and 10% (Fig. 4) the structure is similar to the well known auxetic structure, named a honeycomb. Many papers included simulation of honeycombs, regular in the structure. This is the difference – honeycomb is irregular and it is possible to have a bigger value of Poisson’s ratio.

On the basis of optimization this auxetic structure with a MMA algorithm may be built the real beam or truss composite structure with a very small value of the Poisson’s ratio.

References


Optimization of structures of modern materials using a new hybrid evolution strategy

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Abstract
In the study, a new hybrid evolutionary-gradient algorithm is proposed for a multi-objective topology optimization. The proposed algorithm is developed based on the best compromise between the direct and gradient-based optimization approaches. In order to improve the efficiency of the hybrid evolution strategy the Taguchi method is used to selection of the value ranges of design parameters, which ensure minimal variability of the objective functions with respect to the variation of the design space. The aim of the work is to overcome the problems associated with getting trapped in local minima that may arise during the topology optimization by means of gradient-based optimization methods.

Keywords: Pareto-optimal, topology optimization, topological sensitivity, multi-objective optimization, Taguchi method

1. Introduction

One of fundamental areas of materials science is development of mechanical material properties. Recently, artificial materials known as metamaterials are designed to have mechanical properties, which may not be found in nature [1]. Generally, such materials have extraordinary effective properties inter alia, as negative dynamic modulus and/or density, superior thermoelectric properties, phononic bandgaps and high specific energy absorption. The metamaterials are usually arranged in repeating patterns and are assemblies of multiple individual elements fashioned from conventional microscopic materials such as metals or plastics. Their extraordinary properties are created not by the choice of their chemical composition, but by their exactlying-designed geometric structures constituted by precise shape, size, orientation and arrangement.

The evolution of metamaterials made from micro-/nano-structured materials is intended to develop entirely new, or enhanced material behaviour by rational design. As important feature of metamaterials is the possibility to design their structural anisotropy. It should be stressed that the result of the evolution by natural selection for many of natural structural materials are anisotropic mechanical properties. For instance, human bones can exhibit nearly 20 times higher elastic modulus and tensile strength along the axial direction compared to the transverse direction. On the other hand, structural anisotropy of artificial materials as in the Damascus steel blades, used in Middle Eastern swordsmaking, can also be an essential factor in the improvement of mechanical performance without a change in chemical composition or weight.

For the multidimensional fabrication of metamaterials, there is a variety of printing and pattern processing of well-suited techniques. However, it was demonstrated by Wang et al. [2] that the Connex500 - 3D printer with sub-millimeter feature size can be used to fabricate co-continuous glassy polymer/rubbery polymer materials with enhancements in stiffness, strength and energy dissipation.

In particular, 3D printing as advanced alternative to conventional manufacturing techniques is well-suited to generate complex porous ceramic matrices directly from powder materials [3]. Moreover, this technology allows to manufacture bone grafts with complex shapes including designed internal channel networks to mimic bone structures.

The topology optimization techniques can be applied, to design materials at micro and nano-levels. Numerical methods needed to carry out an efficient topology optimization of continuum structures have been investigated extensively.

2. Proposed method of a hybrid evolution strategy

Since a local search technique of gradient-based optimization methods can be easily trapped in local minima, a hybrid evolutionary-gradient algorithm (HEGA) is proposed to solve global numerical problems of topology optimization with continuous variables [4]. The HEGA combines the multi-objective optimization technique of an evolution strategy based on ideas of adaptation and evolution with the gradient-based algorithm to solve the discrete problem of topology optimization. The hybrid evolution strategy has a powerful global exploration capability and uses natural problem-dependent representations, and primarily mutation and selection, as search operators to find the optimum-design domain for the efficient gradient-based algorithm, which searches one of local optimal topologies. In common with evolutionary algorithms, the operators of HEGA are applied in a loop. An iteration of the loop is called a generation. The sequence of generations is continued until a termination criterion is met. Assuming that an initial population has different individuals, each of them consists of a finite set of points generated randomly, an implicit description of the domain geometry for each individual can be provided by calculating the convex hull area.

As far as real-valued search spaces are concerned, mutation is normally performed by adding a normally distributed random value to each component of a point coordinate vector. The step size or mutation strength (i.e. the standard deviation of the normal distribution) is often governed by self-adaptation, which should keep the evolutionary process within the evolution window. Only fixed points, which are the support points do not change their coordinates in the numerical exploration of the evolution strategy.

The underlying principle behind the class of evolution strategy methods is presented here. First, a population of design
domains (defined implicitly by means of the convex hulls) is generated. The concept of Voronoi diagrams plays a central role in meshing of the convex hulls. Second, the gradient-based algorithm searches local optimal topologies for each design domain. Next some of design domains are modified, and also non-Pareto design domains are eliminated by means of stochastic methods, mutation and selection. Next, the cycle is repeated.

Figure 1 illustrates a potential advantage options of shape improvement of Pareto-optimal curves by means of the proposed method. In order to improve the efficiency of the hybrid evolution strategy the Taguchi method is used to select the value ranges of design parameters, which ensure minimal variability of the objective functions (for minimum compliance and minimum volume problems) with respect to the variation of the design space (represented by the convex hull generated randomly). It can be assumed that for the value ranges of design parameters with minimal variability of the objective functions the hybrid method of topology optimization can only be performed by means of the local search technique of gradient-based optimization methods according to the expectation of a lack of problems of getting trapped in local minima. It should be stressed that for topology optimization of structures with very small volume fractions the possibility to use the ground structure methods [5] or straightforward genetic algorithms should be considered.

3. Conclusions

Topology optimization was implemented through the use of a hybrid evolutionary-gradient algorithm. The proposed method is a compromise between the computational speed of the efficient gradient-based optimization methods and overall accuracy of evolution strategy calculations to determine the global optimum. The calculation results shown in Fig. 1 indicate that the choice of an initial design domain in the task of topology optimization has a significant impact on the optimal shape of structure boundaries computed by means of the gradient-based optimization methods. However, the initial design domains have no significant main effect on the outcome of optimal shape of structure boundaries, if the possibility to choose the initial design domains is very limited within an assumed starting area border.

It should be stressed that the HEGA is an universal method for topology optimization, which is highly susceptible to adequate modifications according to the design objectives.

References

Application of manufacturing constraints method to structural optimization of AEC thin–walled structures

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Abstract

The paper presents a new application of manufacturing constraints in topology optimization in Architecture, Construction and Engineering (AEC). In structural optimization standardized, thin–walled, steel profiles will be used. The standard software for topology optimization was used with a new way of preprocessing the design space. The application of the design methodology is illustrated by an example of topology optimization of a warehouse building structure. It is shown that the methodology is a useful tool for obtaining optimal structure design to fulfil the assumed manufacturing constraints. Topology optimization is a very useful tool for creating conceptual designs in area of AEC. The methodology presented in the paper is prepared for considering the manufacturing requirements of thin–walled building structures.

Keywords: topology optimization, thin–walled structures, manufacturing constraints, building structures

1. Introduction

In the last 20 years, the topology optimization was characterized by a dynamic development, both in research and broad industrial applications. Currently, the topology optimization has significantly developed and with the development of topology theory and an increased number of software tools, it has become an effective method in the design of new structures. As shown in the literature, the topology optimization can generate efficient designs in engineering fields where structures are already considered highly engineered. The aim of the paper is to present an application of design methodology for thin–walled AEC structures that consider the manufacturing requirements. For the preprocessing of design space of topology optimization the procedure of manufacturing constraints will be used. The preprocessing procedure consist of preparation equivalent design space (EDS). The EDS space is discretized using specific size of finite elements and equivalent material properties. A new design methodology is illustrated by an example of warehouse structure optimization. The obtained design should fulfil all the requirements of current building standards [2].

2. The manufacturing constraints procedure

The procedure of manufacturing constraints in topology optimization consists of the discretization of the design space by solid type finite elements of a hexahedral shape and equal dimensions. The element stiffness of solid cube is to be reduced to the stiffness of a thin-walled I–beam of similar dimensions. The stiffness of a thin-walled I–beam was found (e.g. dimensions: 240x240x240x5 mm) by application of the 1kN load and thus obtaining the value of linear deformation. The thin–walled structure has the same dimensions as the dimensions of finite elements of the design space. In order to find the equivalent stiffness of the EDS design space the second FE model was prepared. The stiffness of this model was tuned to reach the value of linear deformation equal to the first model (Fig.1) [3, 4].

Figure 1: The example of numerical procedure of the design space preparation. The first FE model (a,b) and value of the linear deformation for a thin-walled I–beam (c); The second FE model (d,e) and results of linear deformation tuning: before (f), after (g) the modification of the stiffness

3. An example of building structure optimization

A warehouse building structure was selected to illustrate the optimization methodology (Fig. 2). The main parameters of the warehouse are: length 60 [m], width 12 [m], height 6.5 [m]. The optimization problem was to find a minimum mass of the warehouse building structure with static von–Mises stress limits. Minimum value of the yield strength of structural steel was taken into account. In the FEM analysis, the homogenous isotropic linear-elastic material and solid type finite elements of a hexahedron shape was used. For topology optimization the Altair OptiStruct software was selected. OptiStruct solves topological optimization
problems using the SIMP method. A pseudo material density is the design variable in the SIMP method. The material density varies continuously between 0 and 1, with 0 representing void state and 1 solid state [1]. The volume of a typical warehouse was assumed as a design space for topology optimization. The procedure of topological constraints was used in topology optimization. The design space was discretized by finite elements of dimensions equal to the dimensions (240x240x240 mm) of cross-section of standard thin–walled profiles used in building design. Moreover, the EDS tuning procedure was done.

![Figure 2: The design space of building structure](image)

The optimization model contains 5600 elements. In order to fulfil structural recommendations of standards the structure has to be characterized by a suitable static load endurance [2]. In Fig.2, boundary conditions and loads of analysis and optimization were presented. Pressures were assumed according to the requirements of the building standards for the climate zone of southern Poland [2]. The main pressures acting on the structure of the building assumed for the topology optimization are: 1 kN/m² at the roof coming from the snow, 0.5 kN/m² coming from the wind. To speed up the calculations, only 1/4 of the design space for topology optimization was considered with symmetry boundary conditions. The loads were acting in an asynchronous way (3 load cases). Fig.3,4 presents the results of optimization. The solution under manufacturing constraints shown there was achieved after 67 iterations. An initial design outline of a new warehouse building was received by means of optimization. The red areas indicate the most sensitive zones of the structure, in these areas the building main structure beams should be located.

![Figure 3: Obtained layout of the warehouse–top,side view](image)

**4. Conclusions**

A new application of design procedure of thin-walled structures with manufacturing constraints was presented in the paper. As shown in the example of a warehouse building optimization, the manufacturing constraints require a lot of attention at the conceptual design stage. In the methodology, the manufacturing constraints procedure with preprocessing of the design space was presented. As shown in the example, the methodology is an effective tool for optimization of building structures. As a result of topology optimization a new design proposition of the thin–walled structure was obtained. The proposed methodology allows to find a structure easy to be manufactured. The methodology presented in the paper can be easily extended to other types of thin–walled structures.

**References**


Topology optimization as a tool for road pavement structure analysis

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Abstract

This paper presents the analysis of the road pavement structure from the point of view of topology optimization. Analysis of material distribution allows to confirm the requirements for guidelines.

Keywords: topology optimization, road pavement structure, minimum compliance

1. Introduction

In the paper variational formulation of the topology optimization problem is used [1]. A strain energy functional, being equivalent of compliance, was minimized while constraints were imposed on the body mass. This approach was basic to prepare the numerical algorithm and the authors Matlab code, tested and verified [2, 3].

Pavement structures are designed in accordance with the proper guidelines taking into account the planned traffic load (traffic class) by a proper choice of the type of pavement structure (flexible, semi-rigid and rigid structures). The main aim of the paper is to discuss mentioned guidelines from the point of view of topology optimization.

2. Road pavement structure analysis

In Fig. 1a the considered computational model of the road structure loaded by a concentrated force (wheel) is shown. A solution obtained for this model can be considered a reliable basis for dimensioning the structure in the cross section and longitudinal section of the road. In order to maintain correspondence between the recommended pavement structure and the adopted computational model the design domain was divided proportionally to the thickness of the particular road pavement structural layers. At the left side of this figure the road pavement layers are marked by colours: yellow represents the bitumen wearing layer, green – bitumen binder layer, blue – asphalt concrete layer, red – compacted aggregate or roadstone layer. Typical material-void solution is presented in Fig. 1b.

Analysis of the obtained solutions (Fig. 1b, Fig 2) shows that the material is symmetrically distributed in the form of a triangle with a relatively small base. The load is transferred to the ground through base courses such that the material required to carry the load is distributed in the form of a rectangle. By means of computations areas in the design domain exist, in which material should be located when the load is applied to the design domain in the assumed position. Additionally, a similar material amount is noticed in particular layers in the considered cases and the distribution of material for one force (Fig. 1b) is similar to one particular force in a case of three forces (marked in red – Fig. 2).

Next, distributions of a considerable share of shades of grey are considered. In order to facilitate the analysis of such distributions a relevant numerical tool was built. The tool makes it possible to interactively analyze material strain in particular subareas of the design domain due to the fact that the scale of the examined topology can vary in real time during the analysis. The ten-interval grey scale covering the interval of 0-1, is applied e.g., an interval of 0-0.2, as shown in Fig. 1c. As a result, all the elements with a density higher than 0.2, are marked in black.

The viewing scale of 0-1 was adopted in Fig. 1b and in Fig. 3a – for the final topology and for the strain energy distribution in this final step, respectively. Sparse areas with a density intermediate between 0 and 1 (Fig. 3a) are marked with an appropriate shade of grey corresponding to the density value. In Fig. 3a only in the force position in the top wearing course two

Figure 1: Computational model a), 0/1 topology b), grey scale assigned to interval of 0.0-0.2 c)
elements correspond a density of 1 (the black colour). Most of other finite elements detect a density in the interval of 0-0.1 (white). The next solutions in Figs 3b, c, d, e, f, g, and h exhibit density distributions shown in appropriate scales in order to reveal the variation in material density distribution and the most strained places.

Figure 3: Density distribution topologies for $\alpha=0.5$ in scale intervals: a) 0-1; b) 0-0.3; c) 0-0.2; d) 0-0.1; e) 0-0.06; f) 0-0.03; g) 0-0.02; h) 0-0.01

The scale adopted in particular solutions is specified in the figure footnote. Figure 3f shows a topology in the scale of 0-0.03, where densities in the interval of 0.027-1 are marked black and the lower density values are assigned shades of grey and white.

A material with the highest density (e.g. 3d) is distributed in the top course (layer) of the analyzed structure. The next courses should be made of a relatively weaker (lower density) material (e.g. Fig. 3e). A less dense material is present in a larger number of elements (a wider cross section) than more dense material (the narrower cross section). One should also notice that directly under the load (wheel) a material of a certain density, occurs transferring the load to the soil. A certain specific material distribution can be discerned (Fig 3d and e) where the elliptic areas of similarly dense material occur.

A material with the highest density (e.g. 3d) is distributed in the top course (layer) of the analyzed structure. The next courses should be made of a relatively weaker (lower density) material (e.g. Fig. 3e). A less dense material is present in a larger number of elements (a wider cross section) than more dense material (the narrower cross section). One should also notice that directly under the load (wheel) a material of a certain density, occurs transferring the load to the soil. A certain specific material distribution can be discerned (Fig 3d and e) where the elliptic areas of similarly dense material occur.

Figure 4: Density distribution topologies in scale interval of 0-0.06 with plotted isolines

The areas can be distinguished by means of isolines. Figure 4 shows such isolines drawn in Fig. 3e. It is apparent that the load makes it necessary to distribute the material in a specific manner. The distribution is laminar and parallel, but the layers do not run horizontally, so directly under the wheel the strain of the material is relatively higher than in the neighbouring areas. In this way it was demonstrated that the laminar structures are optimal structures are [2]. As confirmed by computations, the larger the distance (downwards) from the load, the relatively weaker, but more widely distributed material is required.

The analysis above deals with the real distribution of material effort under a vehicle wheel. Since the latter moves, each consecutive cross section requires the same distribution. This means that the material requirements for the successive cross sections are identical to those determined directly under the force in the above solution.

Figure 5: Density distribution topology in numerical notation (for topology showed in Fig. 3a)

For a detailed analysis of the material distribution the numerical notation is used (Fig. 5). Fig. 5 presents on its right-hand side the summation rows of the material in following the form: column A - summation over the all the particular row, column G - summation over the grey and black elements, column M - summation over the black elements. The density is proportional to the Young’s modulus of the real material. Fig. 5 shows that the densities in obtained optimal solution should change fluently within each layer, thus decreasing downwards. A detailed material distribution analysis will be shown during the presentation.

Concluding, the correctness of the requirements for the guidelines is confirmed. On the basis of this analysis an alternative road pavement structure may be proposed.

References


Abstract

Topology optimization can be performed with several choices of different design parametrizations. Among other density methods and various interpolation schemes as SIMP and RAMP are very popular due to their simplicity and efficiency. In this approach the element-wise pseudo-density of material is considered a decision variable. In the paper a new interpolation scheme of the effective material properties is proposed. This scheme is based on variational Hashin-Shtrikman bounds of parameters for mixtures. Similar interpolation scheme for the relaxed optimization problem were suggested in [12] but in fact as such, has not been widely applied in practice even though its advantages. The proposed scheme of interpolation (called HSρ) was used in the 2D classical topology optimisation problem (compliance minimization) and to the 2D inverse-homogenization problems to determine topology of microstructure which exhibit the prescribed effective material properties.

Keywords: topology optimization, material model, variational bounds

1. Topology optimization as relaxed 01 problem

Modern topology or shape optimization problems which state the questions on the optimal layout of several materials in given proportions within a feasible domain $$\Omega$$ nowadays are solved by using the Finite Element. The conventional process of the optimization is carried out on usually the rectangular specimen (ground structure) divided into uniform $$n$$ finite element mesh $$\Omega_k$$ ($$k = 1..n$$). In case of the two of component materials $$C_1$$ and $$C_2$$ (of Hooke tensors $$C_1$$, $$C_2$$ or equivalent constitutive matrices respectively $$E_1$$, $$E_2$$) and their proportions in domain expressed by $$0 < \rho_i < 1$$, for each element a variable $$\rho_i$$ is assigned a variable such that

$$\rho_i = \begin{cases} 0 & \Omega_i \in C_1 \\ 1 & \Omega_i \in C_2 \end{cases}$$ and $$\sum_{i=1}^{n} \rho_i = n \rho_k$$ .

(1)

By changing the value of $$\{ \rho_k \}$$ – i.e. by creating a new configuration of the placement of the components the expected result may be described by the objective function of the optimization problem. Such an optimization problem is a difficult, binary integer programming, problem with a large number of variables. Accurate representation of structure requires a large number $$n$$ of variables $$\rho_k$$ i.e. a dense domain division into finite elements, and produce a huge number (at most $$n!/(m!(n-m)!))$$ possible configurations to check, where $$m=\rho_k n$$.

The optimization problems stated as pure 01-element wise problem one can try to solve by using so called hard-kill methods [4] e.g. (Bidirectional) Evolutionary Structural Optimization (ESO or BESO) but this methods does not guarantee the achievement of the global or even a local optimum. Note that for problem (1) the $$E_k$$ of each element are clearly defined. In order to be effectively solved (to reach the global optimum) the topology optimization problem must be relaxed, usually by allowing variation $$\rho_k$$ between 0 and 1 and by making the material characteristics depended of the variables $$\rho_k$$ (fig. 1). Note, that the final proper solution of the optimization problem in case of two materials must contain only

$$\rho_k =1$$ or $$\rho_k = 0$$. The introduction of variable continuum $$\rho_k$$ allows to determine the gradient of the objective function and to search for the minimum [11], using the efficient gradient methods.

Figure 1: Distribution of $$\{ \rho_k \}$$ for 01 and relaxed problem

The variable $$\rho_k$$ determines the proportions by volume of materials in the element $$\Omega_k$$. In fact such type of relaxation of the problem means admitting infinitely fine mixtures of the constituents in each element and requires proper calculating of its effective properties $$E(\rho_k)$$. This point is crucial for the relaxed topology optimization problems. A wrong approximation may not lead to the pure 01 solution or even in case 01 solution the obtained result may be totally unreal. Calculating $$E(\rho_k)$$ can be done by a homogenization approach. For the homogenization method a microscopic structure must be a priori assumed i.e. some type of the underlying periodic composite structure: laminates or Vigdergauz cells etc. This causes great limitations to the considered problem – the resulting solution by definitions is limited to a narrow class of the assumed microstructures. It is clear that the homogenization method involves orthotropic or anisotropic materials. Moreover, the main complications caused by homogenization approach are the additional design variables required to describe the structure of used composites i.e. for example: angle of orientation of layers, inclusion diameters etc. Alternatively, a level-set method can be used [13] or the effective material properties for intermediate $$\rho_k$$ can be determined by adopting so called material interpolation scheme for an artificial isotropic material. The latter method due to their simplicity is very popular.

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2. Isotropic material interpolation schemes

The interpolation scheme usually assumes that the element \( \Omega \) contains an isotropic composite material of a “pseudo-density” \( \rho \). The effective Young’s modulus \( E(E_0) \) can be properly approximated by the functions chosen arbitrary e.g. SIMP scheme [1] (a.k.a power law) or RAMP scheme [8, 9]

\[
E_{\text{opt}}^{\text{SIMP}} = E_i + \rho_i^p \Delta E, \quad E_{\text{opt}}^{\text{RAMP}} = E_i + \Delta E \frac{\rho_i}{1 + p(1 - \rho_i)}
\]

where \( \Delta E = E_z - E_i \). Note, that classical SIMP or RAMP involve only on the Young’s moduli \( E_i, E_z \) while the Poisson’s ratio \( \nu_i = \nu_z \) remains unperturbed. The proposal for case \( \nu_i \neq \nu_z \) can be found in [5]. The power law scheme (SIMP or RAMP) origin from solid-void problem (\( E_1=0 \)) can be easily expanded to 3-materials case with more or less sophisticated dependencies between \( E_i \) and \( \rho_i \) [6]. In above equations the “nature” of the additional parameter \( p \) is two-fold. Apart from ensuring the correct approximation [2] it can be used as “penalization parameter”, aims to penalize the intermediate “nature” of the additional parameter \( \rho \) [10]. From the micromechanical point of view the selected interpolation scheme and range of values of the penalty parameter \( p \) should produce effective \( E_k \) within the area bounded by variational bounds for effective material parameters of mixtures of material i.e. Hashin-Shtrikman (HS) bounds [7] or more precise Cherkaev-Gibiansky bounds [3].

These bounds are expressed by bulk \( k \) and shear \( \mu \) moduli instead of \( E \) and \( v \) as usually. The relationships in 2D between these two sets of moduli are: \( 2k = E/(1-v), 2\mu = E/(1+v) \).

![Figure 2: Comparison of HSp, SIMP and RAMP interpolation scheme for (k1, \mu1)=(1, 0.5) and (k2, \mu2)=(10, 5)](Image)

Similarly to the SIMP, a completely acceptable thermodynamically new interpolation scheme HSp (Fig. 2) can be formulated as

\[
k = (1-p^p)k_{\text{HS}} + p^p k_{\text{HS}}, \quad \mu = (1-p^p)\mu_{\text{HS}} + p^p \mu_{\text{HS}}
\]

The proposed scheme of interpolation (Ref. [4]) was used in the classical topology optimisation problem (i.e. the compliance minimization) and to the inverse-homogenization problems to determine topology of micro-structure. It is noteworthy that using (4) for \( p=1 \) gives at least as good solutions as SIMP with various \( p \) for all problems mentioned above. Using HSp is much easier to get the nearly 01 or even exact 01 solution to the optimization problem than in the case of the SIMP or RAMP interpolation scheme.

References


A new procedure of solution search stabilization for evolutionary topology optimization

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Abstract

The article presents a proposal for improving stability of the evolutionary methods of topology optimization. These methods of sequential elimination and addition of finite elements, usually called Evolutionary Structural Optimization (ESO), are very popular because of their simplicity and ease of software implementation. In the work, the Constant Criterion Surface Algorithm (CCSA) belonging to the ESO family methods is used. The new concept is based on a procedure of determining the value of the parameter of stepwise changing of solution volume that has a significant impact on the stability of the optimization process. In the CCSA algorithm, a parameter of volume reduction $\Delta V$ will be linked with the new parameter of volume material expansion $V_{em}$, which depends on the topological characteristics of the first quasi-optimal solution. Two benchmark problems were used to illustrate how the new procedure of determining $\Delta V$ parameter affects the stability and improves the efficiency of finding the optimal topology solutions.

Keywords: evolutionary methods of topology optimization, numerical stability, Constant Criterion Surface Algorithm (CCSA)

1. Introduction

The ESO method is one of the popular methods of design optimization. Thanks to a simple operation algorithm it allows easy software implementations and obtaining useful solutions [1]. However, from the point of view of the numerical stability and the efficiency of finding the optimal solutions, the ESO algorithm and its enhanced Bi-directional version (BESO) can be further improved [2]. In particular, attention should be paid to the problem of improving the stability of gradual evolution of the topology when optimization procedure is approaching the point of a quasi-optimum.

Huang and Xie proposed improvements of ESO/BESO by modification of the sensitivity parameter responsible for the change of volume at each iteration [3]. To ensure that the newly designated volume will be less than described by "parameter evolution" $ER$, they suggest a simple coupling of parameter $ER$ with the one step backward volume control.

From the point of view of the operation, the algorithm CCSA has been enriched comparing to the original version of ESO algorithm through the procedure of the solution search improvement based on the idea of the simulated annealing. The CCSA algorithm does not have a built-in sensitivity analysis. However, it is characterized by high stability thanks to the procedure of dynamically determining the threshold criterion of elimination the least stressed finite elements. For each iteration, the threshold criterion is separately selected so as to obtain a predetermined value of the volume percentage reduction $\Delta F$ [4].

In the paper, a new concept of a design-specific tuning of the $\Delta F$ parameter is proposed which should positively improve the stability and efficiency of the optimization process. The new method is designed to ensure control of the volume change in each iteration.

2. The procedure of solution search stabilization

Tuning of the value of a $\Delta F$ parameter is realized using a new quantity related to volume expansion procedure. The procedure of adding the layers of material is automatically turned on in the case of exceeding the stress limit value (the problem of local minima). The material is added on the edge of the whole structure, not just in the close vicinity of a spot of a high gradient of criterion function as it is in the BESO algorithm. It makes it possible to change the topology of structure within the bigger design space and to obtain a larger diversity of solutions.

The value of $\Delta F$ parameter will be determined by the equation:

$$\Delta F = \frac{V_{em}}{ntp},$$

(1)

where $ntp$ factor denotes the rate of volume change within the design subspace $V_{em}$.

As examples illustrating the procedure of solution search stabilization, two benchmarks problems was selected. The first example refers to the "Michell cantilever" problem of optimizing truss topology [5] (see Fig. 1). In the numerical example, the rectangular design space is discretized with 6400 elements. The number of elements in the example corresponds to [3].

In the second problem, analytical benchmarks are applied of L-shaped domain optimization with horizontal line support and vertical load is used [6] (see Fig. 2). The FE model consisted of 7500 elements.
Table 1: Numerical results

<table>
<thead>
<tr>
<th>ntp</th>
<th>Number of improvements</th>
<th>Best value / loop</th>
<th>ntp</th>
<th>Number of improvements</th>
<th>Best value / loop</th>
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<td>10</td>
<td>9</td>
<td>48.657 /3837</td>
<td>7</td>
<td>41.733 / 2354</td>
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</tr>
<tr>
<td>20</td>
<td>6</td>
<td>50.156 /421</td>
<td>7</td>
<td>40.306 / 1294</td>
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</tr>
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<td>6</td>
<td>49.8125 / 4143</td>
<td>7</td>
<td>40.520 / 3889</td>
<td></td>
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</tbody>
</table>

In both examples mass minimization with application of compliance constraint is considered. The 4-node plane stress elements, Young’s modulus $E = 1$, Poisson’s ratio $\nu = 0.3$, load value $F = 1$ are assumed. The compliance constraint $g$ was tuned to achieve a certain volume (~50%).

The tests were performed for 5000 iterations, and three values $ntp$ factor of volume rate change. The numerical results are shown in Tab. 1.

3. Conclusions

In the work, a new concept of tuning numerical stability of the evolutionary topology optimization was presented. The new proposal gives possible to customize the optimization process to the topological characteristics of optimized design.

The numerical examples show clear up that the new procedure of determining $\Delta V$ parameter affects the stability and improves the efficiency of finding the optimal topology solutions.

The discussed method can be adapted to other topology optimization methods such as SIMP.

References

Structural optimization of contact problems using piecewise constant level set method

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Abstract

The paper deals with the shape and/or topology optimization of the elastic contact problems using the level set approach. A piecewise constant level set method is used to follow the evolution of interfaces rather than the standard method. A piecewise constant level set function takes distinct constant values in each subdomain of a whole design domain. Using a two-phase approximation the original optimization problem is reformulated as an equivalent constrained optimization problem in terms of the piecewise level set function. A necessary optimality condition is formulated. Finite difference and finite element methods are applied as the approximation methods. Numerical examples are provided and discussed.

Keywords: structural optimization, unilateral problems, piecewise constant level set method, Uzawa method

1. Introduction

The paper deals with the solution of a structural optimization problem for an elliptic variational inequality. This inequality governs unilateral contact between an elastic body and a rigid foundation. The structural optimization problem for the elastic body in unilateral contact consists in finding such topology of the domain occupied by the body and the shape of its boundary that the normal contact stress along the boundary of the body is minimized. The volume of the body is bounded.

The standard level set method [1, 5] is employed in structural optimization in the numerical algorithms for tracking the evolution of the domain boundary on a fixed mesh and finding an optimal domain. This method is based on an implicit representation of the boundaries of the optimized structure. Recently, different modifications of the standard level set method are developed to increase its effectiveness. An arbitrary number of subdomains can be identified using only one discontinuous piecewise constant level set function taking distinct constant values on each subdomain [2, 3, 4, 7, 8].

In the paper an original structural optimization problem is approximated by a two-phase optimization problem. Using the piecewise constant level set method this approximated problem is reformulated as an equivalent constrained optimization problem in terms of the piecewise constant level set function only. Therefore neither shape nor topological sensitivity analysis is required. During the evolution of the piecewise constant level set function small holes can be created without use of the topological derivatives. The paper extends the results contained in [4]. A necessary optimality condition is formulated. The finite difference method is used as the approximation method. This discretized optimization problem is solved numerically using the augmented Lagrangian method. Numerical examples are provided and discussed.

2. Problem Formulation

Consider deformations of an elastic body occupying two-dimensional domain Ω with the smooth boundary Γ. The elastic body obeying Hooke’s law is subject to body forces \( f(x) = (f_1(x), f_2(x)) \), \( x \in \Omega \). Moreover, surface tractions \( p(x) = (p_1(x), p_2(x)) \), \( x \in \Gamma \), are applied to a portion \( \Gamma_1 \) of the boundary Γ. We assume, that the body is clamped along the portion \( \Gamma_0 \) of the boundary Γ, and that the contact conditions are prescribed on the portion \( \Gamma_2 \), where \( \Gamma_1 \cap \Gamma_2 = \emptyset \), \( i \neq j \), \( i, j = 0, 1, 2 \), \( \Gamma = \Gamma_0 \cup \Gamma_1 \cup \Gamma_2 \).

We denote by \( u = (u_1, u_2) \), \( u = u(x), x \in \Omega \), the displacement of the body and by \( v(x) = (v_1(x), v_2(x)) \) as well as by \( \sigma(x) = \{\sigma_{ij}(u(x))\} \), \( i, j = 1, 2 \), the strain field and stress field in the body, respectively. Let us formulate a contact problem in variational form. Denote by \( V_u \) and \( K \) the space and set of kinematically admissible displacements given by \( V_u = \{z \in [H^1(\Omega)]^2 = H^1(\Omega) \times H^1(\Omega) : z_i \equiv 0 \text{ on } \Gamma_0, i = 1, 2\} \) and \( K = \{z \in V_u : z_N \leq 0 \text{ on } \Gamma_2\} \). Let \( \Lambda \) denotes the set of Lagrange multipliers \( \Lambda = \{\zeta \in L^2(\Gamma_2) : |\zeta| \leq 1\} \). Variational formulation of contact problem has the form: find a pair \((u, \lambda) \in K \times \Lambda\) satisfying

\[
\int_\Omega a_{ijkl} e_{ij}(u) e_{kl}(\phi - u) dx - \int_\Omega f_i(\phi_i - u_i) dx - \int_{\Gamma_1} p_i(\phi_i - u_i) ds + \int_{\Gamma_2} \lambda (\phi_T - u_T) ds \geq 0, \quad \forall \phi \in K, \quad \forall \lambda \in \Lambda, \quad (1)
\]

\[
i, j, k, l = 1, 2. \quad \text{The elasticity tensor is denoted by } \{a_{ijkl}\} \text{ and the tangent} \quad \text{normal} \quad \text{displacement} \quad \text{by } u_T(u_N). \quad \text{We use here and throughout the paper the summation convention over repeated indices [4].}
\]

Before formulating a structural optimization problem for (1)-(2) let us introduce first the set \( U_{ad} \) of admissible domains in the form \( U_{ad} = \{\Omega : \Omega \text{ is Lipschitz continuous}, \Omega V o l(\Omega) - V o l(\Omega)^{def} \leq 0, P e r(\Omega) \leq \text{const}\} \) where \( V o l(\Omega)^{def} \int_\Omega dx \) and \( P e r(\Omega)^{def} \int_{\partial \Omega} dx \). The set \( U_{ad} \) is assumed to be nonempty.

In order to define a cost functional we shall also need the following set \( M^* \) of auxiliary functions \( M^* = \{\eta = (\eta_1, \eta_2) \in [H^1(\Omega)] : \eta_i \equiv 0 \text{ on } \Omega \, \forall i = 1, 2, \|\eta\|_{[H^1(\Omega)]^2} \leq 1\} \) where the norm \( \|\eta\|_{[H^1(\Omega)]^2} = \sum_{i=1}^{2} \|\eta_i\|_{[H^1(\Omega)]^2}^{1/2} \). Recalling from [4] the cost functional approximating the normal contact stress on the contact boundary

\[
J_\eta(u(\Omega)) = \int_{\Gamma_2} \sigma_N(u_\eta N(x) dx, \quad (3)
\]
depending on the auxiliary given bounded function \(\eta(x)\) in \(M^{st}\). \(\sigma_N\) and \(\phi_N\) are the normal components of the stress field \(\sigma\) corresponding to a solution \(u\) satisfying system (1)-(2) and the function \(\eta\), respectively. We shall consider the following structural optimization problem: for a given function \(\eta \in M^{st}\), find a domain \(\Omega^* \subset \Omega_{ad}\) such that
\[
J_N(u(\Omega^*)) = \min_{\Omega \in \Omega_{ad}} J_N(u(\Omega)) \tag{4}
\]

3. Piecewise constant level set approach

In [1] the standard level set method [5] is employed to solve numerically problem (4). Consider the evolution of a domain \(\Omega\) under a velocity field \(V\). Let \(t > 0\) denote the artificial time variable. Under the suitable regular mapping \(T(t, V)\) we have \(\Omega_t = T(t, V)(\Omega) = (I + tV)(\Omega)\), \(t > 0\). By \(\Omega^*_t\) we denote the interior of the domain \(\Omega_t\) and by \(\Omega^*\) we denote the outside of the domain \(\Omega_t\). The domain \(\Omega_t\) and its boundary \(\partial \Omega_t\) are defined by a function \(\phi = \phi(x, t) : \mathbb{R}^2 \times [0, t_0) \to \mathbb{R}\) satisfying:
\[
\phi(x, t) = 0, \quad \text{if } \phi(x, t) < 0, \quad \text{if } x \in \Omega^*_t, \quad \phi(x, t) > 0, \quad \text{if } x \in \Omega^*_t. 
\]
Function \(\phi\) is called the level set function.

Let us reformulate problem (4) in terms of a piecewise constant level set function. For hold-all domain \(D \subset \mathbb{R}^2\) partitioned into \(N\) subdomains \(\{\Omega_i\}_{i=1}^N\) such that \(D = \bigcup_{i=1}^N (\Omega_i \cup \partial \Omega_i)\) where \(N\) is a given integer and \(\partial \Omega_i\) denotes the boundary of the subdomain \(\Omega_i\), a piecewise constant level set function \(\phi : D \to R\) is defined as [6, 7]
\[
\phi = i \quad \text{in } \Omega_i, \quad i = 1, 2, \ldots, N. \tag{5}
\]
Consider piecewise constant density function \(\rho : D \to R^2\) defined as
\[
\rho(x) = \begin{cases} 
\epsilon & \text{if } x \in D \setminus \overline{\Omega}, \\
1 & \text{if } x \in \Omega. 
\end{cases} \tag{6}
\]
where \(\epsilon > 0\) is a small constant. We confine to consider a two-phase problem in the domain \(D\) where the characteristic functions of the subdomains are \(\chi_1(x) = 2 - \phi(x)\) and \(\chi_2(x) = \phi(x) - 1\). Therefore
\[
\rho(x) = \rho_1 \chi_1(x) + \rho_2 \chi_2(x) = (1 - \epsilon)\phi(x) + 2\epsilon - 1. \tag{7}
\]
Using (5) as well as (7) the structural optimization problem (4) can be transformed into the following one: find \(\phi \in U_{\rho}^{ad}\) such that
\[
\min_{\phi \in U_{\rho}^{ad}} J_{\rho}(\phi) = \int_{\Omega} \rho(\phi)\sigma_N(u_\eta)\eta_N \, ds \tag{8}
\]
where the set \(U_{\rho}^{ad}\) of the admissible functions is given as
\[
U_{\rho}^{ad} = \{\phi \in H^1(D) : W(\phi) = W^{\rho \eta}_{\text{int}} \leq 0, \ W(\phi) = 0, \ \Per(\phi) \leq \text{const} \}_{1}, \tag{9}
\]
where \(W(\phi) \overset{def}{=} (\phi - 1) - (\phi - 2)\) and
\[
W(\phi) = \int_{\Omega} \rho(\phi) dx, \ \Per(\phi) = \int_{\Omega} |\nabla \phi| \, dx. \tag{10}
\]
The element \((u_\eta, \lambda_\eta) \in K \times A\) satisfies the state system (1)-(2) in the domain \(D\) rather than \(\Omega\).

4. Necessary optimality condition

Making use of a two-phase approximation the original structural optimization problem (4) is reformulated as an equivalent constrained optimization problem (8) in terms of the piecewise constant level set function \(\phi(x)\). Using the Augmented Lagrangian associated to the problem (8) the derivative of the cost functional is calculated and a necessary optimality condition is formulated.

5. Numerical implementation

The optimization problem (8) is discretized using the finite difference and the finite element methods. The discretized optimization problem is numerically solved using Uzawa type iterative algorithm. The minimization of the Augmented Lagrangian with respect to function \(\phi\) is realized using the gradient flow equation.

6. Numerical results

The results of numerical experiments are reported. The optimal domains contain the areas with low values of density function in the central part of the body and near the fixed edges. The obtained normal contact stress is almost constant along the optimal shape boundary and has been significantly reduced comparing to the initial one.

The obtained numerical results seem to be in accordance with physical reasoning. Unlike in the previous papers here the original topology optimization problem is approximated by the two-phase optimization problem. This problem is transformed into the constrained optimization problem where the piecewise constant level set function is variable subject to optimization. Comparing to the standard level set approach the proposed approach do not require to solve Hamilton - Jacobi equation and to perform the reinitialization process of the signed distance function. The proposed method has also a hole nucleation capabilities as the topological gradient based methods do.

References

**Biomimetic Optimisation – New Approach To Aircraft Structural Design**

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**Abstract**

This paper presents the capabilities of the PararelCosmoProjector (PCP) to perform complex optimisation of an aeroplane structure. Complex optimisation is a basis to define full optimisation of shape, size and topology with entire boundary conditions, Fluid Structure Interaction and multiple loads. PCP is a connection of two parts, FSI module and Biomimetical Optimisation module. Biomimetic Optimisation is a new method of optimisation developed in Division of Virtual Engineering in Poznań University of Technology. It bases on the computing model of trabecular bone phenomenon, which is idiosyncratic kind of Topology Optimisation. The continuous development of the system gives the opportunity to use such an important function as parallel computing and mesh generating. The presented paper described the use of the system as to optimise fowler flap of I-22 military jet trainer aircraft and its final adaptation for manufacturing.

**Keywords:** topological optimisation, aircraft design, parallel computing, biomimetics

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**1. Introduction**

A durative reduction of weight is one of the most important directions in aeroplane development. Topological optimisation, although very effective in other cases, cannot be accurate for this task without considering such important parameters like multiload and Fluid Structure Interaction. However, simple adding this features to standard systems can significantly lengthen computing times, that is why creating a completely new system by connecting FSI and Structural Optimization is required.

In the presented study standard SIMP Topology Optimisation method was replaced by Biomimetic Optimisation, whose mechanism is presented in a further part of this paper. The final part includes the presentation of models which were optimised and prepared for manufacturing.

**2. Biomimetical optimisation**

Biomimetical optimisation is an entirely new approach to the optimisation of shape, size and topology. It is based on the phenomenon of adapting a trabecular bone to variable conditions [1]. Unique properties of bones were first noticed in nineteenth century by Julius Wolff, on the basis of his discovery formulated the theory that bones have natural capacity for adaptation its internal shape and topology to external mechanical stimulation and, what is more, this kind of remodelling requires minimal energy.

The trabecular bone is one of the best example for Wolff's law. Under the particular cartilage there is an area of trabeculars which can change its shape, size and number. This change is elicited only by an external mechanical stimulation and the amount of energy delivered to supple the bone.

It is easy to find an analogy between a trabecular bone remodelling and topological optimisation. A specific feature of a trabecular bone adaptation is the fact that the entire process occurs only on the bone surface. Consequently, when stimulated force on a bone increases, then sum of cross sections, number and composition of trabecular will change to create a more stiff structure. Conversely, when force decreases, the structure of the bones evolves to reduce unnecessary tissue so as to safe as much energy as it is possible. This process requires a very long time period, so only regular, long term change in mechanical stimulation can provide this effect.

**2.1. The numerical model of trabecular bone phenomenon**

To adapt this phenomenon to a computational system, the Strain Energy Density (SED) on the surface of optimized geometry was used. Firstly, the simplest example presenting Biomimetical Optimisation method is a two dimensional stick, with a simple bending load on one end, on the other one there is a constrain surface providing support in every length. If the amount of force exceeds the stiffness of the material set, then cross-section of the stick will start to expand as long as the SED level on the surface is outside of the set range. If the lower brink of the set range of the SED is higher then zero, then, after a few iterations the material will be absorbed from less loaded areas.

**2.2. Biomimetical optimisation as a part of FSI system**

The Biomimetic Optimisation method was implemented to computational FSI environment which contains few bootstrap modules connected by shell script. The entire environment is able to run the whole optimisation of aeroplane structure based only on fluid flow around the solid body [2].

The main components of the system are: ParallelCosmo-Projector: Adapted from biological environment, 3D Finite Element Method parallel mesh generator based on 2D cross-sections. Flow code TAU (Deutches Zentrum fuer Luft und Raumfahrt). Structural code Abaqus. AE Tools, specialized procedure to mesh deformation developed in frame of Taurus project and many collateral scripts and procedures.

**2.3. System Operation Loop**

In figure 1 the algorithm of the entire FSI-CSM system work is shown. As working environment is heterogeneous and written in many programming languages, the optimisation process is divided into a few different steps. The first one is

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a Computational Fluid Dynamic simulation of the flow around the optimised solid. Next, the data from the pressure on the surface of the optimised object is collected. This information is exchanged to a coupling surface. The coupling surface is a bridge between CFD and CSM (Computational Structure Mechanic). In the CSM part the coupling surface delivers information about forces affecting the object. This is the stage when Biomimetic Optimisation begins. The Process aligns with Strain Energy Density on the surface and resorbs material to fit in SED set range. After each iteration is finished, the next one starts again from the CFD part. Until the entire Optimized geometry will fulfilled SED sets value. Both, FE mesh generator and Strain Energy Density computation can be realized on a standard PC computer or in parallel environment which provides an opportunity to perform computing on larger and more complicated models.

Figure 1: Parallel mesh generation procedure

2.4. Input and Output data

In order to perform optimisation in the presented system precisely defined input data is required. First, the range of bitmaps illustrating cross-sections of the optimised object is needed. Second, the range of bitmaps illustrating allowable area where optimisation can perform is necessary. Third, a text file with a list of nodes to load and a text file with list of nodes to constrain are required. Finally, quantities of SED range and a planned number of iterations ought to be stated.

After a successful optimisation, the system fabricates 3D mesh files with data from each conducted iteration.

3. The numerical example

In order to show the presented system in use, the authors chose a real industrial example: Fowler flap from I-22 Iryda military jet trainer aircraft. A Fowler flap is movable part of a wing responsible for extending the area of wing during the take off and landing. Because of the location of the Fowler flap the conditions of the flow around the flap are unique. There is no direct pressure operating on the leading edge and therefore the expected results of optimisation should be different from the results of the optimisation of entire wing. The description of Fowler flap is presented in figure 2. It consists of 16 connected ribs, 10 of them are bound to hydraulic cylinders which move the flap.

Figure 2: Fowler flap of I-22

As the initial configuration the basic geometry used in a real aircraft were chosen. Load was based on the pressure occurring during an aircraft taking off at a speed of 245km/h. Also in PCP there is no way to choose a material of the optimised element but the value of the Poisson ratio and the SED can bet set.

All the presented steps were consulted with the National Institute of Aviation and construction engineers of I-22 Iryda. The final step of the entire process was the adaptation of optimized geometry to maintain technological correctness in the evaluations of aeroplane industry.

4. Conclusions

In the paper the features of the Biomimetic Optimisation system were shown. This system is based on trabecular bone phenomenon and is adapted to work in aeroplane industry. The entire environment consists of both FSI solver and CSM part with included Topological Optimisation processor. The system is able to optimise of structural elements of the airplane what was shown on the example of I-22 Iryda Fowler flap.

References


On the decreasing of the optimization problem size

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Abstract
Solution time of nonlinear constrained minimization problem depends on the number of constraints and the number of decision variables. One may try to speed up the solution process by elimination of decision variables least affecting the objective function. Singular Value Decomposition (SVD) of the objective function is intended to support the solution. An idea, to limit the number of the decision variables in case of quadratic objective function, using SVD, is presented in the paper. Results of computational tests are enclosed and discussed, performed on a problem of a quadratic objective function and subjected to quadratic constraints.

Keywords: Singular Value Decomposition, nonlinear constrained minimization, minimization speedup

1. Introduction
Nonlinear minimization problems may be solved by an iterative method falling into one of two broad classes, namely deterministic and stochastic [2,8]. In general stochastic-based methods are efficient when the objective functions having multiple local extremes, while gradient-based search procedures prove in the case of convex objective functions spanned over convex feasible domains [3].

The solution time of a minimization problem by gradient-based search algorithm depends on the number of decision variables, constraints, and the conditioning of the decision variable space. While the number of constraints may not be changed without affecting the final solution, one may try to influence the calculation time by limiting the size of the decision variable space.

Let us consider a quadratic objective function
\[ F(x) = \frac{1}{2} x^T A x + b^T x + c. \] (1)

Shall the objective function be of the type (1), the SVD may be applied in order to decrease the number of decision variables in \( x \), if the application of SVD indicates, that there exists at least one singular eigenvalue in \( A \).

2. Solution approach
An application of SVD algorithm to a square, symmetric matrix \( A \) yields [7]:
\[ A = U^T D U, \] (2)
where \( U \) is an orthonormal matrix \((U^T = U^T)\) and \( D \) is a diagonal matrix, containing moduli of eigenvalues of \( A \), ordered by magnitude. In the case of rank deficient \( A \), at least one diagonal element of \( D \) is singular, i.e. equal to zero. Matrix \( D \) may be decomposed into a product of two diagonal matrices:
\[ D = S J S, \] (3)
where \( S \) is a diagonal matrix, containing square roots of respective diagonal elements of \( D \), with the exception of the singular values, which are replaced by 1’s, while \( J \) is an unit matrix, again with an exception of the singular value locations in \( D \), at which 1’s are replaced by 0’s. In this way the singularity present in matrix \( A \) is transferred into matrix \( J \).

Finally, the objective function (1) is replaced by the following one:
\[ F(y) = \frac{1}{2} y^T J y, \] (4)
accompanied by an appropriate change of variables:
\[ y = S U x. \] (5)
Of course an inverse relationship to (5) holds as well:
\[ x = S^T U^T y. \] (6)

It comes directly from formulas (4) and (5), that 0’s in \( J \), corresponding to singular eigenvalues of \( A \) denote, that certain linear combinations of original decision variables \( x \) do not affect the objective function, and thus may be disregarded during analysis. This, depending on the solved problem, may lead to substantial savings in calculation time and will be a focus of the following analysis.

The minimization problem presented by the objective function (1), and subjected to quadratic constraints \( G_i(x) \):
\[ G_i(x) = \frac{1}{2} x^T M_i x + N_i x + O_i \leq 0, \quad M_i = M_i^T, i = 1, 2, \ldots, m, \] (7)
where \( M_i \) denote square symmetric matrices, \( N_i \) vectors, \( O_i \) constants, while index \( i \) enumerates all these constraints; may be transformed into the following problem, using the new decision variables \( y \): find a minimum of \( F(y) \) given in (4) while subjected to:
\[ G_i(y) = \frac{1}{2} y^T U S^T M_i S^T U^T y + N_i S^T U^T y + O_i \leq 0, i = 1, 2, \ldots, m. \] (8)

Gradients of the objective function \( F \) and constraints \( G_i \) required to find the search direction, and expressed in the new decision variables, may be calculated using the following formulas:
\[ \nabla F(y) = J y \]
\[ \nabla G_i(y) = y^T U S^T M_i S^T U^T + N_i S^T U^T. \] (9)
If required, the gradients of constraints \( \mathbf{V} \mathbf{G} \) may be expressed using the original decision variables as well:

\[
\mathbf{V} \mathbf{G}(\mathbf{y}) = \mathbf{x}^\top \mathbf{M} \mathbf{S}^{-1} \mathbf{U}^\top + \mathbf{N} \mathbf{S}^{-1} \mathbf{U}^\top.
\]

(10)

This may offer some additional time advantage, should it be preferred operate on both the original \((\mathbf{x})\) and new, orthogonal \((\mathbf{y})\) decision variables spaces.

3. Computational approach

In each iteration, when new decision variables vector \( \mathbf{y} \) is found, a corresponding vector \( \mathbf{x} \) is calculated using formulas (6), then values and gradients of constraints are computed using original decision variables, and transformed to the conditioned variables space as required, using formulas (10), to execute next iteration in search of the minimum. The search termination criteria are checked after each iteration in the new \((\mathbf{y})\) decision variables space.

In this approach, at the expense of performing the transformations (6) and (10) during each iteration, there is no need to perform the matrix operations present in (8).

One should note, that the SVD on matrix \( \mathbf{A} \), the single most time consuming step in the whole procedure outlined in the preceding section, is performed only once at the beginning of calculations.

4. Numerical implementation

A set of computer routines in FORTRAN programming language has been prepared to implement the extension, supplemented by an externally developed SVD routine [7].

The implementation is fully transparent to the user of the FDM optimization package [4], i.e. the user needs only to supply the matrix \( \mathbf{A} \) to the code at the beginning of calculations. During subsequent iterations the values of indicated constraints and constraint gradients expressed in terms of original decision variables \( \mathbf{x} \) must be supplied, exactly the same as if the problem was solved without using the extension.

5. Test problem

A search for an estimate of residual stress distribution in an elastic perfectly plastic body subjected to cyclic loads exceeding its elastic bearing capacity may be formulated as a two step nonlinear minimization procedure [1]. In the first step a minimum of a quadratic objective function is to be found, subjected to linear equality constraints [6], while in the second step a solution to the problem of minimization of another quadratic objective function subjected to quadratic inequality constraints is investigated [6]. The matrix \( \mathbf{A} \) is singular here.

6. Numerical results and conclusions

The test problem [1,6] was solved several times for input data selected so, as to get minimization problems differing in size (number of decision variables and constraints) as well as the decision variables to constraint ratio.

During all tests the spectrum of eigenvalues similar to those presented on Fig. 1 was observed. This clearly indicates that the computed eigenvalues split up into two distinct groups of values, separated by at least an order of magnitude.

Based on the results observed on Fig. 1, three levels of reduction in decision variables numbers were proposed, i.e. exactly the same as determined by the examination of spectrum graph (SVD0), decreased by 10% (SVD+), and increased by 10% (SVD-). The results obtained for a sample case are listed in Table 1. with two solutions (STD and SVD [5]) computed without reducing the number of decision variables added for reference purposes.

![Figure 1: Spectrum of eigenvalues for \( \mathbf{A} \) matrix – sample case.](image)

**Table 1: Sample calculation times.**

<table>
<thead>
<tr>
<th>Case</th>
<th>Iterations</th>
<th>Total time</th>
<th>Speed up</th>
<th>Time per iter.</th>
<th>Global error [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD</td>
<td>15261</td>
<td>628.44</td>
<td>-</td>
<td>0.0412</td>
<td>-</td>
</tr>
<tr>
<td>SVD+</td>
<td>1365</td>
<td>112.03</td>
<td>5.610</td>
<td>0.0821</td>
<td>2.08547*10^3</td>
</tr>
<tr>
<td>SVD0</td>
<td>746</td>
<td>59.47</td>
<td>10.567</td>
<td>0.0797</td>
<td>2.08577*10^3</td>
</tr>
<tr>
<td>SVD-</td>
<td>915</td>
<td>83.59</td>
<td>7.518</td>
<td>0.0914</td>
<td>4.03799*10^0</td>
</tr>
</tbody>
</table>

The level of global error [5] indicates that as long as the reduction in the number of decision variables is kept below the threshold level (Fig. 1), the degradation of solution quality is negligible, while substantial gain in total calculation time is observed.

References

Extension of Michell’s classical (1904) truss topology optimization theory to multiple load conditions, stress and displacement constraints, space (3D) trusses, probabilistic design and discontinuous support conditions

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Abstract

This paper describes significant advances in the field of exact structural topology optimization, extending Michell’s 110 years old theory to a number of new design conditions that are listed in the title. Exact optimal topologies serve a highly useful purpose of verifying the validity, accuracy and convergence of numerical methods in topology optimization. The theory presented will be illustrated with examples.

Keywords: structural topology optimization, exact analytical solutions, stress constraints, displacement constraints, space trusses, probabilistic design, multi-load problems

1. Introduction

The historic importance of the material presented is that Michell’s classical (1904) theory [1] has not been extended ever since to the conditions listed in the title, and therefore the work presented fills a significant gap in our knowledge of exact structural topology optimization.

The only exceptions were multi-load truss topology optimization in plastic design (e.g. Spillers and Lev (1971) [2], Nagtegaal and Prager (1973) [3], Hemp (1973) [4], for a detailed review and extensions see the paper by Rozvany, Sokó³ and Pomezanski (2015) [5]).

For compliance-based multiload truss topology optimization some early results were obtained by Rozvany (1992) [6], Rozvany Zhou and Birker (1993) [7], and Rozvany, Birker and Lewinski [8], which were reviewed in detail and extended by Rozvany, Pomezanski and Sokó³ (2014) [9].

For all other classes of exact topology problems listed in the title, there have been no significant results until recently.

2. Solutions for stress, displacement and compliance constraints

For each class of optimization problem, the same strategy is being used, which was tried out on compliance-based problems (see the paper by Rozvany, Pomezanski and Sokó³ (2014) [9]). The general theory is verified, using a simple example of two alternative, antisymmetric loads (see Fig. 1). It was shown by Guo, Du and Cheng (2013) [10], that a conjecture in a paper of Rozvany (2011) [11] is correct, and therefore the solution for such a problem must be symmetric.

The actual research strategy then consists of three steps. In step 1, it was assumed that the optimal solution is a two bar truss, which was optimized for the given design conditions. In step two, all possible three-bar solution were considered, and it was shown analytically, that in all solutions one bar takes on a zero cross-section, resulting in the same solution as in step 1. Finally, in step 3 a general optimality criterion was used analytically to show that the global optimum is the same as in steps 1 and 2.

Figure 1: Optimal truss topology for two alternative point loads and compliance or stress constraints

The solutions for compliance, stress and displacement constraints are compared in Fig. 2. Note that the displacement based optimal solution (dotted lines) is carried out for constrained vertical displacement, and hence the volume can vanish for \( \beta = 0 \) or \( \beta = \pi \). Moreover, it was also verified, that if the constraint is posed on magnitude of maximal displacement then the solution is identical with compliance-constrained solution. Unfortunately, in a two-page abstract it is not possible to show detailed proofs and derivations, but they will be explained in the lecture.

Recently Rozvany and Maute [12] proved that the two-bar truss topology shown in Fig. 1 is also optimal for probabilistic (uncertain) loads (see also the paper by Rozvany at al. [9]).

The paper was prepared within the Research Grant no 2013/11/B/ST8/04436 financed by the National Science Centre, entitled: Topology optimization of engineering structures. An approach synthesizing the methods of: free material design, composite design and Michell-like trusses.
3. Example of an optimal multi-load 3D Michell structure

Figure 2: Comparison of compliance-, stress- and vertical displacement based optimal solutions: a) normalized volume, b) bar inclination

$V_{\text{exact}} = 19.22932421 \frac{P d}{\sigma_0}$  $V_{\text{num}} = 19.2818 \frac{P d}{\sigma_0}$

Figure 3: Example of a 3D Michell structure for two loading conditions: a) an exact analytical prediction using the concept of component loads and b) numerical confirmation using the 3D ground structure with 300mln potential bars

Figure 3 presents an example of a 3D Michell structure for two alternative load conditions. It is probably the first non-trivial multi-load 3D Michell structure in the literature. The exact analytical solution was obtained on the base of component loads [3] and then verified numerically using 3D version of the adaptive ground structure method [13]. Note that the optimal truss presented in Fig. 3 is a statically determinate structure, thus the solution (optimal layout) is valid for both plastic and elastic designs.

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Optimization of dynamic characteristics of composite shells by using genetic algorithms

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Abstract

In the paper a technique of topological optimization by maximum damping criterion of multilayered shells composed of materials with different properties is considered. The semi-analytical method of finite element modeling in integral Fourier transform space is applied to build mathematical models of composite structures. The classic genetic algorithm is used to determine the optimal combination of materials as a sequence of structure layers. It is shown that for multilayered elements of structures, an optimal project can be found by placing the material layers with constant characteristics in a sequence that provides the optimum value of the chosen optimization criterion. The examples of searching for the optimal projects of layer packages for a 15-layered shell by one or several optimization criteria, are presented.

Keywords: optimization, multilayered shells, finite element method, genetic algorithms, damping

1. Introduction

While designing engineering structures for operation under dynamic loads, a problem of optimal choice of materials and their placement within the structure emerges, e.g. when planning the placement of damping and bearing elements for increased efficiency of their functioning. The goals of this optimization can be defined as modification of frequency spectrum, increasing vibration decrements, increasing durability characteristics etc. Solving these problems requires the creation and application of appropriate methods and techniques of optimal design. It is worth noting that the classic optimization methods are ineffective due to large number of project parameters and the specific features of selection. As shown in [3, 5], an efficient process of optimal design can be organized using the evolutionary optimization techniques, especially genetic algorithms, that exploit the methods of nature of selecting the best samples.

2. Problem definition

Two approaches to design of multilayered structures can be applied to obtain the optimal project: the modification of materials [1, 7], and the modification of construction which is called topological optimization, or shape optimization.

The problem of global shape optimization lies in selecting the sequence of material layers with given parameters, specifically reinforcement angles and ratios, to ensure the optimum of optimization criterion. In case of multicriteria optimization the problem is even more complex, as several competing optimality criteria must be satisfied. Thus compromising variants should be considered, and optimal solutions are actually found according to Pareto principle [4].

2.1. Optimization method

In a modification of the classic genetic algorithm, applied in the study, a population of individuals – vectors describing the structure is employed. The vibration decrement, vibration damping speed, frequencies and minimum weight are chosen as the optimization criteria (goal functions).

The result of solving the global optimization problem contains the sequence of material layers with defined properties, that provides minimum or maximum for the chosen optimization criterion. As a result of the multicriteria optimization problem we obtain a multitude of non-dominated solutions that fit the Pareto set for the considered problem, and the respective non-dominated front. To get a single solution we have to attach additional information regarding the relative criteria importance [4], i.e. conditions which mark optimization criteria as more or less crucial.

2.2. Constructing mathematical model of shell vibrations

A shell consisting of layers of a viscoelastic materials reinforced by fibers, is considered. To construct a mathematical model that takes into account the specific features of interaction between the constituent elements (layers in case of shell) and viscoelastic properties of materials, a semi-analytical method of finite element modeling in the integral Fourier transform space, is suggested in [2, 6]. It enables to describe the dissipation of energy in the material correctly using complex moduli, the dependence of the dissipation of energy to the stressed-deformed state of material, the frequency and thermal dependencies of energy dissipation, to define and solve the problems of structure design with maximum damping characteristics and the problems of optimal placement of passive damping elements, including shells, partially covered with damping material.

According to the suggested method, a single layer of the shell (Fig. 1) is considered at the first step, then a synthesis of multi-layered shell is performed according to the constraints of layer bounding (Fig. 2).

To construct the finite element model of a single layer, the approximation of displacements by thickness (z-axis) is made with Lagrange polynomials, and by two other directions (x,y-axes) – with the Fourier orthogonal series.

Figure 1: A computational scheme of a shell layer
Considering this approximation, we determine the matrix of
dynamic stiffness of a layer, describing it as a finite element,
\[ Z(i\omega) = K(i\omega) + (i\omega)^2 M, \]
where \( K(i\omega) \) is a matrix of layer stiffness that depends on the
complex moduli of the material, \( M \) is mass matrix, \( \omega \) is
vibration frequency, \( i = \sqrt{-1} \).

For a multi-layered shell the matrices of dynamic stiffness are
gathered into a global matrix
\[ GZ(i\omega) = GK(i\omega) + (i\omega)^2 GM \]
considering the kinematic bounding constraints of layers by
equating the respective displacements of bound nodal surfaces
(Fig. 2).

To analyze the energy dissipation, a nonlinear eigenvalue
problem is considered
\[ GZ(i\omega) q = 0. \]

Eigenvalues and eigenvectors of matrix \( GZ(i\omega) \) can be
determined using the technique proposed in [6].

The vibration decrement can be determined from the known
eigenvalues (complex frequencies) of the matrix \( GZ(i\omega) \):
\[ \delta_k = \pi \cdot \arctan \frac{\omega' - \omega''}{\omega'}, \]
where \( \omega_k = \omega_k' + i\omega_k'' \) is the complex vibration frequency that
matches the \( k \)-th form.

3. Calculation examples

A 15-layer hollow shell was considered with the following
initial parameters: overall dimensions of shell \( 4 \times 4 \) m;
curvatures \( k_1 = 0.001 \) 1/m, \( k_2 = 0.001 \) 1/m;
thicknesses of layers were taken as equal \( h = 0.001 \) m;
vibration forms: \( n = 1, m = 1 \).

Also the complex moduli and densities of reinforcing
material and matrix materials were given.

The matrix components of complex moduli of composite
material layers were determined by the technique described in
[7], density - by the rule of compounds \( \rho_{ef} = \eta \rho_1 + (1-\eta)\rho_2 \)
(\( \eta \) - reinforcement ratio). Parameters of layer materials are
shown in Table 1.

Table 1: Parameters of materials (\( \phi \) – reinforcement angle)

<table>
<thead>
<tr>
<th>( # )</th>
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<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.25</td>
<td>8</td>
<td>( \pi/3 )</td>
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<td>1</td>
<td>( \pi/12 )</td>
<td>0.25</td>
<td>9</td>
<td>( \pi/4 )</td>
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<tr>
<td>2</td>
<td>( \pi/10 )</td>
<td>0.25</td>
<td>10</td>
<td>( \pi/6 )</td>
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<td>3</td>
<td>( \pi/8 )</td>
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<td>11</td>
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<td>4</td>
<td>( \pi/6 )</td>
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<td>12</td>
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<td>5</td>
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<td>0.25</td>
<td>13</td>
<td>( \pi/12 )</td>
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</tr>
<tr>
<td>6</td>
<td>( \pi/3 )</td>
<td>0.25</td>
<td>14</td>
<td>0</td>
<td>0.75</td>
</tr>
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Having taken the vibration decrement as the goal function,
we obtained the optimal layer package that provides the
maximum decrement at the first bending form:
\[ \chi_{opt} = [7,7,7,7,6,6,5,5,4,4,1,14,14,14] \]
\[ \delta_{opt} = 0.2265. \]

The oscillogram of vibration for the optimal layer sequence
is shown in Fig. 3.

Figure 3: The oscillogram of vibration after optimization by
maximum vibration decrement criterion

The solutions of optimization problems using several vector
criteria, including minimum mass, were obtained in this study
too.

4. Conclusions

Considering the aforementioned arguments it is justified to
state that an optimal package can be obtained for multilayered
structures (beams, plates, shells) by placing the layers of
materials with constant characteristics in a sequence that provides
the optimal values of the chosen optimization criteria.
The necessity of the presented method is caused by the
technological process of producing shells and plates. The
developed technique can be generalized on larger number of
elements, and can also be employed to determine the optimal
placement of passive damping elements in multilayered
structures.

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of Structure Optimization for Composite Shells from
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A new adaptive ground structure method for multi-load spatial Michell structures

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Abstract

In the paper a new method of solving large-scale linear programming problems related to Michell trusses, generalized to multiple load conditions and three-dimensional domains, is proposed. The method can be regarded an extension of the adaptive ground structure methods developed recently by the first author. In the present version both bars and nodes can be switched between active and inactive states in subsequent iterations allowing significant reduction of the problem size. Thus numerical results can be attained for denser ground structures giving better approximation of exact solutions to be found. The examples of such exact solutions (new 3D Michell structures), motivated by the layouts predicted numerically, are also reported and can serve as benchmark tests for future methods of numerical optimization of structural topology in 3D space.

Keywords: 3D Michell structures, multiple load conditions, adaptive ground structure method, linear programming, active set and interior point methods, new exact solutions for structural topology optimization

1. Introduction

The aim of the paper is twofold: a) to develop a reliable and efficient optimization method based on the adaptive ground structure approach, and b) to obtain substantially new exact solutions of spatial Michell trusses subjected to multiple load cases.

The method proposed in the present paper is a natural extension of the adaptive ground structure methods developed by Gilbert and Tyas [1], Pritchard et al. [3] and Sokół [5-7]. The new version allows greater reduction of the problem size by deactivation of unnecessary nodes, which is particularly important for 3D problems because the optimal spatial trusses tend to assume forms of lattice surfaces while most of design space becomes empty.

2. Primal and dual formulations of multi-load plastic design optimization problems

The most concise formulation of plastic design optimization problem for multi-load cases can be written as follows:

\[ \min_{\sigma_c} V = \mathbf{L}^T \mathbf{A} \]

s.t. \[ \mathbf{B}^T \mathbf{S}_{(j)} = \mathbf{P}_{(j)} \] for all load cases (1)

\[ -\mathbf{A} \sigma_c \leq \mathbf{S}_{(j)} \leq \mathbf{A} \sigma_c \]

where \( V \) is the total volume of the structural material in the truss of \( M \) potential bars; \( \mathbf{L} \) is the vector of lengths of bars; \( \mathbf{B} \) is the geometric matrix; vectors \( \mathbf{P}_{(j)} \) define nodal forces for the given load cases \( l = 1, 2, \ldots, K \); \( \mathbf{K} \) is the number of load cases; \( \mathbf{S}_{(j)} \) is the vector of member forces for load case \( j \); \( \mathbf{A} \) is the vector of cross-section areas (the main design variables); and \( \sigma_c \) denote the permissible stresses in tension and compression, respectively.

The primal form (1) is not convenient for direct application of simplex or interior point method and it is recommended to convert it to a more applicable form (see [7] for details). The inequalities (1) can be converted to equality constraints using properly adjusted slack variables \( \mathbf{c}_{(j)} \) and \( \mathbf{t}_{(j)} \), which then allow elimination of original design variables \( \mathbf{A} \) and \( \mathbf{S}_{(j)} \)

\[ \mathbf{A} = \mathbf{t}_{(j)} + \mathbf{c}_{(j)} \] and \( \mathbf{S}_{(j)} = \mathbf{t}_{(j)} - \mathbf{c}_{(j)} \) and \( \mathbf{t}_{(j)}, \mathbf{c}_{(j)} \geq 0 \) (2)

Note that \( \mathbf{c}_{(j)} \) and \( \mathbf{t}_{(j)} \) are the vectors of slack variables which can be interpreted as the additional forces which can be added without violating the restrictions of permissible stresses (1), (i.e. they denote not forces itself but unused reserves of internal forces).

Using Eqn (2) the original problem (1) can be converted to the standard linear programming problem

\[ \min_{\mathbf{t}_{(j)}, \mathbf{c}_{(j)} \in \mathbb{R}^n} V = \sum_{l=1}^{K} \mathbf{L}^T \left( \frac{\mathbf{t}_{(j(l))} + \mathbf{c}_{(j(l))}}{\sigma_c} \right) \]

s.t. \[ \mathbf{B}^T \left( \mathbf{t}_{(j(l))} - \mathbf{c}_{(j(l))} \right) = \mathbf{P}_{(j)} \]

\[ \frac{\mathbf{t}_{(j(l))} + \mathbf{c}_{(j(l))}}{\sigma_c} \geq 0 \] for \( l = 1: K \)

\[ \mathbf{t}_{(j(l))}, \mathbf{c}_{(j(l))} \geq 0 \] for \( l = 1: K-1 \)

which is more complex but in fact two times smaller. Note that the form (3) is new in the literature and more economical than the formulations proposed in [3] or [7]. Moreover, for \( K = 1 \) it automatically reduces to a well-known form used for one-load case problem (see [1] or [5]) thus no additional separate code is needed for this special case.

In order to activate new bars in the adaptive ground structure method we need also dual variables but they are calculated automatically and for free using the primal-dual version of the interior point method. The convenient dual form of multi-load case problem was derived in [7] and is given by

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where the centre of upper square of the bounding cuboidal domain two independent point loads of magnitude compression. They are independent variables for every load displacements and adjoint member elongations for tension and compression. They are independent variables for every load (l) but constrained together by (4), which enables to derive the generalized optimality criteria for multi-load trusses (see [7] for details).

3. The adaptive ground structure method with selective subsets of active bars and nodes

Due to limited space of the abstract we can describe the new method only briefly. The main idea of activating new bars is the same as before [7] but now after each iteration the nodes are splitted into two subsets: active and inactive nodes. Then, in the subsequent iteration the adjoint displacements are updated only for active nodes. Inactive nodes appear in empty regions where no material is needed. Thus before starting the next iteration all bars connected with inactive nodes can also be eliminated (temporary for the current iteration). Consequently, the size of the coefficient matrix of the problem (3) is much smaller in terms of the number of rows and columns. It should be noted that inactive nodes are not removed forever from the ground structure and can be activated if necessary. Moreover, the adjoint displacements of these nodes have to be preserved for subsequent iterations for checking the optimality criteria. Of course this complicates the code but is necessary and crucial to assure convergence to a globally optimal solution.

4. Example of two-load 3D Michell structure

Consider the two-load case problem presented in Fig. 1. The two independent point loads of magnitude \( P \) are applied in the centre of upper square of the bounding cuboidal domain \( d \times d \times 3d \) and directed along the diagonals. These loads have to be optimally transmitted to four fixed supports in the corners of bottom square. In this example, we assume equal permissible stresses in tension and compression \( \sigma_T = \sigma_C = \sigma \).

In Fig. 1a we present the optimal solution obtained numerically using the ground structure with 20x20x60 cells, 26901 nodes and more than 300mln potential bars. The solution was obtained in less than 2 hours using classical computer with Intel i7 processor which clearly indicates a good efficiency of the proposed method. The layout obtained numerically suggests that the exact solution consists of four plane long cantilevers forming a specific hip roof, as shown in Fig. 1b. The exact volume of this complex structure was calculated using the formulae derived by Lewiński et al. [2] and is about 0.3% better than numerical volume.

The numerical solution presented in Fig. 1a was also verified using the superposition principles and the concept of component loads [4].

\[
\begin{align*}
\max_{u, e^+} & \quad W = \sum_l p_i^lu_i \quad \text{s.t.} \quad \sum_l (\sigma_l e_i^l + \sigma_e e_i^+) \leq L \\
& \quad B_u u^l - e_i^+ \leq 0 \\
& \quad -B_u u^l + e_i^- \leq 0 \\
& \quad e_i^l \geq 0, \quad e_i^+ \geq 0 \\
& \quad \text{for} \quad l = 1, 2, \ldots, K
\end{align*}
\]

where \( u, e^+, e^- \) are Lagrange multipliers, called adjoint nodal displacements and adjoint member elongations for tension and compression. They are independent variables for every load condition (l) but constrained together by (4), which enables to derive the generalized optimality criteria for multi-load trusses (see [7] for details).

Note that that the optimal solution presented in Fig. 1 is like a shell structure composed of lattice surfaces. Other interesting new results of 3D Michell trusses will be reported in the full paper.

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Phase trajectories of non-linear noised dynamic system

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Abstract

There is a number of methods which are presently applied for the construction of the mathematical models of dynamic systems – the method of choice, the phenomenological approach and the method of identification. The apparent advantage of the identification is that the process of model selection is based on the objective information obtained experimentally. In spite of the fact that such models suffer a certain error, the results of their application represent, generally, a satisfactory approximation. For this reason the dynamic behaviour of a certain object can be described with several mathematical models differing from each other not only in the values of particular parameters, but also structurally. The article presents the results of experimental studies of vibrations in a buckled rod. In this experiment, values of displacements and accelerations in the system were recorded at both increasing and decreasing frequencies of external excitation. The modes of bending vibrations were determined by simultaneous recording the signals from all five strain gauges. It was demonstrated that for a specified type and parameters of the external excitation in the mechanical system under concern only the first-mode vibrations are feasible. The research was also focused on the development of methods for the parametric identification of dynamic models based on the usage of the phase trajectories on the plane ‘acceleration – displacement’.

It is noted that in many instances, lower accuracy in the experimental data calls for finding new ways not only for the presentation of results of the measurements but also appropriate methods for data processing.

Keywords: identification, dynamic system, phase trajectories, noise, data processing

1. Introduction

Both static and dynamic processes in mechanical systems may detect non-linear elastic and/or dissipative characteristics, which must not be neglected. It is known that this is by no means always that a quite extensive and accurate description of the mechanical systems can be grounded merely on the basis of a direct express physical analysis. As a rule, some of the parameters have to be determined on the basis of experiments. In a number of instances even the structure of the mathematical model turns to be obscure [2-4]. Most of the currently available methods of the nonparametric identification are based on the polynomial approximation of models. Thus, the targets of research of the majority of the modern methods of nonparametric identification are time histories (i.e. the time processes), more specifically, studying the recorded changes, which occurred in the displacement of points in the system under investigation, in the time domain. These methods are based on the application of the wavelet transformation, the series of Wiener and Hammerstein. These approaches are cumbersome to implement, they involve the use of the computer technology [3, 5] and generate a need for storing significant amounts of initial information. These methods are also highly restrictive in terms of types of systems that can be indentified and the modes of external testing excitation.

2. The phase trajectories and their representations in the nonparametric identification of models of mechanical systems

One of the trends in solving direct problems of dynamics of structural elements is associated with the investigations into the behaviour of the phase trajectories on the plane «displacement - velocity». In spite of the fact that the geometrical approach was successfully used for quite a long period of time, it is precisely this approach, which forms the basis of nonlinear dynamics and provides possibilities to predict new effects in different fields of knowledge. The qualitative investigations of dynamic system behaviour are confined to studying the behaviour of trajectories in the phase space. Henri Poincaré laid the foundations of the qualitative theory for studying dynamic processes [1]. The area of application of these techniques is not limited to the problems of autonomous vibrations. There is a number of research where these trajectories were used in solving the inverse problem of mechanics, i.e. identification. The geometric presentation of a single phase trajectory or a set of trajectories enables one to arrive at the important conclusions on the characteristics of vibrations.

As it has been demonstrated by a number of researchers in their works [5, 6], the expansion of the phase space by taking into account the phase planes \((y, \dot{y})\) and \((\dot{y}, \ddot{y})\) substantially improves the efficiency of the analysis of dynamic system behaviour.

3. Experimental study of forced vibrations in a rod

In order to achieve the above mentioned objectives, we have designed and manufactured a purpose-made test bench and constructed a model. The bench body is a rigidly fixed member and provides a support for mounting all other elements. The test bench represents a system of crossbars, struts and braces. At the stage of construction of the model structure, the rod was subjected to the load equal to initial longitudinal compressive force. The value of the axial compression force was accepted greater than the value of the Euler's critical buckling load. After compression, the straight-line shape of the rod centre line of becomes unstable [5]. The parameters of an external excitation have a significant influence to the vibrational modes of the flexible rod systems. During the experiment, special attention
was paid to the feasibility to adjust, record and store the parameters of the external excitation.

The investigations of forced vibrations in the flexible rod were carried out using a complete set of measuring and recording instruments that included the signal recording, conversion and data storing sensors along with a personal computer. Primary data processing, particularly, multiplication and division of the sensor readings into scale coefficients, subtraction of corrections for zero-drift, was realized applying a specially-tailored software.

4. Data processing of time processes

The methodology of applied analysis of discrete time processes abounds with a great assortment of methods and algorithms intended for solving the problems of the identification. Many of these techniques are, as a matter of fact, the additions or generalizations of the previously developed practices for a particular subject area. Therefore, it would be appropriate to classify them not according to their applicability to the certain subject areas, but in conformity with the principal approach (statistical, spectral, wavelet, fractal, nonlinear-dynamic, etc.) that was taken in the analysis of a set of the time processes. One of the most significant tasks is to evaluate information inherent in a time process.

Certainly, the identification of noise requires the expert qualification of the researcher. Moreover, an inexact assessment of noise parameters may result in the low-quality preliminary processing of time histories. In the majority of cases, a priori information about the nature of the noise is unavailable; therefore, the parameters to be used in any particular method of the preliminary data processing of time history have to be selected only by trial-and-error adjustment. That has its effect on the accuracy in further data processing of time processes.

As it was proved by investigations [7], the underestimation of multiplicative noise level results in a greater discontinuity of the object contours in the images, when the contour segmentation is performed. This, in turn, acts on the value of the error in discerning these objects. An excessive evaluation of the level of additive Gauss noise leads to the blurring of the contours of the objects, and that also adds to the error in discerning.

The volume of information that can be extracted from a finite set of points in a time series depends on the properties of the phase trajectories as well as on the properties of a dynamic model, which generated these trajectories. However, while solving the problems of the reconstruction of a time series, the characteristics of phase trajectories are a priori unknown, for this reason, it is impossible to perform any preliminary estimation of these parameters.

The analysis shows that even in the cases of a noise-distorted time process, numerical estimation of model parameters becomes quite an arduous task. This fact prompts the necessity to clear up the time process of a certain quantity of noise. It will allow the investigation of dynamics of the model rather than the dynamics of noise and, ultimately, provide the consistent numerical results.

As seen in Fig. 1, the reconstruction of the time sequence with the use of a band filter has given rise to the characteristic clusters of trajectories; that is attributed to the elimination of the initial discontinuity of trajectories in the course of filtration. It is evident that the performed filtration only slightly changed the dynamic picture inherent in the original time process.

Figure 1: The phase trajectories of fundamental-frequency resonant vibrations in the flexible rod: a) the experimental time processes; b) the time processes with the noise removed

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Dependence of the elastic properties of two–dimensional crystals on their curvature

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Abstract

The work is devoted to elastic properties of two-dimensional materials. As an example of a two–dimensional structure a triangular crystal lattice is considered. Its in–plane and out–of–plane elastic properties are estimated as functions of the curvature radius in the three–dimensional space.

Keywords: two–dimensional materials, membranes, elastic properties, curvature

1. Introduction

Nowadays 2D crystals like graphene, boron nitride, and molybdenum disulfide play important role in science and technology. These materials have an ideal symmetrical microstructure described by their crystal lattice. Different discrete models are used to describe the mechanical properties of such lattices at the micro- and nanoscale. The interatomic bonds between atoms are described with semi-empirical potentials of interaction. Quasicontinuum approaches give an opportunity to connect microscopic properties of the crystal lattice (geometry of the lattice, parameters of the interaction) with macroscopic properties, i.e. elastic moduli. Such approaches are widely used to determine elastic properties of graphene and carbon nanotubes. Usually monoatomic layers are considered as thin flat membranes, and their elastic properties in the basal plane are most interesting for researchers [1, 2]. However, it was shown that it is necessary to consider 2D materials as parts of the 3D world, and the curvature of such membranes may affect on their elastic properties [4]. This investigation considers different crystal lattices to understand the role which the curvature plays.

2. Elastic properties of a simple crystal lattice on the plane

Let us consider a simple crystal lattice. Term “simple” means that an elementary cell of such lattice contains one atom. Following the approach proposed in [3] note Rα as vectors connecting given particle with its nearest neighbours indexed by the symbol α in the reference configuration; the vector R determines the position of the given particle. Then, after uniform deformation the lattice vectors will be changed as

\[ r_α = R_α + u(R - R_α) - u(R) \approx R_α + R_α \cdot \nabla u \]  

Hence the square of the lattice vector can be represented as

\[ r_α^2 = R_α^2 (1 + 2n_α \cdot n_α \cdot \varepsilon), \]

where \( \varepsilon = \nabla u \) is the deformation tensor (\( \{\ldots\}^S \) denotes symmetric part), \( n_α \) is the normal vector directed from the given particle to the corresponding neighbour. A simple crystal lattice can be successfully described by the pair interaction potential \( \Pi(r_α) \), which depends only on the distances between the particles. One of the most important examples of the pair interaction is the Lennard–Jones potential

\[ \Pi(r) = D \left( \frac{a}{r} \right)^{12} - 2 \left( \frac{a}{r} \right)^{6} \]  

Here \( D \) is the depth of the potential well, \( a \) is the equilibrium distance for the given interaction. The strain energy density and the corresponding tensor of elasticity are given by relations

\[ W = \frac{1}{2V} \sum_α \Pi(r_α) \]  

\[ C = \frac{\partial^2 W}{\partial \varepsilon^2} \]  

Here \( V \) is the volume of the elementary cell. The last formula assumes that the derivative is taken in the reference configuration. From Eqn (5) follows

\[ C = \frac{1}{2V} \sum_α \left( \frac{\partial^2 \Pi}{\partial r_α \partial \varepsilon} \right) \left( \frac{\partial r_α}{\partial \varepsilon} \right) + \frac{1}{2V} \sum_α \left( \frac{\partial \Pi}{\partial r_α} \right) \left( \frac{\partial^2 r_α}{\partial \varepsilon^2} \right) \]  

Note, that may be obtained two relations for the following derivative

\[ \frac{\partial^2 r_α}{\partial \varepsilon^2} = 2r_α \frac{\partial r_α}{\partial \varepsilon}, \quad \frac{\partial r_α}{\partial \varepsilon} = 2R_α^2 n_α n_α \]  

The last equation follows from Eqn (2). Comparing Eqns (7) it is possible to obtain

\[ \frac{\partial r_α}{\partial \varepsilon} = \frac{1}{r_α} R_α^2 n_α n_α \]  

*Supported by Russian President’s grant MK-4873.2014.1
Substituting this relation to Eqn (6) gives the explicit dependence of stiffness tensors on the potential of interaction:

\[ C = \frac{1}{2V} \sum_n \left( \frac{1}{r_n^2} R_n \Pi''(r_n) - \frac{1}{r_n^3} R_n \Pi'(r_n) \right) n_a n_a n_b n_b \]  

(9)

If the crystal is considered at the equilibrium state and the interaction is limited by nearest atoms, then \( r_n = a \) and \( R_n = a \), \( \Pi'(a) = 0 \). In this case the tensor of elasticity has a reduced form

\[ C^0 = \frac{Ca^2}{2V} \sum_n n_a n_a n_b n_b \]  

(10)

where \( C = \Pi''(a) \).

3. Simple crystal lattice mapped onto a cylinder

Consider a crystal lattice on the plane as an initial configuration. Following [4] let us map the initial two–dimensional configuration to the three–dimensional surface rolled to the cylinder using the extended Cauchy–Born rule proposed in [5]. Let us consider the triangular crystal lattice as an example. In the two-dimensional cartesian basis lattice vectors will have the form

\[ R_n = X_n i + Y_n j, \]  

namely

\[ R_1 = ai, \quad R_2 = a\left(\frac{1}{2}i + \frac{\sqrt{3}}{2}j\right), \quad R_3 = a\left(\frac{1}{2}i - \frac{\sqrt{3}}{2}j\right), \]

\[ R_4 = -R_1, \quad R_5 = -R_2, \quad R_6 = -R_3 \]

Figure 1 shows the mapping of the flat triangular crystal lattice into the Euclidian three–dimensional space.

![Figure 1: Mapping of the 2D membrane onto the cylinder](image)

The in–plane and out–of–plane elastic parameters of the triangular crystal lattice are calculated for the plane and cylindrical configurations. The dependence on the curvature radius is investigated. The obtained formulae can be applied to simple crystal lattices. The elastic parameters of a triangular lattice with the assumed Lennard–Jones potential is considered as an example.

4. In–plane elastic moduli

Equation (10) allows to obtain the in–plane elastic moduli of the undeformed crystal lattice

\[ C_{1111}^0 = C_{2222}^0 = \frac{3\sqrt{3}C}{4}, \quad C_{1122}^0 = \frac{\sqrt{3}C}{4} \]  

(13)

The in-plane Young’s modulus is given by relation

\[ Y^0 = C_{1111}^0 - \left(\frac{C_{1122}^0}{C_{2222}^0}\right)^2 = 2\frac{\sqrt{3}C}{3}. \]  

(14)

Using the approach proposed in previous section one can calculate the elastic moduli in the deformed configuration. As a result, the relations \( C_{1111}/C_{1111}^0, C_{1122}/C_{1122}^0, C_{2222}/C_{2222}^0 \) can be obtained as a function of the dimensionless parameter \( a/R \). It is possible to show that all this relations tend to 1, when \( R \) increases unlimited. If \( R \) decreases, the elastic constants grow up.

5. Out–of–plane elastic properties

The bending rigidity of a thin membrane can be found in the general case as [5]

\[ C_B = \frac{\partial^2 W}{\partial \kappa^2} \]  

(15)

Here \( \kappa \) is the curvature of the surface we are interested in. In our case it is the curvature of the cylinder. The bending rigidity for the planar graphene was found in [5]. Our goal is to find this parameter for the triangular planar crystal lattice and for the curved membranes.

6. Conclusions

The in–plane and out–of–plane elastic parameters of the triangular crystal lattice are calculated for the plane and cylindrical configurations. The dependence on the curvature radius is investigated. The obtained formulae can be applied to simple crystal lattices. The elastic parameters of a triangular lattice with the assumed Lennard–Jones potential is considered as an example. The future work will be devoted to the consideration of complex crystal lattices and lattices consisting of particles with rotational degrees of freedom.

References


Modified constitutive model of discontinuous plastic flow in intermetallic composites

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Abstract

The paper describes modifications of the previously developed constitutive model of discontinuous plastic flow, observed at extremely low temperatures. The changes are aimed at a better physically-based constitutive description of material behaviour. The stress serration is now evaluated on the basis of change in dislocation density, instead of a phenomenological unobservable function of the state parameters. The modified model is scaled to fit with the experimental stress-strain relation for a copper specimen under tensile loading in liquid helium. A proposal of implementation of the model to a two-phase material of superconducting rods is also included.

Keywords: discontinuous plastic flow, serrated yielding of two-phase material, multi-scale constitutive model

1. Introduction

The problem of constitutive modelling of material, that exhibits instability at extremely low temperatures, was examined by the authors in Ref. [3] and Ref. [4], for uniaxial and multiaxial cases, respectively. Also, some conference presentations addressed the problem, e.g. Ref. [1]. The phenomenon is observed during monotonic tension of samples in cryogenic conditions. First, elastic deformation is followed by regular plastic flow. However, after some plastic strain threshold is exceeded, the yielding becomes discontinuous (serrated). In the macro-scale a repeated sudden drops of stress value is observed while the strain increases monotonically (Fig. 1). After each drop of stress, short relaxation process goes on (stress value decreases asymptotically), and then elastic reloading follows until an updated yield surface is reached. The macro-scale effect is physically explained in the microscopic level, with an assumption of a mechanical nature of the observed instability. Namely, the density of dislocations (ρ) increases significantly due to evolution of barriers (Lomer-Cottrell locks), which block their motion. After a critical interaction for L-C barriers density (B) and shear stress (τ) at the head of dislocation pile-up, the barriers are broken, so a spontaneous motion of dislocations happens. Such a collective effect triggered by an avalanche-like crossing of Lomer-Cottrell locks by edge dislocation pile-ups manifests itself in a macro-scale slip (sudden increase of the permanent plastic strain) and a temperature increase (due to plastic power dissipation). It is observed that the slip takes place at an averaged value of the total strain. Since the plastic part of strain increases jump-like, the elastic one decreases in the same value, and hence the drop of stress is observed. The way how the drop-of-stress value is calculated, is the key point of any constitutive model that includes the phenomenon.

2. Modified approach

In the published papers the plastic strain serration increment was evaluated as

\[ \Delta \varepsilon_{p} = \frac{B}{F_{e}(\sigma, T, \varepsilon_p)} \]  

(1)

with a postulated function in denominator depending, in general, on stress (σ), temperature (T) and current plastic strain (εp). Afterwards, the drop of stress was calculated as elastic decrement

\[ \Delta \sigma_{e} = E \Delta \varepsilon_{p} \]  

(2)

with an elasticity modulus E. A new approach, based on micro scale, is proposed. The relation between after-serration stress (σ) in the lattice and the after-serration density of dislocations (\( \rho_{0} \)) is defined as

\[ \sigma = M \left( \tau_0 + G a b \sqrt{\rho_0} \right), \]  

(3)

whereas just before serration the stress value (\( \sigma_{p} \)) and the dislocation density are related

\[ \sigma_{p} = M \left( \tau_0 + G a b \sqrt{\rho} \right), \]  

(4)

where M is the Taylor factor, \( \tau_0 \) stands for shear stress of internal friction, G is the shear modulus, a is the coefficient of dislocations interaction, b denotes length of the Burgers vector. Hence, the plastic strain serration increment is expressed as

\[ \Delta \varepsilon_{p}^{serr} = M G a b / E \left( \sqrt{\rho} - \sqrt{\rho_0} \right), \]  

(5)

depending on the density of dislocations before and after the drop of stress (Fig. 2). The evolution law for density of dislocations prior to serration, including creation and annihilation terms, is given in Ref. [2,3,4]. It is assumed that the after-serration density of dislocations is not kept on initial level (\( \rho_0 \)) but evolves during the loading process. The evolution function depends on current plastic strain and density of the LC locks

\[ \rho_0 = \rho_0^0 + F \left( B, \varepsilon \right) \]  

(6)

The value of B does not vanish to zero after a particular serration, so the density of dislocations does not return to its initial value. The after-serration values of these parameters are ‘footprints’ of the repeated process of accumulation followed by breakage of the barriers.

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3. Two-phase material behaviour

A modified material model is proposed to describe a two-phase composite material of superconductor composed of a Cu matrix (1) and NbTi fibres (2). For $\beta_1$ and $\beta_2$ being ratios of particular material in section area of a tensioned bar (with $\beta_1 + \beta_2 = 1$), the equilibrium relation takes a form

$$\beta_1 \sigma^{(1)}(\varepsilon) + \beta_2 \sigma^{(2)}(\varepsilon) = \sigma(\varepsilon),$$

(7)

where $\sigma$ denotes a mean stress and $\sigma^{(i)}$ stands for stresses in a particular material; $\varepsilon$ is a control tensile strain, identical for both permanently joined materials. Constitutive relations $\sigma^{(i)}(\varepsilon)$ are modeled for each of components independently, and a resultant stress effect is obtained as a weighted sum with Eqn (7). The approach is tested for a hypothetical composite, as there are no experimental data for NbTi available.

Interesting effect observed in tension diagram (Fig. 3, Fig. 4) is an irregular pattern of serrations. Such a stress-strain relation is well supported by experimental observations. In general, the drops of stress in particular composite materials occur at different values of control strain. Hence, various amplitudes and various frequencies of the mean-stress drops are possible. Discontinuous plastic flow of one material may be either relaxed by continuous deformation in the other, or amplified by simultaneous stress drop in it. Amplitude of a single serration varies among component materials. Hence, the drop of the mean stress depends on the material serrating and its fraction in a section area. In general, the value of mean stress as well as magnitude and frequency of serrations lay in-between of analogical quantities in component materials.

References


The influence of imperfect interfaces on the overall mechanical response of metal-matrix composites

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Abstract

A systematic study of the overall mechanical response of strongly or weakly bonded periodic composites was performed. Results were recovered in the extreme conditions of perfect adhesion and pure friction. The aim of the investigation is to evaluate the sensibility of the overall material behaviour to the interface characteristics and to evidence the different load transfer mechanisms between the matrix and the inclusions. The problem is particularly relevant for metal-ceramic composites, due to a low chemical compatibility and difficult production processes to prevent the desirable coupling between the components.

Keywords: metal-matrix composites, interfaces, macroscopic response

1. Introduction

Metal–ceramic composites have been developed for their potentially interesting thermal, wear and corrosion overall properties. The most promising coupling is often designed on the basis of simple bi-phase mixture laws. However, recent experiences have shown that the actual macroscopic characteristics may not be well predicted even in the simple linear elastic range. On the contrary, the measured effective moduli can even fall outside the classical homogenization bounds [1,2].

These apparent inconsistencies have been attributed to poor adhesion between the reinforcing particles and the matrix due to the low chemical compatibility and difficult production processes of metal-ceramics composite. The presence of micro-cracks or flaws, which do not alter significantly the relative volume content, may also have dramatic influence on the overall mechanical response.

The hypothesis of perfect continuity between the matrix and the inclusions was released in recent research contributions and replaced by different conjectures. In the most common approaches, the interface is attributed a cohesive damaging behavior [3-5]. Still, some investigations suggest that the failure mechanism of reinforced metal-matrix composites consists of plastic strain localization and void formation in a narrow band around the particle inclusions [6,7]. On the other hand, Bonora and Ruggiero [8] assume that the bonding between the constituents is ensured by the contact stresses that develop during manufacturing due to the mismatch of the thermal expansion coefficients.

In most cases, the actual load transfer mechanisms can be identified only indirectly, on the basis of the macroscopic system response.

The study considers simple representative volume elements and aims at evaluating the sensibility of the overall material behavior to the interface characteristics.

Results concerning extreme conditions of perfect adhesion and frictional coupling are briefly summarized in the contribution, which evidences the spectrum of responses originating even from these elementary conditions.

2. The problem

The study considers simple periodic representative volume elements, visualized e.g. in Fig. 1, derived from a square array of cylindrical ceramic inclusions, with a relative volume content varying between 0.1 and 0.7, embedded in a metal matrix. The bulk material response is assumed linearly elastic and isotropic, with constitutive properties characteristic of the Al-SiC coupling considered by Xu and Lu [5], namely: $E_m = 72.4$ GPa and $\nu_m = 0.33$ for the matrix; $E_c = 420$ GPa and $\nu_c = 0.25$ for the ceramic.

The only assumed non-linearity concerns the purely frictional response of the interface, with Coulomb coefficient $\mu = 0.01, 0.5, 1$. The lower value of this series was selected on the basis of numerical stability studies.

Comparison was done with the hypothesis of perfectly bonded components. The case of void inclusion has been also considered, consistently with the observation made in [1,2] that detached or damaged material portions resemble equivalent porous inclusions in the remaining material system.

Uniaxial tensile and compressive and pure shear tests were simulated introducing the due symmetry and periodic boundary conditions. Macroscopic stress and strain components were calculated from the reaction forces to imposed displacement values.

Plane stress analysis was performed by a common finite element code [9]. Some representative results are summarised in the following.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Representative volume elements for 0.1 (left) and 0.7 (right) relative volume content of the inclusion.}
\end{figure}
3. Selected results

The influence of the interface characteristics on the overall mechanical response of the considered composites under compression is summarized in Fig. 2. The curves represent the ratio between the macroscopic uniaxial stress and the corresponding strain for different volume ratios.

A significant drop of the overall modulus is produced as the ceramic content is increased and Coulomb coefficient is reduced. This result reflects the load transfer mechanisms that are activated at the microstructure level, as a consequence of allowed or controlled interface sliding. A different local stress distribution that develops for friction coefficient 0.01 and 1 is for instance visualised in Fig. 3 for 10% particle volume content and 0.1% macroscopic strain.

The consequences of an imperfect interface are even more pronounced in the case of tensile tests, see Fig. 4. While the truly elastic solutions (corresponding to either perfect adhesion or void inclusion) coincide with the compressive ones, as expected, the estimated modulus is otherwise insensitive to the presence of the reinforcement for any friction value and ceramic content. Notably, this result is consistent with the experimental observation reported in Ref. [1,2].

Overall stresses and strains are not purely proportional in the case of frictional interfaces due to the initial relative sliding of the materials in contact. The deviation from linearity is however light and prone to be likely confused with the initial settlement of the equipment in real experiments.

On the contrary, a large sensitivity to the interface adhesion of the overall mechanical response suggests a simple and effective operative procedure for the quality assessment of the coupling resulting from the production processes of metal-matrix composites.

Figure 2: The overall modulus relating the macroscopic stress and strain component in the loading direction in the case of uniaxial compression

Figure 3: The local value distribution of stresses parallel to the loading direction in the simulated case of uniaxial compression for macroscopic strain 0.1%; $\mu=0.01$ (left) and $\mu=1$ (right)

Figure 4: The overall modulus relating the macroscopic stress and strain component in the loading direction in the case of uniaxial tension

4. Closing remarks

A few results concerning the overall behaviour of metallic-ceramic composites with imperfect interfaces were introduced. Other related aspects will be discussed in forthcoming contributions.

References

Effect of temperature rate in modelling non-isothermal fatigue of steel

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Abstract

In the paper the fatigue behaviour of steel in non-isothermal conditions is investigated. Two examples are considered: first the classical Armstrong and Frederick type model is extended to include the temperature effects, and used to model the non-isothermal fatigue of cast steel. As the second example the 2M1C (2 mechanisms-1 yield criterion) model derived by Cailletaud and Sai [1] and identified by Velay et al. [6] to describe the fatigue behaviour of AISI L6 steel at different (but constant) temperatures, is extended to account for the effect of temperature change. Numerical simulations are performed, which indicate the significant influence of temperature rate on the response of constitutive model when cyclic thermo-mechanical loading is considered.

Keywords: non-isothermal fatigue, thermoplastic coupling, constitutive modelling

1. Introduction

The aim of the paper is to investigate the influence of temperature rate on the response of an elastic-plastic material in non-isothermal conditions.

In the available literature the influence of variable temperature on the material behaviour during mechanical loading is often accounted for only implicitly, changing with temperature material characteristics, without any terms related to the temperature rate in kinetic equations. The need for such terms was the subject of discussion in many papers (cf [2]), because they may play a significant role when solving high temperature problems, such as fire conditions or thermal shock problems [4].

2. Material behaviour

In order investigate the influence of temperature rate on the response of the constitutive model of thermo/elastic/plastic material two materials: P91 cast steel and AISI L6 tempered martensitic hot work tool steel are considered. Both steels undergo cyclic softening, regardless of the testing temperature (half-stress amplitude decreases with increasing cumulated plastic strain, see Fig. 1). This softening can be divided into three phases, which are: the rapid softening phase during the initial few hundred cycles, followed by a slow quasi-linear softening phase, and finally again fast softening till rupture. The first phase is generally explained by a rapid change of dislocation density inherited from the quench treatment, the second is related to the formation of dislocation sub-structure and carbide coarsening under the action of time, temperature and cyclic load, while the third phase is a consequence of micro-damage development in the material that ultimately causes failure of a tested sample [5].

3. Constitutive model

Constitutive modelling presented here is based on the framework of thermodynamics of irreversible processes with internal state variables and the local state method. In this approach a central hypothesis is that the state of a material is entirely determined by certain values of some independent

*Acknowledgements. The financial support under grant number 2011/03/B/ST8/05132 from the National Science Centre is gratefully acknowledged.
4. Numerical implementation

4.1. Numerical procedures

The numerical procedures implementing both EAF and E2M1C models were performed with the application of SIMULIA-I-sight package, which provides a platform for automatic optimal selection of material parameters. For this purpose, two components offered by the program: “Data Matching” and “Optimization” were used. A "Data Matching" component allowed to calibrate the model by analyzing different error measures between the experimental data and numerical simulation results. In the present analysis, the error measure used was the square root of the sum of squared deviations. "Optimization" component was applied to find an optimal solution in a user-defined field, with the assumed constraints and the objective function defined. In the present case, in order to find the optimal solution the "Pointer Automatic Optimizer" algorithm was used, composed of four well-known searching algorithms for finding the optimum: simplex linear, sequential quadratic programming (SQP), downhill simplex, and genetic algorithms.

In the case of E2M1C model the parameters were identified by Velay et al. [6] to describe the fatigue behaviour of AISI L6 steel at different (but constant) temperatures. In the present analysis these results were interpolated to obtain temperature functions (cf also [4]).

4.3. Numerical results

Several anisothermal fatigue tests were subjected to numerical analysis, according to different strain and temperature controlling (in-phase, out-of-phase, shifted phase, etc). For each material, P91 and AISI L6 steel two cases were compared: (i) the influence of changing temperature accounted only in changing functions of material parameters (partial thermo-plastic coupling), and (ii) the terms proportional to temperature rate additionally included in the kinetic equations of thermodynamic conjugate forces (full thermo-plastic coupling). The stress-strain loops for both cases are compared in Fig. 2 for AISI L6 steel subjected to out-of-phase strain-temperature control. Qualitatively different results were obtained: without temperature rate terms the response exhibits cyclic hardening due to unreasonable shift of hysteresis loops along the stress axis (cf [2]), while including additional temperature rate terms into kinetic equations of the constitutive model allows for preserving stable behaviour.

Figure 2: Hysteresis loops for out-of-phase strain-temperature program (AISI L6 steel)

5. Conclusions

The results obtained in the presented research indicate that coupling between temperature and dissipative phenomena taking place in the material may have a significant influence on the response of a constitutive model [3]. Disregarding the rate of temperature in the evolution of thermodynamic forces related to hardening effects may lead to erroneous results, especially when solving high temperature problems, such as fire conditions or thermal shock.

References

Viscoplasticity of magnetorheological materials - theoretical description and experimental investigations

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Abstract

The extension of viscoplasticity Perzyna's model for the field of magnetorheological materials is proposed. The model is adopted to identify the mechanisms of microscopic rearrangement of ferroelements producing visible increase of material stiffness, in particular increase of shear modulus. The project of laboratory test stand is presented. It is based on Split Hopkinson Pressure Bar set-up equipped with container for magnetorheological fluid and coil to control it.

Keywords: magnetorheological fluid, magnetorheological gel, Perzyna viscoplasticity model, Split Hopkinson Pressure Bar

1. Introduction

One of functional materials of increasing technical importance is magnetorheological fluid controlled by a magnetic field. This kind of material is a suspension of microsized ferroelements in a carrier viscous fluid. Ferroelements, are mostly made of Fe₃O₄ with diameter 12µm or carbonyl iron particles of 4.5-5.2µm diameter. Mineral oil is usually used as a carrier fluid. In order to avoid aggregation of magnetic sensitive elements, ferroelements are surrounded by silicon coat. According to the experimental data discussed in [1] the properties of magnetorheological fluid can be characterized by nonlinear behaviour of rheological material with magnetic active particles as regards the shear stress - shear strain rate relation. In the literature, the linear Bingham model is commonly used. In many cases the linear relations between shear stress and shear strain rate are not adequate in describing mechanism of magnetorheological fluid flow. On the other hand, the Bodner Partom and the Herschel Bulkely models describe nonlinear relation between shear stress and shear strain rate, cf. [1]. However, the disadvantage of these models is that they are of empirical character, with no firm physical basis are valid only for limited range of variables. Therefore the formulation of a new viscoplasticity model for magnetorheological materials that should be based on physical mechanisms responsible for rate dependency of yield stress remains an open question.

2. The Perzyna viscoplasticity model accounting for magnetic field effect

The linear Bingham model proposed originally in [2] is commonly adopted to describe magnetorheological materials

\[ \tau = \tau_{0H}(H) + \mu \gamma, \]

where the symbols denote: \( \tau \) shear stress, \( \tau_{0H} \) yield stress dependent of magnetic field strength \( H \), \( \mu \) viscosity factor, \( \gamma \) shear strain rate. It does not describe adequately the physical mechanisms in the range of high strain rates [3,4]. It can be observed that the results shown in Fig. 1 deviate from linear Bingham relations for higher shear strain rates Fig. 2. This observation confirms the necessity of the development of adequate viscoplasticity model of magnetorheological materials.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{The own visualization of experimental data published in [3]}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Schematic view of the inadequacy of Bingham model [4]}
\end{figure}

The mechanism of microscopic rearrangement of ferroelements producing visible increase of material stiffness and nonlinear dependence of yield stress - strain rate can be described by the nonlinear viscoplasticity model proposed by...
Perzyna [5] extended for accounting of magnetic field strength 

$$\dot{\epsilon}_{ij} = \frac{1}{2G(H)} \tilde{s}_{ij} + \gamma(H) \left( \frac{\tilde{\nu}}{\sqrt{2}} - 1 \right) \frac{s_{ij}}{\tilde{\nu}}$$  \hspace{1cm} (2)

The symbol \( \Phi \) describes the nonlinear excess stress function

$$\langle \Phi \rangle = \begin{cases} \Phi, & \text{on } \sigma > f(\varepsilon) \\ 0, & \text{on } \sigma \leq f(\varepsilon) \end{cases}$$  \hspace{1cm} (3)

\( \sigma = f(\varepsilon) \) is material characteristic for static tension test, \( \varepsilon \) is nominal small strain, \( 2G(H) \) is elastic shear modulus, depending on magnetic field strength \( H \), \( \gamma(H) \) is the viscosity parameter of material depending on magnetic field strength \( H \), \( \kappa(H) \) is quasi -static yield stress depending on magnetic field strength \( H \), \( \tilde{s}_{ij} \) is deviator of stress tensor, \( s_{ij} \) is deviator of stress rate, \( J_2 \) is second invariant of stress deviator, \( \dot{\epsilon}_{ij} \) is small strain rate deviator.

In order to identify the proposed magneto - viscoplasticity model two kinds of experimental tests are required: quasi - static compression test and dynamic axial compression with use of the modified Split Hopkinson Pressure Bar.

3. Laboratory test stand

The physical mechanism responsible for behaviour of magnetorheological materials subjected to magnetic field is illustrated in Fig. 3. The application of magnetic field produces solidification of material volume.

The testing stand is made on basis of Split Hopkinson Pressure Bar [6]. The main idea is to use the present laboratory device and prepare it to test magnetorheological fluid in magnetic field at high strain rates. Laboratory device to investigate dynamic properties of magnetorheological materials is adopted from present device and adequately modified to work with controlable viscoplasticity model. The idea of proposed modification, which is based on earlier studies with use of Hopkinson Pressure Bar [1,7] is presented in Fig 4.

The new set-up is based on Split Hopkinson Pressure Bar with novel grip (container) for MR fluid. The idea of experiment is to create incident wave inside the MR fluid under the influence of magnetic field. The container is equipped with the seals (to prevent leakage of fluid and keep constant pressure in the specimen), coils (to produce magnetic field) and water cooling system (to keep coils in constant temperature). At the end of transmitter bar the gas accumulator is fixed to prevent too large displacement of the bar. The novelty of the proposed laboratory test stand is the application of coils for the sleeves shown in Fig. 4 to induce the constant magnetic field. Due to this the dynamic axial compression tests of solidified magnetorheological material will be possible.

4. Conclusions

The paper shows that a new methodology of experimental investigation is required to indentify the proposed model accounting for magnetic field effect. Therefore, a new set up of Split Hopkinson Pressure Bar equipped with electromagnetic coils to investigate specimens of magnetorheological material is proposed. The extension of Perzyna viscoplasticity model, which includes influence of magnetic field strength brings a new perspective in advanced mechanics of magnetorheological material. The Perzyna magnetoviscoplasticity model accounting for mechanisms of microscopic rearrangements of ferroelements should better describe the material stiffness and nonlinear dependence of yield stress - strain rate than traditional Bingham model or empirical descriptions discussed in [1].

References


Objectivity of strain measures in the fixed-pole approach

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Abstract

The paper presents a novel geometrically exact spatial beam finite element derived from the so-called fixed-pole approach. While utilising the standard Galerkin discretisation, these elements turn out to have non-standard degrees of freedom which are a combination of translations and rotations. If these unknowns are interpolated using Lagrange polynomials the discretised strain measures turn out to be strain non-invariant. In this work a remedy to this problem is presented.

Keywords: strain-invariance, geometrically exact beam theory, 3D beam theory, fixed-pole approach

1. Introduction

In the context of geometrically exact spatial beam theory, the strain measures are invariant to rigid-body rotation [1]. However, their approximated values do not necessarily inherit this property [2]. Regardless of the choice of the interpolated quantities – being it the iterative [3, 4], the incremental [5, 6] or the total rotations [7] – the interpolation is always applied to the rotations between a particular reference configuration and the current configuration. As a consequence, the rotations interpolated in this way in general include rigid-body rotations, so that the error, introduced by the interpolation, makes the resulting strain measures dependent on the rigid-body rotation [1]. The remedy that Jelenić and Crisfield proposed is logical – elimination of rigid-body rotations from the interpolation of the rotational variables. In [1] it was achieved by decomposing the total rotational matrix $\Lambda(x)$ using a reference orientation matrix and the total orientation matrix, so that

$$\Lambda(x) = \Lambda^h(x) = \Lambda_c \exp \tilde{\Psi}(x).$$

This means that the only interpolated quantities are in fact the local rotations $\tilde{\Psi}$, which results in a strain-invariant and path-independent formulation.

2. The fixed-pole approach

The fixed-pole approach, first described by Borri and Bottasso [8] is based upon the fact that the spatial kinematic quantities may be expressed with respect to a fixed pole (i.e. the origin of the spatial co-ordinate system), rather than the moving pole (i.e. the reference axis at a given cross-section). In that sense, the fixed-pole quantities are merely a special case of spatial quantities. Relating them to their material counterparts results in the so-called configuration tensor

$$C = \begin{bmatrix} \mathbf{I} & \mathbf{r} \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda \end{bmatrix} = \begin{bmatrix} \Lambda & \tilde{\mathbf{r}} \Lambda \\ 0 & \Lambda \end{bmatrix},$$

where $\mathbf{r}$ is the position vector, $\tilde{\mathbf{r}}$ is a skew-symmetric matrix corresponding to the cross-product $\mathbf{r} \times \Lambda$ and $\Lambda$ is the orientation matrix of the cross-section. The fact that $C$ is an element of the special group of rigid motions $SR(6)$ with a closed form of the exponential map [9] enables a unified configuration update, analogous to the update of orientation matrices which are elements of the well-known special orthogonal group $SO(3)$.

The spatial strain measures $\gamma$ and $\kappa$ defined in [10] may be stacked as

$$\{\gamma\} = \{\mathbf{r}'\} - \{\mathbf{t}_1\} = \chi - \chi_N,$$

with

$$\chi = \{\mathbf{r}'\} \quad \text{and} \quad \chi_N = \{\mathbf{t}_1\}$$

as the strain parameter and

$$\chi_N = \{\mathbf{t}_1\}$$

as the “natural configuration parameter” as termed in [8, 9], with $\mathbf{t}_1$ as the unit vector in the direction of the beam reference axis at the cross section. The reason for dividing strain measures in two parts $\chi$ and $\chi_N$ becomes evident when taking the derivative of the configuration tensor with respect to the beam arc-length $x$

$$C' = \frac{d}{dx} \begin{bmatrix} \Lambda & \tilde{\mathbf{r}} \Lambda \\ 0 & \Lambda \end{bmatrix} = \begin{bmatrix} \Lambda' & \tilde{\mathbf{r}}' \Lambda + \tilde{\mathbf{r}} \Lambda' \\ 0 & \Lambda' \end{bmatrix}.$$

Noting that $\Lambda' \Lambda'^T = \tilde{\kappa}$ we obtain

$$C' = \begin{bmatrix} \tilde{\kappa} & \tilde{\mathbf{r}}' + \mathbf{r} \times \kappa \\ 0 & \tilde{\kappa} \end{bmatrix} \begin{bmatrix} \mathbf{I} & \tilde{\mathbf{r}} \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda \end{bmatrix},$$

because for 3D skew-symmetric matrices $\tilde{\mathbf{r}}, \tilde{\kappa}$ we have $\mathbf{r} \times \kappa = \tilde{\mathbf{r}} \kappa - \tilde{\kappa} \mathbf{r}$. The fixed-pole description of the strain parameter is

$$\chi = \{\mathbf{r}' + \mathbf{r} \times \kappa\} \quad \kappa.$$
which allows for recognising that (4) can be written as
\[ C' = \tilde{\mathbf{X}} C \tilde{\mathbf{X}}^{-1} \]
with
\[ \tilde{\mathbf{X}} = \begin{bmatrix} \kappa & \tilde{r}' + \tilde{r} \times \kappa \\ 0 & \kappa \end{bmatrix} \quad \text{and} \quad \tilde{\mathbf{X}} = \begin{bmatrix} \mathbf{K} \Lambda \tilde{r}' \\ 0 & \mathbf{K} \end{bmatrix} \]
as the matrices corresponding to the fixed-pole and material description of the strain parameter, respectively.

When deriving the finite element based on the fixed-pole equations of motion we turn out to have non-standard virtual 
\[ \delta \mathbf{C} \]
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as the matrices corresponding to the fixed-pole and material description of the strain parameter, respectively.

When deriving the finite element based on the fixed-pole equations of motion we turn out to have non-standard virtual quantities 
\[ \delta \mathbf{C} = \begin{bmatrix} \delta \xi \\ \delta \tilde{r} \end{bmatrix} = \begin{bmatrix} I & \tilde{r} \\ 0 & I \end{bmatrix} \begin{bmatrix} \delta r \\ \delta \tilde{r} \end{bmatrix} , \]
where \( \delta \tilde{r} \) is the spin vector. Applying the Galerkin method, this results in non-standard system unknowns. Interpolating these unknowns by means of Lagrange polynomials strain non-invariant and path-dependent results are obtained, even in the planar case. The reason for this is that the rotations are also present in the translational part of the system unknowns.

3. Objectivity of strain measures

Since there are many analogies between the orientation matrix \( \mathbf{A} \) and the configuration tensor \( \mathbf{C} \) [9, 11], the idea of applying the methodology presented in [1, 2] comes naturally. Using this idea the objectivity of the theory is inherited by the actual implementation. The configuration tensor \( \mathbf{C}(x) \) is decomposed using a reference configuration tensor \( \mathbf{C}_r \), which is unique for the whole beam and rigidly attached to it, and a configuration tensor defining a local 6D configuration vector, \( \Phi(x) \) between the reference configuration tensor and the total configuration tensor so that
\[ \mathbf{C}(x) = \mathbf{C}_r \exp \left( \frac{1}{2} \tilde{\Phi} \right) \]
The reference configuration tensor represents the configuration of one of the beam nodes. In the most general case, two nodes \( I \) and \( J \) are selected, and a relative configuration vector between these nodes is calculated using
\[ \tilde{n}_{IJ} = \mathbf{C}_r^{-1} \mathbf{C}_J . \]
\( \tilde{n}_{IJ} \) is then extracted using a 6D version of Spurrier’s algorithm [11, 12]. The reference tensor \( \mathbf{C}_r \) is defined as a midway configuration between \( \mathbf{C}_I \) and \( \mathbf{C}_J \) as
\[ \mathbf{C}_r = \mathbf{C}_l \exp \left( \frac{1}{2} \tilde{n}_{IJ} \right) . \]
The nodal configuration tensor \( \mathbf{C}_i \) is evaluated from \( \mathbf{C}_i = \exp \Delta \mathbf{C}_i \mathbf{C}_i \), and. From (8) we extract the nodal values of the local configuration vector \( \Phi_i \) using
\[ \exp \tilde{\Phi}_i = \mathbf{C}_r^{-1} \mathbf{C}_i . \]
Then the values of the local configuration vector are interpolated along the beam reference axis using Lagrange polynomials
\[ \Phi(x) \cong \tilde{\Phi}(x) = \sum_{k=1}^{N} t^k \Phi_k , \]
\[ \Phi'(x) \cong \tilde{\Phi}'(x) = \sum_{k=1}^{N} t^k \Phi'_k . \]
It should be stressed that an alternative Lagrangian interpolation of \( \Delta \mathbf{C} \) would inevitably result in a non-objective formulation, even in 2D. The described procedure makes it possible to evaluate strain measures (strain parameter) using only the relative configuration vector, and to derive 6D interpolating functions for the configuration spins is [11]. Implementing these two interventions, the fixed-pole formulation becomes strain-invariant and path-independent.

References


Micromechanical modelling of magnesium alloy and its experimental verification

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Abstract

Micromechanical modelling of magnesium alloys is presented. The applied model combines the crystal plasticity framework accounting for twinning with the self-consistent grain-to-polycrystal scale transition scheme. The mechanical response of the material in the experiments involving the strain path changes is studied, together with the prediction of the accompanying texture evolution. It is demonstrated that the evolution of microstructure has an important impact on the overall material behaviour. The model predictions will be verified in experiments performed on the rolled sheets made of AZ31B alloy.

Keywords: micromechanics, crystal plasticity, twinning, texture evolution

1. Introduction

For magnesium alloys, materials of hexagonal (hcp) lattice symmetry, the group of easy slip systems provides only two independent systems per grain. At room temperature, the lack of five independent slip systems, required for a general shape change of volume elements, is compensated by twinning. Activity of this mechanism of plastic deformation is thus beneficial in view of the stress reduction and enhancement of ductility. However, as a result, the development of strong crystallographic textures upon mechanical processing, e.g. rolling or extrusion is observed. The pronounced texture is the source of the strong anisotropy of mechanical properties of the produced components such as, for example, strong asymmetry between in-plane tension and compression for AZ31B sheets. This asymmetry is explained by the activity of different mechanisms of plastic deformation depending on the loading direction. In view of the above observations micromechanics seems to provide a natural tool enabling understanding and description of a relation between the microstructure of the above materials and their macroscopic (overall) properties, cf. [2].

In the contribution, with the use of the recently developed micromechanical model [1, 2], the AZ31B response and associated texture evolution are investigated in tension-compression cycles in view of the experimental results reported in [3]. Additionally, the results of own experimental studies on the material will be reported that will serve for further identification of microstructure-property relationship for this Mg alloy and validation of the proposed model.

2. Micromechanical model

2.1. Crystal plasticity framework

The constitutive model was formulated in [1, 2]. The large-strain crystal plasticity framework accounting for twinning is used. The vital components of the approach are presented below. Twinning is described as a unidirectional slip mode, so that the shear rate on a twin system is calculated as

$$\dot{\gamma}^T = \gamma^{TW} f^T,$$  \hspace{1cm} (1)

where $f^T$ is the volume fraction of the twinned part created by the twin system $f^T$ and $\gamma^{TW}$ is the characteristic twin shear specified by the lattice geometry. Appearance of twin-related orientations in the texture image is accounted for by using the probabilistic twin volume consistent (PTVC) scheme. The method preserves consistency of twinning activity and the reorientation probability, so that the volume effect of twins on texture image is well reproduced. Valid textures are predicted.

The rate of shear on the slip or twin system and the nonnegative resolved shear stress are related by the classical viscoplastic power law. The evolution of the critical shear stress ($\tau_c$) for the subsequent modes follows the hardening law encompassing four types of interactions: slip-slip, slip-twin, twin-slip and twin-twin. In particular, it differentiates between impact of accumulated slip or twinning on the increment of $\tau_c$. The impact of slip activity is described by the Voce-type law that accounts for the athermal statistical storage of moving dislocations and dynamic recovery.

The impact of twinning activity is related to the geometrical effect of twin boundaries on deformation modes activity. The modification of mechanical properties of twins with respect to the matrix is also included in the model. More details concerning the formulation and its physical background can be found in [2].

The overall response of a polycrystal is determined by the self-consistent averaging procedure applied to a representative aggregate composed of individual grains. The self-consistent method makes use of the Eshelby solution obtained for linear elastic materials, therefore linearization of the material response is required. The results presented in the work are obtained with the use of the VPSC model of Lebensohn and Tome, in which the PTVC reorientation scheme and the hardening model have been incorporated.

2.2. Application to AZ31B alloy

Magnesium is a hcp material with $c/a = 1.624$. Following previous studies, cf. [2], three types of slip systems: basal, prismatic and pyramidal ($c + a$), as well as tensile twinning are assumed to be active. Among slip systems the pyramidal ($c + a$) system is the hardest mode, while basal slip and tensile twinning are easier to initiate. Material parameters of the model for AZ31B were identified in [2]. Experiments indicate that the rolled magnesium sheets exhibit the basal texture of axial symmetry in the...
sheet plane, e.g. [3]. Since strain along c-axis can only be accom-
modated by pyramidal (c+a) slip (a hard mode) or unidirectional twinning (only for extension along c axis), magnesium
alloys show high asymmetry and anisotropy in mechanical prop-
erties. Such asymmetry has been observed in uniaxial tension-
compression tests reported in [3]. The proposed model can be
used to explain the observed material behaviour in light of the
predicted active mechanism of plastic deformation. The selected
results are presented in Figs 1-2. They concern samples loaded in
the direction normal to the sheet plane.

Let us shortly analyze the results. For the initial compres-
sion (P0-P1), the strain is accommodated by basal and pyramidal
(c+a) slip, next during subsequent tension (P1-P3) twinning is
active. Its activity has a marked effect on texture: all the grains
have been reoriented due to twinning, so that at the end (P3) all
basal planes are perpendicular to the sheet plane. It enables activ-
ity of twinning mechanisms in the reoriented grains (i.e. detwin-
ing) during subsequent compression (P3-P5). Again almost all
the grains are reoriented, so that the texture returns to the orig-
inal basal texture (P5). It is expected that such behaviour will
be repeated during subsequent cycles. The simulated textures re-
produce well experimental ones reported in [3]. The observed
discrepancy between the experimental and simulated stress-strain
curves stems from the fact that in the model twinning activity
stops earlier than in the experiments.

3. Experimental verification

In order to verify the model the uniaxial tension tests will
be performed on samples prepared form AZ31B sheet of 1 mm
thickness. The preliminary result of tension tests is shown in Fig.
3, together with the microstructure of the as-received material
obtained using Scanning Electron Microscope (SEM). The pre-

cence of initial twins is observed. In order to reduce the presence
of mechanical twins in the initial state the samples were annealed
at 350°C for 2h and cooled slowly in the furnace.

Two-step tension tests will be performed. First the large sam-
ple will be pre-strained up to the pre-defined strain level in order
to induce an asymmetry of the initial texture in the sample plane,
as expected on the basis of modelling predictions [2]. Next, the
smaller samples will be cut from the pretrained material at the
selected angles with respect to the loading direction. The experi-

ments will be performed till rupture and up to the specified strain
with the proceeding unloading of the sample. The initial and final
texture will be measured. The next step will be the study of the
samples response in tension-compression cycles.

Figure 1: Compression-tension cycle in the direction normal to
the AZ31B sheet plane - comparison of the model predictions
and experiment [3]

Figure 2: (a) Relative activities of the deformation modes and
(b) texture evolution (the (0001) pole figure with a sheet plane as
a projection plane) predicted by the model for the compression-
tension cycle in Fig. 1

Figure 3: Preliminary in-plane tension test of AZ31B sheet: (a)
stress-strain curve; (b) initial microstructure (SEM)

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ture and extension twinning on the low-cycle fatigue behav-
Influence of aluminum layer thickness on the fatigue properties of super-nickel alloy

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Abstract

The paper presents the results of fatigue tests performed on the super-nickel alloy after Chemical Vapor Deposition (CVD). CVD process was carried out for the periods of 4 and 12 hours, and as a consequence, a layer of the aluminum of the thickness 20 µm and 40 µm, respectively, was obtained on the surface of nickel alloy. The specimens with layers were tested on the servo-hydraulic testing machine under dynamic loading at high temperature of 900°C. The profiles of micro hardness on the cross-sections of specimens enabled identification of hardness variations. The main aim of the paper was to evaluate the effect of a film thickness on the fatigue properties of the alloy. Additionally, an identification of crack propagation in the layer subjected to cyclic loading was analysed on the basis of deformation changes in the subsequent cycles.

Keywords: aluminium layer, super nickel alloy, fatigue testing, CVD technique

1. Introduction

The extreme operating conditions of modern aircraft engine turbines require an application of the heat-resistant and high-temperature materials. The gas temperature at the combustion chamber outlet reaches 1600°C while the maximum operating temperature of contemporary nickel super alloys (with a monocrystalline structure) is 1100°C. This fact reflects well a response into the question why it is necessary to produce protection covers on the hot parts of the aircraft engine turbines [1]. The most popular method for obtaining diffusion layers is Chemical Vapour Deposition (CVD). In this work an influence of thickness of the diffusion aluminide layers on the high-cycle fatigue strength of nickel super alloy MAR 247 [2] is studied.

2. Methodology

The films for tests were obtained based on the nickel alloy MAR 247. The aluminizing process was carried out by the Chemical Vapour Deposition (CVD) method using AlCl₃ vapours in hydrogen atmosphere as the carrier gas at temperature of 1040°C for the periods of 4 h or 12 h, and the reduced pressure of 150 hPa. Depending on the duration of this process the aluminum layers of 20 µm or 50 µm thickness were obtained on the nickel alloy surface.

The fatigue tests were carried out on the MTS 810 testing machine of the axial force capacity equal to +/- 25 kN and equipped with the FLEX digital controller. The specimen temperature of 900°C during tests was obtained by means of the induction heater mounted on the testing stand as it is shown in Fig. 1a. The geometry of specimens applied in the experimental programme is presented in Fig. 1b.

All fatigue tests were force controlled under the assumption of the zero mean level and constant stress amplitude for a given specimen tested. The frequency during fatigue tests was equal to 20 [Hz], whereas the magnitudes of stress amplitude varied from 380 [MPa] up to 520 [MPa].

Since it was assumed that the diffusion layer is more susceptible to brittle fracture, micro hardness profiles on the cross section of specimens were prepared in order to evaluate a hardness gradient. The micro hardness was measured using the Hysytron device, under load of 0.0005 [N] starting from the edge of specimen. Observation of the fatigue damage development was conducted by means of the light and scanning microscopes in order to confirm whether the layer cracks are generating before the specimen decohesion.

3. Results

An experimental programme contained 14 fatigue tests (7 tests for each layer thickness considered). Microscopic measurements were carried out directly after fatigue tests. The results of fatigue investigations were elaborated in the form of Wöhler diagrams presented in Fig. 2. The figure shows...
variation of the cycles number up to failure depending on the thickness of the aluminum layer deposited on the surface of the super-nickel alloy specimen. It has to be noticed, that the fatigue results are not consistent taking into account the effect of the layer thickness. For three levels (of seven levels considered) of the stress amplitude a longer fatigue lifetimes were achieved for 20 µm layer thickness, for the remaining tests, however, longer lifetimes were obtained for thicker layer.

In the microscopic damage analysis of tests carrying under fatigue conditions usually such damage sensitive strain parameters as the accumulated inelastic strain or accumulated mean strain level well describe a damage development in the subsequent loading cycles. They can be easily determined on the basis of hysteresis loops [3, 4]. In this case however, it has to be emphasized, that the effect observed during microscopic inspections could not be confirmed by the damage sensitive strain parameters mentioned above.

Figure 2: Wöhler characteristics for specimens with surface deposited by the aluminum layer, fatigue in temperature 900°C

The profile of micro hardness variations is shown in Fig.3. It also contains a topography map of the surface tested where the spots of indentation are illustrated. As it is shown in this figure, the values of micro hardness of the aluminium layer are significantly higher than those for the nickel core determined.

Figure 3: Micro hardness profiles and the map of indenter positions for micro hardness measurement

An observation of the fatigue fracture using the light microscope, Fig. 4, and SEM technique, Fig. 5, enabled identification of the layer cracks generated just before the decohesion of the specimens tested.

Figure 4: Illustration of cracks in the layer area close to the specimen failure - photo from light microscope

Figure 5: Illustration of cracks in the layer area close to the specimen failure - photo from SEM

A process of the layer cracking was completely not visible looking on the hysteresis loops evolution for subsequent cycles of loading.

4. Conclusions

The cracks of layer are responsible for damage initiation of the nickel based specimens after CVD process. It has to be noted that the aluminum layer does not affect the fatigue strength of the nickel alloy, also a thickness of layer is not important in this matter. The identification of the layer cracking initiation is impossible by monitoring of the deformation, only. Therefore, in further studies an application of the non-destructive methods to capture the layers cracking start and development is necessary.

References


An improved estimate for threshold fracture energy in solid particle erosion

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Introduction

It has been long observed that loss of material strength during brittle elastic fracture occurs due to radial cracks [1], and that erosion fracture is characterized by short impact pulses [2], which necessitates studying the applicability of various fracture criteria describing the process. There are however still problems with many areas of its qualitative descriptions. The short impact durations mean that the classical critical stress or critical fracture toughness criteria turn out to be unsuitable [3], as a result we cannot explain several important effects in the erosion process caused by the short-time pulses of the initial action.

While several methods to overcome these limitations have been unable to explain some key experimental results, recent approaches based on the notion of incubation time prior to fracture. Here this framework is utilized in order to further the result, obtaining an exact value for the initial energy required for fracture initiation, and comparison with previous work is completed. Additionally the incubation time based fracture criterion is utilized in order to locate the time, with associated radial position, of the initial fracture.

Keywords: erosion, incubation time, threshold fracture energy, blunt impact

1. Problem formulation

We begin with a perfectly rigid indenter described by a power law
\[ z = Ar^\lambda , \quad 0 < \lambda < 1, \quad A \in \mathbb{R} \quad (1) \]
where \( z \) is the vertical coordinate, and \( r \) is the polar radius. As it is assumed that the impact duration, \( T \), is small (< 10⁻⁵ seconds), we can approximate the impacted material with an elastic half-space, and ignore energy wave effects. Therefore the midpoint of the impact, \( t_0 = T/2 \), will be point where the indenter depth, \( w(t) \), achieves its maximum, denoted \( w_0 = w(t_0) \).

Then, following the approach from Argatov et al. [8], the indenter displacement \( w \) takes the following form:
\[ w(t) = w_0 U \left( \frac{\lambda + 1}{\lambda} ; \frac{w_0}{w_0} \right) \quad (2) \]
where the function \( U \), and all components, are explicitly defined.

Additionally, simplifying from [9], the radial stress throughout the surface of the elastic half-space, \( \sigma_r \), is explicitly defined in terms of the contact pressure \( p(r) \), where \( r = r/a \) and \( a \) is the radius of the contact area. It is worth noting that, as the stress function is defined in terms of time dependent variables, this can be utilized in terms of a dynamic, rather than simply quasi-static, based approach.

3. Incubation time based fracture criterion

Let us introduce an incubation time based fracture criterion [10, 11] to predict fracture, using the simplest case for the impact interaction of a superellipsoid particle with an elastic half-space.
It is defined in terms of the criterion function:

\[ \gamma(r, t) = \frac{1}{\tau} \sigma_e \int_{t \to \tau} \sigma_e(r, s) \, ds \]  

(3)

The incubation time based fracture criterion \(G\) can then be formulated as follows [4]:

\[ G(r) = \left\{ \min t \in [0, 2t_0] : \gamma(r, t) = 1 \right\} \]  

(4)

where \(\sigma_e\) is the tensile strength of the elastic material (a parameter evaluated under quasi-static loading conditions), \(\tau\) is the incubation time of the fracture process (a measure of stress-rate sensitivity of the material, which is calculated quasi-experimentally).

Denoting the time of the initial fracture \(t^*\) for any impact, with associated position \(r^*\), we can state these in terms of the time criterion \(G(4)\):

\[ t^* = \min G(r), \quad r^* = G^{-1}(t^*) \]  

(5)

Where \(G^{-1}\) is the inverse of the time criterion function. This simple formulation is only intended to find the initial breakage and can’t be used to predict subsequent behaviour.

4. Key results

With the stress function explicitly formulated, and the incubation time based fracture criterion (4) providing a method for determining the time and location of any initial radial fracture, two clear aims can be achieved:

4.1. Fracture time and position

The creation of an algorithm which can determine the time and radial position of the initial fracture for fixed initial conditions, and to examine the effect of varying initial parameters (i.e. indenter shape, initial velocity). An example of the results of such an investigation is shown in Fig. 1.

Figure 1: Initial fracture time \(t^*\) for fixed indenter geometry over varying initial particle velocity \(v_0\).

The relationships between initial indenter parameters and the resulting time and position of fracture show some clear trends, such as indenter volume playing a larger role than the indenter geometry, and the particle velocity reducing the incubation time prior to fracture.

4.2. Threshold fracture energy

Determining the position, \(r_{\text{max}}\), within the elastic half-space and time, \(t_{\text{max}}\), which, for a given set of initial conditions, the criterion function (3) is maximized, it will be possible to determine whether the fracture criterion (4) will be satisfied. As a result the threshold fracture energy can be explicitly calculated. It is worth noting that this result is a continuation of [8], in which a lower bound on the threshold fracture energy was obtained. A comparison of the two results is shown in Fig. 2.

Figure 2: Initial energy required for fracture \(\varepsilon_0\), for fixed indenter geometry, \(\lambda\), over various impact durations \(t_0\).

It is clear from Fig. 2 that there exists a critical shape parameter \(\lambda = 5.5\), with a unique minimum threshold fracture energy occurring for \(\lambda < 5.5\), while no minimum exists for \(\lambda > 5.5\).

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The energy approach to rate-independent plasticity of metal single crystals

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Abstract

In the modelling of metal single crystals in the framework of rate-independent plasticity, there are known difficulties caused by non-uniqueness in selection of active slip-systems. A related challenge is to predict emergence of non-uniform deformation patterns and the formation and evolution of experimentally observed microstructures. A new constitutive algorithm is presented that tackles those problems using the energy approach. It is based on asymptotically exact formulation of the set of constitutive equations and inequalities as a minimum problem for the incremental work expressed by a quadratic function of non-negative crystallographic slips. The calculated examples of deformation banding patterns and of reduction of multiplicity of active slip-systems in fcc single crystals are compared with the experimental observations.

Keywords: crystal plasticity, slip-systems selection, incremental energy minimization

1. Introduction

There is a long-standing difficulty in the rate-independent crystal plasticity, in the framework of the rigorous theory established by Hill and Rice [1], related to non-uniqueness (or ambiguity) in selection of active slip-systems [2]. The common way to circumvent that problem in calculations is to apply a viscoplastic model where slip-rates are given functions of state and therefore uniquely defined [3]. However, uniqueness of solutions in crystal viscoplasticity may be accompanied by a strong sensitivity to initial imperfections, thus replacing the problem of non-uniqueness by a kind of hidden instability. In the absence of an explicit stability criterion, there are also difficulties in predicting experimentally observed instability phenomena like spontaneous emergence of deformation bands and microstructures.

2. Incremental work minimization problem

It is thus of interest to examine single crystal behaviour within the general framework of stability in materials with rate-independent dissipation [4]. In the presentation, attention is restricted to spatially discretized single crystals without strain-gradient effects. Instability is meant in the extended sense applicable to quasi-static deformation processes (paths), assuming that instability of equilibrium does not occur if the loading device is sufficiently rigid.

Path instabilities either of constitutive origin or with respect to formation of deformation patterns can be addressed in a unified manner using the energy criterion of path instability [4]. This requires the incremental (or rate) problem to be of potential type, which is not true in general in rate-independent plasticity of single crystals deformed by multislip. Recently, the selective symmetrization of the slip-system interaction moduli matrix has been proposed [5], restricted to active slip-systems. It was shown [6] that this makes the energy approach applicable because it admits an asymptotically exact formulation of the set of constitutive equations and inequalities for a crystal as a minimization problem for the incremental work.

3. Algorithm

The incremental work minimization algorithm for rate-independent plasticity of single crystals is presented in detail in [7]. It is based on iterative minimization of the incremental work expressed by a quadratic function of non-negative crystallographic slips. The constrained minimization problem was converted to a smooth unconstrained using the augmented Lagrangian method. The advantage of the incremental work minimization is that it eliminates the solution paths, expected to be of less physical interest, that do not satisfy the energy criterion of path stability. This way the algorithm deals with the long-standing difficulties mentioned in Introduction. Suitability of the algorithm for simulations of large plastic deformation of fcc crystals and polycrystals with multiple changes of active slip-systems was examined by a number of examples.
4. Results

The algorithm allows for numerical simulations of large plastic deformation of single crystals with the automatic crossing of constitutive branching points. Fig. 1 shows an example of uniform simple shear of an fcc crystal with latent-to-self hardening ratio of 1:4, for a kinematically controlled deformation gradient \( \mathbf{F}(t) = 1 + t \mathbf{B} \otimes \mathbf{A} \) starting from \( \mathbf{B} \leftrightarrow [100] \) and \( \mathbf{A} \leftrightarrow [001] \) at \( \lambda = 0 \) up to \( \lambda = 10 \). As indicated in the figure, unstable solutions with more than 5 active slip-systems were eliminated.

For a partially restricted deformation gradient, the number of locally active slip-systems can be much smaller, as illustrated in Fig. 2 for uniaxial tension in the high-symmetry direction with a symmetry-breaking response. Another illustration is shown in Fig. 3 where the energetically preferable pattern of deformation banding was calculated for another fcc crystal subjected to tension with two components of the macroscopic deformation gradient left free. The dependence of the deformation pattern on the loading direction is in qualitative agreement with experimental observations.

Figure 1: (a) Shear stress vs shear strain for uniform simple shear of an fcc single crystal, (b) changes in the orientation of crystallographic axes [7]

Figure 2: Simulations of uniaxial tension of fcc crystal: deformation banding in tension in different directions [6]

Figure 3: Simulations of uniaxial tension of fcc crystal: slip-system selection, stress-strain diagram and changes of crystallographic orientation of the tensile axis [7]

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Application of spherical harmonics to symmetric non-spherical particles description

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Abstract

The problem of particle shape by means of applying spherical harmonics is considered. This type of modelling presents development of semi-analytical model for particles described by a discrete set of points. Modelling technique minimises the order of spherical harmonic expression while maintaining sufficient accuracy. Two discrete sets of points were generated to mimic typical symmetric non-spherical particle shapes - rotational ellipsoidal and the quasi-cube, i.e. smoothed cube with rounded edges. The variation of several criteria (surface area, volume and etc.) against approximation order parameters is illustrated.

Keywords: discrete element method, particle shape, spherical harmonics, smooth cube, ellipsoid

1. Introduction

Discrete element method (DEM) presents the Lagrangian type modelling technique most widely used to study particulate materials. It was found, that the behaviour of particle largely depends on their shapes. Critical review of recent developments in DEM with advances in the formulation and implementation of non-spherical particle models, including representation of non-spherical shape, is presented in [1].

Since the particles of real granular materials are often of irregular shapes, modelling of a particle shape is rather difficult task. Frequently, particles are approximated by spheres, and a well-known Hertz theory is widely used in DEM applications to model the inter-particle contact. However, the simulation of such materials by spherical particles does not always yield the reliable results, since many effects may be missed.

Various shape specific models suitable for approximation of real non-spherical particles were suggested using in DEM, but most of them have appropriate drawbacks. Uptill now there is a lack for a universal modelling tool getting an analytical expression for the shape. The model controlling the accuracy of the results in systematic manner and thus retaining advantages of spherical contact calculation is still desirable.

In this paper spherical harmonics as semi-analytical hierarchical method are introduced. Spherical harmonics are widely used and well-conditioned tool of applied mathematics, physics and engineering. The application samples are encountered in weather and climate modelling, in the representation of gravitational, topographic, and magnetic data in geophysics, in the numerical solution of certain partial differential equations [2,3]. Successful 3D applications of sophisticated spherical harmonics comprising lunar regolith or sand particles is illustrated in [4,5]. In the next section, the spherical harmonics expansion and its formulation as a universal modelling technique are given

2. Spherical harmonics expansion

Spherical harmonics form an orthonormal base, and arbitrary continuous function \( F(\theta, \phi) \) described in spherical coordinates can be expanded into spherical harmonics. In applications, the spherical harmonic expansion of an arbitrary function is limited to a finite number \( N \) as follows:

\[
F(\theta, \phi) \approx \sum_{l=0}^{N} \sum_{m=-l}^{l} a_{l}^{m} Y_{l,m}^{m}(\theta, \phi)
\]

(1)

where \( Y_{l,m}^{m}(\theta, \phi) \) are spherical harmonics of degree \( l \in N \) and order \( m (0 \leq |m| \leq l) \) and \( a_{l}^{m} \) are their coefficients [6, 7]. Formally, spherical harmonics are complex functions, but only real valued functions are applied. They are given by separating imaginary and real parts of complex functions as follows:

\[
Y_{l,m}^{m}(\theta, \phi) = \left\{ \begin{array}{ll}
\sqrt{2N_{l}^{m} \cos(m\phi)} P_{l}^{m}(\cos \theta) & \text{if } m > 0 \\
N_{l}^{m} P_{l}^{m}(\cos \theta) & \text{if } m = 0 \\
\sqrt{2N_{l}^{m} \sin(m\phi)} P_{l}^{m}(\cos \theta) & \text{if } m < 0 
\end{array} \right.
\]

(2)

Using spherical harmonic expansion (Eqns (1)-(2)) there are two basic steps: first – generating spherical harmonics and second – the calculation of expansion coefficients.

3. Modelling approach

Modelling approach considered hereafter is aimed to illustrate description of the non-spherical particle. The particle presents three-dimensional body surface which is described by discrete set of points \((r_{i}, \theta_{i}, \phi_{i})\). A particle surface is assumed to be continuous random function uniquely defined by a discrete set of random as well as determined parameters. Consequently, the surface geometry is extracted from measurements or generated numerically according to specified rules.

Several basic assumptions are employed for the characterization of particle shape. The particle surface presented in a polar form is convex, or star-shaped, i.e. each generatrix ray is uniquely related to the point on the continuous surface. Axial symmetry and/or rotational symmetry are assumed to be imposed.

We assume that the quasi-surface is initially generated in deterministic manner. Definition of geometry is assumed to be hierarchical, and series of models may be developed by their variation. It means that the accuracy of geometry description could be hierarchically improved increasing the number of specified points.
The coefficients are computed using a least square method which minimizes the error between the exact function values and those given by spherical harmonics expansion
\[
\min_a \left[ \sum_{i=1}^{N} \left( r_i - F(a, \theta, \varphi, \rho) \right)^2 \right],
\]
where \( F(a, \theta, \varphi, \rho) \) is defined by Eqn (1). This problem can be formulated as a linear equation system in matrix form. Furthermore, the pseudo-inverse matrix is used for solving the problem in Eqn (3).

4. Numerical results

For the testing of a spherical harmonics technique, two sample particles of a specified geometry have been generated numerically.

The suggested technique is based on numerical data processing technique. The method reads the input data – set of surface points – and generates the spherical harmonics with their coefficients.

Figure 1: Particle shape models generated by spherical harmonics: (a) quasi-ellipsoid; (b) quasi-cube

In Figure 1 models of ellipsoidal and cube shaped particle are given. The order of spherical harmonics expansion for both models is the same \( N=2 \), where total number of functions is 9:
\[
F(\theta, \varphi) = \alpha_0 + \alpha_1 \sin \theta \sin \varphi + \alpha_2 \cos \theta + \alpha_1 \sin \theta \cos \varphi + \alpha_4 \left[ \sin^2 \theta \sin 2\varphi + \alpha_4 \cos \theta \sin \varphi \right] + \alpha_2 \left[ \frac{3}{2} \cos^2 \theta - \frac{1}{2} + \alpha_4 \cos \theta \sin \varphi \right] + \alpha_6 \left[ \frac{3}{2} \sin^2 \theta \cos 2\varphi + \alpha_4 \cos \theta \sin \varphi \right].
\]

For any arbitrarily shaped particle spherical harmonics are the same, only expansion coefficients are recalculated. Below is given coefficients for ellipsoidal and cube shape particles:
\[
\alpha_{\text{ELLIP}} = [2.064 \ 0.7622 \ 0.0772 \ 0.0727 \ 0.0324]
\]
\[
\alpha_{\text{CUBE}} = [1.7024 \ 0.241 \ -0.0691 \ 0.241 \ -0.0991 \ -0.0633 \ -0.653 \ -0.0633 \ -0.203]
\]

The estimation of the models is made by calculating absolute difference between theoretical and experimental values of geometrical characteristics: surface area, volume and inertia of moment. As shown in Fig. 2, error estimation for cube shaped particle gives more accurate results when used more points.

Also accuracy of the model can be changed by increasing the number of spherical harmonics. The more spherical harmonics are used, the more initial points of model should be used. Additionally, it should be mentioned that using more spherical harmonics increases the calculation time.

Figure 2: Error estimation of spherical harmonics model

These spherical harmonics models are used in problem of contact calculation: this analytical expression allows for calculating radius of curvature at any point. The Hertz contact theory is used to determine the forces operating in the contact area. These results are compared to models simulated in finite element software SolidWorks.

5. Conclusion

In this paper spherical harmonics and shown the application of spherical harmonics expansion were introduced. The discussed technique was applied to particles of known geometry in order to have a possibility to estimate the errors. The results show that there is no need to use a great number of spherical harmonics willing to get acceptable results.

References


A unified theory of elastic-plastic-damage material with plastic strain induced phase transformation

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Abstract

In the work, a unified modeling of coupling between plasticity, damage and phase transformation is presented. The well-known formalism of thermodynamics of irreversible processes with internal state variables and the local state method are adopted. The model is derived under the assumption of small strains. A total energy equivalence hypothesis is used to define the effective state variables. The model is intended to use in the case of low stacking fault energy (LSFE) materials subjected to loading at a relatively low temperature where martensitic transformation induced by plasticity appears and has strong influence on the hardening process. A numerical algorithm is proposed and the model is implemented into Abaqus/Explicit, using subroutine VUMAT. Validation of the model is performed on the basis of experimental data available in literature.

Keywords: constitutive modeling, martensitic phase transformation, damage mechanics

1. Introduction

Engineering materials are often characterized by a multiphase microstructure where each phase exhibits different mechanical properties, and also the volume fraction of each phase in the representative volume element (RVE) of the material may be subject to change. The evolution of damage, responsible for the material degradation, may be governed by different mechanisms in each phase: in brittle phases brittle damage is observed, on which the stress state has crucial influence [1], while in soft, ductile phases the damage state is mainly determined by plastic flow. For this reason each phase has to be considered separately when damage evolution is analysed. On the other hand, phases interact with each other, which is a source of substantial difficulty in constitutive modelling. In the work a model based on the material that consists of two different phases is proposed.

The numerical algorithm of the derived constitutive model is also proposed and implemented in FEM program Abaqus via user subroutine VUMAT.

As the examples of engineering materials possible to be described with the use of the developed model, austenitic stainless steels of types: 304, 304L, 316, 316L etc., subjected to mechanical loading at low temperature can be considered. In the present work validation of the model is based on the experiments performed for 316L and 304 stainless steels at cryogenic temperatures [2].

2. Basic assumptions and equations of the model

In the work three dissipative phenomena are taken under consideration: plastic yielding, plastic strain induced austenite to martensite, \(γ \rightarrow α'\), phase transformation and damage evolution in both phases. Initially the whole volume of the material has austenitic structure, but in the course of the plastic deformation the martensitic inclusions appear. It is assumed here that \(γ \rightarrow α'\) transformation is the change of crystallographic configuration but without the diffusion mechanism. It is also assumed that martensitic platelets are randomly distributed and randomly oriented in the austenitic matrix. Thus, initially homogenous material becomes heterogeneous in the process of phase transformation. The volume fraction of the martensite, \(α'\), is reflected in the model by internal variable of scalar type, \(ξ\). Both phases undergo damage evolution; however, it is assumed that the secondary phase inclusions are of rock-like material type, while the matrix is elastic-plastic. Thus, ductile damage evolution is applied to the matrix material and brittle damage evolution is employed to the secondary inclusions. For the description of the current state of damage in the RVE two second order damage tensors are introduced: \(D^p\) - for ductile damage in the parent phase and \(D^s\) - for brittle damage in the secondary phase. A function of the volume fraction \(ξ\) of the secondary phase is introduced to define mixture rule which allows to obtain the average damage tensor \(D^{m}\) in the RVE. Hardening of the material in the course of plastic deformation is reflected by mixed isotropic/kinematic plastic hardening, represented by the two classical plastic state variables \(α^p\) (related to kinematic hardening) and \(ν^p\) (isotropic hardening variable). Both hardening laws are affected by the secondary phase content to include in the model the impact of phase transformation on hardening process. Damage hardening is also possible.

Another effect, included in the present model, is the interaction between the increasing amount of the secondary, martensitic phase and decreasing rate of damage (Fig. 1). Namely, at the first stage of the damage evolution, before the phase transformation threshold \(p_1\) is reached, purely ductile damage develops. In agreement with the experimental observations [2], this stage exhibits accelerated character of damage rate. With the appearance of phase transformation a significant drop of the damage rate is observed (Fig. 1) [2,3].

The total energy equivalence hypothesis is used to define the effective state variables. This approach allows for: (1) the definition of internal state variables as well as the effective thermodynamic conjugated forces, which can be indifferently used in stress space and strain space by the use of the Legendre-Fenchel transformation; (2) obtaining symmetric physical properties of material (symmetric stiffness tensor, compliance

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tensor, strain hardening modules) even in the case of anisotropy induced by dissipative phenomena; (3) modelling of coupling between damage variable and other internal state variables in a natural way [4].

Figure 1: Evolution of damage and martensite content versus plastic strain for 316L stainless steel subjected to uniaxial tension at 4.2K, after [2]

The Helmholtz free energy is used as a state potential and the state relations are deduced from the fundamental Clausius-Planck inequality. The kinetic equations of force-like variables are then obtained taking time derivatives of force functions. The flux variables are obtained based on the hypothesis of normal dissipation in the form of ordinary differential equations of the first order.

In addition to plastic yield surface $F^{\text{plastic}}$, damage surface $F^{\text{damage}}$ and phase transformation surface $F^{\text{transformation}}$ is proposed. All surfaces are subjected to hardening, and kinetic laws are derived by the use of the generalized normality rule with separate consistency multipliers: $\dot{\rho}^\alpha$, $\dot{\rho}^{\text{damage}}$ and $\dot{\rho}^{\text{transformation}}$. This approach allows to describe the kinetics of dissipative phenomena independently (weak coupling).

A numerical algorithm is proposed where the Newton-Raphson scheme is employed to solve the set of nonlinear differential equations, while backward Euler method was used to numerically integrate the equations. The numerical procedure was also built with the use of AceGen program. AceGen was used to generate highly optimized and efficient compiled code in FORTRAN language what allowed to build user subroutine VUMAT. In the next step VUMAT procedure was used in Abaqus/Explicit to simulate simple uniaxial tension test in order to validate the model. The simulation of expansion bellows, made of austenitic stainless steel, was also performed.

3. Validation of the model

The constitutive model of coupled phenomena was validated by means of loading/unloading test performed at the temperature of liquid helium (4.2 K) [2]. The experimental stress-strain curve for 316L stainless steel subjected to uniaxial tension is shown in Fig. 2. The evolution of micro-damage was measured by introducing unloading procedure and tracing variations of the unloading modulus. Parameters included in the evolution equations for thermodynamic forces (isotropic/kinematic hardening laws) and kinetic laws of evolution of internal variables (damage and phase evolution equations) were found with the use of optimization procedure of least squares method with the use of ISIGHT program.

Figure 2: Stress-strain curve with frequent unloading obtained at 4.2 K for 316L stainless steel

Figure 3: Experimental and numerical results of damage evolution

4. Conclusions

In the proposed model of elastic-plastic-damage two-phase material two different types of damage are considered: ductile damage in the primary phase and brittle damage in the secondary inclusions. The average damage state in the RVE is obtained through a superposition of the ductile and brittle parts. Total energy hypothesis is applied instead of commonly used strain energy hypothesis. The advantages of this approach are pointed out. Validation of the model was performed, on the basis of experimental data available in literature.

References

 Thermodynamics of a material model showing creep and damage

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Abstract

Thermodynamical evaluation is performed for a material model describing creep and microcracking assisted damage. The specific Gibbs free energy is written in a format where the effect of non-interacting microcracks in a 2D domain is described as a set of variables called microcrack densities. These variables make computation fast. Entering into the theory as internal variables, microcrack densities form a vital part of continuum thermodynamics. The effective deviatoric stress tensor is modelled to be the driving force for creep, since it describes the averaged shear stress in the matrix material. The material model satisfies the Clausius-Duhem inequality. Stress-strain curves for S2 sea ice are simulated.

Keywords: thermodynamics, microcracks, specific Gibbs free energy, effective stress, material modelling, Abaqus VUMAT, ice

1. Introduction

Many materials, such as ice, metals, composites and concrete show microcracking and creep. Since they are in common use, the response of materials showing these phenomena plays an important economic role. This paper improves the modelling capability by describing creep and microcracking assisted damage by applying continuum thermodynamics with internal variables.

2. Modelled deformation mechanisms and their mathematical representations

Deformations and rotations are assumed small. The (total) strain tensor $\varepsilon$ is assumed separable as follows:

$$\varepsilon = \varepsilon^e + \varepsilon^d + \varepsilon^\alpha,$$

(1)

where $\varepsilon^e$ is the elastic strain tensor for an undamaged material, $\varepsilon^d$ is the damage strain tensor and $\varepsilon^\alpha$ is the viscous strain tensor. If the material model is expressed by the specific Gibbs free energy $g$, the state equations will be

$$\varepsilon - \varepsilon^e = \rho_0 \frac{\partial g(\sigma, a, Q', T)}{\partial \sigma}, \quad \beta = \rho_0 \frac{\partial g(\sigma, a, Q', T)}{\partial a},$$

(2)

and

$$Y' = \rho_0 \frac{\partial g(\sigma, a, Q', T)}{\partial Q'},$$

(3)

In Eqs (2) and (3), $\rho_0$ is the density in the initial configuration, $\sigma$ is the stress tensor and $T$ is the absolute temperature. The internal variables $a$ and $Q'$ are defined later. Variables $\beta$ and $Y'$ are internal forces associated with internal variables $a$ and $Q'$ respectively. The local forms of the first and second law of thermodynamics give the following mechanical part of the Clausius-Duhem inequality:

$$\sigma \varepsilon^e + \beta \cdot a + \sum_{r=1}^{M} Y' \cdot Q' \geq 0.$$

(4)

The physical meaning of quantity $M$ is defined later.

3. Specific Gibbs free energy for a two-dimensional microcracked medium

The specific Gibbs free energy $g$ is assumed to be

$$g(\sigma, a, Q', T) = g^e(\sigma, Q') + g^d(a) + g^\alpha(T),$$

(5)

where $g^e(\sigma, Q')$ is the specific Gibbs free energy for a Hookean material with rectilinear non-interacting microcracks in a two-dimensional body, $g^d(a)$ is the specific Gibbs free energy for creep and $g^\alpha(T)$ is the thermal part of the specific Gibbs free energy.

The quantity $g^e(\sigma, Q')$ requires a more detailed study. It is

$$g^e(\sigma, Q') = g^h(\sigma) + \pi \frac{E}{h} \sum_{r=1}^{N} Q'(a') \times

\times \left\{ \eta' \cdot \sigma \cdot \eta' - \left[ H(\eta' \cdot \sigma \cdot \eta') \right] (\eta' \cdot \sigma \cdot \eta') \right\},$$

(6)

where $g^h(\sigma)$ is the specific Gibbs free energy for a Hookean material, $h$ is the thickness of the body and $M$ is the number of microcrack groups. In each group the sizes and orientations of the microcracks are equal. The quantity $a'$ is the length of the microcrack and the unit normal vector for the microcrack surface is denoted by $\eta'$. The microcrack densities $Q'$ are

$$Q' = m'/ (\rho_0 V'''),$$

(7)

where $m'$ is the number of microcracks within the $r$’th microcrack group, $\rho_0$ is the density and $V'''$ is the volume of the representative volume element.

Equation (6) is based on the work by Basista [2], but the author [3] makes three major enhancements. First, microcracks are collected into groups of the same size and orientation, making computation much faster without causing significant numerical inaccuracy. Second, the introduced microcrack densities $Q'$ enter into the formulation of continuum thermodynamics as internal variables. Microcrack densities $Q'$ have a strong physical foundation. This is a marked benefit over the variable damage $D$; the micro-mechanical meaning is only often motivated verbally. Finally, the Heaviside function $H(\eta' \cdot \sigma \cdot \eta')$ suppresses the microcrack surfaces to penetrate each other in compression.
The damage-elastic strain tensor \( \epsilon^{de} \) takes the form
\[
\epsilon^{de} = \epsilon^e + \epsilon^d \quad \text{(Eqn (1))} \Rightarrow \epsilon = \epsilon^e + \epsilon^d.
\] (8)
Thus, the second part of Eqn (6) gives the damage strain tensor \( \epsilon^d \). Substituting Eqn (6) into Eqn (2) and only writing \( d \epsilon \) gives
\[
\dot{\epsilon}^d = \frac{\pi h}{E} \sum_n Q' ((a')^2 \{ \bar{\eta}^i \bar{\sigma}^i \bar{\eta}^i + (\bar{\eta}^i \cdot \bar{\sigma}^i \bar{\eta}^i) \} - [1 - H((\bar{\eta}^i \cdot \bar{\sigma}^i \bar{\eta}^i))] (\bar{\eta}^i \cdot \bar{\sigma}^i \bar{\eta}^i))^2),
\] (9)
When \( S \) and \( S' \) are the compliance tensors for a Hookean material and for damage, the following is reached at:
\[
\epsilon^e = S \sigma, \epsilon^d = S' \sigma, \text{ and } \epsilon^{de} = S \sigma + S' \sigma, \text{ where } S = S + S'.
\] (10)
Similarly to the above derivation, Eqns (6) and (3) give
\[
Y' = \frac{\pi h}{E} ((a')^2 \{ \bar{\eta}^i \bar{\sigma}^i \bar{\eta}^i - [1 - H((\bar{\eta}^i \cdot \bar{\sigma}^i \bar{\eta}^i))] (\bar{\eta}^i \cdot \bar{\sigma}^i \bar{\eta}^i))^2),
\] (11)

4. Postulate of strain equivalence with the effective stress concept

The postulate of elastic strain equivalence by Chaboche [2] is extended here as follows: If the virgin (undamaged) material obeys the following constitutive equation:
\[
\sigma = f_0(\epsilon^{de}, \text{Virgin}),
\] (12)
then the effective stress tensor \( \bar{\sigma} \) is defined by
\[
\bar{\sigma} = f_0(\epsilon^{de}, \text{Damaged}).
\] (13)
According to the above definition, when the virgin material obeys Hooke’s law, the constitutive equation for the damage is
\[
\bar{\epsilon}_d = C \cdot \epsilon^{de},
\] (14)
where \( C \) is the constitutive tensor for a Hookean material. ‘For damaged material’ is struck out as Eqn (14) also holds when the damage is negligible, i.e. for a virgin material. Eqn (14) yields
\[
\epsilon^e = S \bar{\sigma},
\] (15)
Comparison of Eqns (10) and (15) gives
\[
S \bar{\sigma} = \dot{S} \dot{\sigma} \Rightarrow \bar{\sigma} = M \sigma, \text{ where } M = C \dot{S}.
\] (16)
The notation \( M \) in Eqn (16) stands for the damage effect tensor.

5. Rate equations for the material model

Evolution of the microcrack densities \( Q' \) takes the form
\[
\dot{Q}' = Q_{ecr} \left( (\bar{\eta}^i \cdot (\epsilon - \epsilon^e)) \cdot \bar{\eta}^i \right) \left( k^d Q_{ecr}(\bar{\epsilon}^d) \right) - \epsilon_{ecr}^d) Y'(\tilde{\epsilon}) \quad \text{(Eqn (17))}
\]
where \( Q_{ecr}, \ k^d \) and \( \epsilon_{ecr}^d \) are material parameters, \( \tilde{\epsilon} \) stands for the Macaulay brackets and \( J_{SM}(\bullet) \) is the von Mises operator.

The viscous strain \( \dot{\epsilon} \) modelled here can be e.g. dislocation creep or grain boundary sliding. The driving force for these two deformation mechanisms is the shear force or, when a 3D case is studied, the deviatoric stress tensor \( s \), viz.
\[
s = K \sigma, \text{ where } K = \frac{2}{3} I - S.
\] (18)
In Eqn (18); notations \( I \) and \( I \) stand for the fourth- and second-order identity tensors, respectively. Effective stress tensor \( \bar{\sigma} \) is assumed to describe the ‘average’ stress state between the microcracks and micro-voids. Thus, for a microcracked material the driving force is given by Eqn (16):

The creep rate \( \dot{\epsilon} \) is modelled as follows:
\[
\dot{\epsilon}^e = \epsilon_{ecr} \left( \left( J_{SM}(\tilde{\epsilon} - \bar{\tilde{\epsilon}}) - \epsilon_{ecr}^d \right)^n \right) \nabla \sigma - b \quad \text{(19)}
\]
where \( \epsilon_{ecr}, \sigma_{ecr}, \epsilon_{ecr}^d \) and \( n \) are material parameters and where \( \bar{\epsilon} = K M : \sigma, \quad \bar{b} = M : \bar{\sigma} \quad \text{and} \quad b = K : M : \sigma.
\] (20)
The rates \( \dot{\epsilon} \) and \( \dot{\bar{\epsilon}} \) read
\[
\dot{\bar{\epsilon}} = \dot{\epsilon}^e \quad \text{and} \quad \dot{\bar{\epsilon}} = m \dot{\epsilon},
\] (21)
where \( m \) is a material parameter.

6. Satisfaction of the Clausius-Duhem inequality

By substituting Eqs (11), (17), (19) and (21) into Eqn (4) the Clausius-Duhem inequality is shown to be satisfied.

7. Numerical simulations

The above creep-damage material model was applied to columnar-grained S2 sea ice and coded as an Abaqus VUMAT subroutine. The numerical values in Table 1 gave a good fit with the experimental data obtained, as shown in Fig 1.

Table 1: Values of material parameters for columnar-grained S2 ice at around -10°C.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_s )</td>
<td>843 kg/m³</td>
</tr>
<tr>
<td>( E )</td>
<td>9.5 GPa</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.33</td>
</tr>
<tr>
<td>( h )</td>
<td>5.10 x 10⁻⁵ m</td>
</tr>
<tr>
<td>( M )</td>
<td>15</td>
</tr>
<tr>
<td>( \dot{\alpha} )</td>
<td>3.5 x 10⁻³ m/ s</td>
</tr>
<tr>
<td>( \dot{\eta} )</td>
<td>6.00 GPa</td>
</tr>
<tr>
<td>( k^2 )</td>
<td>2.00 x 10⁻³</td>
</tr>
<tr>
<td>( \dot{\epsilon}_{ecr} )</td>
<td>2.02 x 10⁻⁴ /s</td>
</tr>
<tr>
<td>( \sigma_{ecr} )</td>
<td>0.442 Pa</td>
</tr>
<tr>
<td>( \dot{\sigma}_{ecr} )</td>
<td>2.00 MPa</td>
</tr>
<tr>
<td>( n )</td>
<td>1.17</td>
</tr>
<tr>
<td>( \dot{Q}_{ecr} )</td>
<td>1.50 x 10⁻⁵ kg/m³</td>
</tr>
<tr>
<td>( \epsilon_{ecr} )</td>
<td>1.40 x 10⁻⁴</td>
</tr>
</tbody>
</table>

Figure 1: Stress-strain curve in compression and in tension at \( \dot{\epsilon} = 2 \times 10⁻¹ / s \).

References


Thermal and dissipative effect accompanying discontinuous plastic flow

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Abstract

The paper presents an experimental method for identifying the slip band evolution parameters in FCC materials (such as: austenitic steel, Cu-OFE or cooper-alumini um alloy) during a discontinuous plastic flow (DPF). Experiments were conducted under kinematically controlled uniaxial tension on 304 steel specimens at liquid helium (LHe) temperature (4,2K). Sudden dissipative effects due to slip bands propagation can be observed. The experimental results indicated that the slip band propagate in a regular way along the specimen when no hardening effect occurs in 304 steel. Based on the solution of heat equation by a Fourier transform, convolution theorem and experimentally identified test parameters, it is possible to obtain mathematical description of time response of the thermometer mounted on the specimen (spatially oriented) during uniaxial tensile test of austenitic steel at LHe temperature. The numerical results can be important in the development of thermo-mechanical model of DPF [1].

Keywords: discontinuous plastic flow, tensile test at cryogenic temperatures, slip band evolution, heat equation

1. Introduction

A number of materials exhibit plastic instability under monotonic tension, manifesting as continuous serrated yielding on the stress -strain curves and repeated static or dynamic propagation of strain localization in specimen within a certain range of temperature and strain rate. The most commonly known phenomena are referred to as: Portevin –Le Chatelier (PLC), Lüders bands propagation and discontinuous plastic flow (DPF) - typical for cryogenic temperatures.

Different techniques have been developed to observe the nature of plastic instability under tension. The DPF has been investigated experimentally by many authors [2], however, no-one analysed the strain localization effect. The experimental methods to observe the propagation of slip bands are largely limited due to the extreme conditions. It is worth pointing out that for such materials as austenitic stainless steel the serrated yield occurs below a specific temperature $T_1 < 30 K$. Therefore, experimental evidence of localized deformation remains a challenge.

Cryogenic tensile tests were performed at the LHe temperature (4.2 K). In view of that, the tests at so extremely low temperatures imply specific conditions and constraints, it is worth presenting them in detail in Fig. 1. The heart of the system remains the cryostat equipped with an insert containing sample holder and carrying instrumentation components like the clip-on extensometers, the thermometers and the piezoelectric force sensor.

2. Cryogenic tensile test results

The kinematically controlled uniaxial tensile test was carried out on the 304 steel plate specimen immersed in liquid helium (4,2 K). The stress-strain and temperature-strain diagram is presented in Fig. 2.

Figure 1: Experimental set-up for tensile testing at extremely low temperatures

Figure 2: Tensile test results for 304 steel at LHe temperature
2.1. Experimental identification of slip band evolution parameters

The stress-strain and temperature-strain curves indicate the nature of slip band propagation during the cryogenic tensile test in the 304 steel specimen. Analysing the tests results, it is worth paying attention to the temperature distribution for a range with no hardening effect. The shape of the temperature-strain curve may suggest the nature of the slip band evolution during the tensile test at LHe. The time response of the thermometer in this range has a regular form (a comb shape), such effect can be observed if the slip band during plastic strain propagates from one end of the specimen to the other. On the other hand, the temperature distribution in the hardening range is random. It means that the slip bands may occur randomly and new phase of martensite can influence strain localization effects. This thesis can be confirmed by means of analysis of the temperature sensor and extensometers time responses recorded during the test. Based on this analysis it is possible to determine temporal and spatial characteristics of slip bands.

Figure 3: Time responses of temperature and strain for kinematically controlled test according to the two ranges presented in Fig. 2

Looking at the inserted bars in Fig. 3, it can be found that the plastic deformation was first initiated in a local area near the bottom part of the gage section and it spread over the whole gage section from the initiation site.

3. Solution of heat equation via Fourier transform and convolution theorem

The DPF is related with a periodic dissipation of energy during each serration [1]. It means that for the kinematically controlled tensile test and spatially oriented thermometer, the traveling slip band generates in each serration some amount of heat which is recorded by thermometer as temperature peaks (Fig. 2). Mathematical description of such a phenomenon can be obtained by means of the solution of heat equation via Fourier transforms and convolution theorem- Eqn (1).

\[ u(x,t) = \frac{Q}{c_v \cdot \rho \cdot \sqrt{4\pi k t}} \cdot e^{-\frac{(x-\alpha t)^2}{4kt}} \]  

(1)

The results of numerical simulation of a time response of thermometer during kinematically controlled uniaxial tensile test of 304 steel specimen at LHe temperature are presented in Fig. 4. The basic parameters of test are presented in Table 1.

Table 1: The test parameters and the 304 steel properties [3]

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of test</td>
<td>kinematically controlled</td>
</tr>
<tr>
<td>Cross-head velocity</td>
<td>( V_c = 0.5 \text{ mm/min} )</td>
</tr>
<tr>
<td>Slip band velocity</td>
<td>( V_{sb} = 48.4 \text{ mm/min} )</td>
</tr>
<tr>
<td>Serration frequency</td>
<td>( f_s = 1 \text{ Hz} )</td>
</tr>
<tr>
<td>thermal conductivity</td>
<td>( k = 0.23 \text{ W/m K} )</td>
</tr>
<tr>
<td>specific heat</td>
<td>( C_v = 2 \text{ J/kg K} )</td>
</tr>
<tr>
<td>mass density</td>
<td>( \rho = 7998 \text{ kg/m}^3 )</td>
</tr>
<tr>
<td>thermal diffusivity</td>
<td>( \alpha = 0.0038 \text{ m}^2/\text{s} )</td>
</tr>
<tr>
<td>intensity of internal heat source</td>
<td>( Q = 900 \text{ W/m}^3 )</td>
</tr>
</tbody>
</table>

The thermo-mechanical model of DPF [1] assumes that RVE travels together with the slip bands, it means that thermometer is glued to the slip band. The presented analysis allows for obtaining the numerical results for spatial oriented thermometer when slip band propagates along the specimen during uniaxial tensile test in liquid helium (Fig. 4).

Figure 4: The numerical results of the thermometer time response during DPF for the 304 steel specimen tested at LHe temperature

References


Kinetics of evolution of radiation induced damage

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Abstract

The problem investigated in the work concerns the physical processes involved in radiation damage and the way they affect the mechanical properties of ductile materials. Multiscale modeling of evolution of radiation induced micro-damage in ductile materials subjected to periodic stress states in the inelastic range is presented. The resulting micro-structural and damage evolution causes profound changes of the macroscopic properties and severely degrades the lifetime of the components subjected to irradiation. The evolution of radiation induced damage is combined with the evolution of classical micro-damage of mechanical origin (micro-cracks and micro-voids), within the common framework of Continuum Damage Mechanics (CDM). Two kinetic laws: the Rice and Tracey model and the Gurson model may be conveniently applied to describe the evolution of radiation induced damage in the form of clusters of voids embedded in the metallic matrix. Closed form analytical solutions for the problem of periodic irradiation combined with cyclic axial loads was obtained.

Keywords: radiation induced damage, Frenkel pairs, vacancies, interstitial atoms, displacement per atom, evolution of clusters of voids, Continuum Damage Mechanics (CDM) framework

1. Introduction

Exposure to high energy radiation degrades the microstructure of materials. Energetic particles penetrating a solid displace the lattice atoms from their original positions. In the elastic collisions (initiated when a given atom is struck by a high energy particle), incident particles transfer recoil energy to the lattice atoms. The initial primary knock-on atoms recoil with a given amount of kinetic energy that is dissipated in a sequence of collisions with the other lattice atoms. As a result of cascade process, atoms in the solid can be displaced from their equilibrium lattice positions, creating vacancies and interstitials. These vacancy-interstitial pairs are called Frenkel pairs. The vacancies of the Frenkel pairs often form clusters by means of diffusion.

As an example of a typical problem related to evolution of radiation induced damage, degradation of material properties in a thin-walled irradiated cylindrical part of detector of particles has been investigated. The coaxial target - detector configuration is shown in Fig. 1. The target is hit by high-energy particles beam. The process of beam absorption is associated with emission of secondary particles flux in the radial direction. The secondary particles induce micro-damage in the thin-walled cylinder surrounding the target. As the cylinder is simultaneously subjected to mechanical loads, the fields of radiation and mechanically induced damage occur in the same lattice. This problem is used as an illustration of different nature of both damage types. Thus, the ultimate goal is lifetime prediction expressed in terms of the number of beam cycles for components of particle detectors [1,2].

Two kinetic laws of damage evolution were taken into account: the Rice and Tracey model and the Gurson model. Both of them address the evolution of porosity in the form of spherical or ellipsoidal voids in a different way.

The Rice & Tracey model predicts the growth of an initially spherical void in an infinite, rigid - perfectly plastic material subjected to a uniform remote strain field.

\[
d r_c = r_c \alpha_c \exp \left( \frac{3 \sigma_m}{2 \sigma_y} \right) dp . \tag{1}
\]

The radius increment \(dr_c\) is derived as a function of the equivalent plastic strain \(dp\), the stress triaxiality factor \((3 \sigma_m/2 \sigma_y)\), the current radius \(r_c\) and a scalar multiplier \(\alpha_c\).

On the other hand, the Gurson model is based on the definition of the porosity parameter \(\xi\):

\[
\dot{\xi} = (1 - \xi) \dot{\varepsilon}_{\text{pl}}^{\text{eq}}, \tag{2}
\]

where \(\dot{\varepsilon}_{\text{pl}}^{\text{eq}}\) denotes trace of the plastic strain rate tensor.

The mechanically induced damage is calculated following the Lemaitre-Chaboche model.

Both Rice & Tracey and Gurson kinetics may conveniently be applied to describe the evolution of radiation induced...
damage in the form of clusters of voids embedded in the metallic matrix.

2. Analytical solutions for the problem of periodic irradiation combined with cyclic axial loads

Based on the known dpa in the RVE, the density of defects $q_A$ (clusters of voids) caused by irradiation is computed [3]:

\[
q_A = \begin{cases} 
C_{q_A}(dpa)^n & \text{for } dpa < D_S \\
C_{q_A}(dpa)^n + \text{irr} & \text{for } dpa \geq D_S 
\end{cases}
\]  

(3)

The mechanism of damage evolution for multiple irradiation cycles is shown in Fig. 2.

Figure 2: The mechanism of damage evolution

Single cycle is composed of emission of the particles flux, production of radiation induced damage expressed by the porosity parameter $\xi_0$ and further mechanical loading. The porosity parameter $\xi_i$ increases from cycle to cycle by $\xi_0$. The porosity parameter $\xi_i$, used by Gurson, can be directly recalculated to obtain the classical damage parameter $D_i$.

To find a closed-form analytical solution for one dimensional problem presented in Fig. 2 the recurrence relations are postulated. Integrating the post-irradiation damage evolution equation:

\[
\frac{dD}{d\varphi} = q_A 2\pi \int r \ dr, 
\]  

one obtains the following recurrence relation for damage parameter:

\[
\Delta D_{\text{irr}} = q_A 2\pi (r_{i+1}^2 - r_i^2). 
\]  

(5)

Integrating the Rice & Tracey law, the increase of cluster of voids radius reads:

\[
r_z = r e^{2\tilde{\varphi}}. 
\]  

(6)

Finding the $N$-th term in the sequence indicates that all terms create a geometric series:

\[
D_{\text{irrN}} = ND_{\text{irr}} - Nq_A 2\pi r_0 + q_A 2\pi r_0 e^{2\tilde{\varphi}} + q_A 2\pi r_0 e^{4\tilde{\varphi}} + q_A 2\pi r_0 e^{6\tilde{\varphi}} + \ldots + q_A 2\pi r_0 e^{2N\tilde{\varphi}}. 
\]  

(7)

The following criterion has been used:

\[
D_{\text{irrN}} = q_A 2\pi r_0 e^{2\tilde{\varphi}} \left(1 - e^{2N\tilde{\varphi}}\right) / \left(1 - e^{2\tilde{\varphi}}\right) = D_{\text{cr}}, 
\]  

(8)

where $D_{\text{cr}}$ denotes a critical value of damage parameter, corresponding to lattice failure.

The relation between the number of cycles to failure $N_f$ as a function of dpa on single cycle is presented in Fig. 3.

Figure 3: Number of cycles to failure $N_f$ as a function of dpa

The solid line represents the performance of Rice & Tracey model. For comparison the dashed line represents the number of cycles to failure as a function of dpa corresponding to irradiation of the material not subjected to mechanical stress.

3. Concluding remarks

Finally, the results contained in the present paper can be summarized in the following way:

1) formulation of the problem of radiation induced damage evolution under mechanical loads, including the effect of partial recombination of Frenkel pairs,
2) application of Rice & Tracey and Gurson kinetic laws to the evolution of post-irradiation damage,
3) additive formulation for total damage including both radiation induced and mechanical components,
4) closed form analytical solutions for the problem of periodic irradiation combined with cyclic axial load, and corresponding to Rice & Tracey and Gurson models.

References


Modelling of periodic structures by spectral finite elements

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Abstract

Due to unique dynamic properties periodic structures are widely investigated by researchers. The existence of band gaps in their frequency spectrum implies many potential applications of periodic structures as passive attenuators in engineering. Despite relatively simple manufacturing of such structures, the experimental investigation of their dynamic behaviour as a function of all possible parameters influencing their periodicity is highly inefficient. Therefore various computational methods are used instead. One of such methods is the Time-Domain Spectral Finite Element Method (TD-SFEM). The effectiveness of this method was successfully demonstrated by the authors on selected engineering structures.

Keywords: periodic structures, TD-SFEM, dynamic behaviour, wave propagation, frequency band gaps

1. Introduction

Structures composed of many identical elements in a repeating sequence, called periodic structures, are widely used as engineering structures such as vibration insulators or acoustic filters [1]. The main reason for their use is the inherent ability to attenuate mechanical waves for specific frequencies, also known in the literature as frequency band gaps [2]. Their characteristics are strongly dependent on the parameters influencing structural periodicity [3].

2. Bloch’s theorem and TD-SFEM

Bloch’s theorem, well known from the investigation of crystalline structures may be applied to explain the behaviour of mechanical waves in infinite periodic structures. It can be successfully applied to study one-, two-, and three-dimensional periodic structures. However its main limitation comes from periodic boundary conditions and geometrical limitations to simple structures. In other cases the application of Bloch’s theorem may lead to very complex mathematical descriptions.

In that context the application of the TD-SFEM appears as free of the limitations mentioned and also applicable to periodic structures of finite dimensions – so-called real periodic structures. The TD-SFEM allows one to investigate periodic structures of any boundary conditions (including periodic boundary conditions), any material properties (isotropic and anisotropic, linear and non-linear) as well as complex geometries, all in a wide range of frequencies. It may be added here that the TD-SFEM is much better suited to calculate such high frequency structural dynamical responses than the classical FEM approach [4].

The results presented in Fig. 1 show clearly the agreement between Bloch’s theory and the TD-SFEM based calculations obtained for a one-dimensional periodic medium in a very wide range of natural frequencies. As it can be seen in a wide range of the frequency spectrum the error between the predictions of the TD-SFEM and Bloch’s theory can be practically neglected.

3. Modelling of periodic structures

Numerical models composed of finite elements that are repeating in a certain sequence represent periodic structures. The source of this periodicity lies in the discontinuity of strain/stress fields between the finite elements. As a result of that frequency band gaps are present in their frequency spectra, as presented in Fig. 2, as in the case of typical periodic structures.

The width of the frequency band gaps is closely correlated with the order of approximation polynomials used and mostly manifests in the very upper part of the frequency spectra. Moreover, the presence of the frequency band gaps may falsify structural dynamic responses or even prevent their calculations numerically. Therefore it is necessary to determine precisely the value of the maximal frequency, below which the results of numerical calculations remain reliable. In a general case the value of the maximal frequency stays in the middle of the frequency spectrum calculated [5] – see Fig. 1 for example.

Figure 1: Frequency spectrum of a one-dimensional periodic medium ($c_1 = 5 \text{ km/s}$, $c_2 = 1.25 \text{ km/s}$, $a = 0.04 \text{ m}$, $b = 0.01 \text{ m}$)

Acknowledgments. The Authors of the work would like to gratefully acknowledge the support for their research provided by National Science Centre through the project UMO-2012/07/B/ST8/03741 Wave propagation in periodic structures. All results presented in this paper have been obtained by use of software available at Academic Computer Centre in Gdansk in the frame of a computational project.
In order to model correctly dynamic responses of periodic structures by the TD-SFEM only part of the frequency spectra below this maximum frequency should be used. This guarantees that the results of numerical calculations remain correct, as shown in Fig. 1.

It is worth mentioning that the periodicity may be introduced into a model of a structure in a variety of variants. One possibility is a change in the wave propagation velocities (elastic modulus or material density) in subsequent finite elements. Other possibilities are changes in the geometry (drill-holes or cross-section) or combining all of them.

In this way such numerical models allow to study the dynamic behaviour of periodic structures in an effectively, as a function of selected parameters describing the structural periodicity (periodicity, diameter of drill-holes, material properties, boundary conditions, etc.), as presented in Fig. 3.

Due to this the design process of real engineering periodic structures can be optimised to achieve desirable dynamic characteristics as vibration insulators or acoustic filters.

4. Conclusions

The results of numerical simulations obtained by the authors allow them to formulate the following general conclusions:
1. Real periodic structures can be effectively modelled numerically and studied by the use of the TD-SFEM.
2. Due to strain/stress field discontinuity numerical models may exhibit certain periodic features that can mask the periodic dynamic behaviour of the structures under investigation.
3. The TD-SFEM is superior to the classical FEM approach in terms of modelling structural dynamic behaviour due to higher orders of approximation polynomials used and resulting numerical errors.
4. The efficient modelling of dynamic behaviour of real periodic structures by the TD-SFEM must be restricted to the lower half of the frequency spectrum.
5. The TD-SFEM allows to study the dynamic behaviour of periodic structures as a function of various material and geometrical parameters.

References


MS18

Porous Materials – Theory, Numerical Simulations and Experiments

organized by M. Cieszko, J. Kubik and M. Kaczmarek
Sensitivity analysis of behaviour of sandwich plate with PU foam core with respect to boundary conditions and material model

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Abstract

Sandwich panels consisting of thin metal sheets and a closed-cell structure polyurethane foam core are considered in the paper. A problem of the influence of two main factors on the response of a modelled plate is analysed, mainly: material model of the PU foam and support conditions of plate. Attention is focused on applications and limitations of the most commonly isotropic model used for PU foam, since the foam is definitely an orthotropic material. The orthotropic model of the PU foam is proposed and recommended, alternatively. The results obtained from FE analyses are compared with the experimental results carried out by the authors, conclusions are drawn therefore.

Keywords: sandwich panel, orthotropic model, numerical analysis, numerical investigation

1. Introduction

Sandwich panels made up of two external thin and stiff metal facings separated by a thick, lightweight core are considered in the paper. It is possible to find many literature papers focused on sandwich structures, their applications, experimental tests or different approaches for modelling. A vast majority of them assume that the foam core is an isotropic, linear-elastic and homogenous material [3, 4]. Thus the Kirchhoff modulus of the core material plays a crucial role for structural performance of a sandwich panel. Unfortunately, foams exhibit highly anisotropic properties depending on the direction of measurement [1]. A short elastic range and may exhibit brittle, perfect plastic or hardened responses for the same material, depending on the load direction complicate the determination of mechanical response of foams. Therefore, more advanced analyses are required, with appropriate constitutive modelling of the material to carry out the analysis far beyond the elastic regime. In that case, the set of parameters required by material model reflects the need for numerous experimental tests. Thus, simplifying the foam to an elastic and isotropic model is very attractive and useful for designers. Nevertheless, the degree of anisotropy of PU foam should always be controlled and taken into account if required.

2. Problem formulation

According to code [5] a PU foam used in sandwich panels is an isotropic material, the only parameters to be determined (E₃, G₃₁) are these in the rise direction of foam. A series of tests (tension, compression, bending) were carried out in order to accurately identify the behaviour of foam [2]. The obtained results revealed pronounced anisotropy of an analysed material, shown in Table 1.

<table>
<thead>
<tr>
<th>E₁</th>
<th>E₂</th>
<th>E₃</th>
<th>G₁₂</th>
<th>G₃₁</th>
<th>G₂₃</th>
<th>v₁₂</th>
<th>v₃₁</th>
<th>v₂₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.45</td>
<td>4.41</td>
<td>3.65</td>
<td>2.51</td>
<td>3.00</td>
<td>2.30</td>
<td>0.55</td>
<td>0.27</td>
<td>0.20</td>
</tr>
</tbody>
</table>

The main goal of the work is to show that, despite of large anisotropy of the foam, the application of the simplified isotropic model can be sufficiently accurate. However, in some cases the orthotropic model is more appropriate.

3. Numerical analysis

Numerical models were created in the Abaqus system environment [6]. The first numerical example refers to a structure investigated in real scale during laboratory experiments. In this case the authors analysed elastic behaviour of a sandwich panel loaded by two line forces (four-point bending test). The geometric parameters of the plate are: width B = 0.1 m, total length L = 1.0 m, length of the span L₀ = 0.9 m, thickness of the core dₐ = 94.36 mm, and the thickness of steel facings t = 0.35 mm. It was assumed that the facings are flat and made of steel with Young’s modulus Eₕ = 210 GPa and Poisson’s ratio νₕ = 0.3. The laboratory test were done. The deflection w of the midpoint of the span as a function of the load F is presented in Fig. 1 (No.0). Another lines in the figure represent the numerical solutions and are drawn for 1) isotropic model with real Young’s modulus Eₚ, 1a) isotropic model with real Kirchhoff modulus Gₚ, 1b) orthotropic model.

Figure 1: Four-point bending of the beam B=0.1m

In numerical models, facings were modelled by a shell element, S4 (2D element). The core was modelled using eight node linear brick elements C3D8 (3D element). The panel is supported by two base plates modelled by rigid bodies. For both supporting base plates, only the rotation about the axis perpendicular to the beam axis is free, in order to obtain simply supported boundary conditions. The connection between the
plate and the support is defined as a contact ‘surface to surface’ with a hard normal contact and 0.3 friction coefficient.

The obtained results show that the isotropic model with a real value of Young’s modulus $E_3$ and Poisson’s ratio $\nu_{31}$ (table 1) is an incorrect approach (plot no.1 in Fig.1). If we assume that $G_{31}$ and $\nu_{31}$ (table 1) are the most important parameters, then isotropic model (iso-fraction no.1a) and orthotropic model (no.1b) give similar response comparable with the real behaviour.

The second example differs only in the width of the plate, and $B = 1.0$ m. This example shows that if the panel has a dimensional proportion $L/B = 1$ but the other load and support conditions provide that it works as a beam element then a simplified isotropic model (with $G_{31}$ as a most important parameter) can be used with reasonable accuracy. The relations between load and deflection of the plate are presented in Fig.2.

Figure 2: Four-point bending of the plate $B = 1$m

The next study is focused on the sensitivity of the sandwich plate for numerical approach used in definition of support conditions. Therefore, the “tie” interaction was used between the plate and the support then compared to “contact” interaction already defined. However, no changes were observed in the behaviour of the analysed models ($B = 0.1$ m and $B = 1.0$ m).

The next 2 examples (examples 3 and 4) concern the plate ($B = 1.0$ m, $L = 3.0$ m) supported on three edges. Load is applied on the free edge at mid-span through a rigid 100 mm x 100 mm block. In the example 3 the connection between the plate and the support was defined as a “contact” similarly to the example 1, while in the example 4 plate is tied to the support. The displacements field in example 4 is presented in Fig.3

Figure 3: Distribution of the vertical displacement in plate supported on three edges – Examples 4

The comparison of obtained results presented in Fig. 4, shows that the isotropic and orthotropic models give different results for a plate supported at three edges.

Figure 4: Plate supported on three edges – Examples 3 and 4

Comparative analysis was performed for load $F = 4.0$ kN. In example 3 the difference between “iso” and “ortho” approach is 8.6 %. But in example 4 (“tie” interaction) the scatter of results is higher than 13 %.

4. Conclusions

Various issues constitute the behaviour of modelled sandwich panels. In the article, the main attention was focused on the study of structural sensitivity with respect to the material model, geometric parameters and the type of boundary conditions (load excitations, support).

The range of applicability of classical isotropic PU material model was discussed, based on numerical examples. It was proven, that in some special cases, where sandwich panel starts to act as plate, in contrast to beam behaviour, introduction of a more complex finite element model may be particularly useful. Due to reliable FE model, the evaluation effort of the structural response of a panel subjected to the difficult to realize experimental actions, which are difficult to obtain can be significantly minimized. It mostly refers to the number of laboratory tests, subsequently to the cost of testing procedures.

References

Application of Micro Computed Thomography and Mercury Porosimetry to determination of internal structure of aerated concrete

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Abstract

Two complementary methods: the micro-computed tomography (µCT) and the mercury intrusion porosimetry (MIP) are used in the paper to determine the internal pore space structure of autoclaved aerated concretes (AAC) that belong to materials with double porosity. It concerns the parameters of super-micron pore and super-micron pore porosities and their pore size distributions. Due to limited resolution of the µCT scans they have been applied to investigate the super-micron pore space structure and due to bubble ink effect the MIP data was used to investigate the super-micron pore space structure. A new method is used in the paper for determination of AAC sample super-micron pore porosity based on model of the histogram of 3D µCT scans and optimization procedure. This allowed precise definition of the threshold value of relative density preserving porosity during reconstruction of the binary image of the super-micron pore space. Next, the method of inscription of the largest spheres into the super-micron pore space is used to calculate the pore size distribution in this space. To determine the pore size distribution in the nonpore space contained in porous skeleton of AAC samples the MIP data has been applied. It is possible because two parts of the mercury intrusion curve related to intrusion into super-micron pore and sub-micron pore spaces are distinctly separated. The analysis of this curve was performed basing on the capillary and chain models of the pore space and optimization method. This allows determining the limit pore size distributions in the skeleton pore space.

Keywords: porosity, pore size distribution, autoclaved aerated concrete, micro-computed tomography, mercury porosimetry

1. Introduction

Identification of microscopic pore geometry and macroscopic parameters of the pore space structure of autoclaved aerated concretes (AAC) is a very important issue in the study of their physical properties. The internal pore structure defines mechanical properties of AAC and plays important role in many physical and chemical processes occurring in such materials, e.g. in moisture and heat transport, wave propagation and chemical reactions.

The AAC belong to the group of porous materials with two porosities. It means that their pore space is formed by pores of two classes of sizes: super-micron pores and sub-micron pores. The characteristic sizes of super-micron pores ranges from several microns to millimeter and sizes of sub-micron pores are from several nanometers to micron. The volumes of both types of pores occupied in AAC are comparable. This makes investigations of such materials very difficult. The volume fraction of a given pore type in the pore space of AAC samples has a strong influence on their properties. A large fraction of sub-micron pores decreases the mechanical strength of a sample, whereas large fraction of micro pores has a positive influence on the insulating properties of the material.

The aim of the paper is to present two complementary methods of investigation of AAC pore space structure based on micro-computed tomography (µCT) and mercury porosimetry (MP). Both methods have been used to determine super-micron pore and sub-micron pore porosities of ACC samples and their pore size distributions. Due to resolution of µCT limited to one micrometer this method can be used only for investigation of super-micron pore space. The investigation of this space by MP method is doubtful because of its bubble structure that causes bottle ink effect. The MP method is useful however for investigation of sub-micron pore space. It can be performed measuring disintegrated AAC samples. To determine the mass density and total porosity of AAC samples the hell picnometer has been applied. Results obtained by these two methods are presented in the form of graphs.

2. Investigation of the super-micron pore space structure

The µCT is a modern, non-destructive method used in the identification of the spatial structure of heterogeneous materials and small physical objects. In this method, as in the computed tomography applied in the medical diagnostics, X-rays are used reaching the image resolution of one micrometer. Micromotographic scans of samples of porous materials are the basis for spatial reconstruction of the microscopic pore space geometry or the skeleton architecture.

A new method is used in the paper for determination of AAC sample super-micron pore porosity based on the histograms of 3D µCT scans. In this method, the normed histogram is considered to be density τ(ρ) of probability distribution of voxel mass density ρ in the sample scan. It is a linear combination of two distributions ψρ(ρ) and ψs(ρ) characterizing frequency of voxel occurrence of the pore and the skeleton type, respectively, with various relative mass densities

$$\tau(\rho) = f_s \psi_s(\rho) + (1 - f_s) \psi_r(\rho)$$  (1)

Both functions are defined on the whole range of voxels density values (i.e. for eight bit scan ρ ∈ [0, 255]). This means that attachment of voxel with a given density to the superset of the pore or the skeleton type has a stochastic character, determined by the value of probability.

The volume porosity fs, in this model, defines the probability of pore voxel occurrence in the whole set of voxels in the sample scan. In the case of the ACC sample scan it represent volume fraction of the super-micron pore space (super-micron
pore porosity). This parameter and parameters of both probability distributions are determined by optimisation method. Such approach makes it possible determination of volume porosity directly from the scan of bone sample without necessity of earlier reconstruction of its microscopic pore geometry. Then, the sub-micron pore porosity $f_{vs}^n$ can be calculated from relation

$$f_{vs}^n = f_v - f_{s}^n$$

(2)

where $f_v$ stand for the total porosity of AAC sample measured by the pycnometry method.

The determined super-micron pore porosity allows precise definition of the threshold value of relative density preserving porosity during reconstruction of the sample binary image. The binary image of the AAC super-micron pore space is the basis for determine their pore size distribution. The method of inscription of the largest spheres into the super-micron pore space is used to calculate the pore size distribution in this space. In this method the reconstructed image is transformed into a binary matrix and matrix operations are used for optimum computing time. At each step of this method a pattern of sphere is created and compared in sequence with each part of sample of the same size. As a result, the diameter of the largest sphere which contains this voxel and is completely included in the super-micron pore space is ascribed to each voxel of this space.

3. Investigation of the sub-micron pore space structure

Mercury porosimetry is the standard method for determining the pore size distribution of porous materials. This method enables identification of pores in the range of sizes from 0.3 mm to 3nm. This method was applied to the analysis of the sub-micron pore space structure contained in the porous skeleton only. It is possible because two parts of the mercury intrusion curve related to intrusion into super-micron pore and sub-micron pore spaces are distinctly separated. The analysis of this curve was performed basing on the capillary and chain models of the pore space and the optimisation method. This allows determining the limit pore size distributions in the skeleton pore space.

To determine directly the porosity $f_{vs}^n$ of the sub-micron pore space, the following relation between porosities in double porous materials was applied

$$f_{vs}^n = f_v^s \frac{1-f_s}{1-f_v^-}$$

(3)

where $f_v^s$ denotes the porosity of the skeleton in AAC sample. The value of this porosity is calculated directly from the mercury intrusion curve. Then, the relation (2) can be used again to calculate the porosity $f_{vs}^n$ of the super-micron pore space.

4. Results

Both complementary methods have been applied to investigation of the pore space structure of four classes of AAC samples produced by SOLBET Capital Group. Part of the obtained results are presented in table and illustrated graphically in figures.

Figure 1: Dependence of total, super-micron pore and sub-micron pore porosities on density of AAC samples determined by the pycnometry and MIP method.

Figure 2: Dependence of total, super-micron pore and sub-micron pore porosities on density of AAC samples determined by the pycnometry and µCT method.

References


Stationary flow of non-wetting liquid through layer of unsaturated porous material

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Abstract

Description of a stationary flow of non-wetting liquid (mercury) through a layer of unsaturated porous material is presented. Theoretical considerations are based on the new macroscopic description of the capillary transport of liquid and gas in porous materials formulated within the framework of multiphase continuum mechanics by the first author of this paper. The flow of non-wetting liquid through an unsaturated porous layer is a special case of this model. In spite of one-dimensionality of this problem it is described by three nonlinear coupled equations for spatial distribution of liquid saturation and pressure and for velocity of liquid flow through porous layer. In this case, the fluid flow through the layer of unsaturated porous material is similar to flow through an elastic tube the cross-section of which is being changed depending on the local pressure in fluid. It is shown that the relative permeability coefficient of the layer is harmonic average of liquid saturation in the layer that is a function of the pressures at both surfaces of the layer. Numerical solution of these equations is presented in the paper for various boundary conditions, and dependence of these three fields on parameters of the system are analysed.

Keywords: unsaturated porous materials, macroscopic description, flow through porous layer, non-wetting liquid

1. Introduction

The paper demonstrates results of a new macroscopic description of the capillary transport of liquid and gas in porous materials [1], applied to the relatively simple one dimensional problem of flow of non-wetting liquid through a layer of unsaturated porous material (Fig. 1).

Figure 1: Schematic illustration of non-wetting liquid flow through a layer of unsaturated porous material

The key element of the proposed physical model of the unsaturated medium is division of the liquid in the pore space into two constituents: the mobile and the capillary liquid, which exchange mass and energy in the vicinity of menisci surfaces during their motion. The mobile liquid retains the physical properties of the liquid outside the pore space, while the capillary liquid forms a film on the contact surface with the skeleton. The capillary liquid is immobile, contains the whole capillary energy of the system, and - due to possible negative pressure within it - has solid-like properties. Both liquids and gas are characterized by the mass densities and their distributions are defined by parameters of saturation. For all constituents of the porous medium, balance equations of mass, linear momentum and energy are formulated and constitutive relations for mechanical processes are derived based on the analysis of strain energy inequality and the Lagrange multipliers method. The velocity field of the menisci motion in the pore space induced by changes of the capillary pressure is defined by an additional constitutive relation, similar to Fick’s first law of diffusion.

The flow of non-wetting liquid through an unsaturated porous layer is a special case of this model. In spite of one-dimensionality of this problem it is described by three nonlinear coupled equations for spatial distribution of liquid saturation and pressure and for velocity of liquid flow through porous layer. In this case, the fluid flow through the layer of unsaturated porous material is similar to flow through an elastic tube the cross-section of which is being changed depending on the local pressure in fluid. It is shown that the relative permeability coefficient of the layer is defined by distribution of liquid saturation in the layer and can be expressed as a harmonic average of the layer saturation with liquid and the pressures at both surfaces of the layer. Numerical solution of these equations is presented in the paper for various boundary conditions, and dependence of these three fields on parameters of the system is analysed.

2. Formulation of the problem

2.1. Basic assumptions and equations

The analysis of non-wetting liquid flow through a layer of unsaturated porous material was performed at the following assumptions:

1. Skeleton of the layer is rigid and immoveable, and its pore structure is macroscopically homogeneous and isotropic.
2. Liquid surrounds porous layer, does not wet the skeleton and is incompressible.
3. The process is one-dimensional and quasi stationary. This means that the process of liquid flow through the layer is stationary however the boundary conditions can evolve quasi statically.
4. The pore space of the layer is empty at the beginning of the process.

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At the above assumptions, a quasi-stationary flow of non-wetting liquid is the special case of the theory of capillary transport in porous materials presented in the paper [1]. Equations for pressure $p$, saturation $s_m$ and velocity $v_m$ of the mobile liquid take the form:

$$\frac{\partial s_m}{\partial p} - C_o \frac{d^2}{dx^2} s_m(x, p(x)) = 0,$$  \hspace{1cm} (1)

$$\frac{d}{dx} \left[ s_m(x, p(x)) \frac{dp}{dx} \right] = 0,$$  \hspace{1cm} (2)

$$v_m = -\frac{1}{R_m} \frac{dp}{dx},$$  \hspace{1cm} (3)

where $R_m$ and $C_o$ are constant. The coefficient $R_m$ represents flow resistance of mobile liquid and $C_o$ characterises diffusive transport of menisci in the quasi static process of boundary conditions evolution.

Equations (1) and (2) result from the balance of mass of the mobile liquid in the quasi static processes of boundary condition evolution and in the stationary flow, respectively, whereas Eqn (3) is the constitutive relation. They are coupled and strongly inhomogeneous. In these equations pressure plays the role of dependent and independent variable. For the case when the pressures at both sides of the layer are equal the flow is stopped and the system of Eqns (1)-(3) reduces to one equation that describes liquid intrusion into the layer.

2.2. Initial and boundary conditions

In spite of stationary character of the process of liquid flow through the layer the presence in equation (1) of partial derivatives of mobile liquid saturation with respect to pressure $p$ and spatial coordinate $x$ needs to formulate both initial and boundary conditions for this saturation.

As the initial condition ($p = 0$) for saturation we assume the pore space of the layer to be empty,

$$s_m(0, x) = 0.$$  \hspace{1cm} (4)

Boundary conditions for the considered problem should be formulated both for pressure and saturation. For pressure they take the form,

$$p(0) = p_1, \quad p(L) = p_2.$$  \hspace{1cm} (5)

and their final values are obtained in some quasi static process. During this process the non-wetting liquid is intruded into the porous layer and penetrates only into pores with diameters greater than the critical diameter $D^*$, which is related to the pressure $p$ in the mobile liquid by Washburn’s formula, [2],

$$D^* = -4\sigma \cos(\theta) / p$$  \hspace{1cm} (6)

where $\sigma$ is the surface tension of the liquid and $\theta$ denotes the wetting angle of the skeleton surface.

A random nature of the pore diameter distribution, the saturation with the mobile liquid on the layer surfaces is directly defined by this distribution and the liquid pressure. Boundary conditions on both layer surfaces are:

$$s_m(p_1, 0) = s_m(p_1), \quad s_m(p_2, L) = s_m(p_2)$$  \hspace{1cm} (7)

where

$$s_m(p) = \frac{\int_{D^*}^{D} \delta(D) dD}{D^*},$$  \hspace{1cm} (8)

and $\delta(D)$ defines the surface weighted probability density of pore diameter distribution on the layer surface.

2.3. Relative permeability coefficient

The flow of liquid through a layer of unsaturated porous material on a standard basis is described in extended form of Darcy law, [3],

$$v_m = k \frac{p_1 - p_2}{L \mu},$$  \hspace{1cm} (9)

where

$$v_m = f_v s_m v_m'$$  \hspace{1cm} (10)

is the filtration velocity of the liquid and $\mu$ is its viscosity. The filtration velocity characterizes volumetric flow of mobile liquid through the layer and takes constant value at given difference of pressures on the layer boundary. Parameter $k$ represents permeability of the saturated layer and $k_1$ is called relative permeability. It takes values from the range $k_s \in <0,1>$ and is usually assumed to be a function of the mean saturation of the layer the form of which is determined experimentally.

Applying relations (3), (9) and (10), it can be proved that

$$\frac{1}{k_s} = \frac{\int_0^L \frac{dx}{L_s s_m(x, p(x))}}{R_m}$$  \hspace{1cm} (11)

and $R_m = f \mu / k_m$.

From Eqn (11) it results that saturation with mobile liquid plays the role of local relative permeability of the layer and the relative permeability of the layer is the harmonic average of local layer saturation.

3. Final remarks

The mathematical model formulated here gives an alternative description of mechanical processes of non-wetting liquid flow through the unsaturated porous layer. From the equations obtained, it follows that this one-dimensional stationary processes of liquid flow takes place in two-dimensional pressure-space. Therefore, formulation of the problem requires both initial and boundary conditions to be given. The equations obtained describe both quasi-static process of non-wetting liquid intrusion into the layer caused by changes of pressures on the layer surfaces, and stationary processes of liquid flow through the layer. In contradiction to the standard formulation of this problem, it is described by strongly nonlinear coupled equations allowing detailed description of this process.

References


Application of capillary and random chain models in mercury intrusion porosimetry

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Abstract

In the paper application of capillary and random chain models of pore architecture for determination of pore size distributions of porous materials is proposed. The models estimate the range of pore sizes based on the results obtained from mercury intrusion method. For both pore architectures the expressions describing capillary potential curves were derived and used to determine the pore size distributions in exemplary porous materials. It was shown that capillary model and the chain model are two limit cases of net models of pore architecture for a given pore size distribution.

Keywords: mercury intrusion data, mercury porosimetry, capillary potential curve

1. Introduction

The most widespread method of determination of the pore diameters of porous materials relies on the interpretation of the capillary potential curves obtained from the mercury porosimetry measurements, [1]. Such interpretation is based on the assumption that at increasing pressure the mercury is intruded against the capillary forces into the pores of decreasing diameters. It is equivalent to the assumption that the pore structure of real porous material can be simplified by a bundle of capillaries with random distribution of diameters. In that case the Washburn formula can be directly applied to interpretation of experimental data. It relates the fluid pressure with the diameter of cylindrical capillary in which the meniscus of the fluid is in the equilibrium state. As a result the cumulative curves of volume weighted pore diameter distributions are obtained for a porous material, [1]. The obvious shortcoming of the capillary model is its simple structure, which does not allow to describe complex structure of real porous material (e.g. when the large pores are joined with the other one by narrow necks). In this case it is not possible to fill (to empty) such pores with the mercury at the pressure adequate to their diameter. Therefore, the pore size distribution determined by the capillary model understates the volume of large pores ascribing it to the small pores. As a consequence the obtained distributions are charged by the large error.

The aim of the paper is to demonstrate the possibility of application of the capillary and chain models of the pore architecture to determination of the limit pore size distributions of porous materials based on the interpretation of the capillary potential curves obtained by the mercury intrusion method. It is proved that both models are limit cases of the net model of pore architecture for a given pore size distribution. Therefore distributions determined by the capillary and chain models define the range of pore sizes in the investigated material [3].

The formulas for the capillary potential of a layer of porous media characterized by the capillary and chain random pore architecture were derived for two-sided intrusion of mercury. Four special cases were analysed. It was shown that the pore length distribution in the chain model does not influence significantly the capillary potential curves.

Both capillary model and chain model with constant length were used as a basis for the procedure of determination of the limit pore size distributions. This procedure was used for the evaluation of the range of pore size distribution in exemplary porous materials.

2. Modeling of mercury intrusion into porous materials

2.1. Basic assumptions

We consider the models of the pore space of porous materials in which the individual pores are cylindrical links of random length and diameter distributions. Two independent factors determine the pore space structure of such media: the link size distributions and the way of their connections. The second factor is called the pore space architecture. The pore architecture causes that even for the same pore diameter distribution in the model material its pore space structure can be different. Regarding the pore architecture, in the paper we distinguish three kinds of models of the pore space structures: capillary, chain and network. In the capillary model the links of equal diameter are joined in series and form long capillaries of constant diameters crossing the whole material. In the chain model the links are joined randomly in series and form the capillaries of a step-wise changing cross-section. In the network model a random connected links form the space network.

2.2. Intrusion into half-space

In order to describe the static process of mercury intrusion into a layer of porous medium we first consider a system in which the porous material with initially empty pores occupies the half-space. For the architecture of the skeleton pore space we assume the chain model. Taking into account that mercury does not wet a surface of most materials, under pressure $p$ the mercury will enter the capillaries of porous half-space and its menisci will stop on links of diameter $D$ less than diameter $D'$ defined by Washburn expression

$$D' = 2\sigma \cos(\theta)/p,$$

(1)
where $\sigma$ is the coefficient of the mercury surface tension and $\theta$ stand for the wettability contact angle of the skeleton material by mercury.

Links of diameter satisfying condition (1) will be called the critical links, [2]. Due to random character of the link size distributions, the depth $z$ of menisci placement in capillaries will take a random value and the probability of mercury occurrence, at depth $z$ is given by the following expression

$$F(z) = \alpha_o \int_0^\infty \eta(u)du + \alpha_o \int_0^z F_o(z-u) \eta(u)du,$$

(2)

where function $F_o(z)$ is defined by the Volterwey equation as

$$F_o(z) = \alpha_o \int_0^\infty \varphi(u)du + \alpha_o \int_0^z F_o(z-u) \varphi(u)du,$$

(3)

where

$$\alpha_o = \int_0^\infty \psi(D)dD, \quad \eta(s) = \frac{1}{\alpha_o} \int_s^\infty \varphi(v)dv,$$

(4)

represent the probability of occurrence of the super critical links in the medium, and distribution of truncated links at the surface of the half-space, respectively. Quantity $\alpha$ is the mean value of the links length and functions $\psi(D)$ and $\varphi(v)$ represented probability distributions of diameter and length of links respectively.

2.3. Intrusion into layer

The two-side intrusion of the mercury into a layer of porous media, equivalently, can be considered a process realized in two steps (first step, the left-hand side intrusion and second step the right-hand side intrusion). During the left-hand intrusion the mercury fill the supercritical capillaries (composed of the supercritical links) and subcritical ones (containing also subcritical links) remain partially empty. The left- and right-hand side intrusions into the subcritical capillaries can be considered independent processes. Therefore the function $U(z)$ of mercury distribution in the layer defining the probability of mercury occurrence of the depth $z$ from the surface of the layer takes the form

$$U(z) = F(z) + F(L - z) - F(L)$$

(5)

3. Description of the potential curve

Applying expression (5) for mercury distribution in the layer, the volume $V_0(p)$ of mercury intruded into the layer of limit surface at various pressures can be represented by a following expression:

$$\frac{V_0(p)}{V_0} = \frac{2}{\alpha_o} \left(2F - F(L)\right),$$

(6)

where

$$\alpha_2 = \int_0^\infty \theta(D)dD,$$

(7)

and $\theta(D) = \frac{\psi(D)}{\sigma}$ is the volume weighted distribution of link diameters. Quantities $V_0$ and $F$ represent total volume of pores in the layer of the unit surface and mean value of the probability $F$ in the layer, respectively.

Three special cases of the capillary potential curves of porous layer given by expression (6) are presented in Fig. 1. They concern the chain pore architecture of the layer with the same link diameter distribution and different distributions of link length: capillary model (CM), chain model with constant (PCM) and random (SCM) link length.

![Figure 1: Capillary potential curve of porous layer for capillary model, periodic and stochastic chain model](image)

It is clear that the link length distribution in the chain model does not influence significantly the capillary potential curves. Therefore description of the capillary potential curve for porous material of chain pore architecture can be effectively represented by a model with a constant link length. From (6) we have

$$\frac{V_0(p)}{V_0} = \left(\frac{2}{\alpha_o} \right) \left(1 - \frac{\psi(D)}{\sigma}\right),$$

(8)

where $N = \frac{\alpha_o}{\sigma}$.

For $N = 1$ expression (8) reduces to the simple form

$$\frac{V_0(p)}{V_0} = \alpha_2,$$

(9)

which describes the potential curve for capillary model of pore architecture. It was shown that potential curves for capillary model and chain models are the limit cases of the net model of pore architecture for a given pore size distribution. It means that both models can be used for estimation of the range of pore diameter distribution based on the mercury intrusion data, [3].

References


A new macroscopic description of capillary transport of liquid and gas in porous materials is presented, based on the methods and concepts of multi-constituent continuum mechanics. It is assumed that the liquid and gas filling rigid porous material form a macroscopic continuum consisting of three constituents: gas, mobile liquid and capillary liquid. A mobile liquid retains the physical properties of the liquid outside the pore space, while the capillary liquid forms a film on the contact surface with the skeleton. A capillary liquid is immobile, contains the whole capillary energy of the system, and - due to possible negative pressure within it - has solid-like properties. Both liquids exchange mass and energy in the vicinity of the menisci during their motion. For all constituents of the porous medium, balance equations of mass, linear momentum and energy are formulated and constitutive relations for mechanical processes are derived. The approach is used based on the analysis of dissipation inequality of mechanical energy formulated for the whole system. A new approach is proposed, similar to that used in the rational thermodynamics, based on entropy inequality analysis and the Lagrange multiplier method.

Keywords: unsaturated porous materials; macroscopic description; capillary transport

1. Introduction

The course of phenomena in unsaturated porous materials is generally highly complex. It depends not only on the physical properties of the porous skeleton and the liquid and gas filling its pores, but is additionally complicated by the interaction between constituents of the system during their relative motion and deformation.

The nature of this interaction is determined by the surface phenomena at phase interfaces, and in particular by the capillary interactions. These are determined by the nature of the interaction between the liquid and the skeleton (wettability/non-wettability of the skeleton) and are closely related to the microscopic structure of the skeleton pore space. Due to the very large internal surface area of the liquid filling the pores of a porous material, rarely found in liquids outside the pore space, there are significant differences between the properties and behaviour of the liquid in the two states. These factors cause that the modelling of liquid and gas transport in unsaturated porous materials, at both microscopic and macroscopic levels, is a very complex problem, requiring the development of new concepts and methods of description.

The non-standard nature of the problem is revealed even in a relatively simple problem of the macroscopic description of the process of quasi-static intrusion of a non-wetting liquid (e.g. mercury) into a sample of homogeneous porous material (Fig. 1). The situation here is unusual for hydrostatics, where in spite of the homogeneous pressure distribution in the liquid, the distribution of its saturation in the medium is inhomogeneous. In this case, the non-standard approach to solution of this problem is to formulate balance equations and constitutive relations for processes taking place in the pore space-pressure continuum, [1].

A new macroscopic description of capillary transport of liquid and gas in porous materials is presented, based on the methods and concepts of multi-constituent continuum mechanics. The key element of the proposed physical model of the unsaturated medium is division of the liquid in the pore space into two constituents: the mobile and the capillary liquid, which exchange mass and energy in the vicinity of menisci surfaces during their motion. For all constituents of the porous medium, balance equations of mass, linear momentum and energy are formulated and constitutive relations for mechanical processes are derived based on strain energy inequality and the Lagrange multipliers method. The velocity field of the menisci motion in the pore space induced by changes of the capillary pressure is defined by an additional constitutive relation, similar to Fick’s first law of diffusion.

2. Basic equations

2.1. Basic assumptions

Theoretical considerations are conducted within the framework of multi-phase continuum mechanics. It is assumed that the liquid and gas filling an undeformable porous material form a macroscopic continuum consisting of three constituents: gas, mobile liquid and capillary liquid, while the undeformable skeleton of the isotropic pore space structure forms the space in which transport processes take place. The division of the liquid
into two continua is justified both kinematically and energetically. The capillary liquid forms a film on its contact surface with the skeleton. This liquid is immovable and contains the whole capillary energy. Both liquids exchange mass, linear momentum and energy in the vicinity of menisci surfaces. Mass exchange occurs only during menisci motion in the pore space, which is described by an additional macroscopic velocity field. This enables the modelling of mechanisms of menisci motion in the pore space. Each constituent of the medium is characterised by mass density, and their spatial distribution is defined by parameters of pore saturation.

2.2. Balance equations

For three constituents of the porous medium: mobile liquid, capillary liquid and gas, balance equations of mass, linear momentum and energy are formulated, [2]:

$$\frac{\partial \rho_m}{\partial t} + \text{div}(\bar{\rho}_m \mathbf{v}_m) = -\nabla \varphi, \quad \frac{\partial \rho_g}{\partial t} + \text{div}(\bar{\rho}_g \mathbf{v}_g) = 0,$$

where \(\varphi\) denotes saturations of pores with the given constituent. These saturations are related by an identity

$$s_m + s_g = 1$$

(4)

Partial densities of the constituents \(\bar{\rho}_\alpha\) are linked to the phase densities \(\rho_\alpha\) by the relations

$$\bar{\rho}_\alpha = f, \rho_\alpha s_\alpha$$

(3)

where \(f\) is the volume porosity of the medium, and \(s_\alpha\) denotes saturations of pores with the given constituent. These saturations are related by an identity

$$s_m + s_g = 1$$

(4)

Quantities \(\bar{T}_\alpha\) represent the tensor of partial stresses in the constituents. They are related to the phase stress tensor \(T_\alpha\) by \( \bar{T}_\alpha = f, s_\alpha T_\alpha \). Vectors

$$\bar{\pi}_m = -\bar{\pi}_m, \quad \bar{\pi}_m = -\bar{\pi}_m, \quad \bar{\pi}_g = -\bar{\pi}_g$$

(5)

are the volume densities of interaction forces of constituents.

Using the previously defined quantities describing the state of particular constituents of the unsaturated porous medium, and their mass and momentum balance equations, the dissipation inequality for internal energy for the whole system can be written in the form

$$-\bar{\rho}_m \frac{\partial u^m_m}{\partial t} - \bar{\rho}_m \frac{D u^m_m}{D t} - \bar{\rho}_g \frac{D u^m_g}{D t} + tr(D_m \bar{T}_m) + tr(D_g \bar{T}_g) + c_i^2 (s_m - u^m_m - u^m_g - s_g (w - v_m^2/2)) \geq 0,$$

(6)

where \(u^m_m, u^m_g, u^m_r\) represent the mass density of the Helmholtz free energy of the capillary liquid, mobile liquid and gas, respectively.

2.3. Constitutive relations

Applying the dissipation inequality of mechanical energy (6) the constitutive equations for the proposed model of the capillary transport of liquid and gas in an unsaturated porous medium are derived. The similar procedure to that used in phenomenological thermodynamics, based on the entropy inequality and the concept of Lagrange multipliers is used, [3], [4]. The constitutive relations take form:

$$\bar{T}_m = - f, s_m p_m \mathbf{I}, \quad w = v_m^2/2, \quad \bar{\pi}_m = \rho_m \mathbf{grad}(s_m f_m), \quad (\alpha = m,c,g),$$

$$u^m_m + p_m / \rho_m = u^e_m + p_f / \rho_f, \quad r = \rho_m - \rho_g,$$

$$p_m - \rho_g + (\rho_g - p_g) \frac{ds_g}{ds_m} = c_i^2 s_m,$$

$$\mathbf{v} = -C_m(s_m, \rho_m, \mathbf{grad}(s_m)), \quad s_g = s_g(s_m).$$

Balance equations (1) and constitutive relations (7) form the system of 59 equations describing capillary transport of liquid and gas in unsaturated porous material. These equations contain 59 unknown scalar fields. Therefore, the obtained system of equations is closed and enables formulation of boundary value problems for mechanical processes in unsaturated porous materials.

3. Final remarks

The mathematical model formulated here runs an alternative description of mechanical processes of liquid and gas transport in unsaturated porous materials. From the equations obtained, it follows that processes of capillary transport in such systems take place in five-dimensional pressure-time-space. Therefore, any attempt to describe them in terms of temporal and spatial changes only must always lead to ambiguous relations and inexplicable effects resulting from the reduction of one dimension of their course space.

It can be shown that the equations obtained describe both quasi-static and quasi-stationary processes of capillary transport of liquid and gas in unsaturated porous materials as special cases of the new mathematical model. This concern, among others, the description of quasi-static intrusion of mercury into porous materials and description of liquid or gas flow through unsaturated porous material.

References


Experimental verification of the relationships between Young’s modulus and bone density using Digital Image Correlation

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Abstract

The trabecular bone is a special porous material difficult to predict its mechanical properties. One of the proposed methods is to apply the methodology based on the X-ray microtomography and used the relationship between bone density and mechanical properties. This paper examines five relationships between bone density and Young’s modulus available in the literature. The relationships proposed by Carter and Hayes, Ciarelli et al., Kaneko et al., Keller and Li and Aspden were selected for the tests. Twelve cubic specimens of the trabecular bone from femoral heads were cut out and X-ray microtomography (XMT) scanned. The mechanical compression tests were performed on these specimens with the displacement and strain full field measurements using Digital Image Correlation (DIC) method. On the basis of the XMT numerical models of the tested specimens were generated and numerically tested by simulation of the compression test using finite element method. The results of the experimental tests and results of numerical analysis, where the material data were prescribed on the basis of mentioned relationships, were compared. The DIC method was used to obtain the full field displacement and strain measurements in the form of colour maps which were directly compared with results obtained by means of finite element analysis.

Keywords: digital image correlation, finite elements method, X-ray microcomputed tomography, trabecular bone mechanical properties, bone mineral density

1. Introduction

According to the literature, there are many mathematical relationships between bone density and mechanical properties determined experimentally. Application of these relationships in the finite element analyses (FEA) may result in different accuracy of numerical simulations of the bone mechanical behaviour [1] especially in the case of the trabecular bone structure. The aim of the presented study was to indicate the relationship that assures the best accuracy of numerical simulation comparing with the experiment. The mechanical compression tests with the Digital Image Correlation (DIC) were used to acquire experimental displacements and strain fields of trabecular bone samples and compared with the fields acquired by finite element analysis for the FEA accuracy estimation. The material data for finite element analysis were calculated on the basis of mineral bone density using selected relationships between bone density and its mechanical properties.

2. Materials and method

Twelve cubic trabecular bone specimens (edge length-10 mm) were cut out from the heads of human femur and scanned by means of X-ray microtomography. A hydroxyapatite phantom was used during scanning the specimens to convert the relative Hounsfield Units into real equivalent bone density values [1].

The images obtained from XMT scans were reconstructed using a manufacturer’s software (Datos 2.0). Then, the trabecular bone structures were segmented and 3D surface models of bone samples were created using Mimics software. The Mimics software was also used for the generation of finite element meshes using ten-node tetrahedral elements. The material properties for each numerical model were calculated on the basis of one of five selected relationships proposed in [1,2,3,4,5,6]. The greyscale density calculated from XMT scans using a hydroxyapatite phantom was used as the basis for calculation of ash density \( \rho_{ash} \), apparent density \( \rho_{app} \) and bone volume fraction \( BV/TV \) being variables in selected relationships. Created FE models were virtually placed between two plates.
and the mechanical compression tests were simulated. Numerical models with five different materials property sets were prepared for each of twelve specimens. In total, 60 numerical models were generated and the compression test was simulated for each of them. The results of numerical analysis were compared with the results of the experiments. The experiments consisted of mechanical compression tests with the full field measurements of displacements and strains by means of DIC. Additionally, the nanoindentation measurement of Young’s modulus of single trabeculae was conducted after the compression tests. The mechanical compression tests were performed at room temperature (~25°C) and the displacement rate was 0.5 mm/min. The load range was set up from 0 to 2000 N (or till the specimen failure). The cameras of the DIC system were taking pictures every 100N of the load increment. The displacements fields obtained from numerical calculation and experiments are presented in Fig. 1. It can be seen that the displacement field in numerical solution where the relationship given by Ciarelli et al. (Fig. 1b) was used is the most similar to the field acquired in experiment with the DIC method (Fig. 1a).

Three quantities were compared between results acquired from DIC and FEA: the maximum displacement $\delta_{\text{max}}$, the displacement in control point $A$ $\delta_A$ (Fig. 1), and the average strain value in the selected area $\bar{\varepsilon}$. From these quantities, the average relative errors of numerical solution were calculated and compared to the results of experiments.

The ranges of Young’s modulus values calculated using each relationship were additionally compared with the nanoindentation measurements of Young’s modulus on specimen trabeculae.

3. Results and conclusions

The best accuracy of the numerical analysis was obtained for the relationship given by Ciarelli et al. (2000) $\bar{\sigma}_{\text{max}} = 14.11\%$, $\bar{\sigma}_A = 17.87\%$, $\bar{\varepsilon} = 50.94\%$) as compared to others: $\bar{\delta}_{\text{max}} = 23.20\%$, $\bar{\delta}_A = 25.88\%$, $\bar{\varepsilon} = 60.27\%$) Ref. [5], $\bar{\delta}_{\text{max}} = 29.42\%$, $\bar{\delta}_A = 37.26\%$, $\bar{\varepsilon} = 62.29\%$) Ref. [4], $\bar{\delta}_{\text{max}} = 35.47\%$, $\bar{\delta}_A = 45.18\%$, $\bar{\varepsilon} = 67.99\%$) Ref. [2], $\bar{\delta}_{\text{max}} = 328.81\%$, $\bar{\delta}_A = 36.74\%$, $\bar{\varepsilon} = 216.62\%$) Ref. [6].

The relationship given by Ciarelli et al. [3] allows to obtain the displacement distribution in numerical analysis with a relative error of 17.87%. The relative error for the strain distribution is much higher and equal 50.94%.

This relationship ensures that the range of Young’s modulus calculated and assigned to numerical models (1.461 – 11.829 GPa) is the most similar to the ranges of Young’s modulus measured in nanoindentation tests (2.4 – 15.6 GPa). The dispersion of the Young’s modulus values is natural for the trabecular bone tissue and similar results were several times reported in the literature e.g. 3.27 – 10.58 GPa - Choi et al. [7], 11.4 ± 5.6 GPa - Zysset et al. [8]. The conducted research shown, that there exist relationships between bone mineral density and mechanical that cannot used for the mapping of mechanical properties of trabecular bone. At present, using the subjected method, further research in this field is conducted and new relationships with improved accuracy between experimental and numerical results are worked up.

References


Numerical modelling of aluminium foam based on quasi-static compression test

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Abstract

In the paper numerical modelling of a two-phase composite metal foam (CMF) is presented. In general, foams are highly heterogeneous. Cellular porous structures are made of matrix and gas-filled pores. Together they form a lightweight frame, whose structural response is the resultant of mechanical parameters of ingredients and their geometric determinants. The porosity is the fundamental characteristic, but as insufficient scalar parameter should be complemented by information about the shape, size and distribution of voids. Thus, the network and deformation of foam were reconstructed by digital image correlation (DIC) and mesoscale field generation based on the statistical analysis of cubic samples sides. The aim of the work is to study a research hypothesis and to formulate general conclusions about constitutive relation of highly deformed aluminium foam. In order to achieve these objectives, mechanical phenomenon was investigated by experiments and finite element method (FEM) simulations. Material behaviour was studied in quasi-static uniaxial compression test at 0.001 s⁻¹ and 0.04 s⁻¹ strain rates with the support of the camera recordings.

Keywords: aluminium foam, digital image correlation, finite element method, inverse analysis, material parameters

1. Introduction

The aluminium foam is a heterogeneous material which belongs to ductile CMF family. Many researchers make persistent efforts to investigate, describe and implement constitutive behaviour of foams, due to their high strength-to-weight ratio and energy absorption ability.

The main problem faced during foams modelling is a lack of standards for repeatability of experiments and effective parameters determination. Complex structures of foam causes fundamentally different mechanical response between porous and fully dense metals. To obtain the representative results especially in inelastic range is nontrivial, because of foam large deformations, strain irreversibility, compacting in compression and yielding dependent on pressure.

Presently, various numerical models are used to approximate the foam behaviour as: elasto-plasticity, hyperelasticity, and viscosity. Above material models are based on different formulations and give realistic predictions usually only in specified strain and stress ranges and reflex the structural conditions in average sense. Hence, selecting the suitable model is limited and should be carefully accepted.

The main contribution of this work is to determine trends in homogenized modelling of mechanical response of Al-foam. A simple axial compression test was supported by digital recordings the aid of decision making. The stress-strain curves, deformation modes, statistical analysis of external pores were used to reconstruct the structure and calibrate the main features of foams.

2. Background

2.1. Experiment setup

Quasi-static compression tests were carried out to investigate uniaxial mechanical response of foam. Instron Satec 300DX was used to enforce 90% displacement-controlled deformation at two strain rates 0.001 s⁻¹ and 0.04 s⁻¹. Samples were extracted from sheets of Alcarbon Alcoras-SD aluminium foam. The structure was a closed-cell type with open-cutted surfaces and had a nominal density from 0.2 to 0.4 g/cm³.

![Stress-strain curve of 30x30x30 mm sample.](image)

Figure 1: Stress-strain curve of 30x30x30 mm sample.

The experimental results were recorded as load-deflection points and show the deformation mechanism typical for ductile foams. The nominal stress-strain curves revealed three distinct regimes: linear elasticity $E$ (elastic modulus) caused by bending the cell walls, plateau $\sigma_y$ (yielding stress) induces by the formation of plastic hinges and compacting $\varepsilon_D$ (densification strain), due to collapsing of skeleton phase. As an enhancement of simple data test the DIC of the deformation progress was measured.

2.2. Finite element models

Two approaches were taken into consideration for FEA of foam: structural and homogenized model. The structural modelling consists of a geometrically conditioned scheme and inhomogeneous behaviour defined individually. Foam is
considered in mesoscale, thus the structure was binarized to two-phase composite, without matrix heterogeneity (Al + SiC). Due to its nature, aluminium alloy was modelled as elasto-plastic material based on Huber-Mises criterion and Johnson-Cook law in the form:

$$\bar{\sigma} = H \left( \varepsilon^p \right) \mathcal{G} \left( \varepsilon^d \right),$$  \hspace{1cm} (1)

where $H$, $G$ are hardening and rate sensitivity terms of equivalent plastic strain $\varepsilon^p$ and strain rate $\dot{\varepsilon}^d$. Because of foam was formed under atmospheric pressure conditions, the air-filled pores were treated as voids (material loss) [2].

On the other hand, homogenized approach was used to mimic the smeared structure behaviour. The effective properties of Alcoras-SD foam was investigated by using the purely solid material models as follow:

- linear elastic with plastic hardening varied exponentially or according to power law,
- viscoelastic and viscoplastic terms to include the time-dependent behaviour,
- hyperelastic for large volumetric changes defined in general strain energy potential function,
- crushable foam model with hardening elliptically varied in the deviatoric and pressure stress plane [1],
- nonlinear porous elasticity based on energy conservation and geometrical deformation process [6].

The background assumption of material isotropy was made. In general, this assumption is correct for stochastic structures with high porosity and dense packing.

3. Methods

3.1. Structure recognition

The digital image processing of specimen surface pattern was used to create its 3D representations. The geometry characteristic as phase distinct, porosity, pores size and distribution was studied. The measurement of the planar object surface was determined through statistical and principal component analysis. Blobs labeling and their properties was calculated in Scilab image and video toolbox.

![Figure 2](image)

Figure 2: Specimen and recognition scheme

To correlate the deformation modes of foam the macroscopic displacement field is used. The region of interests (ROI) is sampled from the image as a projection of the specimen front surface [4,5]. Such defined domain was used in the calculation of plain strain components based on the movement of the edges. The requirement is that the deformations from recordings should be the same in FE model.

3.2. Inverse analysis

After the FE model is applied, the unknown material parameters of foam or aluminium matrix can be determined. To solve this problem an inverse analysis based on Trust Region algorithm was performed [3]. The parametric optimization consisted of iterative minimization the cost function through nonlinear least squares algorithm. The solution was obtained in the term of root mean square $\sigma_{RMS}$ of the residues $r_i$ defined as mismatch between experiment $Y_{exp}$ and finite element simulation $Y_{SIM}$.

$$\sigma_{RMS} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} r_i^2},$$  \hspace{1cm} (2)

where $X$ is a vector containing the unknown constitutive constants.

4. Summary

The back-calculation analysis was performed to predict the macroscopic mechanical properties of a highly porous foam. Two approaches were used to build the finite element model: structural and homogenized. A porous system was reconstructed by two-dimensional DIC and geometry statistics. By means of the linking experiment curves with recorded displacement fields the effective tool for quantitative determination of material parameters of foams in elastic and inelastic ranges was presented.

References

Creeping flow of a power-law fluid through a fibrous porous media

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Abstract

Numerical calculations of the velocity field and the component of transverse permeability in the filtration equation for steady, incompressible flow of power-law liquids through the assemblages of cylindrical fibers are presented in the paper. The fibers are arranged regularly in arrays. The flow is transverse with respect to the fibers. The non-linear governing equation in the repeated element of the array is solved using the Picard iteration method. At each iteration step the method of fundamental solutions and the method of particular solutions are used. The non-homogenous term is interpolated by means of the radial basis functions. Then the bundle of fibers is treated as a porous media and on the base of velocity field the permeability coefficients are calculated as functions of porosity.

Keywords: porous media, permeability, power-law fluid, meshless method

1. Introduction

Fluid flow through a fibrous porous media plays an important role in various engineering issues, e.g. industrial filters, resin transfer molding process, flow in spinning artificial fibers or insulating materials. The knowledge of permeability which characterizes the ability of fluid to penetrate the fibers is an important factor in designing of the above mentioned processes and devices.

In the case of parallel arrangement of fibers due to the anisotropic properties of the porous media, the laminar flow through such media is modeled by Darcy’s law in the form:

\[ \mathbf{q} = -K \frac{\nabla p}{\mu} \]

where \( \mathbf{q} \) is filtration velocity, \( p \) denotes the fluid pressure, \( \mu \) is the fluid viscosity, and \( K \) is the permeability tensor. For the fibrous media with unidirectional fiber arrangement the permeability tensor in a coordinate system with the \( z \) axis parallel to the fibers axis has the form

\[ K = \begin{bmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_\ell \end{bmatrix} \]

where \( K_{xx} \) and \( K_{yy} \) are the components of the permeability tensor in the principal directions perpendicular to the fibers, and \( K_\ell \) is the component of the permeability tensor in the direction parallel to the fibers.

There have been many analytical, numerical, and experimental papers in which these components were determined for a Newtonian fluid. The authors usually consider flow in a repeated element of regular array of fibers, as micro structural flow problems, for the determination of the permeability components. Various authors have considered both longitudinal \([5, 7]\) and transverse flow \([3, 6]\) with respect to the fibers.

However using the Newtonian fluid model in many practical applications is unreasonable. It is especially important in considering issues from chemical industry, pharmacological industry, food industry or by simulation of biofluids flows. In such situations the non-Newtonian fluid models are applied \([1, 2, 4]\). One of the simplest and most popular of such fluid models is a power-law fluid.

The paper aims at numerical calculations of the velocity field for steady, incompressible flow of power-law liquids through assemblages of cylindrical fibers by means of the MFS. The fibers are arranged regularly in an array. The flow is transverse with respect to the fibers. Knowing the velocity field, the component of the permeability tensor in the direction perpendicular to the fibers \((K_{xx} \text{ and } K_{yy})\) can be calculated.

2. Problem statement

Let us consider steady, fully developed, laminar, isothermal flow of an incompressible viscous power-law fluid driven by a constant pressure in a system of regular parallel fibers. The flow is transverse with respect to fibers which are arranged in a regular square (Fig. 2(a)) and triangular (Fig. 2(b)) array. The radius of the fibers is equal to \( a \), and the distance between the fibers is equal to \( 2b \).

The Navier-Stokes equations for creeping flow of non-Newtonian fluid takes the form:

\[ \nabla \cdot \mathbf{u} = 0 \quad \text{in } \Omega_R \]

\[ \nabla p = \nabla (\eta(\gamma) \nabla \mathbf{u}) \quad \text{in } \Omega_R \]

where \( \mathbf{u} = [u, v] \) is the velocity, \( p \) is the pressure, \( \eta(\gamma) \) is the viscosity function and \( \Omega_R \) is the 2-D domain of the repeated element of the array. The bulk flow is assumed to occur from left to right.

The power-law model is used in this work to describe the non-Newtonian behaviour of the fluid, consequently, the viscosity function is given by

\[ \eta(\gamma) = K \cdot \gamma^{m-1}, \]

where \( K \) is the consistency factor, \( m \) is the power law index \((m < 1 \text{ for pseudoplastic fluids (shear-thinning)}, m > 1 \text{ for}}

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The work of the second author was supported by the grant 02/21/D5MK/3468 funded by the Ministry of Higher Education and Science, Poland.
dilatant fluids (shear-thickening), and $m = 1$ for Newtonian fluids), and

$$
\gamma = \sqrt{\frac{2 \frac{\partial v^2}{\partial x} + \frac{\partial u^2}{\partial y}}{\frac{\partial v}{\partial x}}} + \frac{\partial v}{\partial y} + \frac{\partial v^2}{\partial y}.
$$

(6)

The relationship between the viscosity function and the shear rate for different of the power-law index $m$ is depicted in Fig. 1.

Figure 1: Power-law fluid for different values of power-law index $m$

We introduce the dimensionless variables:

$$
X = \frac{x}{b}, \quad Y = \frac{y}{b}, \quad E = \frac{a}{b},
$$

(7)

$$
U = \frac{u}{u_E}, \quad V = \frac{v}{v_E}, \quad P = \frac{p}{\mu_0 u_E},
$$

(8)

$$
F(x) = \frac{\eta}{\mu_0}, \quad \chi = \frac{v}{u_E},
$$

(9)

where $\mu_0$ denotes the reference viscosity, while $u_E$ is the reference velocity. Then the governing equations can be written in the following forms:

$$
\nabla \cdot \mathbf{U} = 0 \quad \text{in } \Omega_T,
$$

(10)

$$
\nabla P = \nabla (F(x) \nabla \cdot \mathbf{U}) \quad \text{in } \Omega_T,
$$

(11)

where $\Omega_T$ is the dimensionless considered region and

$$
F(x) = B e^{\chi^m}, \quad \chi = \frac{y}{u_E}, \quad \eta = \frac{\mu_0 u_E}{b},
$$

(12)

$$
\chi = \frac{b y}{u_E} = \left[ 2 \frac{\partial U^2}{\partial X} + \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 + 2 \frac{\partial V^2}{\partial Y} \right].
$$

(13)

The non-linear governing Eqs (10, 11) with the appropriate boundary conditions, yields a micro structural boundary value problem (MVBVP) in a repeated element of an array of fibers.

Symmetry conditions are applied to the boundaries parallel to the flow direction so that no flow will cross these boundaries. On the boundaries normal to the bulk flow direction, homogeneous pressures are specified and a pressure drop ($\Delta p$) is maintained between the two vertical boundary planes. The no-slip conditions at the fiber is considered.

The solution of the MBVP can be obtained by a meshless procedure, e.g. the method of fundamental solutions (MFS) and the method of particular solution (MPS).

Once the boundary solution is obtained solving the pertinent Eqs (10, 11), the non-dimensional permeability of the fibrous porous medium can be calculated on the base of the following expression:

$$
K_{\perp} = \left[ \int (F(x) \nabla \cdot \mathbf{U}) dX \right]^{-1}.
$$

(14)

Figure 2: Two different kinds of a regular array of cylindrical rods (a) square array, and (b) triangular array with marked the repeated element $\Omega_T$

References


Abstract

The paper presents procedures for the identification of parameters of transport, mechanical moduli and coefficients of coupled chemo-osmotic and chemo-mechanical processes in porous organic PVA hydrogels. The studies are focused on application of numerical optimization and data from experiments to calibrate the chemo-mechanical model and also identification of thermal behaviour of PVA.

Keywords: chemo-mechanics, PVA hydrogels, identification of parameters, chemical deformation

1 Introduction

The reactivity of hydrogels causes that the effective specification of structural and physical properties demand the incorporation of complex experimental studies and the application of an adequate mathematical model describing the realistic behaviour of the material [1]. The available results of experimental studies regarding selected material parameters such as permeability, Young's modulus with and without draining, Poisson’s ratio with and without draining, diffusion coefficient, retardation factor and porosity show considerable spread and in some cases discrepancy [2,3]. There is no standards for studies of hydrogels, including determination of deformation and the conditions which must be fulfilled (e.g. temperature) throughout the test. Moreover there is no experimental procedures and models useful for interpretations of coupled behaviour of hydrogels. The paper presents the identification procedures of transport, mechanical, and coupled chemo-mechanical and chemo-osmotic parameters of porous hydrogels based on poli(vinyl alcohol) PVA.

2 Mathematical model

The macroscopic models of chemo-mechanical deformation and reactive transport corresponding to the proposed experimental test are considered. The system of equations useful for solving problems of identification for chemo-mechanical model is as follows:

\[
(1 + \alpha_\alpha M) \frac{\partial p}{\partial t} - (\gamma + \alpha_\alpha M) \frac{\partial c}{\partial t} - \frac{n_i M}{b} \frac{\partial^2 p}{\partial x^2} + \frac{n_i M}{b} \frac{\partial^2 c}{\partial x^2} = 0
\]

\[
R \frac{\partial^2 c - K_c \frac{\partial c}{\partial x} + u_s \frac{\partial^2 c}{\partial x^2}}{\partial t} + D \frac{\partial^2 p}{\partial x^2} = 0
\]

where \( c \) is concentration of solution and \( p \) is pore pressure. The description of the model, parameter and solution of 1-D initial – boundary value problems using Matlab and Comsol environments are given in [4].

The theory used to identify mechanical properties from the creep test is based on Biot’s poroelasticity.

To determine transport parameters linear model of diffusive transport and equilibrium model of sorption are used.

3 Experimental set-up

The studies of mechanical, chemo-mechanical and transport processes were carried out with specially designed experimental set-up. The chemo-mechanical experiment (Fig.1) is based on modified classical reservoir test.

The system includes: diffusion chamber (with hydrogel and reservoir liquid inside), conductometer and thermostat. All measurements were conducted at 25°C. The reservoir liquid solutions were physiological fluid, one and two molar sodium chloride (NaCl) or one and two molar potassium chloride (KCl). The method based on unconfined one dimensional compression shown in Fig. 2 is applied to determine mechanical and rheological parameters.

Figure 1: The experimental system for studies of transport properties and chemical deformation (a) and diffusion chamber (b)

Figure 2: The experimental system for studies of mechanical properties
The chemical, mechanical and thermal deformation of sample were continuously recorded by time of flight method using Optel ultrasonic testing system and pulse signal. The tested material are poly (vinyl alcohol) (PVA) hydrogels.

4 Results

4.1 Chemical deformation of hydrogel

The process of transport of salt into hydrogel material results in both concentration changes in the reservoir and deformation of hydrogel. Figure 3 presents the changes of displacement of upper base of hydrogel for different chemical loading.

![Figure 3: Displacement of the upper base of the chemically loaded samples of hydrogels](image)

4.2 Identification of PVA parameters

The procedure of identification was carried out with the help of the optimization methods, implemented in the Matlab environment. The results of identification are given in Table 1, where transport parameter are: diffusion coefficient (D) and retardation factor (R); mechanical parameters: drained Young’s modulus (Es) and Poisson’s Ratio (νs); hydraulic permeability (K); chemo-mechanical couplings parameters (d, γ) and chemo-osmotic parameters (kc) and (D1).

Figure 4 shows a comparison of time evolution of strain obtained from experimental tests (creep) for hydrogel material with the results of simulations assuming the estimated values of parameters given in Table 1.

![Figure 4: Comparison of strains as the function of time from creep tests with model predictions](image)

### Table 1: Optimized parameters of hydrogels

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Hydrogel 1</th>
<th>Hydrogel 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>D · 10^{-10} [m^2/s]</td>
<td>6.08±0.13</td>
<td>6.97±0.27</td>
</tr>
<tr>
<td>R [-]</td>
<td>1.04±0.07</td>
<td>1.02±0.07</td>
</tr>
<tr>
<td>Es [MPa]</td>
<td>0.902±0.030</td>
<td>0.778±0.040</td>
</tr>
<tr>
<td>Vs [-]</td>
<td>0.31±0.03</td>
<td>0.20±0.03</td>
</tr>
<tr>
<td>K · 10^{-15} [N/m^4kg]</td>
<td>3.29±1.16</td>
<td>5.84±3.01</td>
</tr>
<tr>
<td>d [m^2/s^2]</td>
<td>417.11±67.12</td>
<td>245.00</td>
</tr>
<tr>
<td>γ · 10^3 [m^2/s^3]</td>
<td>-2.88±0.99</td>
<td>5.38</td>
</tr>
<tr>
<td>k_s · 10^{-15} [m^3/kg·s]</td>
<td>2.58±1.24</td>
<td>3.21</td>
</tr>
<tr>
<td>D1 [s]</td>
<td>0±0.00</td>
<td>0.10</td>
</tr>
</tbody>
</table>

5 Conclusions

The obtain results play a crucial role for the recognition of the associated phenomena: mechanical, chemo-mechanical, chemo-osmotic and transport behaviour of organic hydrogels and may form the basis for the broadening of the works towards other chemically sensitive materials like gels and soils.

References


Studies of ultrasonic waves in water or air saturated high porosity materials

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Abstract

The paper presents experimental and theoretical results related to application of ultrasonic technique to studies of fluid saturated porous materials. The analysis is focused on the limit cases i.e. high porosity materials (with porosities ranging from 70% to 95%) saturated with air or water. The commonly used in such cases Biot’s theory is discussed in the context of its potential applicability for prediction of wave parameters: phase velocity and attenuation coefficient as functions of frequency and porosity. The suitability of the model is considered by comparison of results of sensitivity analysis with experimental ultrasonic data obtained for high porosity foams filled with air or water.

Keywords: saturated porous media, ultrasound, wave attenuation, wave dispersion

1. Introduction

The number of natural or artificial porous materials is characterized by their high porosity. For example cancellous bones, porous foams or gels may have porosity close to one and thus the stiffness of the solid frame has very low values as compared with typical low (or medium) porosity materials like rocks. For that reasons the mechanical and structural properties of porous skeleton are not dominating factors for dynamics of the media and the properties of fluid filling the pores become particularly important. In experimental ultrasonic studies of such materials (e.g. cancellous bones) it is observed unusual high dispersion and attenuation of measured longitudinal waves [4].

While the theoretical studies of porous materials belong to strongly developing field in mechanics, relatively little attention is devoted to modeling wave propagation in high porosity media. The aim of the paper is to examine the contribution of the properties of the solid skeleton and pore fluid to velocity dispersion and attenuation in high porosity foams (Sawbones) saturated with air or water. Moreover, the suitability of the macroscopic Biot theory for modeling the wave propagation in such materials is discussed.

2. Modelling

Modeling propagation of ultrasound in high porosity materials saturated with fluid is mostly realized using two-phase theory of fluid-saturated porous materials proposed by Biot [1, 3].

According to the Biot’s theory [1], the linearized linear momentum equations for the solid and fluid phase are:

\[ \rho^s \frac{\partial v^s}{\partial t} - \nabla \cdot T^s = R^s, \]

\[ \rho^f \frac{\partial v^f}{\partial t} - \nabla \cdot T^f = R^f, \]

where \( \rho^s \) and \( \rho^f \) are the macroscopic mass densities, \( T^s \) and \( T^f \) denote stress tensors while \( R^s \) and \( R^f \) stand for interaction forces between solid (s) and fluid (f) phase. Assuming an isotropic elastic porous material fully saturated, Biot postulated that [1]:

\[ T^s = 2\kappa \epsilon^s + (\lambda + \kappa) \nabla \cdot \epsilon^s \]

\[ T^f = (\kappa \epsilon^f + R \epsilon^f) \mathbf{I}, \]

where \( \epsilon^s = \text{tr}(\epsilon^s), \epsilon^f = \text{tr}(\epsilon^f) \) are the dilatations of solid and fluid phase, while \( \epsilon^s, \epsilon^f \) are corresponding strain tensors, respectively. The elasticity constants \( \lambda, \kappa, Q, R \) can be related to physically well defined and measurable parameters of the porous medium: the porosity \( \phi \), the bulk modulus of the solid material \( (K_s) \) and fluid \( (K_f) \) and the bulk modulus of the drained skeleton \( (K_\phi) \). In the case of liquid saturated porous material these constants are given by the formulas proposed by Biot [2].

The simplified form of the elasticity constants can be derived for gas saturated porous medium known as the so-called rigid grain assumption, noting that the material of solid skeleton has a relatively low compressibility.

The interaction forces proposed by Biot [1] and defined by relation (Eqn (1)) include viscous and dynamic couplings, proportional to relative velocities and relative accelerations of phases, respectively:

\[ R^s = -R^f = \eta \frac{\partial}{\partial t} (v^s - v^f) + \rho^f (\alpha - 1) \frac{\partial}{\partial t} (v^f - v^s), \]

where \( \eta \) is the viscosity of fluid, \( \kappa \) and \( \alpha \) parameters describing structure of the porous material called permeability and tortuosity. In the case of gas saturated porous medium, the interaction forces may require incorporation of thermal effects.

3. Ultrasonic Experiments

Ultrasonic studies were performed for 4 samples of highly porous rigid polyurethane foams (Sawbones) frequently used as an alternative test medium for human cancellous bone. The porosities of the samples (obtained by X-ray microtomography SkyScan 1172) ranged from 82% to 95%. Ultrasonic experiments were done in two configurations. First, measurements for air saturated porous foams were performed. In this case non-contact air coupled ultrasonic transducers (The Ultran Group, USA) having center frequencies 100, 200 and 500 kHz, were used. Then, the same samples were vacuum-water saturated and the same experimental protocol as for dry specimens was applied using immersion transducers (Panametrics, USA). Based on the measured signals transmitted through the specimens and reference medium (air/water) the wave parameters: phase velocity and attenuation coefficient as a function of frequency are calculated, following...
the procedure described elsewhere [8].

4. Results

In Fig. 1(a) and Fig. 1(b) the phase velocity and attenuation coefficient as the functions of porosity for air and water saturated porous foams obtained both from ultrasonic studies and predicted by the Biot’s model are presented. The attenuation coefficient increases with frequency, both for air and water saturated samples. Moreover, there is observed a decrease of the absolute values of attenuation as the porosity increases and that the attenuation is higher in the case of air filled materials.

Considering the phase velocity it is visible that the mostly slow wave behavior appears. Interestingly it is observed that in the case of water saturated samples the phase velocity is higher than the SOS in the fluid filling pores, while in the air saturated materials the inverse behavior appears. Moreover, it is visible that as the porosity increases the phase velocities for air and water saturated specimens tends to the values of SOS in air and water, respectively.

It is worth noting that in the case of water saturated specimens (Fig. 1(a)) the experimental results correspond to the predictions to fast longitudinal wave, while in the air saturated materials (Fig. 1(b)) the measured wave parameters reflect the properties of the slow longitudinal wave.

5. Summary and Conclusions

The paper examined the contribution of the properties of the solid and pore fluid to velocity dispersion and attenuation mechanisms for high porosity foams (Sawbones) saturated with water and air. Moreover the suitability of the macroscopic Biot theory, commonly used for modeling ultrasonic wave propagation in low (medium) porous materials, was discussed.

It was found experimentally that the properties of the fluid significantly influence the studied wave parameters. A good quantitative agreement was found between wave parameters (phase velocity and attenuation coefficient as a function of porosity) measured experimentally and predicted by the Biot’s model.

Based on the obtained results one can conclude that the theoretical predictions of the Biot’s theory of wave propagation in fluid saturated materials can be applied for high porosity foams.

References

Experimental and numerical analysis of sandwich panels with composite core

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Abstract

In the paper analysis is conducted of sandwich panels with a composite core. The sandwich panel consists of two external steel facings and a composite soft core. In the paper it is assumed that the core is made of polyurethane foam or mineral wool, or a combination of these two materials. Parameter $\beta$ is defined as the ratio of the thickness of the mineral wool layer to the total thickness of the core, varying from 0.0 to 1.0. Due to the core layer the sandwich panel provides a high load bearing capacity and a high level of insulation. The polyurethane foam core is characterized by a very low weight and high thermal insulation properties, while the mineral wool core is characterized by high acoustic insulation properties and an excellent fire resistance. It is assumed that the combination of both materials in a single core layer makes it possible to take advantage of both materials. In the paper laboratory test are presented. According to the material parameters obtained experimentally, the FE simulations were conducted. A satisfactory correlation was obtained between laboratory tests and FE models for a four-point bending of sandwich beams with a composite core.

Keywords: sandwich panels, composite core, numerical analysis, laboratory tests, four point bending

1. Introduction

Sandwich panels are composite structures consisting of two thin external facings and a thick core. In building industry applications the facings are high-strength steel or aluminium sheets, while the core is a low-density and polyurethane foam, mineral wool or polystyrene layer. Due to the core layer a sandwich panel provides a high load bearing capacity coupled with a relatively low weight. The functional quality of sandwich panels depends on the type of the core material. High thermal insulation can be provided by all materials mentioned, nevertheless the bulk density of mineral wool is twice as high as that of the others. Thus, the requirement of a high acoustic insulation makes, mineral wool the most appropriate to use. When anion resistance is required, only a sandwich panel with mineral wool core layer meets the fire safety requirements.

In the paper it is assumed that the combination of polyurethane foam and mineral wool in one core layer makes a flexible to take benefit of both materials, i.e. suitable fire resistance, acoustic and thermal insulation and a relatively low weight.

2. Problem formulation

In the paper it is assumed that the core was made of polyurethane foam (PUR) or mineral wool (MW) or is a combination of these two materials. The introduced $\beta$ parameter, defined as the ratio of the thickness of the mineral wool layer to the total thickness of the core, varied from 0.0 to 1.0, with a step equal 0.25. Therefore the following cases were considered: $\beta = 0.0$ core made of PUR, $\beta = 1.0$ core made of mineral wool MW, $\beta = 0.25$ core which consists of 25% of MW and 75% of PUR, $\beta = 0.50$ core which consists of 50% of MW and 50% of PUR and $\beta = 0.75$ core which consists of 75% of MW and 25% of PUR, see Fig. 1. The test specimens with composite core layer ($\beta = 0.25, \beta = 0.50, \beta = 0.75$) were prepared in the laboratory of the Poznan University of Technology (Institute of Structural Engineering). The PUR and MW specimens were cut with the assumed proportions longitudinally and glued together with epoxy glue. The total thickness of the specimens was about 100 mm. A similar problem, was presented in [2], for specimens of $\beta = 0.5$.

3. Laboratory tests

The following material parameters of the facings were obtained in the tension test on the rectangular specimens with zinc coat layers: $E_F = 190$ GPa, $\nu_F = 0.3$, $\sigma_{Ft} = 280$ MPa, $\sigma_{Ff} = 370$ MPa, $\nu_{Ff} = 0.0015$.

Material parameters of the core layer the Kirchhoff modulus ($G_C$) and Young modulus ($E_C$) were determined by means of a four-point bending test and the compressive test, respectively. The tests were carried out for each arrangement of the core layer, i.e. for $\beta$ from step 0.0 to 1.0, 0.25. The Poisson ratio of the PUR ($\nu_{PUR} = 0.05$) and MW ($\nu_{MW} = 0.00$) were obtained from the literature [1]. The testing procedures were in accordance with EN 14509 [3], hence, in all tests the loading rate caused a displacement equivalent to 10% of specimen thickness per minute (10 mm/1 min). In the case of test samples with both insulation materials along the core ($\beta = 0.25, 0.50, 0.75$) the two cases were considered, namely PUR material on top (case A) and MW material on top (case B). According to $\beta$ parameter, five independent trials were considered.

The compressive tests were performed on cubic specimens with the side length equal 100 mm. During the test the load-deflection curve was drawn and Young’s modulus $E_C$ of the core was estimated. According to the load-deflection curve the stress-strain relation was obtained. Shear tests (four-point

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bending) were performed on prismatic beams 1000 mm long and 100 mm wide and high. The sandwich beam span of 900 mm was taken. During the test the load-deflection curve was drawn, and both the Kirchhoff modulus $G_C$ and the shear strength $f_{Cv}$ of the core were obtained. For all tested specimens the shear failure mechanism was obtained. It is interesting to conclude that the failure mechanism was always manifested by shear of the PUR layer, see Fig. 2.

Figure 2: Example of a shear failure mechanism of the sandwich beam for case A and B in four point bending ($\beta = 0.25$)

4. Numerical analysis

Another aim of the research was the verification of the numerical model, describing the nonlinear mechanical and kinematical response of a sandwich beam with composite core. Nonlinearity in the sandwich beam performance arises from the assumed nonlinear stress-strain relations of face and core materials. Linear material models were also considered. The numerical model in Abaqus/CEA environment corresponds to the experimental four-point bending test.

The 3D solid model with shell skin faces was implemented. The calculations were performed using the Newton-Raphson procedure. For all three mesh sizes examined: 5, 10 and 25 mm, but the obtained numerical results were almost identical. The differences were less than 0.2%. The finite elements C3D8R and S4R were used for the core and the faces, respectively.

The relationships between the applied force and the vertical deflection of the beam for $\beta = 0.75$ (cases A and B) is shown in Fig. 3. Dotted lines (three for every case) represent laboratory tests, a dashed line refers to the linear FE model, while a continuous line refers to the FE model with nonlinear constitutive law.

Figure 3: Comparison of laboratory and numerical results of four point bending of sandwich beams with composite core ($\beta = 0.75$, case A and B)

Table 1 compares shear stresses for different types of core (different $\beta$ values), obtained from numerical analysis and laboratory trials. These shear stresses correspond to the mean value of the limit force obtained in the experiment. For $\beta = 0.75$, the limit point corresponding to this limit force in the numerical model is indicated in Fig. 3 by a black square.

Table 1: The differences between shear stresses obtained from FE analysis and shear stresses from experiments

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Case</th>
<th>$F$ [kN]</th>
<th>$f_{Cv}$ [MPa]</th>
<th>Diff. [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lab</td>
<td>Lab</td>
<td>FEA</td>
<td></td>
</tr>
<tr>
<td>0.00</td>
<td></td>
<td>2.03</td>
<td>0.118</td>
<td>0.121</td>
</tr>
<tr>
<td>0.25</td>
<td>A</td>
<td>1.04</td>
<td>0.068</td>
<td>0.067</td>
</tr>
<tr>
<td>0.25</td>
<td>B</td>
<td>1.04</td>
<td>0.066</td>
<td>0.060</td>
</tr>
<tr>
<td>0.50</td>
<td>A</td>
<td>0.78</td>
<td>0.041</td>
<td>0.044</td>
</tr>
<tr>
<td>0.50</td>
<td>B</td>
<td>0.97</td>
<td>0.063</td>
<td>0.061</td>
</tr>
<tr>
<td>0.75</td>
<td>A</td>
<td>0.82</td>
<td>0.057</td>
<td>0.055</td>
</tr>
<tr>
<td>0.75</td>
<td>B</td>
<td>0.80</td>
<td>0.055</td>
<td>0.050</td>
</tr>
<tr>
<td>1.00</td>
<td></td>
<td>0.70</td>
<td>0.048</td>
<td>0.043</td>
</tr>
</tbody>
</table>

The corresponding shear stress distributions (for $\beta = 0.75$) obtained from the FE model are visualized in Fig. 4. For cases A and B the shear stress concentration points in the numerical model coincide with the location of the failure visible in Fig. 2.

Figure 4: Distribution of the shear stresses in sandwich beams with a composite core ($\beta = 0.75$) for cases A and B

5. Concluding remarks

The core considered in this paper combines the advantages of polyurethane and mineral wool. A proper prediction of structural behaviour of such composite structure requires a number of experimental studies or a reliable numerical model. There is a satisfactory agreement between the FE model and laboratory tests for all considered sandwich beams with a composite core subjected to four point bending. It is possible to assume that the stress and strain distribution in each layer is correct. Detailed results and further studies will be presented.

References


The energy criteria of plastic flow for aluminum foam in complex load state

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Abstract

The energy-based methods are very important in description of mechanical properties of material. Wide and important group of materials are porous materials like metal foams. In the current paper the phenomenological model, based on energy of nonlinear elastic and porous material, was proposed, and the example for aluminium foam was presented. On the base of phenomenological approach, material homogenization and geometrical interpretation of deformation process, the strain energy density function was constructed and the material stability criteria due to plastic flow were formulated. Because the nonlinearity of material was taken into account, then the stress-strain relations were written in differential form, what is continuation of the investigation carried out in [5], where linear material was assumed for small deformations, and original part of the work. The approximation of stress-strain relation was taken in the shape of parabolic curve, what is sufficient approximation and lets for very easy drafting of the stability regions.

Keywords: energy-based method, strain energy density function, Sylvester’s theorem, energy conservation principle

1. Introduction

Phenomenological modelling of mechanical properties of material is a very important part in design process of structures. The knowledge about behaviour of material in complex load state and possibility of prediction its response are very important sources in assessment process of construction. Interesting group of materials are porous structures, of small density, small weight to be used as a core in beam construction (see [1]). One of the most important methods in modelling such type of materials can be method based on energy. The energy-based methods were successfully used in description of different homogenous materials (see [2], [3], [4]). The main aim of current paper is construction of constitutive relations and energy criteria of stability, due to initiation of plastic flow in nonlinear material, using the example of aluminum foam in complex load state.

2. Geometrical interpretation of deformation process

General assumptions are as follows:
- material is nonlinear and isotropic and has porous structure,
- material can be assumed as homogeneous without unloose of majority (due to the fact that in case of real engineering applications the dimensions of material are very much bigger in comparison with the average dimensions of the material pores),
- stress-strain relation coefficient (the first order derivative of the stress relative to the strain) is not constant value and transversal deformation coefficient (the first order derivative of the transversal strain relative to longitudinal strain) is constant value,
- deformation process takes place in constant temperature and is very slowly, so the temperature influence and kinetic energy can be respectively neglected.

Let’s take into consideration the main relations between stress and strains for nonlinear porous material:

\[ \sigma_i = (1 + \tau_i) \frac{\partial \varepsilon_i}{\partial e} - \sum_{j \neq i} \tau_{ij} \frac{\partial \varepsilon_j}{\partial e} \]  

(1)

and assume that the stress-strain relation and transversal deformation coefficient are given in the form:

\[ \sigma_i = a \varepsilon_i + b \varepsilon^2 + c \]  

(2)

The constructed on the base of energy conservation principle, strain energy density function takes the form:

\[ W(\varepsilon, \varepsilon_2, \varepsilon_3) = \int_0^t \left[ \tau_1(t) \varepsilon_1(t) + \tau_2(t) \varepsilon_2(t) + \tau_3(t) \varepsilon_3(t) \right] dt \]  

(3)

where deformation path, as the set of points in deformation space, which represent subsequent deformation states, is given as:

\[ C: \varepsilon_i(t) = \varepsilon_i^f \text{ for } t \in (0, 1) \text{ and } i = 1, 2, 3 \]  

(4)

After substitution of (2) we receive:

\[ W(\varepsilon, \varepsilon_2, \varepsilon_3) = \frac{2}{1 - \varepsilon} \sum_{i=1}^{3} \left[ \varepsilon_i + \varepsilon^2 \right] \left[ \varepsilon_i^f + \varepsilon^f \right] \]  

(5)

where \( \varepsilon_1, \varepsilon_2, \varepsilon_3 \) are principal stress components.

3. Stability criteria

Material is in stable state if every change of current deformation state needs a work of external loads. In other words material is in stable state if second order variation of its strain energy density function is positive. So material is in stable state when:

\[ \delta^2 W \geq 0 \Rightarrow \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j} \delta \varepsilon_i \delta \varepsilon_j > 0 \]  

(6)

and on the base of Sylvester’s theorem if we have:
The plastic flow takes place if only shape deformation increases and the increment of the relative volume change of elementary material piece equals zero. Hence the first order deformation state is still elastic and stable, and plastic flow of material is not possible if:

\[
\delta^2 W(\varepsilon_1, \varepsilon_2) = \frac{3}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} A_{ij} X \left( \frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j} \right) \delta \varepsilon_i \delta \varepsilon_j > 0
\]

so when:

\[
\begin{bmatrix}
B_{11} & B_{12} & B_{13} \\
B_{21} & B_{22} & B_{23} \\
B_{31} & B_{32} & B_{33}
\end{bmatrix}
\begin{bmatrix}
\delta \varepsilon_1 \\
\delta \varepsilon_2 \\
\delta \varepsilon_3
\end{bmatrix} > 0,
\]

where

\[
B_{ij} = 2A_{ij} X \left( \frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j} \right)
\]

so that:

\[
B_{11} > 0, B_{22} > 0, B_{33} > 0,
\]

\[
B_{12}, B_{13}, B_{23} > 0,
\]

\[
A_1 = (1 + \varepsilon_1)(1 + \varepsilon_2(\varepsilon_1, \varepsilon_2)), A_2 = (1 + \varepsilon_2)(1 + \varepsilon_2(\varepsilon_1, \varepsilon_2)), A_3 = (1 + \varepsilon_3)(1 + \varepsilon_3),
\]

\[
X = \sum_{i=1}^{3} \sum_{j=1}^{3} \frac{\partial^2 W}{\partial \varepsilon_i \partial \varepsilon_j}
\]

4. Example

Let us take into consideration the experimental material characteristic of aluminium foam and its approximation.

The strain energy density function (6) takes the form:

\[
W(\varepsilon_1, \varepsilon_2, \varepsilon_3) = 3.64 \sum_{i=1}^{3} \left[ 0.45(\varepsilon_1 + \varepsilon_2 + \varepsilon_3) + 0.1\varepsilon_i \right]
\]

\[
(\varepsilon_1^2 + 1.0 \varepsilon_2^2 + 1.0 \varepsilon_3^2)
\]

The strain energy density function and the region of stability (according to (7)) in plane 1-2 take the form:

\[
W(\varepsilon_1, \varepsilon_2; \varepsilon_3 = 0)
\]

Figure 2: Plot of strain energy density function for \(\varepsilon_3 = 0\)

5. Conclusions

- The strain energy density function is useful in the determination of material stability region.
- The convexity of function (5) on the base of (9) does not have to mean the stability according to plastic flow.
- The stable region is symmetric due to material isotropy.

References

Probabilistic Methods in Mechanics

organized by M.M. Kamiński and J. Görski
Fatigue life assessment with the use of exponential and power law functions for variable amplitude loading

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Abstract

The paper presents a comparison of two new damage accumulation hypotheses for fatigue life assessment proposed by the authors. The first hypothesis is based on an exponential function, the second on a power law. Both functions are based on theory used in psychology. The hypotheses have been used in the calculation process of the mean damage degree. The calculations have been done, inter alia, for carbon steel, graphite IG-110, P91 and P92 steel, for AISI 1141 steel and many more.

Keywords: damage degree, fatigue, fatigue damage accumulation

1. Introduction

Constructions, machines and their components are often exposed to time varying forces. These forces give rise to the phenomenon of the destruction of the material below the limit of its tensile strength. The impact of fatigue is taken into account in the case of strength calculations using appropriate methods and algorithms. These methods used advanced algorithms counting the cycles causing damage. The most popular are the linear Palmgren-Miner and Sorensen-Kogayew’s hypotheses. The advantage of this solution lies in getting fatigue directly in the number of cycles or time. These algorithms do not take into account the order of occurrence of the amplitudes of damage. Unfortunately, while reviewing the literature not many papers are found which take into consideration the effect of sequence of the loading history on fatigue life. A proposition for a function used in the case of research into memorizing information for the calculation of fatigue life can be found in the work [1]. Another paper, which takes into account the load sequence, is a paper by Ishiyama [2], in which graphite was tested. Due to the characteristics of this material, the loading sequence takes a major role. The work uses experimental fatigue results obtained from the literature for chosen steels and graphite. A comparison of the proposed exponential and power law functions used in the damage accumulation procedure has been done. The proposed hypothesis was used in the calculation of the mean degree of damage, to strive towards the value of 1.

2. New damage accumulation hypothesis

The fatigue damage accumulation is based on the loading history and on the damage accumulation with the use of a factor is dependent on material memory [1]. The Palmgren-Miner hypothesis, which was mentioned in the Introduction, is based on the equation:

\[ D = \sum_{i=1}^{N_f} \frac{1}{N_i} \]  

where: \( N_i \) - the number of cycles determined from the Basquin or Wöhler characteristic.

While it is not perfect, the authors proposed two new hypotheses, to get better results. The first suggested hypothesis is based on an exponential function, where the calculations are made according to the formula:

\[ D = \sum_{i=1}^{N_f} \left( 1 - c \cdot e^{-N_i/N_f} + c \right) \frac{1}{N_f} \]  

where: \( c \) - asymptote - values in the range \( <0÷1> \), \( N_f \) - the current number of cycles to destruction at the instant \( t \), \( N_f = \sum n_i \) - is the total number of cycles until failure.

The second proposed hypothesis is based on the power function, where the calculations are performed according to the relationship:

\[ D' = \sum_{i=1}^{N_f} \left( 1 - \phi \cdot \left( \frac{N}{N_f} \right)^\phi \right) \frac{1}{N_f} \]  

where: \( \phi \) – power exponent – values range \( <0÷1> \).

Both functions (2) and (3) were proposed to describe the human memory, where their main source is psychology [1], and have been adopted for fatigue damage accumulation in materials.

3. Material comparison

The materials used for the comparison are, inter alia: carbon steel [4], graphite IG-110 [2], steels P91 and P92 [3], AISI 1141 steel [5] and others. The results for IG-110 graphite was used for basic calculations. Table 1 presents its basic mechanical properties. The damage degree is presented in Fig.1 for the exponential function, analogously, the same was done for the power law function, presented in Fig. 2.

Table 1: The basic mechanical properties of IG-110 graphite

<table>
<thead>
<tr>
<th>Material</th>
<th>( R_m ) [MPa]</th>
<th>Apparent density [g/cm³]</th>
<th>Compressive modulus [MPa]</th>
<th>Bending modulus ( E_{bend} ) [MPa]</th>
<th>( E ) [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graphite IG-110</td>
<td>25.3</td>
<td>1.78</td>
<td>76.8</td>
<td>37.2</td>
<td>10.2</td>
</tr>
</tbody>
</table>
Analysis of the results of mean damage degree dispersion $D'$ presented in Fig.1 and Fig.2 shows that in the case of an exponential function (2) for $c=1$ we obtained the Palmgren-Miner hypothesis (1), while the mean damage degree approaches the value 1 for $c=0$, and that affects the function (2) and reduces to form

$$D' = \sum_{i=1}^{N} \frac{1}{1 - e^{-e^{-N/f}} \sum_{j=1}^{N} \frac{1}{N_j}}.$$  \hspace{1cm} (4)$$

Whereas, for the power law function for the power exponent $\phi=0$ the Palmgren-Miner hypothesis was obtained (1). The value of damage degree closest to 1 was obtained for $\phi=1$, which affects the function (3) and reduces this function to the form

$$D' = \sum_{i=1}^{N} \left( \frac{1}{\ln(2) \left( 1 + \frac{N}{N_f} \right)^{-1}} \right) \frac{1}{N_j}.$$  \hspace{1cm} (5)$$

For the analyzed case of graphite IG-110 results for the proposed functions were similar. The sequence L-H for the exponential function gives $D'=1.08$, for the power law $D'=1.11$. For the sequence H-L it is $D'=0.65$ and 0.63. A further verification of the proposed models is necessary.

4. Remarks and conclusions

The remaining results can be considered promising. The hypotheses can be widely used because of the simplicity of the exponential and power law functions. The proposed hypotheses can be applied in the case of multi-sequence loads and for different construction materials.

References


The reliability assessment of a steel industrial hall

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Abstract

The study presents a probabilistic approach to the problems of static and stability analyses of a steel industrial hall. Structural design parameters were defined as deterministic values and random variables. The latter were not correlated. The Hasofer-Lind index was used as a reliability measure. The FORM method was employed as a primary research tool. In order to validate the correctness of the computations, SORM, Monte Carlo and Importance Sampling methods were used. The reliability index sensitivity to random variables was defined.

Keywords: random variables, FORM method, Hasofer-Lind reliability index, sensitivity of reliability index.

1. Introduction

Probabilistic methods allow the quantitative assessment of structure reliability. These methods apply the information on types of distribution of design variables and their parameters. Depending on the manner of computing the failure probability, the methods can be classified to the field of approximation and simulation ones [1]. The FORM method, which is one of the approximation methods, was a basic tool employed in the study. In the examples presented below, the STAND software [2] was applied to the reliability analysis.

2. Examples

2.1. Probabilistic approach to the static analysis of steel frame

Reliability analysis was carried out for a steel industrial hall located in Poland. The basic supporting structure of the building consists of steel frames (Fig. 1). All members are made of S235JR steel, the yield point of which is f_y=235 MPa and modulus of elasticity is E=210 GPa. Columns are made from HEB 450 I-shaped beams, and girts from HEA 700 I-shaped beams. Reliability analysis was performed for the most unfavourable load configuration, i.e. permanent load and uniform snow load (Fig. 1).

Figure 1: Supporting structure of a steel industrial hall

The limit functions imposed on the load bearing structure are displacement constraints related to the serviceability limit state and stress constraints related to the ultimate limit state. The initial analysis of the displacement state and internal forces, performed with the Robot software, made it possible to locate the sites of their extreme values to occur. Using the Finite Element Method, the formulas for the maximum vertical displacement and the maximum bending moment were determined with the Mathematica software. Limit functions were defined as functions of random variables grouped in vector: \( \mathbf{X} = \{ X_1, X_2, X_3, X_4, X_5, X_6, X_7 \} \). Below, probabilistic quantities are specified:

- \( X_1 \) – permanent load from the covering self-weight from 1m of the roof plane, \( X_1 \sim N(12.28, 2.46) \),
- \( X_2 \) – permanent load from the cladding self-weight from 1m of the curtain wall, \( X_2 \sim N(4.83, 0.96) \),
- \( X_3 \) – permanent load from a fragment of the roof and wall at the eave reduced to a focused force acting on the structure column, \( X_3 \sim N(8.59, 1.90) \),
- \( X_4 \) – variable load from uniform snow load imposed on the roof plane, \( X_4 \sim N(12.96, 2.60) \),
- \( X_5 \) – variable load from a fragment of the roof and wall at the eave reduced to a focused force acting on the structure column, \( X_5 \sim N(6.48, 1.42) \),
- \( X_6 \) – elastic modulus for S235JR, \( X_6 \sim N(231\cdot10^6, 23.1\cdot10^6) \),
- \( X_7 \) – yield point for S235JR, \( X_7 \sim N(263.2\cdot10^3, 39.48\cdot10^3) \).

Member section areas, member inertia moments, and lengths of columns and girts were assumed to be deterministic quantities. Limit functions were formulated. Those describe the following:

- serviceability limit state

where \( q_{\text{dop}}=L/250 \) denotes permissible displacement in accordance with PN-EN 1993-1-1 standard. For the structure under consideration, value \( q_{\text{dop}}=0.096 \) m.

- ultimate limit state

where

\[
\begin{align*}
M_{\text{ult}} &= 634878X_1 + 825.69X_2 + 275.23X_3 + 631377X_4 + 275.23X_5,
q_{\text{ult}} &= L/250 \text{ denotes permissible displacement in accordance with PN-EN 1993-1-1 standard. For the structure under consideration, value } q_{\text{ult}} = 0.096 \text{ m.}
\end{align*}
\]

\[
\begin{align*}
M_{\text{ult}} &= 25.69X_1 - 0.11X_2 - 0.04X_3 + 25.53X_4 - 0.04X_5,
W_s &= \text{the elastic index of the column strength. For the HEB 450 profile, value } W_s=0.00624 \text{ m}^3.
\end{align*}
\]
The values of the Hasofer-Lind reliability index were determined with the FORM method, and for the sake of comparison, with other methods, i.e. SORM, Monte Carlo and Importance Sampling. The STAND software was used. The results are presented in Tab. 1.

Table 1: Comparative analysis of reliability index computations

<table>
<thead>
<tr>
<th>Type of analysis</th>
<th>Reliability method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FORM</td>
</tr>
<tr>
<td>Serviceability limit state</td>
<td>1.952</td>
</tr>
<tr>
<td>Ultimate limit state</td>
<td>3.794</td>
</tr>
</tbody>
</table>

In addition, graphs that show the sensitivity of the reliability index to random variables for both limit functions were plotted (Fig. 2).

![Graph showing sensitivity of reliability index to random variables](image)

Figure 2: Sensitivity of the reliability index $\beta$ to random variables for a) ultimate limit state b) serviceability limit state

2.2. Probabilistic approach to the stability analysis of steel frame

The frame analyzed in Example 2.2. was made of the material having the same characteristics as those in Example 2.1. The external loads, to which the frame was subjected, were also the same, but the columns were additionally loaded with a compressive force $S$. The latter can be a reaction, transferred from the external structure above the building roof. The value of the critical force was determined using the Finite Element Method with the Mathematica software. In the computations, the geometric stiffness matrices for compression elements were used. The value of critical force obtained was $S_{kr}=3.13 \times 10^4$ kN. The probabilistic approach to the stability analysis involved the determination of the reliability index value for columns loaded with the compressive force, which constituted a certain percentage value of the critical load. In the reliability analysis, deterministic and probabilistic variables were used. These were defined in Example 2.1. When the structure was loaded with successive compressive forces, new limit functions were defined. The general formula for the limit function can be represented as follows:

- for the serviceability limit state, in accordance with Eqn (1), where $q_{\text{max}}$ describes the vertical displacement of roof ridge node
- for the ultimate limit state, in accordance with Eqn (2), where $M_{\text{max}}$ describes the bending moment at the column-to-foundation connection (cross-section $\beta-\beta$).

Tables 2 and 3 present the values of the Hasofer-Lind reliability index which was determined with the FORM, SORM, Monte Carlo and Important Sampling methods for the serviceability and ultimate limit state, respectively. The bending moment, for which the static analysis of reliability was performed, is located in the column-to-beam connection (cross-section $\alpha-\alpha$), and it is the highest value of the bending moment in the analyzed structure. As regards the stability analysis, it is important to take into account the effect of compressive forces on the magnitude and distribution of cross-sectional forces. In frame structures, the greatest impact of these forces on the value of the moments in column-to-foundation connection (cross-section $\beta-\beta$) is observed. The value of this bending moment is twice smaller than that for which the static analysis was performed. Therefore, the stability analysis for the ultimate limit state is restricted to presenting only the extreme cases – the lack of compressive forces and force load of 95% $S_{kr}$.

Table 2: Comparative analysis of the reliability index computations for different values of the critical load for the serviceability limit state

<table>
<thead>
<tr>
<th>Value of the critical load</th>
<th>FORM</th>
<th>SORM</th>
<th>Monte Carlo</th>
<th>Importance Sampling</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%$S_{kr}$</td>
<td>1.952</td>
<td>1.949</td>
<td>1.948</td>
<td>1.922</td>
</tr>
<tr>
<td>70%$S_{kr}$</td>
<td>1.515</td>
<td>1.514</td>
<td>1.518</td>
<td>1.511</td>
</tr>
<tr>
<td>80%$S_{kr}$</td>
<td>1.446</td>
<td>1.445</td>
<td>1.446</td>
<td>1.452</td>
</tr>
<tr>
<td>95%$S_{kr}$</td>
<td>1.339</td>
<td>1.339</td>
<td>1.321</td>
<td>1.332</td>
</tr>
</tbody>
</table>

Table 3: Comparative analysis of the reliability index computations for different values of the critical load for ultimate limit state

<table>
<thead>
<tr>
<th>Value of the critical load</th>
<th>FORM</th>
<th>SORM</th>
<th>Monte Carlo</th>
<th>Importance Sampling</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%$S_{kr}$</td>
<td>4.937</td>
<td>4.930</td>
<td>5.020</td>
<td>4.999</td>
</tr>
<tr>
<td>95%$S_{kr}$</td>
<td>4.784</td>
<td>4.776</td>
<td>5.002</td>
<td>4.835</td>
</tr>
</tbody>
</table>

3. Conclusions

The analysis of the results demonstrates that the FORM method is good enough and definitely much simpler to apply. The relative error of the reliability index was estimated for individual methods at the assumption that the Monte Carlo (where sample size is 10000) served as the reference method. In Example 2.1, concerning the ultimate limit state, the error amounted to: for the FORM method – 2%, for the SORM method – 0.5%, for the Importance Sampling method – 2.6%. As regards the serviceability limit state, the error values were as follows: for the FORM method – 0.2%, for the SORM method – 0.1%, and for the Importance Sampling method – 1.3%. Another important component of the study was to investigate the sensitivity of the reliability index to changes in the probabilistic characteristics of the random variables under consideration. If the reliability index sensitivity due to the random variable $X_i$ is low when compared with other variables, it can be stated that the impact of this variable on failure probability is small. Therefore, in successive computations it can be treated as a deterministic parameter. The probabilistic approach to the stability analysis shows the effect of large compressive forces on the reliability index value. In the serviceability limit state, between the extreme cases (the lack of compressive forces and force load of 95% $S_{kr}$), the reliability index was observed to decrease to 31%.

References

Abstract

The increased interest in telecommunication towers results from the explosive growth of mobile telephony and the Internet. The article presents an original idea of combining the commercial FEA program and created in the spreadsheet module to the first-order sensitivity analysis. The exchange of data between programs is made possible by the technology of COM (Component Object Model). In such an approach a commercial program solves the classical problems of mechanics, while the gradients of the state functional appearing in sensitivity analysis, can be calculated in a spreadsheet program implementing the equations known from the literature. Commercial programs for engineering calculations are increasingly using COM technology, so the above approach can be used widely in a variety of computational problems.

Keywords: structural sensitivity analysis, telecommunication tower, adjoint variable method, COM technology

1. Introduction

Rapid development of mobile telephony and the Internet led to the need to build supporting structures on which the communication systems are mounted. Telecommunication towers are commonly built of steel as spatial trusses. The problems of modeling of this type of structures are described in the publications [4,5]. Due to sensitivity analysis the designer receives additional information where and to what extent the changes to the design to achieve the expected change of its behavior. The results of the sensitivity analysis can be used in the design of new constructions; the reinforcement of existing structures; or in the parametric identification of the structure.

2. Conception

The article presents an original idea of combining the commercial program which is an application of finite element method and created in the spreadsheet module to the first-order sensitivity analysis. In this approach, a commercial program is used to solve problems of static and dynamic mechanics, any structural system (made from rods, shells, solids). The data is exchanged directly with a spreadsheet by using COM technology. The data transfer does not require tools such as "copy-paste", but occurs in real time. The investigated gradients of the state functional are calculated in a spreadsheet by adjoint variable method [1,2,3], and numerical integration is done automatically using the procedures written in VBA programming language.

2.1. FEM model

Considered a bar model of a real telecommunication tower with a height of 50m. Chords are modeled using beam elements and diagonals as truss elements. Wind is a major load, which was determined according to standard [6].

Figure 1: Telecommunication tower a) real structure, b) FEM model

2.2. First order sensitivity analysis – example of static analysis

Bar models of telecommunication towers are systems of dominating axial forces [5]. Basic formulas of the sensitivity analysis for gradients of the state functional can be derived from the principle of virtual work [1,2,3]. Displacement in any node of the lattice can be calculated from the equation of virtual work in the case of a discrete system adjusted to the form

$$\tilde{F} \cdot \tilde{u} = \sum_{i=1}^{n} \frac{N_i}{E_i A_i} \cdot L_i$$

(1)
Assuming that the bars model of antenna tower consists \( n \)-elements, in which dominate normal forces, the components of the sensitivity vector \( W_S \) can be presented as

\[
W_{S,i} = - \frac{N_i N_j}{(EA)_b} \cdot L_i
\]  
(2)

where: 
\( W_{S,i} \) – sensitivity coefficient of \( i \)-th element, 
\( L_i \) – length of \( i \)-th element, 
\( N_i \) – normal force in the \( i \)-th element from primary load, 
\( N_j \) – normal force in the \( i \)-th element in adjoint system, 
\( (EA)_b \) – axial stiffness of \( i \)-th element, 
\( (EA)_b \) – derivative of the rigidity with respect to variable design \( b \).

The first-order sensitivity equation can be represented by a the synthetic formula

\[
\delta S = W^T \delta b \tag{3}
\]

where:
\( \delta S \) – variation of the state variable at the selected point of tested system, 
\( W=S\{W_{S,1}, W_{S,2}, \ldots, W_{S,n}\}^T \) – vector of the sensitivity coefficients of the first-order, 
\( \delta b=\{\delta b_1, \delta b_2, \ldots, \delta b_n\}^T \) – variations vector of the design variables in the selected \( n \)-elements of tested system.

Due to a diverse range of absolute values of the state variables and design variables, in practical applications it is convenient to apply relative expressions

\[
\frac{\delta S}{S} = \delta W^T \cdot \delta b \tag{4}
\]

where:
\( \frac{\delta S}{S} \) – relative variation of the state variable, relative to the initial value of the variable \( S \), 
\( \delta W = [\delta W_{S,1}, \delta W_{S,2}, \ldots, \delta W_{S,n}]^T \) – vector of the relative sensitivity coefficients of the first-order, 
\( \delta b = \left[ \begin{array}{c} \frac{\delta b_1}{b_1}, \frac{\delta b_2}{b_2}, \ldots, \frac{\delta b_n}{b_n} \end{array} \right]^T \) – relative variations vector of the design variables, relative to their initial value in the selected \( n \)-elements of tested system.

In the example, the state variable adopted horizontal displacement of the top of the tower \( (S = u) \), while the design variable is cross-sectional area of the tower chords \( (b = A) \). The main system is the scheme of the wind load perpendicular to one of the walls of the tower. In turn, the adjoint system is a unit force applied at the node and at the direction of the displacement that is looking for. In this case, the relative sensitivity coefficient of \( i \)-th element model of the tower will have the following form

\[
\delta W_{u_i} = - \frac{N_i N_j}{u \cdot EA_i} \cdot L_i
\]  
(5)

Designation of the variables in equation (5) are the same as in the formulas (1) and (2).

The results of the sensitivity analysis (SA), depending on the discretization of the system, shown in Fig. 2. Figure 2a) shows the results of SA in the chords distribution for 15 elements (elements with constant cross-section), while Figure 2b) illustrates the results of SA in the chords distribution for 132 computation elements (elements between nodes on the model).

Figure 2: Distribution of the relative sensitivity coefficients \( \delta W_{u_i} \): a) chords division for the 15 elements, b) chords division for the 132 elements (description in the text)

3. Conclusion

Figure 3 shows typical sensitivity analysis validation test, comparing the results of the parametric analysis and the sensitivity analysis of the tested structure. It can be clearly seen, that for relative variations of the state variables in the range for 20%, approximation the error does not exceed 10% related to the parametric analysis.

Figure 3: Comparison of strict results (parametric analysis) and approximate results (sensitivity analysis)

References


Numerical investigation of the effects of damping uncertainties on Algerian seismic code spectra by Monte Carlo simulation

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Abstract

The Monte Carlo method is used to perform a probabilistic second order analysis of the structural response in order to investigate the effect of critical damping uncertainties on the maximum seismic response of systems with Lognormally distributed random damping. The applicability of the proposed methodology is illustrated using design response spectra of Algerian Earthquake Resistant Regulations (RPA 2003) associated with each of four soil types: rock, firm, soft and very soft. The simulation results are presented in terms of variations of the amplitudes of response spectra and conclusions of engineering importance are given.

Keywords: Seismic design code, Response spectrum, Seismic code, Damping uncertainty, Monte Carlo simulation

1. Introduction

In order to reliably estimate the seismic response of a building, it is essential to quantify not only the effect of the stochastic nature of the seismic motion on the structural response, but also that of the uncertainty of the dynamic parameters [2, 3 and 4]. These uncertainties, mainly caused by the variation of material properties and approximations in the estimation of parameters of the mathematical model used, may introduce, for a given structure, a significant variation in reliability and response, and therefore it is often desirable to consider their effects in the analysis of its behavior and design.

The objective of this study is to estimate the effects of damping uncertainties on the dynamic response of structures. The Monte Carlo (MC) technique is used to numerically simulate the seismic responses and uncertainties are treated by considering the damping as a Lognormal random variable with characteristics obtained on the basis of statistical analysis of a wide range of structures and structural systems. The applicability of the proposed methodology is illustrated using the design response spectra of the Algerian Earthquake Regulations (RPA) respectively for four soil types: rock (S1), firm (S2), soft (S3) and very soft(S4) [5]. The simulation results are then presented for response amplitudes lying in intervals at 68% confidence level to illustrate quantitatively the effect of damping uncertainty on maximum structural response and conclusions of importance for engineers are given.

2. Monte Carlo simulation

In this work, we use the Monte Carlo method to estimate the value \( S = S_d(\mu, T) \) of the acceleration response spectrum associated with a single degree of freedom (SDOF) system with fundamental period \( T \) and uncertain damping factor \( \Psi \) with mean value \( \mu \) and following the probability distribution function (PDF) \( F(\xi) \).

We define the indicator function \( I(\Psi) \) so that \( I(\Psi) = 1 \) if the event \( E = \{ S_\alpha(\xi, T) \} \) falls in the interval \( \{ S - \Delta S; S + \Delta S \} \) occurs and \( I(\Psi) = 0 \) otherwise.

In order to estimate the probability \( P = P(E) \) by the MC method, a sample value for basic variable \( \xi \) with a cumulative distribution \( F^{-1}(\xi) \) must be drawn. The inverse transform method can be used to obtain the random variate, in which a uniformly distributed random number \( u_i \) (0<\( u_i <1 \)) is generated so that \( \xi_i = F^{-1}(u_i) \). Hence the estimate of \( P \) is obtained

\[
\hat{P} = \frac{1}{N} \sum_{i=1}^{N} I(u_i) = \frac{N_E}{N}
\]

where \( \hat{P} \) represents the Monte Carlo estimator of \( P \), \( N_E \) the number of occurrence of the event \( E \) and \( N \) is the number of independent random numbers generated in each cycle.

The accuracy of \( \hat{P} \) can be estimated in terms of its variance. Indeed, assuming that each simulation cycle is a Bernoulli trial with probability \( P \), the number of occurrence of \( E \) in \( N \) tests can be considered to follow a binomial distribution, so that :

\[
Var(\hat{P}) = Var\left(\frac{N_E}{N}\right) = \frac{1}{N^2} Var(N_E) = \frac{(P(1-P))}{N} \cong \frac{(1-P)}{N}
\]

We note that since the exact value of \( P \) is not known, the value \( \hat{P} \) is used as approximation because \( E[\hat{P}] = P \).

Figure 1: Convergence of the probability \( P \) as a function of size of samples

Figure 1. shows the convergence of the probability \( P \) as a function of the size \( n \) of samples. The stability of the simulation was obtained from \( n = 30000 \) which is the value that we used to simulate response spectra values in the present study.
3. Numerical results

3.1. Statistical Characterization of Damping

Evaluation of damping in structures was undertaken by several investigators. In particular, Haviland, [1] shows that log-normal and Gamma distributions provide the best fit to the data. The coefficient of variation (COV) \( \sigma_{\mu} / \mu_{\mu} \) of damping values varied between 42 and 87%.

3.2. Sensitivity of the Response to Changes in the Values of COV

The above simulation technique was applied to generate response spectra compatible with RPA design spectra for each of the four soil types: S1, S2, S3 and S4. We choose to retain for this study the COV values \( C_{R} \) = 0.4. 0.6 and 0.8.

The uncertain damping is assumed to follow the log–normal distribution with mean \( \mu_{\xi} = 5\% \). The design parameters are: \( Q = 1.2, R = 5 \) and \( A = 0.3 \) [5].

We consider only the results obtained for rock (S1) and very soft soils (S4) taking \( C_{R} = 0.4 \) and \( C_{R} = 0.8 \), respectively. Regardless of the soil type and the \( C_{R} \) value, the results, illustrated in Fig. 2, show a low frequency range \([≤ 3Hz]\), corresponding to flexible structures, where the response of the oscillator is controlled mainly by the displacement of the support and there is a strict equality between maximum displacements whatever the value of damping considered.

Another frequency range \([≥ 8Hz]\), where the response of the oscillator is controlled mainly by the acceleration of the support and in which the influence of damping variations is rather small. It is worth noting that the largest fluctuations occur in the intermediate frequency range \([3Hz - 8Hz]\) where the

We also notice that there is a great sensitivity of the spectral response amplitudes when damping fluctuates, indeed, small changes in damping generate relatively large variations in the response.

These observations are summarized in Table 1. We notice first that for a given value of \( C_{R} \) and for both soil types, the values of the COV of spectral amplitudes (noted \( \sigma_{\mu_{CR}} \)) over the frequency range are characterized by approximately the same mean \( \mu_{\mu_{CR}} \) and standard deviation \( \sigma_{\mu_{CR}} \) values as well as the same range of variation. It is clear that values of \( \mu_{\mu_{CR}} \), that of \( \sigma_{\mu_{CR}} \) and those of extremes increase as \( C_{R} \) increases. Indeed, for both soil types, the value of \( \mu_{\mu_{CR}} \) varies from 11.2% to 21.7% while that of \( \sigma_{\mu_{CR}} \) varies from 3.6% to 5.9% when \( C_{R} \) changes from 40% to 80%. These results show also that for both soils, \( C_{R} \) values are ranging from 2.4% to 21.9%.

Table 1: Values of statistical characteristics of the RPA design response spectral amplitudes

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>( C_{R} ) = 40%</th>
<th>( C_{R} ) = 80%</th>
<th>( C_{R} ) = 40%</th>
<th>( C_{R} ) = 80%</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>( \mu_{\mu_{CR}} )</td>
<td>11.2</td>
<td>21.7</td>
<td>11.1</td>
</tr>
<tr>
<td></td>
<td>( \sigma_{\mu_{CR}} )</td>
<td>3.6</td>
<td>5.9</td>
<td>3.6</td>
</tr>
<tr>
<td>S4</td>
<td>( \mu_{\mu_{CR}} )</td>
<td>2.4 - 13.3</td>
<td>4.1 - 19.9</td>
<td>2.4 - 13.3</td>
</tr>
</tbody>
</table>

Similar results were obtained for the soil types S2 and S3 and for real response spectra associated with the record of Parkfield Earthquake of 26 June 1966.

4. Conclusion

In the work we investigated the effect of damping uncertainties on the structural seismic response. The damping was modeled by a random variable following a log–normal distribution with a 5% mean value. The seismic responses of structures, expressed in terms of acceleration response spectra, are then estimated using the Monte Carlo method.

The results show low and high frequency ranges associated with rigid and flexible structures, respectively, where the noticeable influence of damping results in small fluctuations of the response amplitudes. The largest fluctuations are obtained for the intermediate frequency range for which the damping effect is more significant.

It was also shown that small changes in the values of the damping generate relatively large variations of the response and that the fluctuations are more pronounced for higher variability of damping values. These foresee outcomes reflect the fact that the amplitude levels of the response are inversely proportional to the damping values.

This study can be extended while taking into account the effects of systematic uncertainties induced by the engineer in the values used for the dynamic parameters (mass, stiffness, natural frequency of vibration). The study can also be extended to study the influence of variations of the damping on the dynamic response of the structure by considering the aspects of the different seismic regulations in force around the world.

5. References


Reliability of the axisymmetric shell structure by the response function method and the generalized stochastic perturbation technique

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Abstract

The main idea of the paper is to demonstrate an application of the numerical analysis based on the generalized stochastic perturbation method and the response function technique to determine reliability indices for different performance functions concerning frequency of vibration, stresses or vertical deflections. Calculations were performed on the example of the steel shell structure with the randomized wall thickness of all its members. The obtained values of the reliability indices can be used in reliability assessment comparing the minimum one with its limit given in Eurocode and depends upon reliability class, reference period and type of limit states.

Keywords: stochastic perturbation method, response function method, shell structures, reliability indicator

1. Introduction

Computer analysis of civil engineering structures with random parameters has remarkably increasing influence on structural design process, optimization and reliability modeling because of variety of uncertainty natural sources and human activity driven reasons. In shell structures, one of the most important parameter is wall thickness, whose uncertainty may arise in the manufacturing processes or for example as the effect of their corrosion. The solution of the randomness problem contains reliability indicator verification according to the general rules provided by Eurocode [1]. There are plenty methods leading to obtaining the reliability indices, but in this paper, the stochastic perturbation-based method is used, which features are: relatively short computational time, no need for massive computers to provide the analysis and a relatively easy implementation into different FEM computer systems [2,3]. A computational implementation of this method [4] is carried out for dynamics and static problems using the FEM system ROBOT [5], the coefficients of response functions polynomial are computed in the computer algebra system MAPLE [6] from the 11 solutions of the original problem obtained for wall thicknesses taken around their mean value. This form of the response functions leads to determination of all their partial derivatives, with respect to wall thickness coefficient, which can be used for obtaining the probabilistic moments and final values of the reliability indices.

2. The generalized stochastic perturbation technique

The basic idea of the stochastic perturbation approach is an expansion of all random functions of the given problem via Taylor series of the required order about their expectations using the parameter \( \varepsilon > 0 \). In the case of some real function \( f(b) \) of the stationary random variable \( b(\omega) \), the following expression is employed:

\[
\frac{f(b(\omega))}{f(b(\omega))} = \sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!} \frac{\partial^n f(b(\omega))}{\partial b^n} |_{b=b_0} (\Delta b)^n
\]  

Having the input random variable with a symmetric probability density function, in the second order perturbation approach the first two probabilistic moments of the structural response function yield:

\[
E[f(b)|b=b_0] = f(b_0) + \frac{\varepsilon}{2} \frac{\partial^2 f(b)}{\partial b^2} |_{b=b_0} \mu_2(b_0)
\]  

and

\[
Var[f(b)|b=b_0] = \varepsilon^2 \mu_2(b_0) \left( \frac{\partial f(b)}{\partial b} \right)^2 |_{b=b_0}
\]

correspondingly, where \( \mu_2(b) \) denotes second order central probabilistic moment of the quantity \( b \).

3. Finite Element Analysis

Numerical tests were performed on the example of the BGŻ Arena’s steel roof structure presented in Fig. 1. Two independent computational models have been created - the first one (Model I) includes a detailed mesh of the entire shell, while Model II is based upon some surrogate spatial discretization. Therefore, the total number of 3D frame finite elements is reduced from 5785 to 2923 with parallel reduction of the nodal points - from 2120 to 1121, correspondingly. All the structural members have been assigned the properties of S235 steel. The chords were made from rectangular and the rest from circular hollow sections.

Figure 1: Global deformation of the BGŻ Arena’s steel roof structure

Modal analysis was performed in Model I verifying the first mode only, while global deformation of this structure in SLS (Fig. 1) and efficiency ratio of its members in ULS (Fig. 2) were both computed in Model II. The purpose of computational analysis is to determine fundamental frequency of the structure, the maximum of the reduced stresses and vertical deflections in characteristic and frequent combinations. The first magnitude...
has been received from Model I, and the rest – from Model II was.

Figure 2: Efficiency ratio of principal structural elements

4. Computational reliability analysis

Linear response functions (see Fig. 3) of reduced stresses, vertical deflections and first eigenfrequency in addition to the randomized element thickness were adopted after initial determination of their correlation coefficients – their modulus are approximately equal to one ranging from 0.99957 for reduced stresses to 0.99994 for fundamental frequency.

Figure 3: Linear dependence of the response function on wall thickness coefficient on the example of maximum of reduced stresses

The above-mentioned form allows obtain constant functions of expected values and quadratic functions of variance (Figure 4), on the assumption that wall thickness coefficient is the Gaussian input random variables defined by its expected value and coefficient of variation (α). In this case the values of skewness and kurtosis equal 0.

Reliability indices were determined using the second order perturbation-based numerical approach and making the following assumptions: (a) the vertical deflections are limited to 1/250 of the entire span, (b) admissible stress equals 235 MPa and (c) the difference between frequency of forced vibration and fundamental frequency should be bigger than 25%. Their variations upon the input coefficient of variation of wall thickness (α) were presented in Figs. 5a-5d.

Figure 4: Quadratic functions of variance on the example of maximum of reduced stresses

Figure 5: Reliability index of a) maximum of vertical deflections (characteristic combinations) b) maximum of vertical deflections (frequent combinations) c) maximum of reduced stresses d) fundamental frequency of the structure

All of them shows that reliability indicators nonlinearly decrease together with an increase of the uncertainty of wall thickness, thereby the probability of failure raise, which is compliant with the engineering intuition in this matter. The final values are significantly sensitive to the coefficient α and even its small change results in the apparent change of these indices.

References

The lifetime of steel specimens under strain-controlled cyclic bending

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Abstract

In the paper are presented the construction and operating principles of a new machine for fatigue research in conditions of cyclical oscillatory bending. The machine works using a controlled lever deflection amplitude, which, after rescaling the controls the deformation amplitude. Fatigue research carried out on a new fatigue machine may be used for both high and low cycle fatigue. The machine's work was verified on the basis of fatigue studies based on specimens made of 15Mo3 steel. The obtained results were compared with the results of the studies made for the same steel with controlled deformation amplitude for tension-compression. It was shown that the results of research made for tension-compression match the results made for oscillatory bending.

Keywords: lifetime, strain control, bending, push-pull

1. Introduction

Most often fatigue tests for a range of low number of cycles (Low Cycle Fatigue) are performed at a controlled strain and a high range of cycles (High Cycle Fatigue). Tests are characterized by studies in a strength-controlled environment. Such tests are carried out for tension-compression and torsion of thin specimens. However, in the case of bending or torsion (or combinations thereof) of full specimens, as is the case in many experimental studies, the torque causing bending or torsion is controlled, and therefore the tests are conducted in terms of a large number of cycles \([1]\).

In the paper we present a new test stand for fatigue tests under combined bending and torsion, capable of providing experimental studies with controlled strain in the range of a small number of cycles (LCF). The operation of the machine was verified on the basis of experimental tests on specimens made of steel commonly used in the power industry - 15Mo3 \([2]\). Finally, the strain-controlled experimental data for tension-compression and pendulum bending were compared. The test results for both stress \([3]\) and strain amplitude was compared.

2. Test stand for strain-controlled cyclic bending

Our study was performed on a newly constructed machine, as shown on Fig.1.

The idea behind this machine is that using the screw on the eccentric we can set deflection of the machine arm acting on the specimen, and that deflection is set as constant and controlled by the micrometer. This way a constant strain amplitude is obtained on the specimen. One of those specimens is shown in Fig.2. Additionally, during the test bending torque is monitored. At the time when this torque drops significantly, the initiation of fatigue cracking occurs, but further tests allow us to obtain the total fatigue life of the tested specimen.

3. Comparison of fatigue life curves for bending and tension-compression tests

Experimental investigations were performed on specimens made of 15Mo3 steel (16Mo3, 1.5415). This material is applicable in constructions used in the power industry.

The Manson-Coffin-Basquin curve for tension-compression \([2]\) can be written as:

\[
\varepsilon_a = 0.229 (2N_f)^{-0.470} + 766.5 \frac{1}{210900} (2N_f)^{-0.094},
\]

and the Ramberg-Osgood curve as:

\[
\varepsilon_a = \frac{a_s}{21000} + \left( \frac{a_s}{1035} \right)^{1/0.202},
\]
In picture no. 3 change of the time-depending moment at a fixed arm deflection amplitude (2mm), which means a constant strain, is presented. The graph shows that the material slightly weakens cyclically. At a certain number of cycles the moment drastically decreases, and at the same time, on the specimen's surface, a noticeable crack (ca. 1mm) appears. This quantity of cycles is adopted as the initiation of a fatigue crack. In picture no. 4 strain amplitudes versus the number of cycles leading to initiation for material 15Mo3 were established. These data were obtained from literature for tension-compression and Author’s research for oscillatory bending in the function of the number of cycles.

4. Conclusions

1. A newly designed and built test stand is capable to carry experimental studies with a controlled strain for bending in the range of a small number of cycles (LCF).
2. Preliminary experimental studies for LCF pendulum bending, with a controlled strain amplitude, showed that the fatigue life for bending is very close to tension-compression tests.
3. Further verification of the correlation between the bending fatigue curves for other materials is necessary.
4. In the future a strain-controlled test for pure torsion, and a combination of bending and torsion should be performed.

References


A probabilistic analysis of settlement of a non-cohesive soil layer subjected to cyclic loading is presented. The settlement estimation is based on a deterministic compaction model. Originally, the model requires integration of a set of differential equations. Making use of the Bessel functions the settlement of the soil stratum can be analysed by means of simplified formulae. The compaction model parameters were determined for the soil samples taken from the subsoil near İzmit Bay, Turkey, a region hit by an earthquake in 1999. The point estimate method was applied to describe the random behaviour of the stratum. Various sets of random variables were analysed. The results lead to a conclusion which can be useful in the prediction not only of soil settlement under seismic loading but also in the analysis of foundation settlements.

Keywords: Soil settlement, seismic loading, random material parameters, point estimated method

1. Introduction

The number of studies focused on non-cohesive soils subjected to cyclic loading is limited. Most of them concern experimental results and empirical models but only a few analyze the problem theoretically. The theory of compaction proposed by Sawicki and Morland [4] is one of the most advanced. A good agreement between the calculated results with the measured values proves the usefulness of the model.

In the paper application of the model version modified by Sawicki [3] in probabilistic analysis is presented. Moreover, material, load and model uncertainties are included in the calculations. Thus serviceability limit state is assessed in random terms, related to the soil settlement.

The point estimate method (PEM) in the form proposed by Rosenblueth [2] is applied. This method is easy to use, requiring only a crude background in probability theory. Several modifications of the standard Rosenblueth procedure are done in order to optimize the computational accuracy and effort, especially in the case of a large number of variables.

The basic parameters for the compaction model were determined for the soil samples taken from the subsoil near İzmit Bay, Turkey (Sawicki and Świdziński [5]). This region was hit by an earthquake in 1999, causing significant subsoil settlements. Soil parameters and cyclic load parameters were assumed random. In each case the mean value and the standard deviation of the settlement were estimated.

2. Compaction model

The following constitutive equation [3] is applied

$$\frac{d\Phi}{dN} = C_j J \exp(-C_j \Phi)$$  \hspace{1cm} (1)

where $\Phi = (n - n_0)/n_0$ is the relative change of porosity corresponding to irreversible volumetric changes (compaction parameter), $n$ and $n_0$ are the actual and initial porosities, $C_1$, $C_2$ the material constants determined for a given soil from cyclic simple shear tests, $J$ is the second invariant of the deviator of cyclic strain amplitudes, and $N$ is the number of loading cycles.

The second equation describes the relation between deviators of cyclic stress $S$ and strain $Y$ amplitude tensors:

$$S = G Y$$  \hspace{1cm} (2)

where: $G$ is a generalized shear modulus. For strains smaller than $10^{-4}$, a sufficiently good approximation for $G$ may be given in a form $G = G_0 \sqrt{p}$.

The mean effective stress for the homogeneous layer takes the form

$$p = (1 + 2K_o)H(1 - Z)/3$$  \hspace{1cm} (3)

where $K_o$ is the coefficient of earth pressure at rest, and parameters $H$ and $Z$ are shown in Figure 1.

The relationships (1)÷(3) describe volumetric changes of dry non-cohesive soil produced by cyclic shearing. The simplicity of the proposed model is related to the number of four parameters only: $C_1$, $C_2$, $G_0$ and $n_0$ to be experimentally laboratory-determined.

3. Specification of soil properties

The settlement analysis was carried out for the soil taken from 10 m thick sandy layer of a subsoil near the seacoast of İzmit Bay [5]. The values of compaction coefficients occurring in (1) are based on the cyclic simple shear test results. Four series corresponding to four different cyclic shear strain amplitudes were carried out [5]. In each single experiment the settlement of the sample caused by cyclic loading versus the number of loading cycles $N$ was recorded.
Consequently, a common compaction curve as an alternative solution of (1) can be approximated (Sawicki [3])

\[ \Phi = D_1 \ln(1 + D_2 \xi) \]  

(4)

where \( \xi = \frac{\gamma}{\gamma_0} N / 4 \) and \( \gamma_0 \) is the shear strain amplitude.

The compaction parameters \( D_1 \) and \( D_2 \) (4) are assumed random. Application of regression analysis leads to the estimators of mean values \( m_0 \) and standard deviations in the form: \( m_{0\ell} = 9.568, \sigma_{D1} = 1.164, m_{D2} = 0.348, \sigma_{D2} = 0.129 \). Correlation between the parameters was also assessed \( r_{D1D2} = -0.968 \). A similar procedure was applied to the parameters of the shear modulus: \( m_{G0} = 0.518 \times 10^8 \text{N/m}^2, \sigma_{G0} = 0.036 \times 10^8 \text{N/m}^2 \) and to the porosity: \( m_{\alpha_0} = 0.409, \sigma_{\alpha_0} = 0.01 \).

Additionally, physical reasons determine the correlation between \( m_0 \) and \( G_0 \) negative while correlation between \( m_0 \) and \( D_1 \) is positive. Unfortunately, the amount of experimental data is not sufficient for a thorough quantitative analysis. The following coefficients were proposed: \( r_{D1G0} = -0.5, r_{D1\alpha_0} = 0.5, r_{G0\alpha_0} = -0.5, r_{D2G0} = 0.5 \) and \( r_{D2\alpha_0} = -0.5 \).

4. Probabilistic analysis of soil settlement

A non-cohesive elastic soil layer of thickness \( H \) and density \( \rho_0 \) (kg/m³) resting on a rough, rigid base (Fig. 1) is considered. The soil is subjected to an acceleration \( A = A_0 \sin \omega t \) in order to reflect the seismic loading. The exact solution of this problem was given by Przewłocki and Knabe [1], using Bessel functions.

The following loading parameters required for calculations (see Figure 1) were assumed: \( H = 10 \text{ m}, g = 9.81 \text{ m/s}^2, T = 0.5 \text{ s}, k_0 = 0.344, A_0 = 0.2g, N = 100. \) Mean values and standard deviations of the of the soil model parameters \( D \) and \( G \), as well as \( G_0 \) and \( \alpha_0 \), and all required correlation coefficients are given in section 3. The mean values and standard deviations of strain amplitude \( A_0 \) are: \( m_{A0} = 1.962 \text{ m/s}^2, \sigma_{A0} = 0.1962 \text{ m/s}^2 \).

First, using the data described above the deterministic values of the soil settlement was calculated \( s = 0.038 \text{ m} \).

Two sets of probabilistic computations were performed. First, a set of calculations for different numbers of uncorrelated random variables was carried out. In all cases only strong negative correlation between model parameters \( D_1 \) and \( D_2 \) was taken into account \( r_{D1D2} = -0.960 \). An example, consider all parameters random the settlement is described by \( m_s = 0.0433 \text{ m} \) and \( \sigma_s = 0.0092 \text{ m} \). The results proved that the standard deviation of the settlement grows with the increasing number of random parameters.

In the second set of calculations two, almost fully correlated, random variables \( D_1 \) and \( D_2 \) are replaced by a single variable. The variable \( D_2 \) may be obtained from

\[ D_2 = \frac{(e^{D_1} - 1)}{\xi} \]  

(5)

In this case, using PEM computations only 16 samples were considered. The obtained mean value and the standard deviation are: \( m_s = 0.04516 \text{ m}, \sigma_s = 0.00786 \text{ m} \). Reduction of the number of correlation parameters raises the mean values but reduces the standard deviations. However, in engineering meaning the differences between the results of the two models are acceptable.

Further analysis were made to verify the influence of the load amplitude on the settlement. The soil was described using four random variables. The calculations proved that the decrement of the mean value of the settlement makes its standard deviation increase.

In order to assess the correlation impact on statistical parameters of the settlement some additional computations were performed. The highest impact on both random settlement parameters is noted with respect to the \( D_1 \) and \( G_0 \) correlation. Both values decrease while correlation between \( D_1 \) and \( G_0 \) and between \( G_0 \) and \( \alpha_0 \) increases. These values increase with the rise of \( D_1 \) and \( \alpha_0 \) correlation.

A simplified version of the calculation procedure can be also proposed. The variables acting significantly on the statistical characteristics of the settlement are the parameters related to the compaction model and the load, i.e. the strain amplitude. Thus, only these two parameters may be the ones considered random. In such a case there are only four samples analyzed. It causes substantial reduction of computational effort. The obtained mean value and standard deviation are: \( m_s = 0.0438 \text{ m}, \sigma_s = 0.0078 \text{ m} \), respectively. Comparison of the above results and the values of the PEM calculations for four random variables shows their convergence.

5. Conclusion

The comprehensive computations estimate precisely the impact of subsoil parameters, porosity, shear modulus, strain amplitude and other parameters on the mean value and standard deviation of the layer settlement. Statistical parameters of settlement are also affected by correlation between distinct random variables. The direct relation between material parameters lead to a two-variable problem, without losing the accuracy of estimation.

Thus the PEM has proved to be an efficient tool for probabilistic geotechnical engineering applications.

References


Stochastic Finite Element Method SORM study of the corrugated web steel plate girder

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Abstract

The main purpose of this paper is to present a second order reliability, stress and stability analysis of the steel plate girder with the corrugated web subjected to the Gaussian random fluctuations in its web thickness. Such an analysis is carried out using the Stochastic Finite Element Method (SFEM) based on the generalized stochastic perturbation technique and discretization of structure with the volumetric tetrahedral and hexahedral finite elements. It is numerically implemented using the FEM system ABAQUS and the symbolic algebra system MAPLE, where all the probabilistic procedures are programmed. We compare the perturbation-based results with these obtained from the traditional Monte-Carlo simulation and, separately, an analytical solution calculated by a symbolic integration carried out in MAPLE. The reliability index for deformations is calculated according to the First Order Reliability Method (FORM) or Second Order Reliability Method (SORM) and can be further used in durability prediction of such structures. A stochastic variable is the web thickness, whose randomness comes from an extensive corrosion or manufacturing imperfection, for instance.

Keywords: SFEM, corrugated web, SIN web, FORM, SORM, reliability

1. Introduction

The corrugated web beams increase their influence in civil engineering practice since their first appearance principally due to their high transverse rigidity and small weight. They tend to substitute the classical I-beams and columns and are extensively used as supports, roof trusses and columns in hall constructions, homogeneous steel bridge girders with large spans, as well as in composite box bridge girders, where the flanges are made of a concrete. Contrary to the beliefs of the producers and practitioners, theoretical foundations and numerous computational issues connected with the corrugated, and particularly SIN web, remain still unresolved and may contribute to their failure. Furthermore, the current state of the art is insufficient to distinguish a response of the corrugated web beams to the elevated temperature and reliability of such girders is still questionable. This is principally due to longitudinal waviness and vertical slenderness (and extreme thinness) of their webs, which additionally makes them exceedingly predisposed to the stochastic processes like corrosion. That is the reason why the corrugated-web beams recently constitute a topic of the scientific interest on numerical and experimental basis, especially in terms of its stability, reliability, response to the elevated temperature and behavior with web openings.

2. Numerical analysis

A numerical analysis concerns the simply supported girder with a span of 40 m, the web height of 2.5 m, flange width of 1.4 m and loaded by the uniform pressure of 107.14 kN/m$^2$ on its upper flange and by the self-weight.

2.1. Details of the FEM model

An analysis is made in the FEM system ABAQUS with 573 043 linear hexahedra (C3D8R) and 152 460 second-order tetrahedral finite elements (C3D10). Geometry includes the web and flanges, as well as the transverse ribs and welds.

A constitutive model used to simulate both, steel and welds is linear with a plastic limit of 460 MPa and connections between these materials are defined as ties of a total amount of 1392. Details of the girder geometry and discretization are provided on the Figure 1.

Figure 1: A discretization of the FEM model.

2.2. Von Mises stress

The numerical results include a von Mises stress, whose distribution is presented on Figure 2. In addition to the stress distribution revealed by the beam theory, some substantial periodic stress peaks are placed on the weld and nearby the support. Interestingly, the highest stress is reported in the middle of the girder span on the weld, and the (shear) stress pattern nearby the support is nowhere near to proportional with the main axis of the girder as the beam theory states. An additional remark is, that the transverse ribs on the support and, particularly, in the middle of the girder span are majorly unused. Some additional insight in this matter is available in [2].
2.3. Stability

Considering the stability of such a girder, its mode is strictly related to the web thickness. When web is thick, a girder fails for the moment in the middle of its span and the web is perfectly stable. Only when the web becomes 5 mm and less (ratio of web thickness to its height reaches 1:500), it starts to behave locally unstable on the support. Such a behaviour is presented on Figure 2, showing reaching of a girder plastic limit nearby the support. This scheme is obtained with use of an additional, shell model with 77 422 rectangular 4-noded shell elements and under the load equivalent to its first critical value revealed by the buckling FEM analysis.

Figure 2: A von Mises stress of the volumetric model with ribs and welds.

Figure 2: A von Mises stress pattern at the critical load for \( t_w = 5.0 \) mm.

3. Reliability analysis

A reliability analysis is made for the three different kinds of weighting scheme of the Least Squares Method (LSM), Dirac, uniform and triangular with polynomial response function and according to the FORM and SORM.

3.1. Theoretical background

General formulas used for determination of the reliability index are the following ones [1]:

\[
\beta_{\text{FORM}} = \frac{E[g]}{\sigma(g)},
\]

\[
\beta_{\text{SORM}} = -\Phi^{-1}(P_f),
\]

where \( E[g] \) denotes the limit state function, \( \sigma(g) \) is its variance and \( P_f \) means the probability of failure according to the chosen probability distribution, which can be calculated according in the following manner

\[
P_f = \Phi\left(\frac{\beta_{\text{FORM}}}{\sqrt{1 + \beta_{\text{FORM}}^2}}\right).\]

where \( \kappa \) denotes the curvature approximating the primary surface for the given probability distribution.

3.2. Reliability index

The resulting graphs of reliability given on Figures 3-4 firstly show, that the index is non-linearly decreasing with an increase of the input uncertainty for both, FORM and SORM and is basically independent of the probabilistic method type. An additional observation is that its value is much higher than the limit one (3.3-4.3) up to a high input uncertainty of more than \( \alpha = 0.20 \) and that the type of the weighting scheme selected for the analysis has a noticeable influence on both the character and magnitude of the index, especially for the moderate input uncertainty.

Figure 3: A reliability index of the corrugated I-beam for the different types of a weighting scheme according to the FORM and based on the limit function of displacement.

Figure 4: A reliability index of the corrugated I-beam for the different types of a weighting scheme according to the SORM and based on the limit function of displacement.

References


Influence of separation gap on the response of colliding models of steel structures under seismic and paraseismic excitations

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Abstract

As a result of high urbanization, the need to erect closely-spaced buildings forces designers to consider collisions between structures taking place during ground motions. Experimental and numerical studies have confirmed that such collisions (often referred to as structural pounding) may cause serious damage to the structural elements and even lead to total collapse of colliding structures. The main reason of earthquake-induced structural pounding, besides insufficient in-between gap size, is related to the differences in stiffness and/or mass of colliding structures. The aim of the study is to present the results of experimental investigation focused on dynamic behavior of closely-separated steel structures under seismic and paraseismic excitations. To perform this study, models of three steel towers, with different dynamic parameters, have been constructed and mounted to the platform of a shaking table. The influence of the size of separation gap ($\Delta_1 = 0$ mm; $\Delta_2 = 30$ mm; $\Delta_3 = 60$ mm) on the response of colliding structures has been investigated. Experimental study confirms that earthquake-induced pounding may lead to significant changes in the structural behavior. The smallest structural response has been observed for the case of zero gap size as well as when the gap size is large enough to prevent pounding.

Keywords: steel structures, structural pounding, earthquakes, shaking table

1. Introduction

The safety and reliability of steel structures under seismic and paraseismic excitations is among a number of different aims during the design stage. Under moderate to strong ground motions, collisions between adjacent, insufficiently separated buildings have been repeatedly observed. After the Athens earthquake of 1999, a significant damage due to pounding between adjacent parts of school buildings was observed [9]. Collisions resulted in total collapse of a number of structures during the Loma Prieta earthquake in 1989 [4]. Also, the San Fernando earthquake in 1971 caused serious damage at the places of interactions between the main building of the Olive View Hospital and the stairway tower [5].

A major reason leading to structural pounding results from the differences in dynamic parameters of the structures [1,2,6]. These differences lead to the out-of-phase vibrations and to interactions during the time of ground motion.

The phenomenon of structural pounding has recently been analyzed using various structural models and different models of collision (see, for example, [1-3, 6,7]). However, most of the research works concerned reinforced concrete buildings, the studies on steel structures were limited (see, for example, [8]). The aim of this paper is to present the results of experimental investigation focused on dynamic behavior of closely-separated steel structures under seismic and paraseismic excitations. The influence of the size of separation gap on the response of colliding model structures has been investigated by conducting tests on a shaking table.

2. Experimental study

To perform the experimental study, models of three steel towers, with height of 1000 mm and different dynamic parameters, were constructed and mounted to the platform of a shaking table (see Fig. 1). Towers were made of four steel columns (rectangular box section 15 x 15 x 1.5 mm) with spacing of 480 mm in the longitudinal direction (corresponding to the load direction) and 571 mm in the transverse direction. Additional skew bracings (also rectangular box section 15 x 15 x 1.5 mm) were used to prevent transverse and torsional vibrations. In order to obtain different dynamic characteristics of structures, additional mass (concrete plates with the dimensions of 50 x 50 x 7 cm and 42.2 kg weight) were used. In the configuration analyzed, two concrete plates were mounted at
the top of the middle tower (tower no. 2), only one concrete plate was mounted at the top of the external towers (tower no. 1 and 3) – see Fig. 1.

The first stage of the study has been focused on identification of the dynamic properties of each structure by conducting free vibration tests. Four accelerometers have been used during each test to measure structural vibrations of towers. Three of the sensors have been located at the top of the towers and one has been placed at the platform to control its movement (see Fig. 1). Values of natural frequency for each tower, as obtained from the free vibration tests, were summarized in Table 1. As it can be seen from the table, the natural frequencies of the models are very similar to the parameters of small, few-storey buildings. This fact justifies the acceptance of the scaled structural models and allows us to draw more general conclusions related to real structures.

The results of the shaking table experimental study indicate that pounding may have a significant influence on the response of colliding structures under seismic and paraseismic excitations. In the experimental study of the behavior of three structures presented in this paper, earthquake-induced pounding resulted in the decrease as well as in the increase in the structural responses. However, even if the peak response is reduced, the effect of collisions may result in substantial damage in the places of interactions. The smallest structural response has been observed for the case of zero gap size as well as when the gap size is large enough to prevent pounding.

### References


Discrete random variables in reliability calculations of a reticulated shell

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Abstract

Implementation of the point estimation method (PEM) in reliability analysis of a three-dimensional truss structure is presented in the paper. The method can be applied for a variety of engineering problems. The main advantage of PEM, in contrast to other probabilistic routines is a small number of calculations required for a relevant solution. Moreover, this method requires a vast background neither in theory of probability nor in reliability. The PEM is sufficiently accurate to estimate probabilistic moments of engineering importance. The PEM approach is illustrated by limit load case of a reticulated shell including its geometric and material random parameters. The analysis was performed for different combinations of basic variables. Both geometrical and material nonlinearities were taken into account. Symmetric and asymmetric cases of snow load were taken to assess the structural reliability. Sensitivity of the structural response to cross-sectional area was also taken into account.

Keywords: reliability, point estimate method, reticulated shell

1. Introduction

The problem of cost minimization in construction industry is always important and up-to-date. The cost reduction can be obtained by a careful structural design, employing sensitivity analysis or optimization methods. A number of key factors should be taken to solve the considered task. A proper selection of structural element cross-sections is such a high importance issue.

Engineering calculations are usually conducted for an idealized structural model and deterministic loads. However, many structures are highly sensitive to variations of their material and geometric parameters. These fluctuations are usually random, so a probabilistic description should be used in engineering design. It is a rare situation up till now, due to a shortage of easy-to-use, straightforward algorithms for the problem. Applicable methods and related software usually take a solid theoretical background on probability theory to deal with the random variables used and their moments. The Monte Carlo simulation method is crucial in the field, usually used for the result verification. A number of the so-called variance-reduction techniques, e.g. Stratified Sampling and Latin Hypercube sampling was formulated to reduce the computational effort.

In the paper a reticulated shell structure described by random parameters is analyzed. The point estimate method (PEM) is applied (Rosenblueth [4]). The method is easy in its use, requiring little theoretical probabilistic background. It was widely applied in geotechnical design, only a few papers took other structural reliability problems into account.

2. The point estimate method

The point estimation method maps the continuous random variable of a given probability density function into a discrete variable of a probability mass function containing N points. The following formula holds

\[ p_x(x) = \sum_{i=1}^N \delta(x - x_i) p(x_i) \]  

where \( \delta(x - x_i) \) is Dirac delta, and \( p(x_i) \) is a probability corresponding to a fixed value of random variable \( x_i \).

The number of points \( N \) selected for the estimation depends on the order of probabilistic moments to be analysed. The standard PEM is inefficient in cases of a large number of random variables used in reliability. The Rosenblueth method [4] is one of the simplest and the most effective procedures, reducing the number of samples only to \( 2n + 1 \). In turn the algorithm proposed by Hong [1] reduces substantially the number of evaluation points in the case of multivariate random problems. This method is dedicated only to uncorrelated random variables, their dispersion may be assessed by means of skewness.

3. Reticulated shell

A typical example of a reticulated structure described by Rakowski and Kačprzyk [3], was taken into consideration (Fig. 1). The radius of a three-dimensional truss was 50 m, the truss was 8.216 m high. Tubular sections RO 647.8×20 were designed for the structural elements. All elements were made of S355 steel. The elements were connected by means of ball joints.

The critical load for the ideal structure was calculated \( \lambda_{dad} = 0.26123 \). The limit state function (performance function) was defined as the value of the admissible load multiplier, its non-exceedance standing for a safe state. Several preliminary examples made using the MSC Nastran code [2] proved that even small changes in geometric description (node displacements) resulted in considerable changes of the load multiplier. For example, displacement of the highest structure point (no. 13 in Fig. 1) only 0.10 m down led to a 14.3% load multiplier drop.
4. Reliability calculations

In the first step of the probabilistic analysis a random geometric discrepancy was represented by its mean value $m_a = 0.0 \text{ m}$ and the standard deviation $\sigma_a = 0.08 \text{ m}$. Thus the majority of generated imperfections covered the interval $(-0.13, 0.13 \text{ m})$. Initial vertical displacements of nodes 7–12 were computed proportionally to node 13 discrepancies (Fig. 1). The direct Monte Carlo method was applied. The convergence analysis proved that the results obtained for 55 realizations can be a proper reference level. It is worth noting that only two realizations are required using PEM. In this case the errors of computed mean values and standard deviations are 0.3% and 17%, respectively (Sorn et al. [5]).

A similar analysis was performed using two random variables, the geometric and material parameters. The random node displacements were identical to the case of one-dimensional analysis. The second random parameter, Young’s modulus, was also assumed Gaussian ($m_E = 210.0 \text{ GPa}$, $\sigma_E = 4.0 \text{ GPa}$). In this case slight errors of the mean value and standard deviation errors occurred only, 1.23% and 1.35%, respectively.

Following the satisfactory conclusion of the PEM applications advanced analysis was performed next. The space truss model was defined using the following random variables: the initial vertical displacement of node 13, the initial vertical displacements of nodes 7–12 proportional to the displacement of node 13 displacement, Young’s modulus and cross-sectional area of truss elements. The standard, Rosenblueth and Hong versions of the PEM were taken into consideration. Each case brought about the mean value and standard deviation of structural limit load-carrying capacity. The Hong approach (8 realizations) gave almost the same estimators of the limit load and the standard deviations as provided by the classical Rosenblueth method (16 realizations), see Sorn et al. [5].

A problem of snow-loaded space truss reliability was investigated further on. Two variants of snow load: a uniformly distributed load and an asymmetric load are provided in the PN-EN 1991-1-3 code. The Gumbel distribution was applied to describe the snow load variability. The space truss parameters were described in two variants, using four and seven random values. The Hong version of the PEM [1] was applied. The obtained results show a considerable impact of a load class (symmetric, asymmetric) on the probabilistic response parameters.

Next the sensitivity of the truss reliability to the variation of cross-sectional parameters was estimated. According to the Standard PN-EN-1990 the level of structural reliability depends on the assumed lifetime of the structure. A classical Hasofer and Lind definition of reliability index was applied. Assuming a 50-year service and the RC3 reliability class the minimum reliability index is $\beta = 4.3$. The results in Fig. 2 indicate that the cross-sectional area of the elements cannot be less than 300 cm².

5. Conclusion

The Point Estimation Method can be effectively used in the reliability assessment of a real structure. The PEM advantage lies in its possibility to reduce the number of evaluation points due to multi-dimensional random variables analysed.

References

On bending of the steel bisymmetric I-beam profile exposed to fire

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Abstract

The main issue in the analysis is an application of the generalized 10th order stochastic perturbation technique implemented as the Stochastic Finite Element Method to carry out numerical simulation of thermal stresses and deformations of the steel I-beam profile exposed to a specific fire scenario. This approach is based on the general order Taylor expansion of all input random parameters and the resulting state functions about their mean values as well as on the Least Squares Technique employed to determine the analytical functions in-between design parameters and structural thermo-elastic responses. Transient distribution of a temperature inside the given steel I-beam as well as the resulting stresses and deformations are found using the Finite Element Method system ABAQUS CAE using fully coupled thermal-stress analysis. This work includes specifically a validation of the reliability index for ultimate and serviceability limit states for this beam under fire conditions. Computer simulation of the fire consists in a solution of the radiative heat transfer problem of uniform increase of the temperature according to the so-called fire curve adopted after civil engineering Eurocodes relevant to steel structures design.

Keywords: stochastic perturbation technique, Stochastic Finite Element Method, fire simulation, coupled thermal-stress analysis

1. Introduction

Reliability of engineering structures under fire conditions remains always very challenging and practically important knowledge. It is possible to use it for an efficient prediction of the failure time for the structures to get specific information about their fire resistance and evacuation time. The well-known Monte-Carlo simulation needs huge computer time effort for the large scale Finite Element Method (FEM) models like the one used below. In this case it is better to use higher order perturbation techniques implemented with the FEM originating from the Second Order Second Moment method [3] relevant to small initial random fluctuations. We compute probabilistic coefficients of up to the fourth order to gain more information about a probability density function itself. The basic probabilistic moments as well as skewness and kurtosis are calculated by full symbolic expansion of integral definitions and partial differentiation of the nodal response functions with respect to given random input variable. Finally, we determine the reliability index of the structural element under fire using its basic definition after Eurocode 0.

2. Theoretical background

We consider a Gaussian random variable \( b \), its expectation \( E(b) \) and their certain algebraic function \( f(b) \). Probabilistic moments of this function are calculated using the general 10th order expansion via Taylor series with random coefficients [2]

\[
f(b) = f^n(p^n) + \sum_{n=1}^{10} \frac{p^n}{n!} \frac{\partial^n f(b)}{\partial b^n} \Delta b^n
\]  

and the classical integral \( n \)th central probabilistic moment of this function \( f(b) \) introduced as

\[
\mu_n(f(b)) = \int_{-\infty}^{\infty} (f(b) - E(f(b)))^n p_b(x)dx.
\]  

Our method is based on the iterative solution of fully coupled deterministic FEM equations given as follows

\[
\begin{align*}
C^{(i)} (\theta^{(i)}) + R^{(i)}(\theta^{(i)}) &= R^{(i)} \\
K^{(i)} (\theta^{(i)}) \theta^{(i)}(\theta^{(i)}) &= R^{(i)}
\end{align*}
\]  

where \( C \) is heat capacity matrix, \( \theta \) is nodal temperature vector, \( K \) is heat conductivity matrix, \( K \) denotes stiffness matrix, \( R \) is a matrix of nodal loads, \( R \) is a matrix of thermal loads, \( q \) is the displacement solution vector. The upper index \( i \) stands for the test number relevant to the Response Function Method [2], where the basic value of our input parameter being randomized is slightly modified. Further determination of the probabilistic moments proceeds from their integral definitions extracted from Eq. (2) thanks to the symbolic derivation of all partial derivatives with respect to the given random variable \( b \).

3. Numerical example

Numerical experiment consists of the simply supported and 6,0 m long steel I-beam profile - IPE200. A floor concrete plate is attached on the top flange; it causes that fire flames can surround the cross-section from its left, right and bottom sides (Fig. 1). Mechanical boundary conditions are equivalent to the simply supported beam, initial temperatures are set equal to 20°C for all the nodal points, while fire simulation is performed by setting surface temperature equal to the value obtained from the so-called fire curve (ISO 834-1 statements [5]).
Therefore, as it can be expected, a distribution of the resulting steady-state temperatures throughout the entire cross section is not uniform (Fig. 2).

Our FEM beam model consists of 51600 eight-node coupled solid brick finite elements (C3D8RT) with the edge equal to 10 mm. Additionally, we assume the overall load uniformly distributed over the top midline of this I-beam as equal to $q = 7 kN/m$. Material properties of the steel (Young’s modulus, thermal expansion, specific heat and conductivity are all temperature-dependent and defined according to experimentally driven diagrams [4]. We use fully coupled thermal-stress analysis with overall time of fire accident equal to 15 minutes and time step equivalent to 1 second. The resulting comparison of the gas temperature according to ISO-834-1 [5] and steel extreme temperature is shown in Fig. 3.

The resulting gas temperature is also random variable in this case with its input coefficient of variation $\alpha(T_g)$ in the range of [0.0,0.2]. Figure 4 shows the reliability index calculated for the extreme displacements of beam under fire conditions. We may see time fluctuations of this index obtained for four different values of the input coefficient of variation $\alpha(T_g)$. It is quite natural and expected that the higher the input coefficient of variation, the lower the resulting reliability index (larger stochastic variations result in smaller probability of the survival). Obviously, this reliability index almost exponentially decreases together with time of the fire accident - a distance in time from fire initiation to catastrophic failure decreases rapidly.

We performed fully coupled transient analysis to determine the critical time when the value of reliability index exceeds extreme acceptable level provided in [1]. This limit value is equivalent to $\beta = 3.8$, reached after about 150 seconds. This computational example was made to determine the reliability indices for this steel structure with random parameters subjected to a fire with random temperature using the stochastic perturbation method associated with the Finite Element Method.

References


Dynamic response of the steel chimney by the stochastic perturbation-based Finite Element Method

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Abstract

The main aim of the paper is a presentation of the results obtained for the stochastic analysis concerned steel chimney with the height equal to 40 meters and subjected to a random wind loading. Analysis was performed using generalized stochastic perturbation technique based on a Taylor expansion and traditional Finite Element Method. Polynomial response functions of the observed design parameters such as deflections of the top of the structure, bending moments or transverse forces are numerically determined via symbolic algebra system MAPLE with the Least Square Method embedded in this system. The presented computational procedure including both dynamic and stochastic analysis can serve as a tool for the reliability estimation of slender structures.

Keywords: stochastic perturbation method, elastodynamics of structures, steel chimneys, reliability analysis

1. Introduction

Steel chimneys [4] are used widely in the industry as structures that allow to remove the exhaust into the higher layers of the atmosphere. Their main advantages comparing to concrete structures are weight, strength (resulting as a high value of the strength/weight ratio), simplicity of the assembly and dismantling. Considering bearing capacities of the chimneys, very important factor is the influence of the main load, namely wind in the context of the structural responses. The case with the observation of the deflections of the top of the structures, transverse forces and bending moments with reference to the wind loading has been studied. The main attention was focused to changes of the wind loading as the most natural source of the randomness in the exploitation of this kind of structures. In the context of stochastic analysis, large random variations of the input uncertainty sources and a necessity of third and fourth probabilistic characteristics determination, the Second Order Second Moment (SOSM) perturbation-based method is still extended to include as long stochastic Taylor expansions as necessary. The second issue here is the replacement of the Direct Differentiation Method (DDM) consisting of successive formation of the increasing order equilibrium equations obtained via partial differentiation with respect to the random input by the Least Squares Method (LSM). The series of iterative deterministic solutions to the given boundary-initial problem using FEM with the given random input varying around its expectation enable to get the local polynomial approximation at each degree of freedom separately thanks to the weighted version of the LSM itself. All the calculations are carried out due to the interoperability of the computer algebra program MAPLE with a commercial FEM system ROBOT. We determine up to the fourth probabilistic characteristics of the structural random response.

2. Governing equations

Stochastic perturbation method [3] is based on an expansion of the random functions into Taylor series of a required order. In this particular formulation, assumptions of the Gaussian probability density function is not necessary, because we can implement such an approach to the non-symmetrical density distribution as well. Expansion for the random deflection \( u \) with respect to the random wind velocity \( v(o) \) can be expressed as:

\[
\begin{align*}
u(v(o)) &= u^0 + \varepsilon_1 \sigma_1 (v(o)) + \varepsilon_2 \sigma_2 (v(o)) + \ldots \\
&+ \frac{e^n}{n!} \frac{\partial^n u(v(o))}{\partial v^n} \bigg|_{v=v^0} \Delta v^n 
\end{align*}
\]

and classical integral definitions like for the \( M \)th central probabilistic moments

\[
\mu_M(u(v)) = \int_{-\infty}^{+\infty} (u(v) - E(u(v)))^M p_{u(v)}(v) dv.
\]

A full symbolic approach assures the expansion with \( a \) priori given length and its \( a \) posteriori modifications according to the numerical errors noticed for higher than the second probabilistic moments of structural deformations. The method in the structural context is based on the iterative solution of the discrete equations of motion

\[
M_{ij} \ddot{q}_i(t) + K_{ij} q_j(t) = f_i(t)
\]

using the Hilber-Hughes-Taylor \( a \)-method [2] to get the series of deterministic nodal responses \( q_i(t) \), \( i=1,...,n \), where \( n \) is a total number of the LSM tests. It enables for a recovery via the Least Squares Method of the polynomial response functions \( q_j(t, b) \), where for a given \( \tau \in [0, \infty) \) one may get...
\[ q_j(\tau, b) = A_j(\tau)b^j ; j=1,...,m \]

where \( m \) is the approximation order such that \( m \leq n \). Further determination of the probabilistic moments proceeds from their integral definitions extracted from Eqn (2) due to the symbolic derivation of all partial derivatives with respect to a given \( b \).

3. **Computational analysis**

The main aim of the work is to apply a higher order Stochastic perturbation-based Finite Element Method. Motivation of this work comes from the necessity of the reliability index determination for the thin-walled steel structures subjected to dynamic excitation of the wind blows of a naturally stochastic character. The steel chimney presented in Fig. 1 is modelled with the use of 4-noded thin shell finite elements in the FEM system ROBOT and the dynamic response functions are found due to 11 tests with varying basic velocity of the wind acting on the chimney surface, while the dynamic spectrum of a wind blow is adopted after [1] and shown below in Fig. 2.

![FEM discretization of the chimney](image1)

![Dynamic excitation of the steel chimney](image2)

The expected values and coefficient of variation of the displacements of the top of the chimney are shown in Fig. 3 and Fig. 4, respectively (obtained for input \( \alpha \) equal to 0.15). Execution of the full-time analysis allows for an observation of probabilistic parameters fluctuations – in both cases these variations are significant, especially in the context of the serviceability limit state. A dispersion of the displacements expectations leads to a conclusion that not only a full dynamic study is necessary too. A proper wind excitation spectrum must be assumed. In the analysis time responses of the displacements in addition to input random wind velocity are nonlinear in some time instants.

Skewness and kurtosis of the displacements at the top of our chimney are presented in Fig. 5 and Fig. 6. The collected results allow to validate the probability distribution of the random output as non-Gaussian. These plots conclude that the complexity of the structural response is rather high, especially in the context of the displacement skewness. It also confirms that a full dynamic time analysis is appropriate for the considered class of the design issues.

**References**


Second order statistics of dynamic response of structures with lognormally distributed damping

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Abstract

A computational method is applied to evaluate second order statistics of the response of structures with lognormally distributed damping subjected to general dynamic loading. Numerical results show that excellent agreement is obtained using this method and the Monte Carlo simulation for values of covariance of damping equal or less than 60%. However, for larger values, more important difference in results between both methods is observed for structures with light damping. Moreover, computer time savings achieved by both methods and time domain solution strategies to evaluate sensitivity functions for dynamic systems with large number of degrees of freedom are discussed.

Keywords: Random damping, dynamic response, structures, second order statistics, Monte Carlo simulation

1. Introduction

The problem of dynamic response of multi-degrees of freedom (MDOF) systems with random damping has received significant attention in recent years (e.g. \cite{1,2,3,4,5}). This problem is of importance in relation to the design of dynamically sensitive structures such as tall buildings and industrial chimney stalks that strongly rely on damping for their performance under wind and seismic induced vibrations.

However, selecting an appropriate damping value to be used for the design of a tall structure is to some extent a subject of controversy because of paucity of measured data from real buildings, large variance errors in many of the few measurements that were taken, and the occasional misuse of measurement techniques (Ref. [6]). Damping estimates have an intrinsic variability and may depend on a wide range of factors including vibration amplitude, natural frequency, structural dimensions and local site conditions (Ref. [5]). For excitation frequencies close to the resonant frequencies, the sensitivity of the response to damping becomes critical. Errors in the estimation of the damping matrix will generally result in a large error in the response.

The article presents the main results of a numerical investigation on the sensitivity of dynamic response of MDOF systems with random damping. A statistical linearization technique coupled with the use of sensitivity functions has been utilized to determine second order statistics of the system response. The numerical results obtained from the application of this methodology to a typical industrial building structure have been checked by Monte Carlo simulation (MCS) method (Ref. [7]). The significance of damping randomness and its implications on the sensitivity of system response in the neighborhood of a resonant frequency are discussed in light of considerable ranges of damping uncertainties.

2. Statistical linear model (SLM)

The matrix system of differential equations of motion governing the displacement \{x(t)\} an n-Multi-Degree Of Freedom (n-MDOF) dynamic system subjected to an external excitation \{F(t)\} can be written as:

\[ \begin{bmatrix} M \end{bmatrix} \ddot{x} + \begin{bmatrix} C \end{bmatrix} \dot{x} + \begin{bmatrix} K \end{bmatrix} x = \begin{bmatrix} F(t) \end{bmatrix} \]  

The damping coefficients \( c_i \) are assumed lognormally distributed with mean value \( \bar{c} \) and standard deviation \( \sigma_c \). The total number of dampers, in the general case, is \( p \geq n \). Arranging the damping coefficients in a p dimensional vector \( \{c\} \), denoting by \( \{x^c(t)\} \), the value of \( \{x\} \) when \( \{c\} \) takes on its mean value \( \{\bar{c}\} \) and introducing sensitivity functions

\[ \xi_k = (\partial x / \partial c_k)_{x=x_0}(k=1,...,p) \]  

where the vector \( \{\xi\} \) can be expanded in a Taylor series as follows:

\[ \Delta x_i \approx \left( \frac{\partial x_i}{\partial c} \right) \{\Delta c\} \]  

from which it can be shown that

\[ \sigma_i^2 = \left[ \frac{\partial x_i}{\partial \bar{c}} \right] \text{cov} \left[ \frac{\partial x_i}{\partial \bar{c}} \right] \]  

where \( \sigma_i^2 \) denotes the variance of the random variable \( x_i \), and \( \text{cov} \) is the covariance matrix of the damping coefficients \( c_i \). Its \( ij \)th element is defined by

\[ \text{COV}_{ij} = \rho_{ij} \sigma_i \sigma_j \]  

The sensitivity functions \( \xi_i \) are available by differentiating Eqn.1 with respect to \( c_i \) as follows:

\[ \begin{bmatrix} M \end{bmatrix} \ddot{\xi} + \begin{bmatrix} C \end{bmatrix} \dot{\xi} + \begin{bmatrix} K \end{bmatrix} \xi = -\left( \frac{\partial \begin{bmatrix} C \end{bmatrix}}{\partial \bar{c}} \right) \ddot{x} \]  

The left side of Eqn (7) is identical to Eqn (1) and the right side \( -\left( \frac{\partial \begin{bmatrix} C \end{bmatrix}}{\partial \bar{c}} \right) \ddot{x} \) can be interpreted as a fictitious forcing vector. In the case of general loading \( \{F\} \), the right side of Eqn (7) can be obtained from the time derivative \( \dot{x} \) of the response \( x^c \). For large MDOF systems, the vector \( x^c \) can be computed by solving Eqn (1) with nominal damping \( c = \bar{c} \) using mode superposition analysis (Ref. [8]). Alternatively, the vectors \( \{x\} \) and \( \dot{x} \) can be obtained systematically using step by step integration methods of structural dynamics. In order to obtain the vectors \( \{\xi\} \) \((i=1,...,p)\), Eqn (1) must be solved first with the forcing vector \( \{F\} \), after which Eqn (7) is solved p times.
Thus, the global bulk of computations involves essentially, a computer simulation for the evaluation of the covariance matrix \([\text{cov}]\), the solution of Eqn (1) for \(\{\mathbf{x}\}\), \(\{\mathbf{i}\}\) and finally the solution for \(\{\mathbf{z}\}\), \(p\) times.

In Fig. 2, the standard deviation of building response with light damping at the second floor is presented for both SLM and MCS. The results suggests that differences in standard deviation of building response obtained for both methods are insignificant up to COV of damping values less than 60%. However, for larger values, the errors introduced by the linearization technique, increase concomitantly with an increase in COV of damping. Moreover, it should be noted that a large dispersion in results between the SLM and MCS methods is observed for dynamic systems with light damping and large values of damping variability.

4. Summary and Conclusions

This paper investigated the significance of damping variability on the dynamic response of MDOF building structures with uncertain damping in the neighbourhood of resonant frequency using a statistical linear model.

The numerical results obtained from the application of this methodology to a typical industrial building structure were checked by MCS method and excellent agreement was obtained even for considerable ranges of damping uncertainties. The limits of validity of the statistical linearization technique were also determined. In addition, the significance of damping uncertainty on building response in the neighborhood of a resonant frequency was illustrated for commonly associated damping uncertainties with structural damping values. It is shown that depending on the mean values of damping, the effects are more pronounced for higher variability of damping values. The methodology used in this study is applicable, not only to damping but also to other structural parameters. The authors are currently expanding the present methodology to include the contribution of higher order terms in the statistical response model and to assess the sensitivity of building response in light of ranges of damping uncertainty with covariance values exceeding 80%.

References


Structural reliability of overhead power lines by means of Monte Carlo Method and RSM

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Abstract

The article discusses the issue of reliability assessment of overhead power lines by means of both Monte Carlo Method and Response Surface Method. The suspension tower OS24 ON150+10 is considered, as the weakest element of the line. A comparison is made between deterministic and probabilistic approaches to wind and icing loading. The PDFs for wind and icing loads are proposed. Reliability is assessed for both critical atmospheric loading conditions of the structure and tension produced in the conductors.

Keywords: overhead power lines, critical wind and ice loading, reliability, Monte Carlo Method, Response Surface Method

1. Introduction

Reliability of power lines is often related to structural failure of support towers, because their breaking is one of the major causes of electrical outage. It can be proved by numerous examples published along the last years around the world, including the outage due to the excessively fallen wet snow in Western Poland in April 2008 [2]. In a study made by CIGRE in 1991-1996 [1] regarding the failure of transmission lines worldwide, the following observations were made:
- most of the documented failures had their origin in statistically determined events, mainly weather related events such as wind, icing and combination of both,
- the use of probabilistic methods in the design of the lines could improve their reliability, nevertheless the lack of data regarding the icing events is a major obstacle,
- the towers were in most cases the weakest element of the line.

Therefore the main objective of the study is to determine reliability of a support tower of an overhead power line subjected to wind and icing loads. For this purpose, analysis is made of structural reliability of a tower supposed to be situated in the surrounding lowlands of Gdańsk, Poland.

2. Structural analysis

In the paper, a lattice transmission tower (OS24 ON150+10, by SAG Elbud Gdańsk), depicted in Figure 1, is considered. The passage tower, 32.15m in height, is situated on a straight line, and sustains 6 AFL-6 240 conductors and 2 AFL-1,7 70 earth wires, with spans of 280m. The structure is erected from 14 different L-shape profiles, each one made of S355 steel.

The model was implemented in Autodesk Robot Structural Analysis software. It comprises of 3891 nodes and 4395 beam elements. Geometrical and material nonlinearity is assumed. In order to determine the invariant loading conditions, PN-EN 50341 [6] is taken into account. Wind and icing loading is considered both deterministic and random.

Figure 1: Geometry of the considered transmission tower

Eleven load cases are considered, based on PN-EN 50341. The most unfavourable load case corresponds to the load case in which all the conductors on one side are broken – synonymous to a collapse of a neighbouring tower, leading to a maximum stress of 333MPa. The load case regarding the combined action of standard-based wind and ice produces a maximum stress of 228 MPa, which supposes a reserve of bearing capacity of 35%.

3. Wind and icing load considered random

3.1. Random approach to wind load

A deterministic approach to wind load considers load dependent on the geographic location and the altitude above sea level only. On the other hand, the probabilistic approach may additionally deal with the actual wind measurements from meteorological stations, the distribution of azimuthal angle of the wind pressure and the actual period of wind load returnwith. A set of 36 real-time measurements of the abovementioned factors was obtained from Gdynia Oksywie Airport vicinities [3]. On their basis, the mean value of is used, and the standard deviation of is calculated. Next, random load is simulated, using Weibull distribution with parameters: .
3.2. Random approach to icing load

Analogically to the previous remarks, the deterministic approach to icing load considers only its dependence on the geographic location and the diameter of the cable. The probabilistic approach may also provide with the actual icing quantity and density measurements and the period of icing load return with. In the paper, the probabilistic icing load is derived based on observations of [5]. The load is simulated again using Weibull distribution with parameters: \( \alpha = 0.38 \) and \( \beta = 0.103 \), as in Figure 2, giving \( \mu = 0.34mm \) and \( \sigma = 1.11mm \).

![Figure 2: PDF of wind (left) and ice radial thickness (right)](image)

4. Reliability Assessment

4.1. Determination of Ultimate Limit State Function of the tower against atmospheric loads

To facilitate the assessment of the reliability of the tower, some combinations of wind and icing leading to the structural failure (without taking into account the broken conductors load case) are determined. Ten different series of numerical calculations are performed to find a Limit State Equation (LSE). Next, using interpolation, the LSE can be approximated. A quartic fit of the LSE is adopted, and the equation is given:

\[
\bar{g}(s,w) = 80,708 - 1,693 \cdot s - 0,950 \cdot w
\]

The approximated LSE (line) and the numerical realizations (squares) are presented in Figure 3. It resembles strongly the reference solutions, proposed by the standards or in [4].

![Figure 3: Limit combinations of wind and ice thickness, along with the cloud of 1% of generated MC samples](image)

4.2. Critical atmospheric loading conditions of the structure

First of all, a crude Monte Carlo variant is performed to assess the reliability of the structure. A test, consisting of 60000 samples (50 years \( \times \) 12 months \( \times \) 100 realisations, shown as dots in Figure 3) is conducted, giving the probability of failure \( P_{\text{MC}} = 0.00008 \) , and the resultant Cornell reliability index of \( \beta_{\text{conP}} = 3,765 \). The Hasofer-Lind (HL) reliability index is also calculated, equal to \( \beta_{\text{HLRF,con}} = 3,211 \), resulting in a difference of 17.66%.

Afterwards, a RSM approach is adopted, and the tensile response of a cable subjected to atmospheric loads is being approximated by the 1st and 2nd order polynomials as:

\[
\bar{g}(s,w) = 66,868 - 1,150 \cdot s + 0,153 \cdot w - 0,008 \cdot s^2 - 0,017 \cdot w^2 - 0,014 \cdot s \cdot w
\]

On the basis of the obtained response surfaces, the reliability indices are calculated, by means of the author’s software RSM-Win, equal to: \( \beta_{\text{con}} = 4,004 \) (1st order) and \( \beta_{\text{HLRF,con}} = 3,692 \) (after 20 iterations), \( \beta_{\text{HLRF,con}} = 3,696 \) (after 5 iterations, both for 2nd order approach), resulting in a maximum difference of 5.97%, in comparison to MC results.

5. Conclusions

It can be clearly shown that the probabilistic assessment of structural reliability using standard techniques (MC, RSM) is possible and trustworthy. In order to obtain results of a satisfactory accuracy, few numerical calculations of the model are required. However, the model can introduce its own uncertainty. Unlike wind data, the icing events are only recorded in few places worldwide and it is hard to contain them. The location of the measuring stations is also very important.

References

Reliability assessment of truss towers using Monte Carlo Method, PEM and RSM

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Abstract

The paper discusses the reliability assessment of simple random truss structures using three different probabilistic methodologies: the Monte Carlo method, the Point Estimate Method and the Response Surface Method. A benchmark truss structure example is analyzed. A set of numerical calculations of critical load is performed and the results are taken as the basis of advanced probabilistic calculations. Using the samples, numerical efficiency and accuracy of these methods are compared. A discussion is proposed to point out the advantages and drawbacks of PEM and RSM in comparison to MC method.

Keywords: simple random truss structures, reliability, Monte Carlo Method, Point Estimate Method, Response Surface Method

1. Introduction

Nowadays, the simulation-based methods of reliability assessment through repetitive numerical model calculations regain their importance, mainly due to a swift development of computer-aided design tool performance. Probabilistic structural reliability analysis is conducted to determine the impact of relevant input parameters on the change of model safety. An appropriate identification of methodologies to deal with simple engineering problems may be helpful both in design and optimization of the new and improving of the existing constructions.

This paper critically discusses the behaviour of three most popular probabilistic techniques used for reliability assessment, on a benchmark example of a simple truss tower.

2. Structural analysis

The paper deals with a 25-member truss, proposed in [1, 3], presented in Figure 1. The supports have a spacing of 508cm. The width of the intermediate and upper storeys connectors is equal to 190.5cm. The height of each storey is 254cm. The tower is made of aluminum (\(E=69\)GPa) and the following loads are taken in each of the top two nodes: \(P = (P_x, P_y, P_z) = (19.2kN, 19.2kN, 2400kN)\). The initial cross section area of the members is \(A=3,225cm^2\).

Figure 1: Geometry of the considered simple truss tower

It was assumed, that the random variables will be associated with the possible change of the cross sectional area of bars during design process, similarly to an optimization approach.

It was noted, that the bars can be steel profiles only. For 32 steel sections found in the catalogues with a sectional area close to that of the initial value, a histogram of the cross-sectional area availability is shown, given in Figure 2. It fits Gaussian PDF more than the uniform type. Based on this observation, a PDF of the cross sectional area was proposed, resulting in two basic parameters: \(\mu_A=3.246cm^2\) and \(\sigma_A=0.334cm^2\).

Figure 2: A histogram based on the bar’s cross-sectional area availability in steel catalogues

However, considering the 25 cross-sectional area values of each element random is suboptimal and inconsistent with the methodology of truss tower erection process. Five distinct random variables were eventually proposed – 2nd storey struts (\(x_1\)), 2nd storey columns (\(x_2\)), connectors (\(x_3\)), 1st storey struts (\(x_4\)), and 1st storey columns (\(x_5\)). This number was also selected because the reduction of the numerical problem is desirable, and due to the author’s software RSM-Win is most efficient for a maximum of 5 variables.

The sensitivity of structural response to the change of random cross-sectional area is presented in Figure 3.

Figure 3. The sensitivity of structural response (\(m_l\)) to the change of random variables (cross-sectional area of bars \(A\)).
3. Formulation of the SLS requirement

In the paper the critical multiplier \( m_0 \) of the applied set of loads at the stability loss of the tower is analysed. Assuming all cross-sections as mean (the proposed PDF mean value), a multiplier equal to \( m_{id} = 1.066 \) was obtained.

 Afterwards, three discrete values, representing the cross-sectional area variable PDF were chosen specifically for the task – the mean value \( A_{mean} = 3.246\,\text{cm}^2 \) and the boundary values related to \( +3\sigma \) and \( -3\sigma \) (giving \( A_{min} = 2.252\,\text{cm}^2 \) and \( A_{max} = 4.240\,\text{cm}^2 \)). Due to this choice, a set of \( 3^3 = 243 \) samples is created and then computed by means of SOFiSTiK software.

A minimum value of the multiplier, equal to \( m_{min} = 0.738 \) is found for the design case of maximum connectors and minimal columns and struts. The maximum value of the multiplier, equal to \( m_{max} = 1.399 \) was found for the opposite design case.

It was decided to define structural failure as the occurrence of buckling under a load less than 80% of the initially applied set of loads.

4. Reliability assessment

4.1. Monte Carlo Method calculations

In the first approach, slightly similar to a Stratified Sampling Monte Carlo methodology, all 243 samples are taken into consideration.

This leads to the probability of failure equal to \( P_{f,id} = 0.05761 \) and a resultant value of reliability index equal to \( \beta_{id} = 1.57513 \). The Monte Carlo method results were taken as reference values for the further calculations.

4.2. Point Estimate Method calculations

The second approach incorporated the Point Estimate Method (PEM) in the form proposed by Rosenblueth [2].

In this methodology, each variable is assigned to be either \( +3\sigma \) or \( -3\sigma \), whereas the remaining variables are set at their mean values. This approach gives 10 or 11 computational samples, the latter when the sample of every variable set at its mean value is also taken into account (enriched PEM, XPEM).

The probability of failure is equal to \( P_{f,PEM} = 0.05983 \) in both cases, giving a resultant value of reliability index equal to \( \beta_{PEM} = 1.55620 \) and an 1.21% error, when compared to the MC results.

4.3. Response Surface Method calculations

Subsequently, reliability will also be assessed using the Response Surface Method, which applies an approximate, simplified function (in this case, the 1\textsuperscript{st} and 2\textsuperscript{nd} order polynomials) to represent the structural response.

The approximations are performed by an author’s software RSM-Win, with four sets of sample points. Three input sets are taken from previous calculations and the fourth is an Importance Sampling variant of the MC method.

The Importance Sampling (IS) technique is based on selecting crucial samples for the combinations of \( \pm 3\sigma \) values due to each random variable of the task, giving 2\textsuperscript{5} = 32 points of interest.

On the basis of the obtained response surfaces slope factors, probabilities of failure and reliability indices (based on the Hasofer-Lind-Rackwitz-Fiessler approach) are calculated. The results are summarized in Table 1.

<table>
<thead>
<tr>
<th>RSM approx.</th>
<th>SS-MC (243 samples)</th>
<th>IS-MC (32 samples)</th>
<th>PEM (10 samples)</th>
<th>XPEM (11 samples)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{RSM} ) (1\textsuperscript{st} order)</td>
<td>0.04527</td>
<td>0.05705</td>
<td>0.06173</td>
<td>0.06173</td>
</tr>
<tr>
<td>( \beta_{RSM} ) error</td>
<td>6.94%</td>
<td>0.31%</td>
<td>2.20%</td>
<td>2.20%</td>
</tr>
<tr>
<td>( P_{RSM} ) (2\textsuperscript{nd} order)</td>
<td>0.05761</td>
<td>0.11836</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>( \beta_{RSM} ) error</td>
<td>0.00%</td>
<td>24.88%</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

5. Conclusions

Probabilistic reliability assessment of simple trusses, using the standard probabilistic techniques is clearly possible. However it should be noted, that the truss analysed in the paper is nothing more than an illustrative example, as its probability of failure is very high, significantly higher than those of real-life engineering structures.

Moreover, PEM and RSM proved to be a good alternative for the MC calculations of such simple engineering structures.

The Response Surface approximation is an effective way to reduce the MC sample population, combined with a PEM-based choice of sampling points is an efficient tool to assess reliability of geometrically nonlinear models.

However, it should be pointed out, that such a small number of samples as used in PEM+RSM approach gives completely erroneous results for the quadratic approximation, indicating the fact, that for more advanced approximations, the number of points of approximation should be adequate.

As mentioned in the previous studies made by the authors, given in [4], the Importance Sampling approach is not intended for the analysis of simple models. While the IS-based 1\textsuperscript{st} order approximation is satisfactory (presumably circumstantially), the 2\textsuperscript{nd} order approximation is showing a large stand-off error, nearly ten times greater than the average error result.

As prompted by the sensitivity trend lines (Figure 3), the best approximation is obtained for the 1\textsuperscript{st} order polynomial regardless of the used method which seems to be a curious phenomenon.

Finally, it is shown, that such an analysis can be an excellent starting point for sensitivity analysis of simple models of engineering structures.

References


MS20

Safety and Reliability of Structures

organized by T. Łodygowski, A. Glema and J. Małachowski
Buckling curve for reinforcing steel bars

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Abstract

The paper presents the results of numerical analysis dealing with elastic and elasto-plastic buckling of reinforcing bars. Bars of different geometrical parameters made of steel of various mechanical characteristics were analysed. The system COSMOS/M based on FEM was used. It follows from the obtained results that slenderness is not the only factor to affect elasto-plastic buckling of reinforcing bars. Such parameters as the length of plastic plateau, the ultimate stress and the module of plastic hardening are important as well.

Keywords: stability, buckling, inelastic buckling, reinforcing steel bar, slenderness.

1. Introduction

Some accidental loads e.g. seismic loads, acting on structures, can result in the appearance of postcritical states. Experience from past earthquakes and results of experiments for example by Bayrak Sheikh [1], and Korentz [2] indicate that the destruction process of beams and columns takes place at the instant of inelastic buckling of reinforcing longitudinal bars. Other results of experiments by Monti and Nuti [3], Korentz [4], show that for steel bars buckling occurs for stress greater than the yield strength.

The Euler theory describes the critical load for elastic buckling and is valid only for long columns. The ultimate compression strength of the column material is not geometry-related and is valid only for short columns. In between, for a column with an intermediate length, buckling occurs after the stress in the column exceeds the proportional limit of the column material and before the stress reaches the ultimate strength equal to the yield strength. This kind of situation is called inelastic buckling.

The Euler hyperbola describes the buckling curve in the elastic range. However, the Johnson parabola describes the buckling curve in the post yielding range, wherein the critical stress does not exceed the yield strength. The point at which the shift is made from the Euler formula to the Johnson formula is usually taken as half the yield strength of the material. In subject literature there are no formulae which describe buckling curves for thickset bars.

The paper presents buckling curves for reinforcement bars in the elastic range and post-yielding deformations.

2. Description of the research

Grades of steel from which reinforcing bars are manufactured are characterised by a great variability of mechanical properties after yielding. Differences lie in the length of the yield plateau, the strengthening ratio and the shape of a hardening curve (Fig. 1a)

Calculations were performed for different material curves σ-ε presented in Fig. 1a and described by the parameters α=5,10,15, β=1.25,1.50,1.75 and γ=0.025, 0.050, 0.075.

Material curves were the same for compressed and stretched fibres. Such parameters as the yield limit (fsu=400 MPa), the strain instant of yielding (fsy=0.002), Young’s modulus (Es=200000 MPa) and final strains (e=0.160) were the same for all material curves. In order to describe analytically the hardening curve the Mandera model presented and described by Eqn (1) was adopted:

\[
\sigma_s = f_{su} + (f_{sy} - f_{su}) \left( \frac{\epsilon_{su} - \epsilon_s}{\epsilon_{su} - \epsilon_{sh}} \right)^p, \quad \text{where} \quad p = E_s \frac{\epsilon_{su} - \epsilon_{sh}}{f_{sy} - f_{su}} \quad (1)
\]

One of the buckling modes of inelastic buckling of a reinforcing bar embedded in an RC column is shown in Fig. 1b. Most often bars of a longitudinal reinforcing bar buckle between two neighbouring ties. This phenomenon is preceded by a cover failure. Calculations were performed for a bar clamped at both ends and with the radius φ=16mm (Fig. 1c).

The ratios between the distance of the supporting points s, and the diameter φ ratio were as follows: s/φ=5÷40, for the adopted supporting manner it corresponds to the classical slenderness definition 0.5s/φ=10÷80.

Figure 1: a) Models of steel, b) mode of inelastic buckling of reinforcing bar, c) physical model of the bar

In the numerical analysis of large elasto-plastic deformation of compressed reinforcing bars, the commercial system Cosmos/M [5] was used. The finite element BEAM2D with the elasto-plastic option, the Huber-Mises-Hencky yield criterion, the associated plastic flow law and the isotropic hardening were used in the numerical model. The nonlinear material characteristic σ-ε with the option PLASTIC was included into the analysis. Nine material curves of parameters α, β, γ given above were used in a form of consecutive points of coordinates (σi,εi), i=1,...,N. The system Cosmos/M allows to introduce as
many as 200 points of the material curve.

3. Analysis of the results

The influence of a length of the plastic plateau \( \alpha = \varepsilon_{pl}/\varepsilon_y \) of the hardening coefficient \( \beta = f_{su}/f_{y} \), and the stiffness of the hardening branch defined as \( \gamma = E_{sh}/E_s \) on the critical stress \( \sigma_{cr} \) in bars is illustrated in Fig. 2a, Fig. 2b, Fig. 2c respectively.

The critical stress \( \sigma_{cr} \) is defined as a quotient of the maximum force, carried by bar \( F_{\text{max}} \) and the bar cross-section area \( A \).

A parameter decisive on the critical stress is the slenderness of the bar. The slenderness increment \( s/\phi \) causes the critical stress \( \sigma_{cr} \) to decrease. If the bar slenderness of the bar is less than 5, the critical stress can be equal to the strength \( f_{su} \). If the slenderness of the bar is smaller than 9, for a small slenderness, between 5 and 9, the critical stress does not depend on slenderness only. In this case the critical stress depends in a crucial way on steel mechanical properties of steel, expressed by \( \alpha \), \( \beta \) and \( \gamma \). The critical stress decreases as the plastic plateau \( \alpha \varepsilon_y \) increases (Fig. 2a). The critical stress increases as steel strength \( f_{su} \) increases (Fig. 2b). The critical stress increases critically with the stiffness of the hardening curve \( E_s \).

If the slenderness of the bar varies within the range 10 to 15, then the critical stress remains at the yield strength. After crossing the slenderness 15, the critical stress decreases and is smaller than the yield strength, the intensity of a decrease in the critical stress grows as the slenderness increases.

If the slenderness of bars is higher than 9, then the steel mechanical properties do not have an impact on the critical stress.

4. Recapitulation

On the basis of the calculations conclusion is made that not only geometrical properties of bars act on the inelastic buckling, there are also mechanical properties of steel after yielding.

If the bar slenderness is smaller than specific limit values then after reaching the yield limit a bar can work in the state of stable equilibrium. The mechanical properties of steel after yielding described by the length of plastic plateau, the ratio between the ultimate stress and the yield stress and the shape of the hardening branch influence on the value of the limit slenderness. So creating the accurate models of behaviour of compressed bars regarding their inelastic buckling refers to the shape of an entire material curve.

References


Advances in the computation of fires interacting with structures

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Abstract

Numerical simulations of fires interacting with structures are mainly performed as a part of the performance-based design process. Fire simulations have been utilized in the design of steel structures for long, but the need to simulate the interactions with other types of structures, such as compartment barriers and individual building components, is emerging. In this presentation, the recent or ongoing developments of the fire simulation code FDS are presented, and the means to couple the simulations with the calculation of structures are discussed. Finally, the newly developed methodology for probabilistic barrier failure assessment will be presented.

Keywords: CFD, fire simulation, verification and validation, risk assessment, fire resistance

1. Introduction

The adoption of performance-based design codes and the availability of efficient computational tools have increased the use of numerical fire simulations in fire safety engineering. While the majority of such simulations are made to assess the life safety and evacuation conditions, the applications for structural fire safety are also becoming common. Models for the localized and natural models have been applied in the design of steel structures within EU since the development of the Natural Fire Safety Concept and the Eurocodes where the possibility for using advanced modelling has been explicitly allowed.

In addition to the load-bearing capacity assessments, the need for the a performance-based analysis of other types of building components and systems is arising. The energy efficiency requirements, for instance, create a need to evaluate the performance of insulation systems, considering both material issues and installation details, such as protective layers. In addition, the air tightness requirements can increase the importance of the ventilation network component performance in preventing the smoke spreading through ventilation network. The adoption of risk-based approaches, in turn, is currently driving the development of probabilistic methods in the field.

The advances in the field of fire simulations in the context of structural fire safety can be classified into three development categories: improvement of fire modelling accuracy, methods for the interaction between the fire and structures, and the capabilities for new types of applications, such as the risk assessment of structural failures and the incident management applications serving the first responders. All three categories will be next discussed, but the advances in the structural analyses following the fire analyses are not included.

2. Fire simulation accuracy and scope

Fire Dynamics Simulator (FDS) is a CFD-based tool for reactive, low-Mach number flow with LES turbulence modelling and physical models for the heat and mass transfer processes, such as two-phase flows, thermal radiation and solid pyrolysis [1]. The main improvements during the last few years have been done in the numerical implementations aiming at increasing accuracy of the transport phenomena. The development philosophy is based on the continuous and automated verification and validation process. The understanding of this process and utilizing its results can and should be applied in the engineering quality assurance procedures. At best, the validation results are presented as quantity-specific biases and standard deviations, thus enabling quantitative confidence assessment of the computational results [2].

Regarding the physical models, more focus were put in modelling the boundary layers and convection heat transfer, focusing on the rigorous validations and the associated needs, such as the inflow turbulence level. Spectral properties of the absorbing and emitting gases were added to the RadCal narrow band calculation to account for a wider range of different fuel compositions.

Much work was done in the field of material characterization for the purposes of pyrolysis and flame spread modelling. The purpose of this work is to enable the prediction of the fire heat release rate (HRR) instead of prescribing it, which is the usual approach in the current design fire fires of the performance based approach. Moving from the traditional fire testing methods, which were usually design for pass/fail type of approvals, towards the chemical analysis techniques, such as the thermogravimetric analysis (TGA), the materials can be described in terms of model parameters, such as the kinetic parameters of degradation reactions. Several methods have been developed for the estimation of these parameters from the small scale experimental data [3]. While these methods are still topics of active research, their adoption in engineering work is becoming possible as well.

3. Fire-structure interaction

Although transferring the fire model results, such as temperature, heat flux or velocity, to the boundary conditions of the thermo-mechanical model of the structure is understood well in theory, the availability of practical tools is important for the feasibility of the coupled simulations. One of the tools for this purpose is FDS2FEM which takes the FDS results and creates input data for Finite Element Method (FEM) tools such as ABAQUS and ANSYS [4]. It interpolates the fire conditions from the FDS mesh to match the temporal and spatial discretization of the FEM model. This procedure is illustrated in Fig. 1.
While using the automated tools for model interoperability, it is possible to transfer the boundary conditions rigorously, considering the three main ingredients: gas phase velocity and temperature and the incident radiative heat transfer. For hand calculations, a convenient way of transferring the fire exposure is the Adiabatic Surface Temperature $T_{AST}$ [5] which is solved iteratively from the zero net heat flux requirement

$$\dot{q}_{net}^\prime + \dot{q}_c = \varepsilon (\sigma (T_{AST}^4 - T_w^4) + h(T_{AST} - T_w)) \quad (1)$$

where the left hand side contains the net radiative and convective heat fluxes, as predicted by the fire model, and the right hand side contains the convection and radiation corrections to achieve zero net heat flux at position which has temperature $T_w$ in the fire model. In the FEM code, the net heat flux would be calculated as

$$\dot{q}_{net}^\prime = \varepsilon (\sigma (T_{AST}^4 - T_{FEM}^4) + h(T_{AST} - T_{FEM})) \quad (2)$$

where $T_{FEM}$ is the FEM solution for the surface temperature.

In the analysis of steel structures, the failure criterion providing the fire resistance can usually be formulated in terms of the critical temperature. It is more difficult to define the failure or design acceptance criteria for some other types of building components. Computational assessment of fire resistance could have scenario specific acceptance criteria for the insulation and integrity properties. Considering long tradition of test-based determination of fire resistance, adopting of the computational methods can change the design of building components more than what the change of assessment methodology alone would imply.

4. Probabilistic assessment of barrier failures

Fire barrier failure probabilities are required when one needs to estimate the likelihood of losing the fire compartmentation. The challenge is then the unknown relation between the fire resistance ratings and the failure risk under natural, i.e. non-standard fire development. A practical challenge is also the computational cost of carrying out a probabilistic analysis for different structural elements.

In recent projects on the nuclear power plant fire risk assessment, methods have been developed for estimating the probability of fire-induced barrier element failures in large compartments and significant fire loads. The estimation process consists of several phases:

1. Different alternatives of the scenarios are combined using event trees, starting from the ignition frequency and propagating through the fire detection, suppression and fire service intervention to the un-availability of the barriers (e.g. open fire doors) and a barrier failure.

2. A range of possible fire exposures is obtained using Monte Carlo –simulation with Latin Hypercube Sampling and CFD-fire model as a deterministic tool.

3. The comparative barrier failure assessment methods is based on the French EPRESSI-method [6], modified for the use in probabilistic framework. First, a numerical model is created for each barrier element type (door, damper, penetration seal, etc.). The model, which can be just 1-D heat transfer model or a FEA model with more dimensions and details, is calibrated using the known fire resistance rating. The failure criterion in the standard test setup is the cold-side temperature. Using the model, a small number of analytical time-temperature curves are searched for which the failure criterion is not met. To estimate the failure probability, the Monte Carlo –simulation results are compared against these ‘performance curves’ and the number of random fires where the exposure exceeds the performance curves even at one time instance gives the failure probability.

The approach was tested carrying out a set of simulations for a nuclear power plant cable room. The difficulty of choosing the performance curves was caused by the ‘travelling’ nature of the fires, causing pre-heating phases with wide range of different lengths.

References


Application of the Discrete Wavelet Transform to defect localization in plates

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Abstract

The paper presents the defect detection in plates considering the influence of external static and dynamic loads. The aim of this work is to detect the localization of defect provided that damage exists in the considered plate structure. The Kirchhoff plate bending is described and solved by the Boundary Element Method. The plates are of rectangular shape, different boundary conditions and additionally, they rest on the internal column supports. The analysis of a structural response is conducted with the use of signal processing tool namely Discrete Wavelet Transformation (DWT). Defects in plates are modelled as slots near the plate boundary.

Keywords: defect detection, structural identification, wavelet transformation, the Boundary Element Method

1. Introduction

Defect detection in engineering structures is significant for monitoring of a structural behaviour. There are different non-destructive techniques which enables the identification of defective part of a structure. This problem is extensively investigated by scientists and some approaches based on e.g. optimization of loads [1], information of natural frequencies [2], heat transfer [3], inverse analysis [4,5], soft computing methods such as evolutionary algorithms [6] or artificial neural networks [7,8] are applied. Defect (damage) can be effectively detected using a relatively new method of signal analysis called wavelet transformation (WT) [9,10] also in its discrete form [11,12], [13]. Combining this method with, earlier mentioned, ANN or inverse analysis one can precisely identify defect (damage) details. The paper presents the issue of defect detection in thin plates excited by external static and dynamic loads. Numerical examples are presented.

2. Problem formulation

The aim of the presented work is to detect the localization of defects provided that they exist in the considered plate structure. Numerical investigation is conducted basing on signal analysis of structural static and dynamic response. The plate bending is described and solved by the Boundary Element Method. The boundary and boundary-domain integral equations are derived in singular and non-singular approach [14]. Rectangular plates are considered, supported on boundary and resting on the internal column supports. Internal supports are introduced using Bézine approach [15]. The analysis of a structural response is carried out with the use of Discrete Wavelet Transformation (DWT). The multiresolution signal analysis using Mallat pyramid algorithm [16] is applied too. Defects in plates are modeled as slots near the plate boundary.

3. Selected numerical examples

Rectangular plates, supported on boundary and resting on the internal column supports are considered. Defects are introduced by additional edges forming a crack in relation to the basic plate domain. A static external concentrated load \( P \) is imposed at selected points along direction parallel to the one plate dimension. The measured variables are: deflections, curvatures and bending moments. The data are gathered in one measurement point, located near and slightly distal position in relation to the loss of plate basic domain, in equal time intervals. Decomposition of the obtained signal is carried out using DWT and Daubechies 4 wavelet. The plate properties are: \( E = 205 \) GPa, \( \nu = 0.3 \); plate thickness \( h = 0.05 \) m; the dimensions of the slot are: \( d = 0.005 \) m, \( e = 0.25 \) m; external load \( P = 10 \) kN; the internal column supports have the square cross-section of the side dimension \( 0.08 \) m and they are treated as additional sub-domains of the constant type [15]; all of plate edges are divided into 30 boundary elements of the constant type.

3.1. Example 1

The plate shown in Fig. 1 is considered.

\[ N = 64 \] number of measurements

Figure 1: The plate structure resting on 8 column supports with all edges free

Figure 2: DWT, signal: static deflection measured in D point, \( N = 64 \) number of measurements

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3.2. Example 2  

The plate shown in Fig. 3 is considered.

Figure 3: The plate resting on 4 column supports with two opposite edges simply-supported

![Figure 3](image)

Figure 4: DWT, signal: curvature $\kappa$ measured in D point, $N=64$ number of measurements

4. Concluding remarks

The implementation of discrete dyadic wavelet transformation to identification of signal discontinuity in the analysis of plates is presented in the paper. The thin plate bending is described by the boundary (static analysis) integral equations and solved using the BEM. The dynamic analysis of the structure response is also carried out. Although the considered issue is two-dimensional from the point of view of deformation description, applied one-dimensional discrete wavelet transform (DWT) leads to efficient results in defect detection. The analysis is carried out without any signal noise reduction. It discovers small disturbances in response signal of defected structure and does not require the reference to a signal from undefected structure. The convenience is that in the present approach the data are gathered in one measurement point in equal time intervals. The distance of the measurement point from damaged area is crucial for proper defect localization. Considered examples, quite correctly identify the presence and position of defects (Figs. 2 and 4). The existence of additional column supports inside a plate domain does not substantially impair the response signal. Effectiveness of the proposed method is indicated by previously presented numerical experiments. Effectiveness of the proposed method is indicated by previously presented numerical experiments, where the defected plates are properly localized, even for a relatively small number of measurements ($N=64$).

References


Progressive collapse of a reinforced concrete flat slab frame under column removal

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Abstract

The paper reports selected results of a 3-year EU-funded project carried out at European Laboratory for Structural Assessment at JRC Ispra, Italy and devoted to investigations of the potential for progressive collapse of a reinforced concrete flat-slab frame building. A central first storey column is suddenly removed causing the structure to find an alternative load path. To check whether the progressive collapse can happen, the nonlinear dynamic analysis is performed. The analysis takes into account the reinforced concrete fiber section approach, uniaxial nonlinear relationships both for concrete and steel, and time-history transient calculations.

Keywords: progressive collapse, reinforced concrete structures, nonlinear dynamic analysis

1. Introduction

The role of the accidental loads in civil engineering, as defined in [3] is becoming more and more important. Particularly the effects of explosions are under intensive research recently. This subject of investigations is interdisciplinary and covers such areas as explosion effects on structures [2, 8], as well as, in general, robustness and progressive collapse of structures [5, 1].

There were a few world-wide known examples of progressive collapses to name only a few: in 1968, a gas explosion caused a partial collapse of the Ronan Point residential apartment building, London, the UK. In 1995, a bomb detonated near the Alfred P. Murrah Federal Building (Oklahoma, the US) led to its major collapse. The same year, a gas explosion in a prefabricated residential building (Gdansk, Poland) made the total collapse of three floors. In 2006, the steel truss roof of Katowice Trade Hall (Poland) collapsed due to the weight of snow which overloaded the improperly braced plane trusses.

The eurocode [3] does not specifically deal with accidental loading such as internal or external bomb explosion, instead here an approach related to the unspecified cause can be followed. In particular, Eurocode [3] requires that the load-bearing elements (e.g. columns, girders or a fragment of wall) are removed anywhere in the structure one at a time and check if progressive collapse could occur.

The sudden removal of a structural element can make the structure collapse or develop an alternate load path. Many documents such as [2, 3, 4] recommend that the prescriptive rules can help to achieve the expected behaviour in the design process and then during the occurrence of accidental loading. These prescriptive design rules include: providing sufficient ductility to the structure, ensuring static indeterminacy (redundancy), providing continuous rebars both in the top and bottom zones of the RC cross-sections.

2. Experiment and numerical simulation

We analyse a reinforced concrete flat-slab frame building (Fig. 1) which is described in detail in [7]. First, the structure was tested for medium seismicity (0.25g peak ground acceleration). The building had survived the design earthquake, tested at the reaction wall facility at the European Laboratory for Structural Assessment, Joint Research Centre, Ispra, Italy, with minor damage and transported out of the laboratory for demolition. Taking this opportunity, the structure was devoted to controlled slow demolition using a concrete crunching machine with the goal of investigating its safety against progressive collapse. The experiment has shown not only that the structure survived the removal of two central columns, but also how challenging the structural testing against collapse is.

Figure 1: Front view and building after two columns being removed

However, buildings can be exposed to fast dynamic abnormal events, such as bomb explosions or impacts, so the dynamic nature of the loading must be considered. Therefore, to answer the question: what would have happened if the columns had been destroyed dynamically in an almost instantaneous manner?, the structure is re-evaluated by the author using dynamic linear and nonlinear simulations in SAP 2000. The results are reported in [7, 6].

The simulation of the fast column removal is performed by suddenly cancelling the reaction forces standing for the column. The rate of the column removal is specified by a time function - linear ramp to maximum value. For actual bomb explosions, the time in which a structural member is destroyed is very short (some milliseconds). In the FE calculations, the removal time is
closer to zero, which means a quasi instantaneous removal. The
dynamic effects of the removal rate on the dynamic response of
the structure have been analysed in Report [5] and the results
shows that the most unfavourable dynamic effects occur when
the column is destroyed within 5 ms.

The linear analysis is quite straightforward, on the other hand,
the nonlinear analysis can be done in many ways, depending on
how the nonlinear part is incorporated in the finite element soft-
ware. For example, in SAP 2000, the material nonlinearity is
modelled through the assignment of plastic hinges at discrete
points of finite elements. There is a question, what are ade-
quate plastic hinge properties? Frequently used default plastic
hinge parameters for progressive collapse analysis are those of
US ATC-40 for reinforced concrete structures and FEMA-273
NEHRP for steel structures. However, those parameters are spec-
ified for horizontal pushover analysis rather than vertical analysis.
Therefore, in the next step the plastic hinge nonlinear approach
is compared to the fibre section approach and their nonlinear ma-
terial properties as implemented in a research-oriented finite ele-
ment software such as Opensees. Opensees allows to use realistic
nonlinear uniaxial stress-strain relationships both for concrete
(confined and unconfined zones) and reinforcement steel (see Fig.
2) with hardening, softening and residual strength capabilities.

This paper presents the selected results of a linear and non-
linear dynamic analyses of a reinforced concrete flat slab frame
building under column removal. The structure resists both ex-
perimental quasi-static column removal as well as a numerically
simulated dynamic fast column removal. The numerical calcula-
tions take into account nonlinear material relationships for both
concrete and steel.

Figure 2: Stress-strain relationships for concrete (Kent-Park
model) and steel (Menegotto-Pinto model)

Therefore, the structure is remodelled in Opensees and linear
and nonlinear dynamic analyses were performed as described ear-
lier. The specific options used to obtain a FEM solution include:
the modified Newton-Raphson algorithm with initial stiffness
and the norm of the left hand side solution vector of the matrix equa-
tion to determine whether convergence was reached.

An example plot of the response time history at critical node
where the missing column is attached to the rest of the frame is
presented in Fig. 3. Figure 4 shows the bending moment time
history at critical cross-section of the first floor right girder. It
can be observed that the analysis in Opensees which used more
credible nonlinear material parameters gives larger deformation,
however still the structure survives the sudden removal of the cen-
tral column. Other numerical cases will be presented during the
Conference.

3. Conclusions

This paper presents the selected results of a linear and non-
linear dynamic analyses of a reinforced concrete flat slab frame
building under column removal. The structure resists both ex-
perimental quasi-static column removal as well as a numerically
simulated dynamic fast column removal. The numerical calcula-
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Rheology of wooden beams reinforced by CFRP with discontinuities

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Abstract

The paper concerns the rheological bending problem of wooden beams reinforced by CFRP types with discontinuities. The reasons for the discontinuities are assembly mistakes and specific defects of wood. A numerical model of these beams is presented, in which the rheological properties of the component materials have been described with the constitutive relations of linear viscoelasticity in the form of the five-parameter model. Numerical model was made in ABAQUS and INTEL FORTRAN. A positive influence of CFRP reinforcement on rheological increments of deflections was stated. These increments are less than in the case of non-reinforced beams. A significant redistribution of stresses in the cross-section occurs in the reinforced wooden elements. The CFRP type is further loaded, stresses in the wood decrease. The concentration of stresses in the glue joint surrounding discontinuities may lead to a dangerous failure mode called delamination.

Keywords: ABAQUS, CFRP, creep, delamination, displacements analysis, finite element method, rheology, UMAT subroutine, wooden structures

1. Introduction

This paper concerns the analysis of deflections and stresses in viscoelastic wooden beams reinforced by CFRP with discontinuities. The reasons of the discontinuities are assembly mistakes and specific defects of wood. In these beams a redistribution of stresses may occur due to different rheological properties of the component materials, which leads to changes in the location of the neutral axis, strains and displacements in time. Assembly defects and exploitation failure in the glue layer between wood and CFRP can onset a delamination process which can lead to ultimate damage.

2. Analyzed specimen

Analyzed wooden beam reinforced by CFRP tape and non-reinforced beam is illustrated in Fig. 1.

The beams of natural dimensions made of wood (pine, *Pinus*), CFRP tape (Sika CarboDur H514) and epoxy adhesive, were examined in the four-point bending test. Mechanical parameters of these materials have been determined from additional separate tests and literature data.

3. Rheological model of wood

The rheological properties of the polyester glass, wood and the epoxy adhesive are described with the five-parameter model (Ref. [2,3,4]) shown in Fig. 2.

![Figure 2: Five-parameter rheological model of component materials](image)

Figure 2: Five-parameter rheological model of component materials

The material parameters $E$ and $\eta$ of the model have been determined from additional separate tests. The mathematical formula for the relaxation function of the applied model was derived from the constitutive relation of linear viscoelasticity in differential form by making use of the Laplace transformation (Ref. [4]).

4. Delamination

In literature several techniques are found for the prediction, i. e. the onset, and the propagation of cracks (Ref. [5]). One of them is the cohesive zone technique. It assumes a cohesive zone between two components wood and CFRP in a composite structure. The cohesive layer has a very small but finite thickness relatively to the thickness of the CFRP layer. The model relates stresses to relative displacements in the interface. This stress-separation model assumes initially a linear elastic behaviour of the interface followed by an initiation and an evolution of the failure. The used stress-separation-based model...
assumes three different types of separation well-known from the fracture mechanics, whereby one is normal to the interface (opening mode) and the two others are parallel to it (sliding or shearing modes). The failure initiation depends on the interfacial strength and can be estimated by a quadratic stress interaction function, originally proposed by Hashin. This criterion can be represented as follows:

\[
\left( \frac{\sigma_{33}}{R_{33}} \right)^2 + \left( \frac{\sigma_{23}}{R_{23}} \right)^2 + \left( \frac{\sigma_{13}}{R_{13}} \right)^2 \leq 1
\]  

where \( \sigma_{33} \) is the interlaminar normal stress and \( \sigma_{23} \) and \( \sigma_{13} \) are the corresponding shear stresses in the interface. The stresses are calculated in integration points of a cohesive element. The Hashin criterion is presented in terms of Cauchy stresses. The quantities \( R_{33} \), \( R_{23} \) and \( R_{13} \) are the tensile strength in the thickness direction and the shear strengths, respectively. The symbol \( \mathbf{(*)} \) denotes the Macaulay brackets. It means that only tensile stress \( \sigma_{33} \) is included into Hashin criterion.

Interface damage evolution is expressed in terms of energy release \( G \). In Figure 3 the relation is presented graphically. \( \delta_0 \) is the relative displacement corresponding to the strength \( R \) in this direction (for one of the modes) and \( \delta_f \) corresponds to the final displacement for separation.

![Figure 3: Stress – relative displacement law](image)

6. Conclusions

A positive influence of CFRP reinforcement on rheological increments of deflections was stated. Reinforcement of wooden beam by CFRP tapes can decrease the process of growth of rheological displacements (38% wooden beam, 23% -wooden beam with CRFP), when full adhesion is assumed (Fig. 3).

![Figure 4: Comparison of rheological deflection curves](image)

5. Numerical model of wooden beams reinforced by CFRP with discontinuities

Numerical model of wooden beams reinforced by CFRP with discontinuities was made in ABAQUS and INTEL FORTRAN. The UMAT (User-defined MATerial) subroutine is used to implement the rheological five-parameter model (Ref. [2]). This subroutine calculates the stress increment and the Jacobian matrix \( C \) of the constitutive model for each time increment

\[
\frac{\partial \Delta \sigma}{\partial \Delta \epsilon} = C
\]

A differential form of constitutive equations of five-parameter model and the central difference operator were used to calculate the Jacobian matrix

\[
p_0 \cdot \sigma + p_1 \cdot \dot{\sigma} + p_2 \cdot \ddot{\sigma} = q_0 \cdot \dot{\epsilon} + q_1 \cdot \ddot{\epsilon} + q_2 \cdot \dddot{\epsilon}
\]

All elements of reinforced beams were modeled as parts (PART) joined using TIE function. An finite element C3D8R (8-node linear brick, reduced integration with hourglass control) was used.

References

Numerical analysis of the influence of temperature on the dynamic characteristics of structures with viscoelastic dampers

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Abstract

In the paper, the effect of change of temperature in viscoelastic (VE) dampers on the dynamic characteristics of structures having such dampers installed on them are presented. Both classic and fractional rheology models are used to describe the properties of VE dampers. The time-temperature superposition principle is used to describe changes in the properties of VE dampers due to temperature changes. The dynamic properties of structures with dampers are determined from the solution to an appropriately defined nonlinear eigenvalue problem. Two numerical procedures are used to determine changes of dynamic characteristics of structures with temperature.

Keywords: dynamics of structures, viscoelastic dampers, dynamic characteristics, temperature influence

1. Introduction

Viscoelastic dampers can effectively reduce the amplitudes of vibrations of building structures excited by the wind or earthquakes [1]. Descriptions of the dynamic behavior of the dampers of this type are complex, because the properties of the viscoelastic (VE) materials the dampers are made of depend on the excitation frequency and temperature [2, 3]. In previous papers, only the influence of excitation frequency was taken into account [4, 5].

The changes of temperature of a VE damper can be caused by changes of environmental temperature or by so-called self-heating, induced by dissipation of energy. The influence of temperature on the properties of VE dampers has been described in several papers (see [3, 6]). However, the influence of changes of temperature in VE dampers on the dynamic characteristic of structures having such dampers installed on them has not been analyzed systematically.

In this paper, the computation method is presented allowing for the analysis of the influence of temperature changes on natural frequencies, dimensionless damping ratios and eigenvectors (modes of vibration) of structures with VE dampers.

2. Influence of temperature on parameters of models of VE dampers

In this research, the classic and fractional rheological models are taken into account. The generalized Maxwell model is used as the classic model. The fractional Zener model is the non-classic one. This allows analysis of a number of simpler models, such as viscous model, simple Kelvin model, simple Maxwell model, classic Zener model etc.

Figure 1: A diagram of fractional Zener model of damper

In order to determine the VE material response to changes of temperature, the time-temperature superposition principle can be used, as given by the following relationship (1):

\[ E(t, T) = E(\alpha_T t_0, T_0) , \]

where \( E \) is the complex modulus, \( t_0 \) and \( T_0 \) are the reference time and the reference temperature, respectively. The symbol \( \alpha_T \) denotes the so-called shift factor. This principle can be applied to the frequency domain and sometimes named as the frequency-temperature correspondence principle [6]:

\[ E(f, T) = E(\alpha_T f_0, T_0) , \] (2)

where \( f_0 \) is the reference frequency.

The shift factor usually is calculated from the Arrhenius formula or from the following William-Landel-Ferry formula:

\[ \log \alpha_T = \frac{-C_1 \Delta T}{C_2 + \Delta T} , \] (3)

where \( C_1 \) and \( C_2 \) are constants and \( \Delta T = T - T_0 \).

According to relationship (2), the calculation of material response to any temperature \( T \) consists in shifting the argument of function \( E(f_0) \) for reference temperature \( T_0 \) in the following way:

\[ f_0 = \alpha_T f . \] (4)
3. Equation of motion of structures with VE dampers and the nonlinear eigenproblem

3.1. Equation of motion of VE damper

The equation of motion of the Zener model of damper is:

\[ u(t) + r^\alpha D^\alpha u(t) = k_s \Delta q(t) + r^\alpha (k_d + c_d) \Delta q(t) \]

(5)

where \( u \) is the force in the damper, \( q_j \) and \( q_i \) denote the nodal displacements of the damper model and \( \Delta q(t) = q_j(t) - q_i(t) \). \( D^\alpha(\bullet) \) denotes the Riemann-Liouville fractional derivative of the order \( \alpha \) with respect to time \( t \). After applying the Laplace transformation with zero initial conditions, the transform of force in damper can be written as:

\[ \mathbf{D}(s) = \frac{k_s + (k_d + c_d)(s \tau)^\alpha}{1 + (s \tau)^\alpha} \mathbf{D}(s), \]

(6)

where \( \mathbf{D}(s) \) is the Laplace transform of \( \Delta q(t) \) and \( s \) denotes the Laplace variable. For \( \alpha = 1 \), Eqs 5 and 6 describe the classic Zener model.

On the basis of the time-temperature superposition principle, it can be proved that the following relationship holds true:

\[ r_0 = \alpha \tau, \]

(7)

where \( \tau^\alpha = (c_d/k_d) \) could be understood as the relaxation time and the symbols \( k_d \) and \( c_d \) denote the stiffness and the damping factor of the damper, respectively (see Fig. 1).

The relationship (7) is crucial to the proposed approach because, in this simple way, it is possible to determine the parameters of VE dampers for different temperatures.

3.2. Equation of motion of structure with VE dampers

The equation of motion of a structure with VE dampers can be written in the following form (compare [7]):

\[ \mathbf{Mq}(t) + \mathbf{Cq}(t) + \mathbf{Kq}(t) = \mathbf{p}(t) + \mathbf{f}(t), \]

(8)

where \( \mathbf{M} \), \( \mathbf{C} \) and \( \mathbf{K} \) denote the mass, damping, and stiffness matrices, respectively. Moreover, \( \mathbf{q}(t) \) is the vector of displacements of the structure, \( \mathbf{p}(t) \) is the vector of the excitation forces and \( \mathbf{f}(t) \) is the vector of the interaction forces between the frame and the dampers. After applying the Laplace transform, the equation of motion (8) can be written as follows:

\[ (s^2 \mathbf{M} + s \mathbf{C} + \mathbf{K}) \overline{\mathbf{q}}(s) = \overline{\mathbf{p}}(s) + \overline{\mathbf{f}}(s). \]

(9)

The vector \( \overline{\mathbf{f}}(s) \) is formed using Eqn (6) (compare [7]).

After assuming \( \overline{\mathbf{p}}(s) = \mathbf{0} \), the final form of the equation of motion in the frequency domain for the structure with VE dampers is as follows:

\[ (s^2 \mathbf{M} + s \mathbf{C} + \mathbf{K}_d + \mathbf{G}_d(s)) \overline{\mathbf{q}}(s) = \mathbf{0}, \]

(10)

where

\[ \mathbf{K}_d = \sum_{j=1}^{m} k_{0j} \mathbf{L}_j, \quad \mathbf{G}_d(s) = \sum_{j=1}^{m} \frac{k_{0j} s^\alpha}{s^\alpha + \omega^2} \mathbf{L}_j, \quad \omega = \frac{k_{0j}}{c_{0j}}. \]

(11)

The symbol \( \mathbf{L}_j \) denotes the location matrix of the \( j \)-th damper and \( m \) is the number of dampers on the structure.

3.3. Nonlinear eigenproblem

Equation 10 constitutes the nonlinear eigenproblem. In this research work, the solution to the problem is obtained using the so-called continuation method [7]. The calculated values of \( s \) are the eigenvalues and \( \overline{\mathbf{q}}(s) \) are the eigenvectors. The knowledge of the eigenvalues \( s_k = \mu_k + j\eta_k \), where \( i \) is the imaginary unit, allows for the calculation of the natural frequencies of the structure \( \omega_k \) and its dimensionless damping factors \( \gamma_k \). It can be done in the following way:

\[ \omega_k^2 = \mu_k^2 + \eta_k^2, \quad \gamma_k = \mu_k / \omega_k. \]

(12)

4. Example results

The computations were made for the shear frame model of two-storey building structure. The mass is lumped, the same at every floor: \( m = 2000 \text{ kg} \). The bending rigidity of each storey is \( k = 4000 \text{ kN/m} \).

The VE dampers are located on each storey. The reference values of parameters of the damper are following: \( T_0 = 0.2 \text{ °C} \), \( k_d = 18.73 \text{ kN/m} \), \( k_d = 3443.68 \text{ kN/m} \), \( c_d = 396.02 \text{ kNs/m} \), \( \omega_0 = 0.609 \). The shift factor \( \alpha_\tau \) was computed using the constants \( C_1 = 19.5 \) and \( C_2 = 80.2 \).

In Fig. 2 the dimensionless damping factor \( \gamma_k \) of the first mode of vibration versus temperature is presented.

Figure 2: The dimensionless damping factor \( \gamma_k \)

References

Verification and validation of thermal and mechanical response of steel beams according to selected fire exposures and various numerical approaches

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Abstract

In the paper, the problem of modelling of steel structures exposed to fire is considered. The modern design codes, like Eurocodes [1], give an advantage for designers by allowing the performance based design, where the designed structure needs to fulfil the performance criteria, in contrast to old approaches based on prescriptive rules. Therefore several approaches and procedures can be chosen to model the structural behaviour in fire. This paper focuses on the influence of adopted approach on the results of solution of engineering problem with special consideration of the temperature distribution inside the cross-section and finite element model used for evaluation of performance criteria. The numerical simulations are compared within each other and finally validated against experimental results. At the end, conclusions about the physical phenomena occurred the beams in fire and recommendations for modelling and design are made. During the conclusion process, particular interest is put in setting together the results from analytical methods, simplified numerical models and complex 3D approaches.

Keywords: Performance based design, fire engineering, numerical simulation, finite element method, heat flux, heat transfer

1. Introduction

The paper considers the problem of modelling of steel structures exposed to fire. Modern design codes, like Eurocodes [1], give an advantage for designers to allow for the performance-based design, where the designed structure needs to fulfil the performance criteria, in contrast to old approaches based on prescriptive rules. The beneficiaries are structural fire engineers, who are now able to check the real safety requirements addressed to a particular structure. On the other hand, performance-based approaches need a comprehensive treatment of each analysed case. The main problem here is to adopt the complexity of modelling into the design goals. The purpose of the paper is to answer the question on the usefulness of possible formulations of the heat transfer problem between the fire and structural element in order to get the mechanical response with certain level of reliance.

The experimental results published in [2] will be compared with numerical computations for 1D (beam), 2D (shell) and 3D (brick) models. The problem of heat transfer between gas and solid will be defined by a simplified zero-dimensional approach proposed by Eurocode 1993-1-2 [3] and an approach proposed by the authors, which takes into account the influence of radiative and convective heat fluxes on different parts of a cross-section.

2. Heat transfer problem formulation

The main contribution of the work is the working model for heat transfer between hot gases and a cross-section of structural element, where a cross-section is parametrized and divided into a finite number of parts corresponding with integration points from finite element method model (Fig. 1b). Each part is receiving heat by its surfaces and transferring it by the link connections described by material conductivity and specific heat. Each type of section is parametrized and its characteristics (eg. view factors for radiation between surfaces and gas) are calculated apriori and stored in a database dedicated for each cross-section.

The model is based on: the procedure developed by authors and devoted for calculation of heat flux into structural sections exposed to fire, and the classic theory about radiation, convection and conduction of heat. Because of the numerical problems in calculation of view factors for adjacent surfaces in cross-section, the method developed by Mitalas and Stephenson [5] and furthermore used in computer code called FACET [6] is used for approximate computation of view factors. In this procedure the classical integral [6,7] with the problematic distance $r$ in the denominator

$$ F_i = \frac{1}{A_i} \oint_{A_i} \int_{A_i} \frac{\cos \beta_i \cos \beta_j}{\pi r^2} dA_i dA_j $$

(1)

is transformed using Stokes theorem into an integral over a perimeter

$$ F_i = \frac{1}{2\pi A_i} \sum_{j \neq i} \phi(p,q) \int_{A_j} [(T \cos \theta \ln T + S \cos \theta \ln S + U/\omega - R) dA_j] $$

(2)

where $S,T,U,\psi,\omega$ are the functions of side on the perimeter and $\phi(p,q)$ is a dot product between the normal direction of surfaces.

Figure 1: Model for heat transfer problem: a) the idea, b) division into sub-elements
The core of developed method is the solution of radiation interchange between 3 arbitrary surfaces, where two of them are related to the solid phase surfaces and the third one is a virtual surface corresponding to gas (Fig. 1a). For such a triplet it is possible to formulate the equilibrium equation in the form:

$$ \begin{bmatrix} \dot{q}_i^T \nabla T^T \end{bmatrix} + C \begin{bmatrix} T^T \\ \nabla T^T \\ T_{dir}^T \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} $$

(3)

where in matrices $A, B, C$ the apriori calculated section parameters are stored.

3. Test cases

In order to get the information about the influence of numerical model complexity on the thermal and mechanical response of steel beams in fire, 5 real test referred in [2] as 1, 2, 5, 6, 10 are simulated, verified and validated. All of them are partially exposed for fire I-beams with dimensions, span, loads and fire resistance time summarized in Table 1.

Table 1: Test cases

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>6</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-section</td>
<td>254 x 146 mm</td>
<td>254 x 146 mm</td>
<td>356 x 171 mm</td>
<td>406 x 178 mm</td>
<td>254 x 146 mm</td>
</tr>
<tr>
<td>Span</td>
<td>4.5 m</td>
<td>4.585 m</td>
<td>4.5 m</td>
<td>4.5 m</td>
<td>4.5 m</td>
</tr>
<tr>
<td>Loads</td>
<td>44,15 kN</td>
<td>46,73 kN</td>
<td>72,90 kN</td>
<td>72,62 kN</td>
<td>33,38 kN</td>
</tr>
<tr>
<td>Fire resistance</td>
<td>35.5 min</td>
<td>23.0 min</td>
<td>27.0 min</td>
<td>23 min</td>
<td>21 min</td>
</tr>
</tbody>
</table>

The verification and validation processes are carried out with respect to the section temperature and deflection history.

4. Numerical analysis

All of the selected tests are simulated using 1D beam, 2D shell and 3D brick finite elements. Since in 2D and 3D modelling standard S4 and C3D4 elements are used [8] with linear shape functions and full integration, 1D modelling uses shear flexible B31 elements (Timoshenko beams) what is more appropriate for slender cross-sections, like for example class 4 cross-sections. In the case of mentioned examples it has a minor influence on results.

Together 15 finite elements models are prepared and for each of model temperature is applied as the boundary conditions in two ways: according to 0D Eurocode model providing the uniform temperature distribution within steel section; and the procedure proposed by authors in p.2 of this paper. That results in total 30 analyses, where each of them is also verified with respect to the mesh size.

The exemplary output from one of the analyses is shown in Fig. 2. In that case the difference between the beam modelled using beam and shell element is shown. It is clearly seen, that results differs in about 3-4 minutes, while the model with shell elements have been fully validated against the experiment, where the failure occurred in 27 minute.

5. Conclusions

In the work the mechanical response of beam structural elements is taken into consideration. The main attention is put into the influence of modelling onto the results, what aims in getting the recommendations for methodology used by structural fire engineers during the performance based design. The models are compared within each other, verified and validated according to experimental results. The conclusions are driven both with respect to simple design recommendation and explanation of physical phenomena which can, or cannot, be caught by different approaches. Especially non-uniform degradation of the cross-section parameters caused by temperature gradient inside the section and lateral torsional buckling phenomena at elevated temperature is taken into consideration.

References


Effect of non-linear elastic support of a rotating flexible shaft on bifurcating behaviour of the system

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Abstract

In the paper, dynamic stability of a rotating slender shaft imperfectly supported at one end is analyzed. The imperfection is modeled by a spring with a nonlinear cubic characteristic. Flexible rotating shafts are known to exhibit dynamic loss of stability due to a presence of internal friction in the shaft material. At a certain rotation speed, called the critical speed, the static equilibrium position bifurcates into a new periodic solution. In the paper, the effect of non-rigid mounting on the critical angular velocity is examined. Additionally, the near-critical response of the shaft is thoroughly investigated in terms of its orbital stability. It was confirmed that a digressive restitution force in the support may lead to particularly hazardous self-excitation.

Keywords: rotating shaft, elastic support, stability, critical speed, flutter, bifurcation

1. Introduction

The analyzed system is a slender Euler-Bernoulli flexible beam having length \( l \), cross sectional area \( A \), geometric moment of inertia \( J \), Young's modulus \( E \), density \( \rho \), and is simply supported on one end. The other support is not perfectly rigid but elastic. Non-linear elasticity occurs—the restitution force obeys the following expression:

\[
S(w) = \kappa_1 w + \kappa_3 W^3 \tag{1}
\]

where \( w \) denotes lateral displacement of shaft at the place of elastic mounting. The shaft rotates with quasi-static angular velocity \( \omega \). The material is assumed to exhibit some damping and is simply elastic. Non-linear elasticity occurs—the restitution force obeys the following expression:

\[
S(w) = \kappa_1 w + \kappa_3 W^3
\]

where \( w \) denotes lateral displacement of shaft at the place of elastic mounting. The shaft rotates with quasi-static angular velocity \( \omega \). The material is assumed to exhibit some damping and is simply supported on one end. The other support is not perfectly rigid but elastic. Non-linear elasticity occurs—the restitution force obeys the following expression:

\[
S(w) = \kappa_1 w + \kappa_3 W^3
\]

and where \( k \) is the eigenvalue of the beam first eigenform. Equations (2) are already discretized partial ordinary differential equations of the rotating shaft derived, among others, in Ref. [2]. The method of discretization was based on Galerkin's orthogonalization in which only the first eigenfunction \( W(x) \) corresponding not to the simply supported beam, but parameter-dependent function sensitive to the coefficients of elasticity \( \kappa_i \) and \( \kappa_j \) is introduced in the form of the boundary conditions of the analyzed beam, namely \( EJW''(l) = \kappa_1 W(l) + \kappa_3 W^3(l) \). The functions \( Y \) and \( Z \) appearing in Eqn (2) are time variables constituting components of the predicted unimodal solution:

\[
y(x,t) = Y(t)W(x) , \ z(x,t) = Z(t)W(x) . \tag{2}
\]

Let us introduce new variables \( Y = u_i \), \( \dot{Y} = u_2 \), \( Z = u_3 \) and \( \dot{Z} = u_4 \) in order to transform Eqn (2) into a system of four differential equations of the first order:

\[
\begin{align*}
\dot{u}_i &= u_2 \\
\dot{u}_2 &= -(a_1^2 + \kappa_1)u_i - a_1^2 \beta u_2 - a_1^2 \omega \beta u_3 - k_1 u_4 - k_3 u_4^3 + u_1^2 u_4 \\
\dot{u}_3 &= u_4 \\
\dot{u}_4 &= a_1^2 \omega \beta u_2 - (a_1^2 + \kappa_1)u_4 - a_1^2 \beta u_3 - k_1 u_3 - k_3 u_4^3 + u_1^2 u_3 
\end{align*}
\]

or briefly:

\[
\dot{\mathbf{u}} = \mathbf{f}(\alpha, \mathbf{u}) = \mathbf{A}(\alpha) \mathbf{u} + \mathbf{N}(\alpha, \mathbf{u}) \]  

where \( \mathbf{A} \) is the matrix of linearized system (4), \( \mathbf{N} \) — vector of the nonlinear part of Eqn (4), \( \mathbf{u} = [u_i, u_2, u_3, u_4] \).

2. Critical rotation speed

Let us examine stability of the analyzed system investigating the eigenvalues of the matrix \( \mathbf{A} \) in (4), which correspond to the eigenproblem \( \det[\mathbf{A}(\alpha) - \mathbf{rI}] = 0 \). This leads to the characteristic equation of the fourth order in \( r \):

\[
r^4 + 2a_1^2 \beta r^3 + r^2(2a_1^2 + a_1^2 \beta^2 + 2k_1) + 2a_1^2 \beta r(a_1^2 + k_1) + a_1^4 (1 + \omega^2 \beta^2) + 2a_1^2 k_1 + k_3 = 0 \tag{5}
\]
By applying Hurwitz criterion to Eqn (5), a conclusion holds, that the critical rotation speed is equal to:

$$\omega_{cr} = \sqrt{a^2 + K_1}$$

(6)
The course of the critical speed versus stiffness of the elastic support is shown in Fig. 2. The stiffness coefficient $K_1$ in Fig. 2 is expressed in terms of a multiple of the stiffness $\kappa_d$ which denotes rigidity of a perfectly simply supported beam undergoing loading applied to its middle point $\kappa_d = 48 E J / l^3$.

3. Nonlinear analysis

After exceeding the critical angular velocity, the system loses its stability. The trivial and static equilibrium position bifurcates into a new equilibrium state manifest by self-excited near-critical vibration. Before constructing the newly occurred bifurcating solution, the check is made first whether the dynamical equations of motion satisfy conditions of the Floquet theorem necessary for such a solution to exist. Analyzing Eqn (4), one finds that the function $f(\omega, u)$ disappears at the equilibrium position $u = 0$, i.e. $f(\omega, 0) = 0$, furthermore $f(\omega, u)$ is differentiable with respect to $\omega$ and $u$ and the matrix $A(\omega)$ has a pair of eigenvalues such that

$$\text{Re}\{r(\omega_c)\} = 0$$

and

$$\text{Re}\left(\frac{dr(\omega_c)}{d\omega}\right) = \frac{\alpha^2 \beta \omega_c}{4 \omega_c^2 + \alpha^2 \beta^2} \neq 0,$$

and the rest eigenvalues have negative real parts

$$\text{Re}\{r_{\lambda}(\omega_c)\} = -\alpha^2 \beta < 0.$$ Thus, all the necessary conditions of the Floquet theorem are fulfilled.

3.1. Construction of the bifurcating solution

The bifurcating solution is predicted in form of an infinite series, see Ref. [1], which when truncated at the first term can be expressed as

$$u^0(t) = \text{Re}\left[Q \exp\left(i \left(\Omega_0 + \Omega_1 \frac{\omega_c - \omega_c}{\omega_2} - \frac{\omega_c - \omega_c}{\omega_2} \right) t\right)\right],$$

(7)

$$\epsilon = \frac{2 \omega_c - \omega_c}{\omega_2}$$

where $Q$ is the eigenvector corresponding to eigenproblem $\{A(\omega_c) - i \Omega_0\}Q = 0$, $\Omega_0$ is the initial flutter frequency ($\omega_c$ in the considered system) and the coefficients $\omega_2$ and $\Omega_1$ are found to be

$$\omega_2 = \frac{84 K_1 (4 \omega_c^2 + \alpha \beta^2)}{3 a^2 \beta \omega_c}, \quad \Omega_1 = \frac{8 a \beta K_1}{3 \omega_c}$$

(8)

which finally enable construction of the bifurcating solution.

3.2. Amplitude of self-excited vibration

It is known that the amplitude (more precisely, the norm) of the bifurcating solution is proportional to the small parameter $\epsilon$ in the infinite series, which in turn, can be expressed in terms of $\omega_c$, $\omega_c$, and $\omega_c$, i.e.

$$|u^0| = \sqrt{2 \epsilon} |q| = \sqrt{\frac{\omega_c - \omega_c}{\omega_c}} |q|$$

(9)

Accordingly to the calculations carried out, the amplitude strongly depends on the nonlinear part of the elastic support stiffness $\kappa$, see Fig. 3.

3.3. Stability of the bifurcating solution

The orbital stability of the determined bifurcating solution is ruled by the Floquet quotient which is defined as follows:

$$\sigma = -\text{Re}\left[\frac{dr(\omega_c)}{d\omega}\right] \omega_c \epsilon^2 + O(\epsilon^4)$$

(10)

where $O(\epsilon^4)$ describes negligible terms of higher orders. Analyzing the nonlinear part $N(\omega_c, u)$ of Eqn (4) one finds that:

$$\sigma = -\frac{16}{3} K_c \epsilon^2 + O(\epsilon^4)$$

(11)

As can be seen, stability depends on the nonlinear part of the stiffness only. The self-excited vibration stays orbitally stable as long as the coefficient $K_c$ remains positive.

4. Conclusions

The analyzed system is strongly sensitive to the stiffness of the elastic support. Compliance of the bearing diminishes critical rotation speed which endangers the shaft with self-excited vibration. If the loss of rigidity is sudden and unintentional, it may lead to large-amplitude vibrations of the shaft eve at moderate rotation speeds. It the compliance exhibits some nonlinear properties, and the nonlinearity is digressive ($K_c < 0$), the loss is catastrophic.

References


Reliability analysis of reinforced concrete structures: a comparative study

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Abstract

A full probabilistic assessment of reliability of reinforced concrete structures holds the key to proper calibration of current safety formats for nonlinear finite element analysis of concrete structures. This current study attempts to establish the best possible stochastic finite element scheme based on numerical trials on a simple reinforced concrete beam. The focus of this present work is to develop a method that takes into account spatial variability, performs well in the right side tail of the probability density function of the resistance and takes into account a model uncertainty.

Keywords: Reinforced Concrete, Reliability, Finite Element Method

1. Introduction

Studies concerning the assessment of the reliability of existing reinforced concrete structures, which makes up the bulk of infrastructures around the world, are of paramount importance. The safety of RC structures can be assessed through nonlinear finite element analysis (NLFEA). The source of non linearity includes cracking, crushing, shearing and de-bonding of reinforcement bars. The geometry, loading pattern and material properties are expected to vary significantly from one structure to another and within a structure, so the quest of assessing the safety of these structures can be best attempted within the framework of probability and statistics. The basic philosophy is to generate a distribution of resistance of the structure as an output based on the input distributions of model, loading and geometry. This output distribution of the resistance, particularly around the right side tail region, leads to the reliability calculation for these structures.

2. Brief summary of available methods

NLFEA for full scale structures are often computationally intensive; so invoking a large number of NLFEA in order to find out the output distribution, which is typical of Monte Carlo method, is not an available option. It is with the intention to avoid time consuming computations that semi-probabilistic safety formats for NLFEA have been designed. The fib Model Code 2010\(^1\) devises several global resistance based safety formats, suited for NLFEA. Among others, Pimentel et al.\(^2\) and Cervenka\(^3\) elaborates on the background and use of several such formats. These semi-probabilistic safety formats include factors that capture the structure to structure variability in the material parameters of the model and loading. The accuracy of safety formats for relatively complex systems might be questioned; it is assumed that they produce results which are on the conservative side. A full probabilistic analysis is required to improve the safety formats for NLFEA.

2.1. Random variable approach for discretization of random parameters

There are two ways to incorporate the stochasticity of the input parameters into the NLFEA formulation. The first is the Random variable approach, where the stochastic parameter varies from sample to sample, but for any particular sample the input parameters are considered to be same for the entire domain. In other word, the input parameters are homogeneous in random variable approach. The advantage with this method is the relative ease with which the stochasticity can be included in the finite element formulation, particularly with commercial FE code DIANA. The obvious disadvantage is that this approach ignores the inhomogeneity of input parameters, so the resulting reliability information is not a vast improvement over semi-probabilistic method.

2.2. Random field approach for discretization of random parameters

Discretization of stochastic parameters of the structure as random field in particular presents a better opportunity to simulate the effects of randomness. Random field models are more robust and can model the spatial fluctuations in the input parameters, as well as the ensemble variations. Sudret et al.\(^4\) presents a detailed review of various types of random field discretization methods concerning stochastic finite element based approaches to structural reliability. The inhomogeneity in the spatial extent needs to be incorporated at the element level in the finite element formulation. This may not be convenient while using commercial packages. One of the earlier method of random field discretization is the mid-point method\(^5\). Here, the same finite element discretisation is used for discretizing the random input parameters. For each element, the randomness in the input is represented by a random variable at the mid-point of that element. As a consequence, there is no variation of input parameter within...
an element. The random variables at the mid-point of the elements are simulated such that they conform to the spatial correlation of random input parameters, obtained through experiments. Even though this method is bit crude by nature, it is possible to implement this mid-point method through commercial packages without making any changes in the source code. Another, better, method for random field discretization is Optimal Linear Expansion method (OLE). In this method a stationary multidimensional random field $f(x, \xi)$ can be discretized as,

$$f(x, \xi) \approx \tilde{f}(x, \xi) = f_0 + \sum_{k=1}^{N} S_i(x) \phi_k(\xi)$$  \hspace{1cm} (1)

where, $\tilde{f}(x, \xi)$ is the discretized random field along spatial extent $x$ of dimension $p$, $N$ denotes the number of nodal points used for the random field discretization, $f_0 = \langle f(x, \xi) \rangle$ denotes the mean of the process, $S_i(x)$ are deterministic shape functions and $\phi_k(\xi)$ represents a vector of random variables associated with the $N$ nodal points. The shape functions $S_i(x)$ are determined by minimizing the variance of the error of discretization, subject to the condition that the expectation of the discretization error is zero. Even though OLE is a very sophisticated method of expressing the random field, its implementation through commercial package may require significant changes in the source code. Once the input parameters are discretized, the next step is to implement it through a NLFEA procedure to compute the structural reliability. There are few available methods which can be executed.

2.3. Directional Sampling method

The Directional Sampling (DS) method samples combinations of parameter values randomly. Commercial NLFEA codes can make use of DS to compute failure probability. The more samples that are made, the more accurate the failure probability. Calculating the reliability very accurately will be very time consuming.

2.4. Response surface approach

A response surface based approach to structural reliability has been described in [4]. This approach uses finite element analysis to generate a response surface on which computationally cheap reliability calculations can be performed. If $\tilde{g}(\xi)$ is an approximate reliability surface, then

$$\tilde{g}(\xi) = a_0 + \sum_{i=1}^{N} a_\xi_i \xi_i + \sum_{i=1}^{N} a_{ij} \xi_i \xi_j$$  \hspace{1cm} (2)

represents the response surface, where $\xi$ are the input random variables, $a_\xi$ and $a_{ij}$ are the coefficients to be determined through the minimization of the error,

$$\text{error}(a) = \sum_{i=1}^{N_P} (y_i - \tilde{g}(\xi_i))^2$$  \hspace{1cm} (3)

Here $k$ denotes the sample points.

$$\tilde{g}(\xi) = [1, \xi_i, \xi_j]^T [a_0, a_\xi, a_{ij}] = V(\xi) a$$  \hspace{1cm} (4)

Minimization leads to,

$$a = (V^T V)^{-1} V^T y$$  \hspace{1cm} (5)

Once the response surface is constructed, the failure probability (and reliability) can be computed by a straight forward Monte Carlo method. The accuracy of this method depends on the choice of the sample points. Directional Adaptive Response Surface (DARS) Sampling is one such method of selecting sample points [6]. DARS is a refinement on DS in the sense that sampling occurs as in DS, but instead of performing a complete calculation for each sample point, calculations are only carried out for sampling points that are nearly critical. For non-critical sample points the response of the structure is approximated. This method is more efficient than DS due to the fact that fewer time consuming calculations have to be performed.

2.5. First Order Reliability Method

The principle of the First order reliability method (FORM) is to compute the reliability index $\beta$ through a Taylor series expansion till the first order derivative term in the neighborhood of the design point. The design point is calculated through an iterative procedure which requires information about the response and the first order derivative of the response. Once $\beta$ is found, the probability of failure can be found as,

$$P_f = \Phi(-\beta)$$  \hspace{1cm} (6)

Where $\Phi$ is the joint probability density function of the standard-Normal distribution. The derivative of the primary response of the structure, which is required to compute the design point, can be found from the nonlinear finite element system of equations

$$K(U(\xi), \xi) U(\xi) = F(\xi)$$  \hspace{1cm} (7)

The derivative of the response can be found as,

$$\nabla_x U = \frac{T(U(\xi), \xi)^{-1}}{\nabla_x F(U(\xi)) - U(\xi) \nabla_x K(U(\xi), \xi)}$$  \hspace{1cm} (8)

where $T(U(\xi), \xi)$ is the tangent matrix given by,

$$T(U(\xi), \xi) = K(U(\xi), \xi) + U(\xi) \nabla_x K(U(\xi), \xi)$$  \hspace{1cm} (9)

FORM approximates the response of a structure by varying a parameter value. This method is very efficient if a structure has one predominant failure mode, i.e., one combination of parameter variables that causes failure. This method is therefore most suitable for relatively simple structural components. However, if the structure has more than one failure mode, this method can only find one mode and therefore gives unreliable estimates of the reliability index.

3. Outlook

In a comparative study the available methods will be investigated for suitability and effectiveness. They will be compared on the basis of accuracy, ease of implementation and computational cost.

References


Online prediction method of the blast pressure loading for structural and personnel safety

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Abstract

The document provides a detailed methodology of a rapid prediction method of the blast pressure loading. The online code was prepared by the authors using the Python programming language, and was called SapperBlastModule. The authors prepared the initial version of the three modules. The first one is responsible for the prediction of the blast pressure during the free air or surface explosions. The second module includes the inverse procedure, where the charge mass and stand-off distance are calculated based on the overpressure and positive impulse values. The third module produces the pressure vs. time curves for all walls and the roof of a rectangular civil engineering structure. All algorithms were prepared based on the Unified Facility Criteria standard and authors experience. Additionally, this code generates the report file. Moreover, the blast loading schemes were compared with the FEM solutions. The numerical detonation process, evolution of the blast wave and the loading of the structure were obtained including Abaqus Explicit v.6.14 for a rigid obstacle.

Keywords: blast loading, prediction method, structural safety

1. Introduction

It is possible to perform safety assessment of a structure under explosive loading, however, this aspect of safety is not considered in many worldwide standards. Nevertheless, there are many methods which give a wide range of results. These depend strongly on the insight of the designer or researcher and on the fields of interest, such a highly dynamic behaviour [1,2,3,4,5], which are taken into account. In fact, only a few countries introduced explosive loading in official codes. Nevertheless, the most popular is the US standard Unified Facility Criteria [3,4]. This work presents a rapid assessment methodology which is based on Unified Facility Criteria in particular. However, the authors will focus on the credibility and the real factor of safety resulting from this approach. There are many factors which influence strongly the proper assessment of the blast loading. The shape of the charge, location of the ignition point or weather conditions are important for blast outcomes. Because of these reasons the safety factor e.g. the mass increasing must be taken into account. Moreover, a complete profile of blast pressure with time always has the classical form, as presented in Fig. 1. This loading is generated by pressure changes which initially occur over only a few microseconds, released during chemical conversion inside the explosive volume.

2. Blast Loading

The variation in overpressure in free air space for a surface explosion can be affected by the presence of ground reflections. In Figure 1, \( P(t) \) is the overpressure history, also called the air blast pressure. The term stand-off is used to denote the separation from the charge centre of any particular measurement point. When the process is initiated, following the explosion at the time of arrival \( t_{oa} \), the pressure suddenly increases to a peak value \( P_{so} \) which exceeds the ambient pressure equals to \( P_0 \). Hence, the pressure decays to \( P_0 \) in time \( t_{dp} \), and again reaches \( P_{so} \) pressure in order to finally reach again the barometric value, at time \( t_0 \). The sum of times of over and under pressures is called the duration time \( T \). The value of \( P_{so} \) is usually referred to as the peak side-on overpressure or incident peak overpressure, see Fig. 1.

Figure 1: Standard blast pressure as a function of time

The knowledge of the instantaneous pressure changes and the duration of the positive phase allows us to calculate the blast impulse of an explosion \( I_s \). It is noticed that the positive phase and its impulse are highly important for the structural strength for any kind of obstacle. In fact the blast resistance capacity of any structure can be presented as lying along a curve drawn in \( P-I \) space [2,3].

3. Results

The final results base on the empirical formulas showed in UFC standards [2,3,6]. The exemplary results are presented in Fig. 2, where the pressure vs. time relation is obtained using US standard. Moreover this solution is compared with FEM outcome, according to the set of equations presented by Brode [2,6].
The blast pressure evolution on the cut-surface of the ambient air and frontal surface of the obstacle after free air explosion

The additional part of this work is an upgraded version of the Johnson-Wilkins-Lee equation. The solution is coded in Fortran programming language and is changed especially for a negative phase. This effect is presented with an example of a spatial flexible steel structure loaded by a different behaviour of the negative pressure [7].

The last feature of the online prediction code is the feature that allows to predict the loading scenarios of a blast pressure curves for the separate outside surfaces of a rectangular civil engineering structure.

References

The influence of utilizing different materials and their configurations on ballistic panels blast resistance

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Abstract

The article is focused on materials and their configurations to provide protection against explosive blasts of lightly armoured vehicles. The authors consider elastomeric material, aluminium and polyurethane foam which are easily available on the local market at a reasonable cost. A computer simulation method was chosen to solve the problem in an efficient way. The Finite Element Method (FEM) implemented in the LS-DYNA [2] commercial code was used with an explicit (central difference) time integration algorithm. Space discretization for each option was built with three dimensional elements ensuring satisfying accuracy of calculations. The investigation has proved that the panels made of energy consuming materials limit the maximum level of reaction force. The highest reaction is observed when polyurethane foam interacts with a blast wave. The simulation has shown that ballistic panel supports rebound from a massive block that is critical for the achieved results.

Keywords: computational mechanics, ballistic protection, armour

1. Introduction

This article is focused on materials and their configurations to provide protection against explosive blasts on light armoured vehicles. Traditionally, the term “armour” connotes a thick layer, typically steel which can protect an object regardless of the nature of a threat. Such images accurately depict traditional protection systems utilized in past decades but, according to present vehicle mobility requirements, they are not suitable for the modern army. Versatility of an armour is strongly recommended, however, is often hard to achieve. Threats may be divided into two main groups. The first of them are all types of projectiles or blast debris the energy of which is concentrated on a relatively small area during impact. The other danger, which is a subject of this paper, is a blast wave which, while interacting with a large vehicle surface, can injure a crew due to high acceleration of their bodies.

Typical materials for blast absorption are fibre composites, foams, magneto-rheological fluids, rubbers and porous materials [1]. In this research, the authors are focused on a few solutions which are easily available on a local market at a reasonable cost. The first one is elastomeric material in which energy consumption is caused by its hysteresis behaviour. Moreover, aluminium and polyurethane foams were used.

2. Investigation

The geometry and dimensions of the ballistic panels and the test stand were chosen in such a way that they could be easily tested experimentally. A steel stand (fig. 1) was subjected to a blast wave from a 120 g TNT charge placed 430 mm above the ballistic panel support. Such value was chosen because it is the distance between the ground and the chassis of KTO Rosomak vehicle (armoured personnel carrier) and is considered to be a good example of a light armoured vehicle. The height of the ballistic panel was set to 50 mm.

Figure 1: Physical model of a blast test

Four kinds of simulation tests have been conducted. Each calculation variant included the same type of a charge mass and a different material or materials structure. In the reference variant (V1) no ballistic panel was used. Two kinds of initial boundary conditions for a massive block were used. In the first one, it is simply placed in a space (V1a), in the second one its rear wall is fixed (V1b). In V2, V3, V4 variants rubber material, aluminium foam and polyurethane foam were used respectively. In V5 and V6 variants, two-layer sandwich structures containing rubber and alumina foam were utilized. All the examined structures are depicted in fig. 2. The most important factor when deciding the ability of ballistic panel protection is force transferred to the massive steel block.

3. FEM model description

A computer simulation method was chosen to solve the problem in an efficient way. The Finite Element Method (FEM) implemented in the LS-DYNA [2] commercial code was used with an explicit (central difference) time integration algorithm. The boundary conditions were defined by supporting the test stand at its back wall. In order to minimize computation time, only a quarter of the model was analysed and the symmetry was defined.
To describe the contact between the ballistic panel components and the projectile core, a penalty method was utilized. To estimate a wave blast impulse, a widely known method, ConWep, developed by [3], is used. All components in the ballistic panels were modelled using hexagonal elements only. Element size was the same for the whole volume, of each component. In fig. 3, a mesh for a chosen V5 variant is depicted.

Figure 3: Mesh of two-layer panel (V5)

4. Simulation results

The main purpose of this investigation is an assessment of ballistic panels’ ability to absorb blast energy. Their layer structure and material used make them easy to manufacture. The assessment is based on the analysis of reaction force which acts on the ballistic panel support.

The results presented in fig. 4 show that maximum forces and work which is performed by those forces during the process differ substantially for each variant. The comparison of V1a and V1b variant indicates the importance of a proper boundary conditions description (fig. 4). Although the mass of the test stand is much bigger than the mass of the panels, it is crucial to represent the way it is fixed in the space or, more adequately, to model the supporting material. In reality, such a test stand would be placed on the ground. Therefore, it is crucial to use a proper soil material.

The highest reaction force level is observed in the case of V4 variant, where polyurethane foam interacts with a blast wave (fig. 5). It is quickly compressed and after this process is finished the force increases rapidly. Similar results are achieved for V2, V3 and V6 variants. However, in the panel where the blast interacts with aluminium foam supported by elastomeric material, relatively higher forces can be observed. It is important to notice two significant facts. Firstly, short reaction impulse duration is caused by the fact that a ballistic panel support rebounds from the massive block. If those two parts were connected with each other, the simulation results would be substantially different. Secondly, each test was carried out for a different areal density. In this paper, this fact is neglected according to prior assumptions, however, in reality the mass of the ballistic panel may be considered as important.

Figure 4: Force versus time for each analysed panel

Figure 5: Force versus time for each analysed panel

5. Conclusions

The obtained results indicate that the discussed ballistic panels may be utilized as vehicle crew protection. The investigation proves that panels made of energy consuming materials limit the maximum level of reaction force. The highest reaction is observed when polyurethane foam interacts with a blast wave. In other variants both the forces maximum values and curve shapes are similar. Simulations show that ballistic panel supports rebound from the massive block quickly after the blast reaches its target, which is crucial for obtained results.

Achieved values of reaction forces depend on numerical aspects, especially on properly defined boundary conditions. Although the mass of the test stand is much bigger than the mass of the panels, it is important to represent the way it is fixed in the space or, more adequately, to model the supporting material.

References

Assessment of the biomechanical parameters of the disabled drivers during frontal crash

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Abstract

The paper is related to safety issues of disabled drivers. Based on results of numerical analyses the authors have described behaviour of disabled drivers who use special equipments during frontal crash. To perform analysis numerical model of MPV car (Multi-Purpose Vehicle) using 3D scanning was built. Adaptive devices in the form of knobs on the steering wheel and control device for acceleration and braking pedals were added. Driver with disabilities was modelled using the modified Dummy Hybrid III 50th model, developed by Humanetic company, which was seated using gravity load before simulations. As the result of performed numerical analysis the charts of dummy’s mass centre movement, angle of body rotation, head accelerations, forces between dummy and seatbelts are presented. Based on those charts several injury criteria factors were estimated (e.g. HIC and NIC).

Keywords: Disabled driver, knob on the steering wheel, frontal crash

1. Introduction

The world population surpassed seven billion [8]. Nearly 15% of them are disabled persons, among which are a large group of people with movement disabilities. These people are often well-educated and have unique skills that could be successfully used in a professional or social life. Unfortunately, lack of mobility is the huge obstacle preventing participation in the normal life. This is particularly noticeable in smaller towns, where there is no well-organized public transport with the fleet of low-floor buses or trams.

One of many possibilities to increase the level of mobility of people with disabilities is to adapt cars to their needs. In Poland there is an association Spinka [7], whose goal is to create a nationwide network of specialized Disabled Driver Training Centres, and financial support programs aiming to help in the purchase and adaptation of vehicles. These activities results in growing availability of both used and new cars that can be driven by disabled.

There is a lot of adaptive equipment for people with disabilities available on the market [2, 5]. Choice of a specific solution depends on the type of disability. However, mounting of any type of the equipment in the car changes the position of the driver and kinematics of his movements compared to that anticipated by car manufacturer. In particular, such devices affects behaviour of the driver during collision, and thus its safety. Therefore, each new type of equipment for disabled driver (including a combination of existing solution and new vehicle) should be tested before passing to the user. One of the methods to verify the safety of the mechanical system, which is the additional adaptive equipment, are numerical simulations [3].

The aim of the article is to determine the behaviour of the disabled driver during frontal crash, and estimation of biomechanical parameters and criteria characterizing his safety. The paper presents a methodology of car geometry data collection, acquisition of material data, numerical model preparation and numerical analysis algorithm.

2. Car interior geometry

The assessment of a disabled driver safety it the presence of additional equipment was carried out for the MPV (Multi-Purpose Vehicle) car type, which is currently being produced. Its interior geometry for the numerical analysis was obtained using the 3D F5 Mantis Vision scanner [1, 6]. The result of the scan was the cloud of point (Fig. 1), which has been further processed to obtain set of triangles. In the next stage of the work these triangles had been converted into four-node shell elements forming surfaces of the car interior.

![Figure 1: Interior of the car – point cloud after 3D scanning](image)

3. Numerical model

During the numerical simulation three initial velocities were analysed: 30, 40 and 50 km/h. Curves defining velocity change during crash were taken from additional numerical analysis aimed at projection of the frontal car impact into rigid wall. Obtained curves (Fig. 2) were used to control the body of the car only. All other parts of the model have been associated with the initial velocity and movement was implemented thanks to the interaction with a car body.
In the numerical analysis the Hybrid III 50th dummy model was used. This is a very accurate model of Anthropomorphic Test Device (ATD), that gives results comply with the experimental test in about 95% [4, 6]. Preparation of the numerical model included settling of the dummy under gravity load and fastening it by the seat belts. Seat belts model featured tensioner, retractor and scrolling through the lugs [6].

Numerical analysis was carried out in two stages. In the first stage seating phase of a dummy was simulated. In the second stage seat belts and initial velocity were added and then frontal crash was simulated.

4. Results

The measured parameters based on numerical results included displacement of the dummy centre of mass and other points in longitudinal, transverse and vertical direction. Based on these movements, change of ATD body angle was calculated (Fig. 3).

Based on the calculated history of forces and moments NIC (Neck Injury Criterion) was calculated according to the equation:

$$ N_{IC} = \frac{F z}{F_{int}} \frac{M_f}{M_{int}} $$

(2)

where: $Fz$ is the axial load, $F_{int}$ is the critical intercept value of load used for normalization, $M_f$ is the flexion/extension bending moment, $M_{int}$ is the critical intercept value for moment used for normalization.

References


The main goal of this paper is to analyse the difference in the structural behaviour of composite steel and concrete columns in case of standard and localised fire. A set of concrete filled steel tubular columns is analysed in terms of their response to mechanical loading when exposed to fire. Heat transfer from the gas into solid is described by the use of adiabatic surface temperature. The fire resistance of elements is obtained taking into account both material and geometrical non-linearities and non-uniformly distributed fire along length and circumference of the columns. The comparison of the structural behaviour of the columns in terms of vertical and horizontal displacements and fire resistance time is provided.

Keywords: structural fire engineering, composite columns, localised fire, adiabatic surface temperature, CFD

1. Introduction

Two common approaches are widely employed when the effect of fire on structures is considered, i.e. heating conditions according to standard temperature-time curve or parametric fire, both described by EN 1991-1-2. A different approach can be applied for large compartments, where a fully developed fire is unlikely to occur. This motivation led to a study presented in the following paper concerning fire performance of composite hollow steel columns filled with plain concrete exposed to localised fire.

2. Localised fire

The study of behaviour of concrete filled composite columns exposed to localised fire is based on a previously validated mechanical model possible to reflect the structural behaviour of concrete-filled steel tubular columns exposed to heating conditions according to standard temperature-time curve. In order to perform the mechanical analysis in the natural fire conditions the FDS (Fire Dynamics Simulator) software [4] is used.

2.1. FDS and Abaqus coupling

The coupling between FDS and Abaqus is done in sequential way, which means that the simulation of fire is carried out a priori the heat transfer and mechanical analyses. After that the results of the fire simulation are transferred into the finite element code as boundary conditions to the heat transfer step.

The cross-sectional dimensions of analysed structural elements (columns) are great enough to reproduce the column in fire simulation performed in FDS. That means, the boundary data needed for the heat transfer analysis can be transferred directly from the selected surface in fire simulation model prepared in FDS. Nonetheless, because of the rectangular mesh in FDS, the actual shape of circular columns cannot be reflected. Thus, the approximation of cavities has to be made in external scripts which connect the CFD software (FDS) and FEM software (Abaqus). The approach similar to the nearest-neighbour mapping algorithm [5] is used to deal with this particular problem.

Heat transfer from the gas into solid is described by the use of adiabatic surface temperature described in [10]. This allows the reduction of the output from CFD computations without loose of accuracy [8]. Adiabatic surface temperature is the parameter allowing to the actual exposure of the surface to the energy coming from the convection and radiation phenomena.

2.2. Impact on the structural behaviour

Non-standard fire exposition both along the length of a column and its circumference causes additional effects on the element. The non-uniform temperature distribution inside the cross-section might result in significant thermal bowing along with additional eccentricities combined with second order effects. These effects have to be taken into account as far as the structural behaviour and fire resistance time of the columns are concerned.

3. Mechanical response to standard fire

The analysed columns consist of steel tubes filled with plain concrete. The typical response presented in terms of evolution of vertical displacement in time is shown in Fig. 1.

![Figure 1: Typical CFT column response under elevated temperature, column loaded to its 40% load-bearing capacity](image)
shortening of the column occur. From the point when the loading plate comes into contact with the concrete core (24th minute), both steel tube and concrete core are load-bearing members. Due to further degradation of mechanical properties, after a certain time failure occurs.

3.1. Fire resistance time of CFT columns

The results of a more detailed study on the prediction of fire resistance time are shown in Figure 2, where measured and calculated fire resistance times for 41 analysed specimens are compared. Specimens included in the validation process are taken from the research conducted and published in the recent years (1992-2013) by various researchers in different fire testing laboratories [1,2,3,5,7,9]. The sequentially coupled thermal mechanical analysis was applied to the models with initial imperfections corresponding to the first Eigenmode with a magnitude of L/1000. The utilisation ratio of the analysed columns varied from 20 to 70%, while the length varied from 3180 to 3810 mm and Am/V ratio varied from 8.37 to 28.31.

Figure 2: Comparison of calculated and measured fire resistance time for all tested CFT columns

3.2. Material model

The results shown in Fig. 2 were obtained using a linear Drucker-Prager yield criterion for concrete, with friction and dilation angles equal to 36°. It was combined with stress-strain relationship for concrete taken directly from the Eurocode 1992-1-2. The mechanical properties of steel and thermal properties of both concrete and steel were taken from Eurocodes as well. In terms of normalised resistance ratio FRR(calculated) / FRR(test), the mean value for all analysed columns was 1.00 with standard deviation equal to 0.26.

4. Conclusions

The comparison of structural behaviour of the columns in terms of vertical and horizontal displacements is provided. The difference in fire resistance time and the development of temperatures inside the cross-sections is presented and compared to values obtained from standard tests and analysed specimens exposed to localized fire. Additional effects resulting from non-standard exposition of the columns to fire both along the length and circumference are emphasised.

References

Parallel computing using Multi Processor System-on-Chip (MPSoC) for structural damage detection in real time

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Abstract

The numerical example for the local damage detection in real time of the plane steel truss structures is presented. In the paper a new computing model is described. The model has the following characteristics: Big data, Real-time, Multitasking and Parallelism. The combination of multiprocessing systems and parallel programming techniques are used to solve the problem. The combination makes it possible an efficient automation of mapping of parallel software into parallel hardware. The interest is put on determining the optimal feed forward NN architecture from a net of NN, NoNN, which reduces the time required to learn from training examples. The proposed architecture allows to predict reactions of organizations quickly when problems appear or to predict new trends in the near future.

Keywords: damage detection, big data, artificial neural network, parallel computing, multiprocessor systems on chips

1. Introduction

Most civil structures are constantly exposed to diverse natural and environmental conditions, including strong winds, earthquakes, and others, and therefore, to abnormal ultimate loads, which may even cause structures to collapse. Monitoring of building structures provides the way to compare actual structural responses and characteristics with values used in design assessments in order to obtain, in real time, an advanced warning of the onset of durability/structural problems at a stage when preventative action is possible. It refers to a broad concept of assessing the ongoing in-service performance of structures using a variety of measurement techniques, e.g., RTK-GPS, WSN, and others emerged creating an exciting new field within civil engineering. Damage detection is usually performed in an unsupervised learning mode using neural computing [3]. Generally, with the static displacements U under prescribed load F, the static equilibrium of a structure in a finite element formulation is represented by: \( \mathbf{KU} = \mathbf{F} \), where, \( \mathbf{K} \) denotes a stiffness matrix; \( \mathbf{K} = f(E, S, L, 0) \), where each structural element has modulus of elasticity E, cross-sectional area S, and length L which is inclined with an angle \( \theta \) measured counterclockwise from the positive global X axis. On one hand, incomplete, insufficient data/small available data may lead to solutions, which are beyond the permissible area. On the other hand, NN effect is guaranteed only if this network has, an optimal architecture (good enough), among various required conditions. In order to solve the problems, a new computing model is required of the following characteristics: Big data, Real-time, Multitasking and a lot of parallelism.

2. Proposed solution

Finding \( \mathbf{U} \) from the available information of \( \mathbf{K} \) and \( \mathbf{F} \) can be considered as a direct problem, which presents a Feed-forward approach for structural damage detection. It is presented by the direct mapping \( \Psi \) as follows: \( \Psi: \mathbf{S} \in \mathbb{R}^m \rightarrow \mathbf{U} \in \mathbb{R}^n \), where, \( n \) expresses a number of nodes; \( m \) denotes a number of structural members. Next, damage is defined as the maximum stiffness reduction at one or more local elements. The structure is described as a set of finite elements categorized into undamaged and damaged states in different degradation levels. Introducing of damage is expressed by the loss of structural stiffness through a reduction in the cross-section area of an element used in computing \( k_{ij} \) of the stiffness matrix \( \mathbf{K} \). It leads to the formulation of the damage-detection problem as follows: for a given vector \( \mathbf{U}_{obs} \), find the vector \( \mathbf{S}^* \), for example, such that \( \mathbf{K}(\mathbf{S}^*)\mathbf{U}_{obs} = \mathbf{F} \), where \( \mathbf{S}^* \) denotes a threat element of structure. It is mathematically equivalent to obtaining the mapping, \( \Phi \), that shows a feed back problem between the known response of a structure and the physical state of them. \( \Phi: \mathbf{U} \in \mathbb{R}^n \rightarrow \mathbf{S} \in \mathbb{R}^m \), then, we have to overcome the following issues: Information processing with big data originated from the RTK-GPS, WSN and the virtual data generated from the MCM-FEM (Monte-Carlo Method and Finite Element Method) integration; determining the optimal architecture of ANN referred as a condition for the optimization of faster training – Resilient Back Propagation, RBP; determining the final solution in real time and developing fast response for early warning. We require the following: considerable processing power (fast computing, computation-intensive) for complex ‘map-reduce’ computations; no-standard parallel programming and processing paradigms suitable for handling large collections of data and multitasking problems requiring multiprocessing systems. We use multiprocessor architecture in a single chip in the work (MPSoC) [2], endowed with complex communication infrastructures, such as hierarchical buses or networks on chips (NoCs). One key aspect for MPSoC users is the ability to rapidly program a variety of different combination of parallel software that run on parallel hardware in an automated fashion. For example, we can write a program open n cores for openMP using MATLAB program [4]. The reduction operation is used in order to integrate operations of all processors, for example, see Fig. 1.

This embedded computing is all about tuning the hardware and software architecture of a system to meet the specific requirements. In this paper, the plane truss used for damage identification is presented, in which, feedforward networks of sigmoidal neurons, two neural layers is applied. The emphasis is
on the preparation of massive information and optimization of architecture of ANN through parallel computing, in which the SIMD technology for multi-core computing is used for multiprocessor system on chip. An abstract view of MPSoC is represented in the Fig. 2.

The error training, EE, obtained from different workers/processors (CPUs) after training process is presented in the Fig. 3. We receive, in the final result, a reduction in errors of 10% in comparison with the calculation using one processor. The SIMD idea of Monte-Carlo simulation using neural network vector is presented in the Fig. 4.

3. Obtained results

Integrating of NN computing and FEM based on MPSoC allows us to recognize the threat of this structure, which is located in the seventh element having the cross-sectional area, $S_7 = S^* = 0.0028 \text{ m}^2$ that does not belong to the training dataset (compatible with FEM computing with error, $e = 0.0032 - 0.0028 = 0.0004 \text{ m}^2$). Note that this error will be reduced depending on the increase of training dataset and engineering experiences.

References


An analytical study of electromechanical buckling of micro spherical thin film bonded to a spherical compliant substrate

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Abstract

The work presents analytical study of electromechanical buckling and postbuckling of a spherical elastic thin film electrode bonded to a linear elastic compliant substrate that is bonded to inner rigid electrical grounded electrode. If the applied voltage reaches a critical value the spherical thin film buckles in an ordered periodic pattern. The principle of minimum total potential energy was implemented to analyse the electromechanical buckling and postbuckling states. It was found that for low applied voltages the hexagonal periodic pattern is characterised by the lowest energy thus is preferred among the others. However, for high applied voltages the hexagonal mode become unstable and it bifurcates to the herringbone mode pattern.

Keywords: Electromechanical Buckling, Postbuckling, micro elastic sphere, elastic foundation, föpple-Von karman equations

1. Introduction

The study of buckling of a planar elastic thin film bonded to an elastic compliant substrate attracted recent attention during the last decade [1-2]. It was shown that elastic films buckle in ordered periodic patterns if the in-plane compression stress reaches a critical value [2]. Most of these studies investigated the buckling response that stems either from the effect of thermal mismatch of a planar thin film and a compliant substrate or from constrained contraction in-plane strains. Experimental studies of buckling of planar films revealed that checkboard, hexagonal and herringbone periodic pattern modes are most reasonable and preferable [1]. Figure (1) presents a 3D view of these three periodic mode patterns. The checkboard, hexagonal and 1D models are characterised with identical critical stresses and wave length at the critical buckling state.

Buckling of micro SiO₂ spherical thin film bonded to Ag elastic core was studied experimentally by Cao G. et al. (2008) [2]. The radius and thickness of the micro spherical film were 3-7 [μm] and 150 [nm], respectively. It was found that at low compression stresses the hexagonal pattern is preferred. However, for high compression stresses the hexagonal mode becomes unstable, it bifurcating into the herringbone mode.

The work presents an analytical study of the electromechanical buckling and postbuckling of a spherical thin isotropic elastic film. Figure (2) illustrates schematically the spherical elastic electrode thin film bonded to a dielectric compliant substrate that is bonded to an inner rigid electrically grounded electrode. Applying voltage on the elastic electrode subjects an inward radial attracting electrostatic force and consequently generates in-plane compression stresses in the film.

2. Formulation

A periodic mode pattern of the buckling response allows to execute local analysis of a single representative unit cell. In this local analysis, the electromechanical behaviour of one unit cell determines the electromechanical response of the spherical film. The normalized nonlinear equilibrium equations of a spherical thin film in a local coordinate system (x,y) are given by:

$$\kappa V^4 w - r^3 V^2 f - r^2 H(f, w) + \frac{r^4}{g} E_g w = \frac{r^4 V^2}{(g - w)^2}$$ (1)

$$V^4 f = -\frac{1}{r} V^2 w - \frac{1}{2r^2} H(w, w)$$ (2)

The nonlinear equilibrium equations (1) and (2) are equivalent to the well-known Föppl von-Karman plate equations and presented in the form of the displacement w(x,y) and Airy-stress function f(x,y). The normalization rules of eqns. (1-2) are in according to: x=X/R, y=Y/R, r=R/h, g=G/h, w=W/h, f=F/(EhR²), κ=1/(12(1-υ²)) and E₅=E₅/E₆. The right hand side of eqn. (1) describes the attraction electrostatic force,
where $v$ is the normalized applied voltage, $E$ is the substrate/film modulus ratios, $E_r$ and $v$ are the elastic modulus and the Poisson’s ratio of the film and $r$ is the normalized radius of the spherical film. In addition, $g$ is the normalized gap between the two electrodes, equal to the substrate thickness. In this study $g << r$, hence, the compliant substrate is modelled as a linear elastic foundation. $H(f,w)$ is a nonlinear operator and is given by $H(f,w) = f_{xx}w_{yy} - 2f_{xy}w_{xy} + f_{yy}w_{xx}$.

3. Critical Electromechanical Buckling Analysis

If the applied voltage $v$ is lower than the critical value, the film deflects in a uniform radial displacement $w_0$ and contracts into a smaller axisymmetric spherical shell. However, if the applied voltage reaches a critical value ($v_{cr}$), buckling occurs and the film generates periodic elastic wrinkles in addition to the radial uniform contraction. The critical electromechanical buckling equations of the film are derived substituting the postulated admissible displacement $w(x,y) = w_0 + w_1(x,y)$ and Airy-stress function $f(x,y) = n_0(x^2 + y^2)/2 + f_1(x,y)$, where $w_0$ and $n_0$ are associated with the pre-buckling state. For a checkboard mode $w_1(x,y) = A\cos(kx)\cos(ky)$, where $A$ is the amplitude and $k$ is the number of generated elastic wrinkles. The critical buckling state is governed by the following two equations derived directly from eqns. (1-2).

$$\frac{\kappa^2}{(g-w_0)} + n_0 - \frac{r^2}{g} E_r w_0 = 0$$

$$\frac{\kappa^4}{(g-w_0)} - r^2 \kappa^2 f_0 + r^4 \kappa^4 w_1 + \frac{1}{g} E_r w_1 - \frac{2\kappa^2}{(g-w_0)} w_1 = 0$$

At the verge of buckling the pre-buckling state eqn. (4) and the critical buckling state eqn. (5) are valid. Accordingly, the buckling voltage ($v_{cr}$) and number of elastic waves $k_{cr}$ at the critical state, are derived. The transcendental equation derived from the 1D and hexagonal modes is identical to the equation of a checkboard mode. Thus, all three modes are characterised with identical critical states and solutions.

4. Results and discussion

Figure (3) plots the critical buckling voltage in the function of $r$ for different substrate/film modulus ratios $E_r$. As anticipated, the critical buckling voltage increases with increasing $E_r$ and initially decreases with the increasing $r$. At small values of $r$ (i.e. high curvature) the mechanical bending stiffness is more dominant than the stiffness of an elastic foundation. However, with increasing $r$ bending stiffness decreases and the elastic foundation stiffness becomes dominant, explaining the increasing pattern of the buckling voltage. For $r \to \infty$, the curvature and compression in-plane stress diminishes. As a result, buckling voltage asymptotically converges to the well-known pull-in voltage of a planar elastic film bonded to an elastic foundation as shown in the zooming curves in Fig. (3). The number of elastic wrinkles $k_{cr}$ at the critical buckling states monotonically increases with increasing $r$ and $E_r$, as presented in Fig. (4). Figure (5) plots the elastic energy $U$ of the film normalized by the elastic energy at the unbuckled state $U_0$ for $E_r = 10^{-4}$, $v = 0.33$ and $g = 50$. The zooming curves plots the associated applied voltage as function of the amplitude.

References


Application of quasiconvex analysis: enhanced micromechanical modelling of martensitic phase transformations and numerical implementation

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Abstract

In the contribution a physically and mathematically sound framework is shown for the modelling of martensitic phase transformations based on energy relaxation concepts. The micromechanical model considers parametrisation of an underlying microstructure in terms of laminates of the 1st and 2nd order. Thus, an effective energy density for the phase mixture consisting of austenite and a twin of martensite is derived. The incorporated internal state variables act as energy minimisers yielding the optimum microstructure configuration for prescribed boundary conditions. With this approach, a suitable approximation of the desired quasiconvex hull is achieved.

Keywords: quasiconvexification, energy relaxation, shape memory alloys, solid to solid phase transformations

1. Introduction

The application of quasiconvex analysis—also referred to as energy relaxation methods—has turned out to be a very promising method in the context of micromechanically well-motivated material models, in particular for phase transformations in, e.g., shape memory alloys. In this contribution, two main aspects related to this method shall be worked out. First, an enhancement of a former laminate-based framework is shown which allows for a suitable approximation of the desired quasiconvex energy hull of the underlying multi-well energy density. Second, numerical problems known to accompany energy relaxation schemes and their solution are addressed.

2. Quasiconvexification

As shown by [1], the weak lower semicontinuity of (energy) functionals coincides with quasiconvexity of such and thus, the existence of minimisers can be proved. Moreover, the oscillating minimising sequences required for the mathematical proof can be identified with experimentally observed, physical microstructures. These microstructures can be modelled by a C¹-smooth displacement fluctuation fields \( \mathbf{w} \) applied to a representative volume element (RVE) on the material microscale \( \Omega \). A functional \( \psi \) is quasiconvex, if

\[
\psi(F) \leq \min_{w} \left\{ \int_{\Omega} \psi(F + \nabla w) \, dV \right\} \quad \text{=: } \psi^Q(F)
\]

subject to

\[
\int_{\Omega} \nabla \mathbf{w} \, dV = 0 \quad \text{and} \quad \int_{\partial \Omega} \mathbf{w} \, dA = 0
\]

holds. The quantity \( \psi^Q \) is defined as the quasiconvex energy hull. Equation (1) states, that the homogeneous deformation \( F \) is stable if no admissible fluctuation field \( w \) can further reduce the functional. On the other hand, the homogeneous solution will decompose into a heterogeneous field or, in other terms, a microstructure, if energetically favourable. Analytical results for the quasiconvex hull are only possible in special cases. However, the quasiconvex hull may be approximated by parametrisations and/or discretisations of \( w \) yielding upper or lower bounds to the quasiconvex hull. In this context, the use of rank-one-convexification and therewith laminates of different order is a frequently applied, promising method.

3. Laminate-based modelling framework

Taken from a domain of significant contributions, the work of [2] serves as a basis for the present contribution. In the above mentioned paper, a material model for shape memory alloys (SMA) is established where, besides the parent austenite phase, an arbitrary number of martensite phases is considered. Each of these martensite subregions is associated with micromechanical properties, in particular elastic constants and transformation strains, the precise values whose are governed by the different orientations of the crystallographic variants, see Figure 1. The respective total strains are rank-one-connected which means that the Hadamard compatibility is ensured with respect to total strains. The underlying internal state variables can be summarised as follows: The volume fraction \( \xi_i \) for martensite variant \( i = 1, \ldots, n_v \) (where the total number of considered martensite variants \( n_v \) is arbitrary in general), the orientation \( \eta_i \) of the 1st order laminate, the orientation of the 2nd order laminate \( \eta_n \), the projected jump of strains \( \alpha \) between austenite and averaged martensite, and the projected jumps of strains \( b_j \) between the different pairs of martensite variants with \( j = 1, \ldots, n_v - 1 \). Together with a sophisticated numerical treatment, this framework

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has shown to provide a physically and mathematically sound framework for the modelling and simulation of martensitic phase transformations.

4. **Enhanced micromechanical model**

The above mentioned basic model, however, exhibits one major drawback: the experimentally observed microstructures in SMA rather consist of a mixture between austenite and *twinned martensite*, which cannot be captured by the model of [2]. In order to further increase the physical plausibility of this model, an enhanced version has been established in [3]. Therein, the number of martensite variants is reduced to two. Moreover, additional microstructural degrees of freedom are introduced, related to the crystal orientations of the martensite crystals, see Figure 2. These rotations are considered energy minimisers yielding the optimum configuration of the microstructure at the onset of phase transformations. In this context, “dynamic” martensite variants occur, while those of the previous models may be referred to as “static”. The internal state variables of the enhanced model are given by two volume fractions of martensite $\xi_1$ and $\xi_2$, the laminate orientations $m$ and $n$, the projected jumps of strains at the interfaces $a$ and $b$ as well as the aforementioned rotations symbolised by the generalised rotation tensors $R_1$ and $R_2$ in both of the martensite phases. It could have been shown ([3]), that this enhancement results in a significant reduction of the effective energy density during inelasticity and a better approximation of the quasiconvex energy hull. Furthermore, it facilitates to model twin formation continuously and by energetic considerations. In this contribution, the model developed in [3] is enhanced in terms of a full 3d implementation.

5. **Parametrisation and numerical stability**

The concept of energy relaxation offers a comprehensive and suitable framework for the micromechanical modelling of, e.g., phase transformations. However, the numerical solution of the underlying minimisation problem subjected to several equality and inequality constraints is known to suffer from numerical pitfalls which lead to severe instabilities. Even the use of well-established minimisation schemes available in scientific software cannot guarantee proper results. In this contribution, the effect of different parametrisations of the related quantities such as the laminate orientation unit vectors on the numerical stability is elaborated. Moreover, a sophisticated algorithm based on the aforementioned available numerical schemes is presented which allows stable numerical implementations. As an outlook, the robust algorithm developed shall serve as a basis for Finite Element simulations of materials such as magnetic shape memory alloys or magnetostrictive materials proving an unconditionally stable local material routine necessary.

**References**


Two-scale elastic-plastic model of RPC in the plane stress state

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Abstract

The work is concerned with the determination of effective material parameters of reactive powder concrete (RPC) in the range of its nonlinear response using a two-scale modelling technique. We have also carried out a series of experimental tests which allow us to validate the proposed numerical model of RPC. The behaviour of a RPC concrete on a macro scale is described on the basis of phenomena occurring in the microstructure of material. The material microstructure is taken into account by means of a representative volume element (RVE), whose the structure is generated in a stochastic way with data from the designed recipes of RPC. It is assumed that the microstructure of RPC is composed of isotropic ideally-elastic-brittle constituents and the material is homogenized at the macro scale. This approach is a sufficient basis for a simple modelling of microcracks that cause the nonlinear behaviour of the material at the macro level. The numerical analysis is done here for the plane stress state problem and at each level of analysis the finite element method is applied.

Keywords: two-scale modelling, numerical homogenization, FEM, RPC, experimental tests, elastic-plastic model

1. Introduction

Reactive powder concrete (RPC) is currently one of the most modern building materials produced on the basis of cement, and belongs to the class of Ultra-High Performance Concrete (UHPC) with its strength and high ductility comparable to steel [2]. Reactive powder concretes are also classified as cement matrix composites with ultrahigh resistance properties and are often called the low-temperature ceramics. Thanks to the ultra high-strength and ductility of RPC, the weight and dimensions of cross-sections of structures built from RPC can be significantly reduced, with a simultaneous large freedom in providing the structure an architectural fit and crossing over significant spans. Owing to its physical and mechanical properties, reactive powder concrete finds a wide interest not only as the construction material but also as a cladding one and even as a material for furniture. Our aim is to develop a model that allows to study the influence of the composition of RPC concrete on its macro mechanical properties and that could be used in a static strength analysis of buildings and engineering constructions made of RPC. Concrete is a structural composite material and its hierarchical structure may be analysed in a multiscale approach, starting from the molecular dynamics simulation of hydrated cement solid nanoparticles [4]. In the paper we present a two-scale model of reactive powder concrete (Fig. 1) and make use of the numerical homogenization technique. In this method, the response of the medium on a macro-scale to external loads is determined on the basis of structural analysis on a micro-scale. On the micro-scale level the distributions of micro-strains and micro-stresses are determined, which by the way of homogenization provide information about the macro-quantities. The whole micro-analysis is carried out on the so-called representative volume element (RVE). This is a volume assigned to a material point which is representative for a small vicinity of the point. When the characteristic microscopic length is one order smaller than characteristic macroscopic length, we are able to take into consideration only effects of the first order. In the case of the RPC concrete, this condition is fulfilled. We may assume that the characteristic dimension on the micro-scale is a fraction of ground quartz, 0.2 mm. While on the macro-scale this will be the dimension of the cross-section of a structural element, e.g. 0.2 × 0.2 m.

Figure 1: Two-scale modelling technique

2. Two-scale elastic-plastic model of RPC

Creating the RVE structure of dimensions 10 x 10 mm consists in the random selection of an element (from a 50 × 50 grid) and also the random assignment of the component (pores,
crushed quartz, sand, cement matrix) to the selected position (Fig. 2) and [1]. The concept of the RVE is a delicate one, especially for concrete with cracks. The issue of approximation of the random microstructures is discussed in detail in [3]. The boundary value problem of mechanics for the specified RVE after FEM discretization is obtained as the minimization of the energy function with additional constraints

$$\min_u \varphi(u) = \frac{1}{2} u^T Ku - u^T f \quad \text{s.t.} \quad Cu - g = 0$$

(1)

Figure 2: Representative volume element of RPC: a) microstructure [5], b) finite element model of RVE

The problem of homogenization over the RVE of volume $V$ is to find a displacement field $u(X)$ such that $\text{div} \sigma = 0$ in $V$, while satisfying the boundary conditions on the boundary $\Gamma$ so that the Hill energy criterion is fulfilled:

$$\mathbf{\sigma} \cdot \mathbf{n} = (\mathbf{\sigma} - \mathbf{\varepsilon})$$

(2)

The nonlinear material behaviour is modelled taking into account degradation of the microstructure (RVE). Calculations at the macro level are carried out iteratively (Fig. 1). At each load increment a reduced stress condition by the Burzyński hypothesis (3) is checked for each element of the RVE. If the reduced stress exceeds a limit value, then the element is damaged

$$\sigma_{cut} = \min \left\{ R_t \left(\sigma + \sigma_t\right)^2 - R_c \left(\sigma - \sigma_c\right), 0 \right\} + R_s \left(\sigma - \sigma_c\right)$$

(3)

where:
- $R_t$ - tensile strength of a microstructure component,
- $R_c$ - compressive strength of a microstructure component,
- $R_s$ - shear strength of a microstructure component.

It is assumed that the microstructure of RPC is composed of isotropic ideally-elastic-brittle constituents and at the macro scale the material is homogenized. This approach allows for a simple modelling of microcracks that cause the nonlinear behaviour of the material at the macro level. The numerical analysis is done here within the assumption of the plane stress state, and at each level of analysis the finite element method is applied.

3. Numerical example

The first test was to check the behaviour of the homogeneous material at the macro level during the compression test and a control of the degradation of microstructure. The next test was planned for the same task but with a degradation of the microstructure enforced in a different way and a heterogeneous material was simulated. In the last test we simulated the behaviour of the RPC (Mixture I) by using the Burzyński criterion (3) to control the degradation process of the RVE (Fig. 3).

Figure 3: Response of RPC (Mix. I) by the Burzyński criterion

4. Laboratory testing

The main aim of the conducted laboratory tests was to validate our computational model. Some of the obtained experimental results are displayed in Fig. 4. Further results of our numerical analysis and experimental tests will be presented at the conference.

Figure 4: Beginning of a damage process: beam B4-M2

References

Abstract

The article presents a new method of evaluating the efficiency of strongly non-linear vibroisolation with constant reaction force (VCRF) in terms of the flow of energy generated by the power of structural forces. The method is based on the analysis of energy flow in the dynamic structure of a strongly non-linear mechanical one degree of freedom system equipped with VCRF. The analysis uses the The First Principle of Power Distribution in a Mechanical System to obtain a distribution of the energy transferred into the system into three kinds of energy inputs related to inertia, dissipation and elasticity. The sum of these three kinds of energy gives the total energy input introduced into the mechanical system by the driving force. The specific energy inputs and the total energy input were calculated depending on the dimensionless frequency ranging from 0 to 5. They were then used to calculate the value of the function of energy efficiency of strongly non-linear VCRF defined as a ratio of the total energy input introduced into the system by the driving force to the energy input flowing through elastic elements depending on the dimensionless frequency of vibration. The study revealed that VCRF is very efficient and vibroisolation was effective within the range of dimensionless frequency \( \delta \) (6 – 150) with values ranging from 35 to 909, respectively.

Keywords: strong nonlinear vibroisolation, energy efficiency

1. Introduction

A possible way of measuring the efficiency of classical vibroisolation, used to reduce forces generated by machines during their operation and to minimize displacements in vehicles resulting from uneven road surfaces, is the function of vibroisolation efficiency. It is calculated as an inverse of transmissibility. As a dimensionless quantity, the efficiency function is defined by the formula (1) for a mechanical system with one degree of freedom [8,9]

\[
EE(\delta, \xi) = \frac{1 + \delta^2 \xi^2}{1 + 4\xi^2 \delta^2}
\]

(1)

where

\[
\delta = \frac{f_0}{f_n} = \frac{\omega_n}{\omega_0} = \sqrt{\frac{m}{k_z}}
\]

(2)

– dimensionless frequency of vibrations in a mechanical system,

\[
\xi = \frac{c_z}{2m_k z}
\]

(3)

– the damping ratio in a mechanical system,

\( m_0 \) – reduced, alternative mass coefficient,

\( c_z \) – reduced, alternative damping coefficient,

\( k_z \) – reduced, alternative stiffness.

The function is represented graphically in Figure 1 for \( \xi = 0.1 \) and 0.01. The value of the function of vibroisolation efficiency depends strongly on the value of dimensionless frequency \( \delta \).

Values of the function of vibroisolation efficiency from that point onwards are less than 1 and represent the degree of reduction in force or displacement amplitudes after passing through the vibroisolation system. The efficiency of vibroisolation improves with increasing value of dimensionless frequency \( \delta \). For example, at frequency \( \delta = 3.7 \) efficiency reaches the value of 10, which is regarded sufficient for passive vibroisolation. This value was obtained for the damping ratio \( \xi = 0.1 \), which is considered optimal in vibroisolation systems.
3. Energy efficiency of vibroisolation as a new method of its assessment

By applying the First Principle of Power Distribution [3,5] to the mechanical one degree of freedom system with CRF+k₁ vibroisolation, the following equation of power is obtained (4):

\[
\frac{1}{2} \int_{0}^{t} P \cdot \text{sgn}(x) + 2 \sqrt{S_{0}^{2} + P k_{x} + P k_{x}} \cdot x \cdot \text{sgn}(x) + k_{x} \cdot x = P_{\text{loss}}(t); \quad [W]
\]

where

\[
\left[M + \frac{0.5 P m_{x}}{S_{0} + P k_{x} + P k_{x}} \right] x - \frac{0.75 P k_{m}}{(S_{0} + P k_{x} + P k_{x})^{2}} \dot{x} + k_{x} x = P_{\text{loss}}(t); \quad [J]
\]

– is the momentary power of inertia,

\[
\frac{1}{2} \int_{0}^{t} P \cdot \text{sgn}(x) + 2 \sqrt{S_{0}^{2} + P k_{x} + P k_{x}} \cdot x \cdot \text{sgn}(x) = P_{\text{loss}}(t); \quad [J]
\]

– is the momentary power (of friction) dissipation,

\[
k_{x} x = P_{\text{loss}}(t); \quad [J]
\]

– is the power of elasticity of spring \( k_{1} \),

\[
F_{0} \cdot x \cdot \text{sin}(2 \pi f_{0} t); \quad [W]
\]

– is the momentary power of the driving force causing the motion of mass \( M \).

It is proposed that energy efficiency of vibroisolation of vibration isolation systems be defined as a ratio of the total energy input in \([J]\) introduced into the mechanical system with vibroisolation by the exciting force to the amount of energy in \([J]\), which simultaneously flows through the elastic elements during the period of simulation \( t \). The elastic energy input is an energy measure of the mechanical load of the support structure of the mechanical system. Thus defined, energy efficiency can be expressed as [7]:

\[
\text{EEF}_{\text{vibroisolation}} = \frac{\text{DE}_{\text{elas}}}{\text{DE}_{\text{elas}}}, \quad (9)
\]

where: \( t_{s} \) – the time of simulation of total energy flow in the system with vibroisolation,

\[
\text{DE}_{\text{elas}} = \int_{0}^{t} P_{\text{elas}}(t) \cdot dt + \int_{0}^{t} P_{\text{elas}}(t) \cdot dt + \int_{0}^{t} P_{\text{elas}}(t) \cdot dt \quad (10)
\]

– total energy input flowing through the dynamic structure of the mechanical system during a simulation period \( t_{s} \),

\[
\text{DE}_{\text{elas}} = \int_{0}^{t} P_{\text{elas}}(t) \cdot dt \quad \text{– the amount of energy flowing through elastic elements during the system’s operation transferred from the source of energy. Figure 3 illustrates the dependence of calculated energy efficiency of CRF+k₁ vibroisolation on dimensionless frequency. This time the X axis represents dimensionless frequency \( \delta \). This values of \( \delta \) is easy to obtain for the WOSSO vibroisolation system analysed in the paper because natural frequency is equal 0.2 Hz.

\[
\text{Figure 3: Energy efficiency of CRF+k₁ vibroisolation vs dimensionless frequency \( \delta \), at the frequency of the driving force equal to 30 Hz}
\]

The figure indicates a high energy efficiency of the CRF+k₁ vibroisolation ranging from 35 to 909.

\[
\text{References}
\]


Numerical modelling of the rate-dependent polarisation switching in ferroelectric materials based on a sequential laminate approach

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Abstract

The work focuses on modelling and simulation of microstructure evolution and polarisation switching in ferroelectric single- and polycrystals based on a multi-rank sequential laminate approach. These materials, at microscopic level, are composed of several regions of uniform polarisation distribution called domains, which evolves to a form stable configuration to minimise the total energy with respect to external electromechanical loads. The energetics and kinetics of the microstructure evolution can be modelled considering, e.g., mixture theory, where the mixture consists of a number of electromechanical phases or variants with arbitrary volume fractions. Considering kinematic and polarisation compatibility conditions between ferroelectric domains, this laminate-based formulation is governed by a mixed energy-enthalpy function and a rate-type, convex dissipation potential, which determines the evolution of the multi-rank volume fractions.

Keywords: microstructure evolution, domain switching, rate-dependencies, laminate-based modelling

1. Introduction

Ferroelectric crystals and ceramics, e.g. barium titanate (BaTiO\(_3\)), are functional materials exhibiting complex electromechanical coupling behaviour that can be applied in a variety of applications. When such materials are subjected to a sufficiently high macroscopic cyclic electric field, polarisation reversal along the direction of the applied electric field occurs resulting in a characteristic hysteretic behaviour. Experimental observations confirm that the strain and polarisation hysteresis is dependent on the frequency of the electric load and the magnitude of the external compressive stresses, acting along the direction of the electrical load. Furthermore, the strains obtained in single crystals were observed to be of higher magnitude than these obtained with commercial polycrystalline ferroelectric materials; cf. \cite{1, 2}.

In order to predict the dissipative behaviour of ferroelectric single- and polycrystals, laminate-based formulations are of particular interest in the area of micromechanical modelling approaches. The concept of sequential laminations to generate so-called engineered domain configurations to minimise the total energy of the ferroelectric crystal was examined in \cite{3, 4}, including the compatibility restrictions as introduced for ferroelectric materials by \cite{5}. Following the research works of \cite{6, 7} to capture the domain microstructure and switching effects in ferroelectrics, this contribution deals with the formulation of a mixed energy-enthalpy laminate-based model to study the rate-dependent behaviour of ferroelectric single- and polycrystals subjected to external electromechanical loads.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Experimentally observed butterfly curves (left) and hysteresis loops (right) in a (001)-oriented BaTiO\(_3\) single crystal subjected to varying magnitudes of external compressive stresses and cyclic electrical load at a frequency of 0.2 Hz; cf. \cite{1}}
\end{figure}

2. Laminate-based model

Below the Curie temperature \(T_C\), a bulk ferroelectric crystal exhibits a number of domain types, or rather variants, separated...
by domain walls. Each variant is characterised by a set of spontaneous strains and polarisation of that particular variant. Under electromechanical loading conditions, the ferroelectric crystal is considered to form compatible domain configurations wherein the compatibility conditions across the interface, introduced in [5], between any two variants \( r \) and \( s \) can be expressed as

\[
\varepsilon^{(r)} - \varepsilon^{(s)} = \frac{1}{2} \left[ \mathbf{a}^{rs} \otimes \mathbf{n}^{rs} + \mathbf{n}^{rs} \otimes \mathbf{a}^{rs} \right]
\]

and

\[
\left[ \mathbf{P}^{(r)} - \mathbf{P}^{(s)} \right] \cdot \mathbf{a}^{rs} = 0,
\]

where \( \mathbf{n}^{rs} \) denotes the unit normal vector related to the interface between the two ferroelectric variants \( r \) and \( s \) with \( r, s = 1, \ldots, nv \), and where \( \mathbf{a}^{rs} \) is any arbitrary vector.

Let the volume fraction of each variant \( i = 1, \ldots, nv \) be denoted by \( \lambda^i \). Then, the average macroscopic transformation strains and polarisation of the crystal can be expressed as

\[
\varepsilon_M(\lambda) = \sum_{i=1}^{nv} \lambda^i \varepsilon^{(i)}, \quad P_M(\lambda) = \sum_{i=1}^{nv} \lambda^i P^{(i)}; \quad \lambda^i \geq 0,
\]

together with \( \sum_{i=1}^{nv} \lambda^i = 1 \). Instead of weighting the average quantities by the \( i \)-th variant volume fraction \( \lambda^i \), a representation based on the volume fractions of the \( j \)-th rank laminate \( \mu^j \), with \( 1 \geq \mu^j \geq 0 \) where \( j = 1, \ldots, nv-1 \), can be used; cf. [3], where

\[
\lambda^i = \left\{ \begin{array}{l}
\mu^1 \prod_{j=1}^{i-1} \left[ 1 - \mu^j \right], \quad r = 1, \ldots, nv-1, \\
\prod_{j=1}^{r-1} \left[ 1 - \mu^j \right], \quad r = nv.
\end{array} \right.
\]

For a sequential laminate-based model, the mixed energy-enthalpy \( H \) is defined as a function of total strains \( \varepsilon(u) \), electric field \( E(\phi) \) and a set of internal variables, here the multi-rank laminate volume fractions \( \mu \in \mathbb{R}^{nv-1} \), which account for the dissipative response of the ferroelectric material considered. The particular \( H \) chosen is

\[
H(\varepsilon, E, \mu) := \frac{1}{2} \left[ \varepsilon - \varepsilon_M(\mu) \right] : E \cdot \varepsilon(\mu(\mu)) : \left[ \varepsilon - \varepsilon_M(\mu) \right] - \frac{1}{2} E \cdot E - P_M(\mu) \cdot E + \frac{1}{2} \mu^{j+1} \left( f_j^l \right)^{m+1}, \quad \lambda \geq 0,
\]

wherein \( E \) is the isotropic elasticity tensor of fourth-order and \( \varepsilon \) is the isotropic dielectric modulus of second-order. The third-order piezoelectric tensor \( \varepsilon(\mu) \) is considered transversely isotropic with respect to the polarisation director \( \alpha(\mu) \).

The laminate volume fractions may evolve, and suitable evolution equations for the volume fractions are formulated in the form of a scalar, convex, non-negative dissipation potential \( \zeta(\mu) \). The rate-dependent dissipation potential is written in terms of convex switching or yield functions \( \Phi^l(f^l) \), as

\[
\zeta(\mu) = \sum_{j=1}^{nv-1} \sum_{l=1}^{m} \left\{ \frac{1}{\eta[\mu]} \left( \Phi^l(f^l)^{\eta[\mu]} \right) \right\}.
\]

wherein the Macaulay bracket, \( (\bullet) = \max[\bullet, 0] \), is used. The material parameters \( \eta > 0 \) and \( m > 0 \) influence the time-dependent evolution of the laminate volume fractions. Solving the necessary condition of (6) particularises the Perzyna-type evolution equations as

\[
\mu^i \leq \frac{1}{\eta} \left( \Phi^l(f^l)^{\eta[\mu]} \right)^{\eta[\mu]} f_j^l(\mu) f_j^l(\mu) \right\}.
\]

A Fischer-Burmeister complementarity function-based algorithm along with a standard Newton-Raphson method based on Euler-backward integration scheme is employed to solve the scalar-valued evolution equations for the laminate volume fractions; see [8]. This algorithmic approach satisfies the constraint automatically due to the restricted range of the volume fractions.

3. Numerical examples

A cyclic electric field and a constant compressive stress are applied to a three-dimensional body such that, e.g., the strains and the electric field are homogeneous. The material parameters for the single crystal BaTiO\(_3\) are taken from literature and the viscous parameters are set to \( \eta = 0.01 \) and \( m = 2 \). The hysteresis plots with different loading conditions are shown below.

![Figure 2: Simulated butterfly (left) and hysteresis loops (right) for varying stresses and cyclic electrical load at 0.1 Hz.](image)

![Figure 3: Simulated butterfly (left) and hysteresis loops (right) for a stress level of 2.86 MPa and varying electrical frequencies.](image)

References


Influence of active elements on the pendulum’s rotational motion for energy harvesting

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Abstract

The paper proposes an active pendulum system for energy harvesting recovery. The system consists of a pendulum attached to the main system (oscillator). The structure is controlled directly by non-linear damping of a pendulum (magnetorheological) or by changing suspension parameters of a main system (magnetorheological damper or/and shape memory spring). The dynamics of an autoparametric pendulum system operating in rotational regimes have been investigated with a view to energy harvesting. The influence of active parts on pendulum’s rotation is investigated in detail.

Keywords: pendulum, energy harvesting, rotation, MR damper, SMA spring, control

1. Introduction

Vibration energy harvesting (VEH) has gained immense popularity in recent years. This topic is motivated by the possibility of energy recovery from ambient vibrations via non-linear dynamics of the parametric pendulum. The harvesters are designed for different kinds of motion of the energy source. One of the most popular are harvesting devices based on rotational motion. Usually, such type of harvesters are characterised by significantly higher kinetic energy than oscillations and thus have a potential of delivering more energy.

In the field of vibration energy harvesting, research has shifted focus from linear systems [1] with or without tuning capabilities to non-linear systems with widened usable bandwidth, multiple attractors, and chaotic oscillations [2]. Non-linearities in VEH systems usually are used to improve efficiency in the changing ambient conditions. Additionally, the region of ambient vibration frequency can be broadened by non-linear inclination of the resonance curve and the existence of additional solutions. Therefore, the main advantage of introducing non-linear elements over the common linear devices is the possibility of increasing the frequency range of operation.

Therefore, the paper proposes a concept of a pendulum system dedicated to energy harvesting which is controlled by smart elements, mounted in the pendulum pivot and installed in the suspension of a main system.

2. Model of an active pendulum system

2.1. Description of the system

The investigated dynamical system consists of an oscillating main system and a vibration pendulum, as shown in Fig.1. Such systems in practice are used as vibration absorbers or energy harvesters. The system has three active elements. The first is a magnetorheological damper (MR) mounted in the pendulum pivot. The second and third are installed in the suspension of the main mass. The active suspension consists of a MR damper and a shape memory alloys spring (SMA). The MR dampers are used for controlling the damping of the system, while the SMA spring is used to change the system stiffness. Dynamics of a two-degree-of-freedom system is described by vertical displacement of the oscillator (X) and angular position of the pendulum (φ).

![Figure 1: An active pendulum system for harvesting energy.](image)

The system is vertically excited by harmonic force y(τ) = qcos(ωτ), where q is the amplitude, and ω is the frequency of the excitation.

2.2. Equations of motion and active elements

Differential equations of motion of a system presented in Fig.1. are obtained by application of Lagrange’s equation of the second kind, and presented in dimensionless form:

\[
\ddot{X} + F_{MR2}(\dot{X}; X) + F_{SMA}(X, T) + \mu \ddot{\phi} \sin(\phi) + \phi^2 \cos(\phi) = y(\tau),
\]

\[
\ddot{\phi} + F_{MR1}(\dot{\phi}; \phi) + \lambda(\dot{X} + 1) = 0.
\]

The non-linear damping in the pendulum pivot has a form

\[
F_{MR1} = \alpha_2 \ddot{\phi} + \alpha_4 \tanh(\delta_1 \dot{\phi} + \delta_2 \phi),
\]

where \(\alpha_2\) represents the viscous damping, \(\alpha_4\) dry friction, and \(\delta_1, \delta_2\) are constants describe hysteresis shape. The MR function of oscillator’s damper is described by the function

\[
F_{MR2} = \alpha_1 \dot{X} + \alpha_3 \tanh(\delta_1 \dot{X} + \delta_2 X),
\]

where \(\alpha_1\) and \(\alpha_3\) characterize dimensionless viscous and...
3. Exemplary results

The regular anti clock-wise direction rotations of the pendulum under dry oscillator damping ($\alpha_3$) in Fig.2 is presented. This 3D bifurcation diagram shows maximal angular velocity ($\dot{\phi}_{max}$) obtained for rotations of the pendulum. Continuous lines denote stable solutions, while dashed lines denote unstable solutions. The results are obtained with the help of a continuation technique using Auto07p software.

Figure 2: Positive regular rotations of the pendulum under non-linear damping of the oscillator, for the following parameter set: $\alpha_3 = 0.26, \alpha_2 = 0.1, \mu = 17.2, \lambda = 0.12, q = 2.45, \alpha_4 = 0, \delta_1 = 10, \delta_2 = 1, \theta = 2, \beta_1 = \beta_2 = 0$.

The black line denotes pendulum rotation for linear $\alpha_3 = 0$, while: red ($\alpha_3 = 0.1$), green ($\alpha_3 = 0.2$), blue ($\alpha_3 = 0.3$) and gray ($\alpha_3 = 0.4$) non-linear damping, respectively. The label $L$ represents saddle-node bifurcation. The pendulum system can execute rotational motions in both directions i.e. in the clock wise (negative) and anti-clock wise direction (positive).

Figure 3: Influence of non-linear damping on the positive and negative pendulum’s rotation, for $\theta = 0.9$.

The results are obtained with the help of a continuation technique using Auto07p software.

Figure 4: Basins of attraction identified positive and negative rotation of the pendulum, for $\theta = 0.9$.

Influence of parameter $\alpha_3$ on maximal angular velocity in Fig.3 is shown. Note, that for negative rotations, the minimal values are shown. Introducing dry friction in the suspension of the oscillator, we reduced the rotation region, but maximal angular velocity of the pendulum changes slightly, what is important from the energy harvesting point of view. The direction of rotation depends on the initial conditions. The basin of attraction for frequency $\theta = 0.9$ showing set of initial conditions for positive (red) and negative (black) rotations in Fig.4 is shown.

During the conference, detailed dynamics analysis and influence MR rotatory damper and SMA spring on the rotations of the pendulum will be presented.

4. Conclusions and remarks

The paper shows influence of non-linear damping of the pendulum and non-linear damping and stiffness of suspension on the pendulum rotations. The dry friction comes from MR damper reduces rotational region, but its angular velocity (energy harvesting) remains practically unchanged. The highest velocity was obtained for frequency $\theta \approx 0.95$.

Application of smart elements can improve energy recovery and can maintain solution on demand attractor, or change one into another.

References


Opitmally tuned fuzzy control for smart, possibly damaged piezocomposites

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Abstract

Various models of smart composite beams and plates with piezoelectric layers, with glue were created. The finite element method has was for the numerical solution of related static and dynamic problems. Furthermore, delamination between the various layers was modelled, in order to estimate one of the most commonly appearing damage in composites. Finally, active control was applied by means of optimally tuned fuzzy inference rules. Tuning was achieved using genetic optimization. The results demonstrate a viable way of modelling and optimal design of active smart piezocomposites.

Keywords: piezocomposites, delamination, vibration control, fuzzy control, genetic optimization

1. Introduction

Finite element modelling of the dynamic response of smart piezocomposites with adhesive and piezoelectric layers is firstly outlined. Smart composites may suffer from fatigue, damage and delamination, so the nominal model of the structure changes. Robust controllers are required, in order to work even in the presence of these deviations. Fuzzy controllers are generally intended to have robust properties. Therefore a fuzzy controller is used in order to study the effect of delamination on the control of a smart piezoelectric beam.

2. Electromechanical model

Consider a plate structure consisting of three layers: The upper and lower layers composed of a piezoelectric material, the elastic core and thin adhesive layers. Each layer is modelled by Mindlin theory. For the thin adhesives peel and shear strains can be assumed to be constants through its thickness and other strains are negligible. A finite element model based on the layerwise displacement theory which incorporates the electromechanical coupling effects and the adhesive layer flexibility has been used, Ref. [1]. The equations of motion and charge equilibrium of the system can be written as

\[
\begin{bmatrix}
M & [K_m] & [K_a] \\
[K_m] & [K_m] & [K_a] \\
[K_a] & [K_a] & [K_a]
\end{bmatrix}
\begin{bmatrix}
\ddot{d} \\
\ddot{\phi}
\end{bmatrix}
+
\begin{bmatrix}
[K_m] & [K_m] & [K_m] \\
[K_m] & [K_m] & [K_m] \\
[K_m] & [K_m] & [K_m]
\end{bmatrix}
\begin{bmatrix}
d \\
\phi
\end{bmatrix}
=
\begin{bmatrix}
F_m \\
F_m
\end{bmatrix}
\]

\[d = d_0 + \varepsilon x \]

where \(d\) and \(\phi\) are the global mechanical and electrical loads vectors, \(M\) is the global mass matrix, \([K_m]\) and \([K_a]\) are the global mechanical stiffness, mechanical–electrical coupling stiffness and dielectric stiffness matrices respectively. \(F_m\) and \(F_a\) are the respective global mechanical and electrical loads vectors.

3. Delamination modelling

A non-linear delamination law is considered in the previously outlined electromechanical model, Fig. 1:

\[
\sigma = E \varepsilon
\]

Figure 1: Stress-strain behaviour in the adhesive

For each finite element, the element average strain \(\varepsilon\) is calculated as follows:

\[
\varepsilon = \frac{w_{ii}\phi_{ii} - w_{i}\phi_{i}}{h_{ii}}
\]

where \(w\) is the vertical displacement of the upper and lower layers, and \(h\) is the thickness of the adhesive layer.

The iterative solution follows classical Newton-Raphson techniques and specialized algorithms, see Ref. [2].

Delamination between the middle and the upper plate layer was considered in a composite plate with dimensions 100x100mm, fixed on the left side. Each element consists of 12 nodes: 4 nodes belong to the lower layer, 4 nodes to the middle and 4 nodes to the upper layer. Every node has 5 degrees of freedom (three translational and two rotational). Therefore, the structure has 363 nodes and 1815 degrees of freedom.

When the mechanical load is increased, delamination occurs almost instantaneously across the upper layer, Figs. 2, 3, as it is expected, cf. Ref. [3].

Since the softening behavior cannot be depicted by a load control procedure, a displacement controlled procedure is used to capture partial delamination effects, see Figs. 4, 5.

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5. Numerical example

The fuzzy controller, initially created for the control of the non-delaminated structure, has been applied on a structure with 50% delamination of the structure’s lower, sensor layer, with satisfactory results, Fig. 7. It is concluded, that the functionality of the controller in an extended delamination is quite satisfactory.

4. Optimally tuned fuzzy controller

A Mamdani-type fuzzy logic controller is developed, using Matlab, with two inputs (displacement, velocity) and one output (control force), collocated at the free end of the plate. Trapezoidal and triangular membership functions are used. The decision is based on a set of 15 if-then rules, the AND operator and the centroid defuzzification method. Genetic algorithms can be used for the optimal tuning of the parameters, Ref. [4]. The nonlinear control feedback resulting from the optimized fuzzy logic controller is shown in Fig. 6.

References

Subgroups of jet groups and material symmetries

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Abstract

R. A. Toupin, in his paper on couple-stress elasticity [10], considered certain types of second-grade material symmetry. This symmetry is described by subgroups of second order jets groups, which are in [3] called Toupin subgroups. In the talk, we explain the mathematical background and present ways of generalizations and applications of methods of differential geometry in material science.

Keywords: jet groups, material with microstructure, conjugate subgroups, symmetry

1. Introduction

Materials, in which changes occur in the molecular or crystalline texture at various microscopic scales (substructure) and influence the macroscopic behaviour through peculiar interactions are commonly known and used. Materials such as liquid crystals, ferroelectrics, quasicrystals, polymeric fluids are paradigmatic examples. In our paper, we use differential geometry as the mathematical framework to describe the two fundamental entities of our discourse: the space-time manifold and the body manifold. Even though Charles Ehresmann in his original papers from 1951 underlined the conceptual meaning of the notion of an r-jet for differential geometry, jets have been mostly used as a purely technical tool in certain problems in the theory of systems of partial differential equations, in singularity theory, in variational calculus and in higher order mechanics. The theory of natural bundles and natural operators clarifies once again that jets are one of the fundamental concepts in differential geometry. In spite of the fact that the jet groups lie at the core of considerations concerning geometric objects and operations, they have not been studied very extensively.

Such a differential geometry approach also includes non-orientable manifolds. In this context, it is good to ask whether they are really useful for creating physical models of nature and whether we can directly observe them. First, we can somewhat provocatively quote ([1], page 446): The unorientable surfaces are never discussed in the literature since the primary interest of mathematicians in surfaces is in the study of one complex variable, number theory, algebraic geometry, etc. where all surfaces are oriented. Nevertheless, at present, there are already scientific papers (outside mathematics itself) with interesting occurrence of nonorientable manifolds. For instance, in the material science (see [7]) knots and nonorientable surfaces in chiral nematics are studied, as an example of a phenomenon that topological concepts have come to play an increasingly significant role in characterizing materials across of diverse range of topics, e. g. in the study of defects, [8]. We recall that also so called exotic manifolds exist, i. e. manifolds with inequivalent differentiable structures. This was first shown by John Milnor [9] that, for the 7-dimensional sphere having usually both orientation preserving and orientation reversing maps, it is possible to construct such differentiable structure which has no orientation reversing maps (it proves the inequivalence of two different structures).

Our intention is to clarify the role of jet groups in the classification of material symmetries. We focus on subgroups of jet groups.

2. The configuration

Let $B, S$ be two smooth manifolds ($\dim B = b, \dim S = s, b \leq s$) and $\kappa: B \to S$ a smooth embedding (i.e. an injective smooth mapping such that $\kappa(B)$ is a submanifold of $S$ and the (co)restricted mapping $B \to \kappa(B)$ is a diffeomorphism, see [6]). We will call $B$ the body, $S$ the space and $\kappa$ the configuration. As $B$ and $S$ are manifolds, they are endowed with local maps $\varphi^B_i: U_i \to \mathbb{R}^b (U_i \subseteq B, i \in I)$ and $\varphi^S_i: V_i \to \mathbb{R}^s (V_i \subseteq S, i \in \bar{I})$. In classical situations, we meet the case $\dim B \leq 3$ and $S = \mathbb{R}^s$ where $B$ can be covered by a single map, so both index set $I$ and $\bar{I}$ are singleton. Let us mention as a curiosity that some authors, cf. [5], consider local maps inversely — as mappings from the real space to a manifold, which is not a problem. The maps provide local coordinates, let points $p \in B$ have coordinates $(\xi^j), j = 1, \ldots, b$ and points $p \in S$ have coordinates $(\chi^i), i = 1, \ldots, s$. The local coordinate expression of the configuration $\kappa$ is a map $\tilde{\kappa}: \mathbb{R}^b \to \mathbb{R}^s$ such that $\tilde{\kappa} \circ \varphi^B_i = \varphi^S_i \circ \kappa$ and it can be expressed by

$$x^i = \kappa^i(\xi^j).$$

(1)

Of course, it is an infinite number of configurations in general. One of them is labeled as significant for us, so called reference configuration $\kappa_0: B \to S$.

We will now assume that different configurations are smoothly parameterized. Let us consider one-dimensional manifold $T$ (with local maps $\varphi_T^i: W_T \to \mathbb{R}, W_T \subseteq T, i \in I$ giving a local coordinate $t$ to a point of $T$) and a smooth map

$$\chi: B \times T \to S$$

endowed with the property that its restrictions to specific $t$ are configurations, in particular

$$\chi_{|t=t_0} = \kappa_{t_0}.$$  

(3)

Usually, $t$ is interpreted as time and the map $\chi$ is called the motion. Its local coordinate expression is

$$x^i = \chi^i(\xi^j, t).$$  

(4)

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The microstructure is expressed by \( k \) linearly independent vectors, \( k \leq s \), called directors (cf. e.g. [11]) assigned to points \( p \in \kappa(B) \subseteq S \). Thus, we consider frames over points of \( \kappa(B) \subseteq S \), in particular we do not exclude the case \( k > b \).

3. Jet groups. Toupin subgroups

We will describe jet groups acting on frames. A group \( G \) is called a split extension of a group \( N \) by a group \( H \) if \( N \) is a normal subgroup of \( G \) and \( G \) contains a subgroup \( H \) such that \( H \cong H \), \( N \cap H = \{e\} \) and \( NH = G \). Alternatively, one says that \( G \) is a semidirect product of \( N \) by \( H \). The notation is \( G = N \rtimes H \), [2].

Alternatively, given any two arbitrary groups \( \hat{N} \) and \( \hat{H} \) and a group homomorphism (such a group homomorphisms can be induced by an action of \( H \) on \( N \), for details see [6]) \( \tau : \hat{H} \rightarrow \text{Aut} \hat{N} \), we can construct a new group \( G = \hat{N} \rtimes_{\tau} H \) through its operation \( * \) defined by

\[
(\bar{n}_1, \bar{h}_1) * (\bar{n}_2, \bar{h}_2) = (\bar{n}_1 \tau(\bar{h}_1) \bar{n}_2, \bar{h}_1 \bar{h}_2)
\]  

Then pairs \((\bar{n}, e_R)\) form a normal subgroup \( N \) of \( G \) isomorphic to \( \hat{N} \), while pairs \((e_R, \bar{h})\) form a subgroup \( H \) of \( G \) isomorphic to \( \hat{H} \). This semidirect product is consistent with the definition above, namely \( \hat{N} \rtimes_{\tau} \hat{H} \cong \hat{N} \rtimes H \).

Let \( r \)-jets of smooth maps \( \mathbb{R}^n \rightarrow \mathbb{R}^n \) with non-zero Jacobian determinant in \( \mathbb{R}^n = (0, \ldots, 0) \) and sending \( 0 \) to \( 0 \) together with the jet composition form a group which is called the \( r \)-th jet group and denoted by \( G^r_n \), for details see [6]. Moreover, for \( 0 \leq s < r \), we have a canonical epimorphism \( \pi^{r-s} : G^r_n \rightarrow G^s_n \). (We consider \( G^0_n \) as the trivial group.) Let us write \( B^{r-s}_n = \ker \pi^{r-s} \). Groups \( B^{r-s}_n \) are normal subgroups of \( G^r_n \), see [6], Proposition 13.11.

Thus, in the sense discussed above, we have \( G^r_n = B^{r-s}_n \rtimes G^s_n \).

We remark that \( G^r_n \) is the general linear group \( GL(n, \mathbb{R}) \) for which a structure of subgroups is extensively studied for many years and includes i.a. triangular and diagonal subgroups, orthogonal and symplectic subgroups, etc. So, let \( M \) be a subgroup of \((e_{B^{r-s}_n}, G^r_n)\) (e.g. induced from classical groups just mentioned), \( n \in M \) and let \( b \in B^{r-s}_n \). Then elements \( bmb^{-1} \) generate the conjugate subgroup \( H = bM b^{-1} \) of \( G^r_n \), which is called the Toupin subgroup of \( G^r_n \) associated with \( M \) and \( b \).

For a subgroup \( K \) of \( G^r_n \), let \( \pi^{r-k}_K \) denote the restriction of \( \pi^{r-s} \) to \( K \). Then, for \( a \in G^r_n \), we will examine the fiber \((\pi^{r-k}_K)^{-1}(a)\).

First, for a Toupin subgroup \( T \), we observe \((\pi^{r-k}_T)^{-1}(id_{C^1}) = id_T\). This leads to the following definition. We say that a subgroup \( K \) of \( G^r_n \) is the generalized Toupin subgroup of \( G^r_n \) if it has the property \((\pi^{r-k}_K)^{-1}(id_{C^1}) = id_K\).

4. Current research and new results that will be presented

We generalize the results about subgroups of jet groups known for second order up to now for any order \( r \). We intend to confront results with the real material symmetries. Reflections on smart materials will also be taken into account.

References

Double-source flutter in a discrete-continuous rotor/bearing system with magnetic fluid

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Abstract

A discrete-continuous model of a rotor is studied consisting of a flexible shaft simply supported on its one end and linked with a rigid journal of a slide bearing filed with a magnetic fluid as a lubricant. It is a model of the real Rotor Kit RK-4 by Bently Nevada, being used for demonstrations of dynamic phenomena in rotors. The model exhibits flutter-type instability under increasing rotation speed with dual mechanism of flutter – one related to the destabilizing effect of the internal friction in the shaft, the other being induced by non-conservative fluid film forces in the bearing. Both mechanisms show different dynamics what is manifested by a different behaviour of the eigenvalues on the complex plane. The internal friction drives a fast eigenvalue while the fluid film-driven one is much slower. A semi-active control via magnetically induced changes of the fluid viscosity affects the interaction between both flutter mechanisms and influences the stability of the rotor and its near-critical behaviour.

Keywords: discrete-continuous rotor, internal friction, slide bearing, flutter instability, magnetic fluid

1. Introduction

The paper is aimed at the recognition of the dynamic interaction between two physically different mechanisms of the flutter-type instability and bifurcation appearing in a transversely loaded discrete-continuous model of a rotor under increasing rotation speed. The rotor consists of a flexible element in form of a slender shaft and a massive rigid part as a journal of a slide bearing. Such structure is used in Rotor Kit K-4 by Bently Nevada.

The dynamic behaviour of both flexible and rigid parts of the rotor is explained with attention focused on the interaction of two flutter mechanisms, critical rotation speed and possible scenarios of the rotor near-critical behaviour.

2. Models of rotor and MF

The considered discrete-continuous model of the rotor is shown in Fig. 1.

Figure 1: Discrete-continuous model of the rotor

The flexible and rigid parts of the rotor are linked with a Cardan-like joint providing transmission of torque and ensuring plane transverse motion of the rigid journal within the bearing clearance, without inclinations with respect to the rotor axis. Both parts – the shaft and the journal are subjected to transverse unidirectional loads, distributed \( q \) and concentrated \( Q \), respectively. They can be identified with the gravity forces but may be of different nature as well.

Experiments with MFs show their rheological behaviour corresponding to the Bingham model with the relation between the shear stress and shear rate (Fig. 2) with a characteristic yield stress dependent on the applied magnetic field.

Figure 2: Bingham model of a magnetic fluid

As a result of the yield stress \( \tau_0(H) \) a slide bearing shows non-zero load capacity at \( \omega = 0 \) thus, the Bingham model is not valid for \( \omega \to 0 \). A bilinear model of MF would be closer to reality. However, flutter appears at high rotation speeds so the Bingham model will not lead to errors. It should be noted that the effective MF viscosity is increased by the magnetic field and reduced by the rotation speed.

3. Equations of motion

The equations of motion of the shaft element can be written down as follows:

\[
\begin{align*}
\frac{\partial^2 u}{\partial t^2} + \frac{1}{l} \frac{\partial u}{\partial t} &= -\frac{z}{l} \left( \bar{X} + h \bar{X} \right) \frac{EI}{\rho A} \left( \frac{\partial^2 u}{\partial z^2} + \beta \frac{\partial^2 u}{\partial z^2} + \beta_0 \frac{\partial^2 u}{\partial z^2} \right) \\
\frac{\partial^2 v}{\partial t^2} + h \frac{\partial v}{\partial t} &= -\frac{z}{l} \left( \bar{Y} + h \bar{Y} \right) \frac{EI}{\rho A} \left( \frac{\partial^2 v}{\partial z^2} + \beta \frac{\partial^2 v}{\partial z^2} + \beta_0 \frac{\partial^2 v}{\partial z^2} \right) + \frac{Q}{\rho A} \tag{1}
\end{align*}
\]

where \( (u,v) \) are deflections of the shaft at the longitudinal coordinate \( z \), \( (X,Y) \) - coordinates of the journal centre, \( E, I, \rho, A \) - shaft stiffness, unit mass and length respectively, \( h \) - external damping, \( \beta \) - loss factor in the K-V model, \( \omega \) -
rotation speed. Equations (1) are linear but they can be easily supplemented with non-linear terms resulting from the geometric curvature non-linearity of the shaft. They are important in bifurcation.

The equations of the journal transverse motion in the Cartesian coordinates are as below:

\[
\begin{align*}
mx &= P_0 \cos \alpha - \dot{P}_0 \sin \alpha + EL \left[ \frac{\partial^2 u}{\partial z^2} + \beta \frac{\partial^2 u}{\partial z^2} + \beta \frac{\partial^2 v}{\partial z^2} \right] + Q \\
my &= P_0 \sin \alpha + \dot{P}_0 \cos \alpha + EL \left[ \frac{\partial^2 v}{\partial z^2} + \beta \frac{\partial^2 u}{\partial z^2} - \beta \frac{\partial^2 v}{\partial z^2} \right] + Q
\end{align*}
\] (2)

where \( m \) is the journal mass, \( \alpha \) - angle of the journal centre in polar coordinates and \( P_0, P_1 \) - denote film force components in the radial and circumferential directions, adopted from [1]. The dynamic film forces depend on the journal centre displacements and velocities and describe both wedge and squeeze effects of isothermal fluid film.

\[
P_r = 2C \left[ \frac{e^2(\omega - 2\alpha)}{pq} + \frac{e^2}{p} + \frac{2e}{p\delta^2} \arctan \left( \frac{1+e}{1-e} \right) \right]
\] (3)

where \( e = r/c \) is the relative journal eccentricity, \( (r, \alpha) \) - journal centre polar coordinates, \( C = 6uR/\delta^2 \) and \( R, L, \delta \) denote journal radius, length and relative clearance, respectively.

Equations (1) and (2) undergo a single-mode Galerkin’s discretization assuming:

\[
u(z,t) = U(t) \sin \frac{\pi z}{L}, \quad v(z,t) = V(t) \sin \frac{\pi z}{L}
\] (4)

with the basic sinus functions satisfying all the boundary conditions of the simply supported shaft. The resulting coupled ordinary equations in the Cartesian coordinates \((U, V, X, Y)\) are:

\[
\begin{align*}
\ddot{U} &= -2 \frac{P^2}{\pi^2 m} \cos \alpha - \frac{P^2}{\pi^2 m} \sin \alpha + hY - \gamma(U + \beta U + \beta v) - hU \\
\ddot{V} &= -2 \frac{P^2}{\pi^2 m} \sin \alpha + \frac{P^2}{\pi^2 m} \cos \alpha + \frac{4q1}{\pi^2 m} \gamma(U + \beta U + \beta v) - hV + 4q1 \gamma \frac{1}{m} \\
\ddot{X} &= \frac{P^2}{\pi^2 m} \cos \alpha - \frac{P^2}{\pi^2 m} \sin \alpha + \chi(U + \beta U + \beta v) \\
\ddot{Y} &= \frac{P^2}{\pi^2 m} \sin \alpha + \frac{P^2}{\pi^2 m} \cos \alpha + \chi(U + \beta U + \beta v) + \frac{Q}{m}
\end{align*}
\] (5)

where \( \chi = \frac{1}{m} \Omega^2 \), \( \gamma = \left( 1 + \frac{1}{\pi^2 m_c} \right)^2 \Omega^2 \), \( m_c \) is the shaft mass and \( \Omega \), denotes the first shaft bending frequency. Equations (5) have a nontrivial solution \( U_0, V_0, X_0, Y_0 \), which depends on the transverse loads \( q \) and \( Q \), loss factor \( \beta \) and on the rotation speed being the bifurcation parameter. It is convenient to express the equilibrium in the dimensionless polar coordinates of both the shaft and the journal - \((w_0, \phi_0)\) and \((e_0, \alpha_0)\), respectively. They are described as follows:

\[
\begin{align*}
\tan \phi_0 &= -\frac{1}{\beta \omega} - \frac{1}{\omega} \frac{W_0}{W_{man}} + \frac{1}{\sqrt{1 + (\beta \omega)}}, \\
\tan \alpha_0 &= \frac{2}{\pi} \frac{e_0}{\sqrt{1 - e_0^2}} + \frac{c^2 \omega^2 (\pi/\omega_0 + 4e_0^2)}{P^2/\omega_0} + Q + 4q1 \frac{1}{m^2}
\end{align*}
\] (6)

The coupled normalized curves of equilibrium of the shaft and journal, under increasing rotation speed \( \omega \) are shown in Fig. 3.

\[\text{Figure 3: Shaft and journal equilibria under increasing } \omega\]

4. Stability analysis

The stability of the non-trivial equilibrium of the discrete-continuous rotor is studied using the matrix \( A(\omega, Q, q, \beta) \) of the system (5) locally linearized around equilibrium \( U_0, V_0, X_0, Y_0 \). The qualitative results are shown in Fig. 4 in form of the eigenvalue trajectories under increasing rotation speed \( \omega \).

\[\text{Figure 4: Eigenvalue behaviour – double source of flutter}\]

The eigenvalue related to the shaft internal friction moves much faster than that associated with the film effect and the and resulting bifurcation scenario depends on which of them intersects the imaginary axis as first. This scenario can be changed by the magnetically induced increase of viscosity which causes speed-up of the slower eigen-frequency.

5. Conclusions

Two physically different flutter mechanisms interact in a discrete-continuous rotor, associated with the internal friction and the non-conservative fluid film forces. One of them is usually dominant but in particular situations a double bifurcation may occur when two pairs of eigenvalues simultaneously intersect the imaginary axis producing bifurcation into two independent periodic solutions. This structure can be broken or avoided by an external magnetic control of the MF bearing lubricant.

References

Frequency response function of structures with viscoelastic dampers and its design sensitivity analysis

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Abstract

In the paper, the frequency response function of planar frames with viscoelastic dampers mounted is considered. The Adhikari superposition method is used to compute the dynamic response of structures. Based on this solution, design sensitivity analysis of the frequency response functions is presented. The direct differentiation method is used to determine design sensitivity of the frequency response functions.

Keywords: frequency response function, sensitivity analysis, viscoelastic dampers, fractional derivative model

1. Introduction

The frequency response function (FRF) is one of the most important tools to evaluate the dynamic response of structures. It is very important in many areas, such as model updating, vibration control, optimization, and other ones. The FRF can be calculated using a direct frequency response method solving the inversion of dynamic stiffness matrix. In the case of a great number of exciting frequencies, it involves high computational costs. In such cases, the Adhikari superposition method is often used to calculate the FRFs them as a sum containing all modes. It is described in detail in [1,3]. Some authors studied this approach to find the dynamic response for systems with dampers, and for a large-scaled model [5].

The design sensitivity analysis of the FRF was studied by several authors. The direct differentiation method and the adjoint variables method is described in [2]. The sensitivity of FRF of structures with viscoelastic dampers determined by the superposition method is used to compute the dynamic response of structures. Based on this solution, design sensitivity analysis of the frequency response functions is presented for the first time. Next, the direct differentiation method is applied for design sensitivity analysis of the FRF.

2. The equation of motion of structures with viscoelastic dampers

2.1. Forces in dampers

In the paper, the fractional Kelvin model of a damper is considered (see Fig. 1a). The model consists of a spring of a stiffness \( k_0 \) connected parallel with a fractional dashpot described by damping factor \( c_0 \) and fractional parameter \( \alpha \) (\( 0 < \alpha \leq 1 \)).

![Figure 1: Models of dampers a) the fractional Kelvin model, b) the fractional viscous model](image)

The equation of motion of the Kelvin damper is:

\[
\ddot{u}_i (t) = k_0 \Delta q_i (t) + c_0 \Delta q_i (t) + \sum q_j, \quad q_i \text{ and } q_j \text{ denote the nodal displacements of the considered damper and } i \text{ the damper number. }
\]

The equation of motion of the Kelvin damper is:

\[
\ddot{u}_i (t) = k_0 \Delta q_i (t) + c_0 \Delta q_i (t) + \sum q_j, \quad q_i \text{ and } q_j \text{ denote the nodal displacements of the considered damper and } i \text{ the damper number. }
\]

The equation of motion of the Kelvin damper is:

\[
\ddot{u}_i (t) = k_0 \Delta q_i (t) + c_0 \Delta q_i (t) + \sum q_j,
\]

where: \( u_i \) is the force in the \( i \)-th damper, \( \Delta q_i (t) = q_i - q_j \), \( q_i \) and \( q_j \) denote the nodal displacements of the considered damper model, and \( i \) is the damper number. \( D^{\alpha} (\bullet) \) denotes the Riemann-Liouville fractional derivative of the order \( \alpha \) with respect to time \( t \) [7].

Application of the Laplace transform to Eqn (1) leads to:

\[
\bar{u}_i (s) = G_i (s) \Delta \bar{q}_i (s)
\]

where: \( \bar{u}_i (s) \) and \( \Delta \bar{q}_i (s) \) denote the Laplace transforms of \( u_i (s) \) and \( \Delta q_i (s) \), respectively, \( s \) is the Laplace variable and \( G_i (s) = k_0 + c_0 s^{\alpha} \). The considered damper model contains the fractional viscous model (see Fig. 1b) when \( k_0 = 0 \) in Eqn (2). The classical models can be obtained by introducing \( \alpha = 1 \).

2.2. Equation of motion of structures with dampers

The equation of motion of a structure with dampers can be presented in the following form [4]:

\[
M \ddot{q} + C_2 \dot{q} + K_2 q = p(t) + f(t)
\]

where: \( M \), \( C \), and \( K \) denote the mass, the damping, and the stiffness matrices, respectively. \( q(t) = [q_1, \ldots, q_n]^T \) is the vector of displacements of the structure, \( p(t) = [p_1, \ldots, p_n]^T \) is the vector of the excitation forces and \( f(t) = [f_1, \ldots, f_n]^T \) is the vector of the interaction forces between the frame and the dampers. Taking the Laplace transform the equation of motion can be written in the form:

\[
\mathcal{s}^2 \bar{M} \bar{q} + s \bar{C} \bar{q} + \bar{K} \bar{q}(s) = \bar{p}(s) + \bar{f}(s).
\]

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Having formed the vector \( f(t) \) and substituting it into (4), the equation could be written as:

\[
\mathbf{D}(s)\tilde{\mathbf{q}}(s) = \tilde{\mathbf{p}}(s)
\]

(5)

where:

\[
\mathbf{D}(s) = s^2\mathbf{M} + s\mathbf{C} + \mathbf{K} + \sum_{i=1}^{m} \mathbf{G}_i
\]

(6)

is the dynamic stiffness matrix, \( \mathbf{G}_i = \mathbf{G}_s(s)\mathbf{L}_i \), \( \mathbf{L}_i \) is the matrix of location of dampers, and \( m \) is the number of dampers.

If the vector of external forces is a zero vector \( \mathbf{p}(s) = 0 \), Eqn (5) leads to the following nonlinear eigenproblem:

\[
\mathbf{D}(s)\tilde{\mathbf{q}}(s) = 0
\]

(7)

3. Frequency response function

3.1. Direct frequency response method

From Eqn (5) the vector \( \tilde{\mathbf{q}}(s) \) can be obtained as:

\[
\tilde{\mathbf{q}}(s) = \mathbf{D}^{-1}(s)\mathbf{p}(s) = \mathbf{H}(s)\mathbf{p}(s)
\]

(8)

where \( \mathbf{H}(s) = \mathbf{D}^{-1}(s) \) is the frequency response matrix.

3.2. Adhikari superposition method

From the residue theorem \cite{1,8} the frequency response matrix can be expressed in terms of poles and residues:

\[
\mathbf{H}(s) = \sum_{j=1}^{s} \mathbf{R}_j s^{-j} , \quad \mathbf{R}_j = \frac{\tilde{\mathbf{q}}_j}{\tilde{\mathbf{q}}_j} \frac{\partial \mathbf{D}(s)}{\partial \mathbf{s}_j} \tilde{\mathbf{q}}_j
\]

(9)

where \( n \) denotes the number of eigenvalues of eigenproblem (7).

After substituting \( s = i\lambda \), where \( \lambda \) is the excitation frequency, the frequency response matrix can be obtained.

4. Design sensitivity analysis

In order to determine the relationship describing the sensitivity of FRF, it is necessary to use the following equation:

\[
\mathbf{H}(\lambda)\frac{\partial \mathbf{H}(\lambda)}{\partial \mathbf{p}} = \mathbf{I}
\]

(10)

Differentiating Eqn (10) with respect to the design parameter \( p \) leads to:

\[
\frac{\partial \mathbf{H}(\lambda, p)}{\partial p} = -\mathbf{H}(\lambda, p)\frac{\partial \mathbf{D}(\lambda, p)}{\partial p} \mathbf{H}(\lambda, p)
\]

(11)

where \( \mathbf{D}(\lambda, p)\frac{\partial \mathcal{D}}{\partial p} \) can be obtained by differentiating Eqn (6).

5. Numerical example

In order to illustrate the method presented in this paper, a two-storey building with fractional viscous damper situated on the second storey is considered. The following data are adopted: the mass of every floor \( m = 1000.0 \text{ kg} \), the storey stiffness \( k = 1000000.0 \text{ N/m} \), and the damper parameters: \( k_0 = 0 \), \( c_0 = 2500.0 \text{ Ns/m} \), and \( \alpha = 0.6 \). Damping of the structure is neglected. The frequency response matrix \( \mathbf{H}(\lambda) \) is determined for the excitation frequency taken from the range \( \lambda \in (0, 20 \text{ rad/s}) \). The results are presented in Fig. 2, pointing out real and imaginary parts of the function \( H_{11}(\lambda) \). Two graphs overlap, obtained from both direct frequency response method and Adhikari superposition method.

Figure 2: The frequency response function \( H_{11} \)

6. Conclusions

In the paper, two methods of FRF calculation are compared: Adhikari superposition method and direct frequency response method. The obtained results confirm the possibility of using the presented method for calculating the FRF for structures with viscoelastic dampers. Such analysis for shear frames with viscoelastic dampers described by fractional derivatives was carried out for the first time. Formulae to assess the sensitivity of FRF with respect to variation of design parameters are also presented. The method used to assess the sensitivity is easy to formulate, systematic to apply, simply to code, staying in good agreement with the exact results.

References

Actuation by reconfiguration—modular active structures to create Programmable Matter

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Abstract

We examine, analytically and numerically, forces produced by collective actuators—possible future building blocks of Programmable Matter. The actuators are composed of tiny spherical robotic modules which can strongly attach to their neighbors, and move by rolling over one another using electric or magnetic local propulsion mechanisms. An actuator works through collective reconfiguration—a coordinated motion of its constituent modules—which results in a global deformation of the structure. The simulations are performed using specially adapted discrete element method software, and account for friction and elastic deformations of modules.

Keywords: programmable matter, active materials, actuators, mechanical strength, modular robots

1. Introduction

Programmable Matter (PM) is a class of hypothetical future meta-materials whose properties and behaviour, e.g., the ability to autonomously change shape, can be programmed and controlled \([1]\). One of the approaches to creating PM is based on the concept of self-reconfigurable robots, in which the material is assumed to be composed of small interacting robotic modules. Each module is supposed to possess an energy source, to be able to communicate and process information, and to mechanically interact with its neighbours through attachment and actuation devices and sensors. An object made of a large number of modules could change shape by making its module move relative to one another, into a desired target configuration. Usually, the modules’ connection topology changes during such reconfiguration, which does not happen in simpler active structures—like the actuated truss \([2]\).

A huge research has been done on the problem of reconfiguration planning for modular robots—finding a geometrically admissible sequence of movements, transforming an initial arrangement of modules into a target one. In the paper, by contrast, the PM is viewed as a mechanical system, capable of exerting forces and doing work, subject to the limitations of the actuation mechanisms of its modules. We expand on our previous results \([5]\) and investigate the forces produced by a selected class of modular systems in the presence of friction and elastic deformations.

2. Active microstructures

Many module designs have been proposed as building blocks of three-dimensional self-reconfigurable robots. We base our analysis on a particular one—the spherical catom \([3, 4]\). Catoms use controllable electric or magnetic fields to attach to their neighbors and roll over them, Fig. 1(a). This type of actuation, however, has unsatisfactory strength. Therefore structures made of catoms, as they are, are not expected to be large or capable of exerting significant forces \([5]\)—their strength is proportional to the number of electric or magnetic connections in their cross-sections.

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3. Analysis of active microstructures

We simulate active microstructures using the Yade DEM software [6]. Strong connections are modeled as cohesion, with all six degrees of freedom at the connection constrained. Therefore, strongly connected modules form one elastic structure. Weak connections are also modeled as cohesion, but with only translational degrees of freedom constrained. Weakly connected modules can therefore roll over one another. The actuation mechanism is simulated by applying torques to active modules, which make them roll in desired directions. The laws of cohesion in Yade have been modified to allow controllable attachment and detachment of modules.

We analyze quasi-static forces produced by actuators in the presence of friction and elastic deformations. An actuator is placed between two bounding walls, and the force it exerts on them is computed at different elongations. Any time-dependent effects are excluded from the analysis.

4. Numerical results

Examples of force-elongation plots for square linear actuators are shown in Figs. 5 and 6, for the module radius $r = 65 \mu m$ and actuation torque $\tau = 16 pNm$. Figure 5 shows the decreasing force of the actuator, as the number of active modules pushing the structure decreases with increasing elongation. It also displays the cyclic discrepancies between the analytical and numerical results, caused by elasticity in the presence of friction.

Figure 6 presents variation of force produced by an elastic actuator, per active module, with the actuator height. The build-up of elastic deflections along the height of the actuator, combined with friction, diminishes the efficiency, especially for higher actuators.

The presented drawbacks can be overcome, to some extent, by designing actuators with increased spacing between certain modules.
Influence of geometrical and physical irregularities on dynamic characteristics of a passively damped structure

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Abstract

The focus of the paper is determination of dynamic parameters for structural systems with viscoelastic (VE) dampers, when a building is irregular or simply non-symmetric. The structure considered as an elastic linear system. The dynamic characteristics of a structure with VE dampers are determined from a solution of the appropriately defined eigenvalue problem. The solution of the equations of motion is given in the frequency domain. The problem of qualitative differences between the solutions derived for regular and irregular structures is discussed. Several conclusions concerning torsional modes of vibration are formulated as a basis for numerical test results.

Keywords: viscoelastic dampers, irregular structures, non-classically damped system

1. Introduction

In civil engineering passive damping systems are mounted on structures in order to reduce excessive vibrations caused by wind and earthquake [1,2]. Different kinds of mechanical devices, such as viscous dampers, viscoelastic dampers (VE), tuned mass dampers or base isolation systems, are used in the passive systems. The force displacement characteristics of VE dampers are represented by rheological models (i.e. the Kelvin or Maxwell model).

The equations of motion for multi-degree-of-freedom structures with installed VE dampers are derived. The analytical solution of these equations is given in the frequency domain. For regular structures we assume the shear frame model with flexible columns and rigid beams (Fig. 1a). For irregular structures we adopt a 3D model is adopted with additional masses concentrated at floor levels (Fig. 1b).

![Figure 1: The model of structure: a) the plane shear frame model, b) the spatial frame model](image)

Both, mass and stiffness irregularities in the floors planes are considered. An uneven weight distribution at floor level leads to different location of the centre of mass (CM) for each storey (Fig. 2). Similarly, arrangement of stiffening elements in a considered storey determines the position of the centre of stiffness (CS).

![Figure 2: Plan view of a selected structure floor](image)

The dynamic characteristics of the structural system with VE dampers (i.e. the natural frequencies and non-dimensional damping ratios) are determined after solving the appropriate nonlinear eigenproblem [3,4]. The results of selected numerical tests by the proposed method for structures with irregularities exhibit a certain relation between the stiffness eccentricity and modes of vibration. Based on these results the considered structures are classified as torsionally rigid or torsionally flexible [5].

2. Irregular structures with VE-dampers

The non-symmetric floor plan or irregular live load distribution may generate the mass eccentricity varying between storeys. The uneven mass distribution leads to different forces and corresponding cross-sections of the structure members. Thus, the building is both, mass and stiffness eccentric. Further irregularities in structure stiffness are caused by VE dampers installed with a system of bracing.

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The properties of VE dampers can be captured using Kelvin or Maxwell rheological models, where a dashpot with the constant $c_j$ is connected with a spring of stiffness $k_j$ in parallel or in series, respectively.

The force $u(t)$ which acts in the Kelvin model can be given as:

$$u(t) = k_j \Delta q(t) + c_j \dot{\Delta q}(t),$$

while the force in the Maxwell model of damper is governed by the equation:

$$v_c u(t) + u(t) = k_j \Delta q(t),$$

where $v_c = k_j / c_j$, $\Delta q(t) = q(t) - \bar{q}(t)$. The dot over a symbol denotes the derivative with respect to time, $t$.

An approximate position of the centre of stiffness $(e_{x0}, e_{y0})$ may be computed for a storey in the auxiliary coordinate system in the following way [5]:

$$e_{x0} = \sum_{i=1}^{n} K_{c,i} x_i + \sum_{i=1}^{n} K_{b,i} x_i, \quad e_{y0} = \sum_{i=1}^{n} K_{c,i} y_i + \sum_{i=1}^{n} K_{b,i} y_i,$$

where

$$K_{c,i} = \frac{12E I_i}{h^3}, \quad K_{b,i} = \frac{12E I_i}{h^3}, \quad K_{b,i} = \left[ \frac{L}{A E} + \frac{1}{k_j} \right] \cos^2 \beta,$$

$E$ is the modulus of elasticity, $I_j$, $I_x$ are the column cross-section second moments of area, $h$ is the storey height, $L$ is the brace length, $A_j$ is the area of the brace cross-section, $\beta$ is the angle between the bracing member and the horizontal plane (Fig. 1) and $k_j$ is the stiffness parameter of the damper.

3. The equation of motion

The equation of motion of a structure with VE dampers could be written in the following form:

$$M \ddot{q}(t) + C \dot{q}(t) + K q(t) = f(t) + p(t),$$

where the symbols $M$, $C$ and $K$ are the mass, damping, and stiffness matrices of the structure, respectively. Moreover, $q(t)$, $p(t)$, $f(t)$ are the vector of displacements, the vector of excitation forces and the vector of interaction forces, which act between the structure and the dampers, respectively.

Applying the Laplace transformation with zero initial conditions to the equations (1) and (2), the transform of the damper force can be written as:

$$\mathcal{L} = k_j \Delta q(t) + s c_j \Delta \dot{q}(t), \quad \mathcal{L} = \frac{k_j}{v_c + s} \Delta q(t),$$

where quantities such as $\mathcal{L}$ and $\Delta q(t)$ denote the Laplace transforms of the force $u(t)$ and the displacement $\Delta q(t)$, respectively, and $s$ is the Laplace variable.

The equation of motion (4) after Laplace transformation may be written in a general form, which enables to take into account various models of dampers in one structure:

$$\left[s^2 M + s C + C_j(s) + K + K_j + G_j(s)\right] \mathcal{L} = \mathcal{0},$$

where: $\mathcal{L} = 0$, $K_j = \sum_{i=1}^{n} K_i L_i$, $C_j(s) = \sum_{i=1}^{n} C_i(s) L_i$, $G_j(s) = \sum_{i=1}^{n} G_i(s) L_i$, $\mathcal{L}_i$ is the location matrix. The functions $K_j$, $C_j(s)$, $G_j(s)$ must be defined for each damper $r$ (i.e. for Kelvin model $K_j = k_j$, $C_j(s) = s c_j$, $G_j(s) = 0$).

Equation (6) constitutes the nonlinear eigenvalue problem. In the case of small damping, eigenvalues and eigenvectors can be obtained as complex conjugate numbers and vectors, respectively. The natural frequencies and non-dimensional damping ratios are determined from the eigenvalues, in a similar way to the viscous damping case.

4. Numerical test

In order to validate the effectiveness of the proposed approach we derived the dynamic characteristics for a structure with VE dampers. The regular five-storey steel frame was tested. The experimental results from shaking-table studies described in [6] coincide with the analytical ones obtained using the proposed method.

Further on, the dynamic behaviour of an irregular steel frame building with a L-shaped plan view was examined. The non-linear analyses were carried out using the program Abaqus. Predictions associated with the stiffness eccentricity were confirmed.

5. Conclusions

The results of sample numerical calculations confirm that the estimation of mass and stiffness centre enables to predict the dynamic behaviour of irregular structure with dampers. It is possible to determine which form of vibration is fundamental, and whether the torsional movement dominates. Moreover, one may be designed appropriate dampers and bracing system distribution to improve the torsional stiffness of a building structure.

References


A framework for the simulation of phase-transforming elasto-plastic SMA and TRIP steel polycrystals

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Abstract

A new framework is introduced for the simulation of shape memory alloys (SMA) and transformation induced plasticity (TRIP) steel undergoing martensitic to austenitic phase-transformations with simultaneously evolving plastic deformations. The derivation and elaboration of a generalised model facilitating the reflection of the characteristic macroscopic behaviour of SMA and TRIP steel is presented. The foundation of the overall formulation is a scalar-valued, thermodynamically consistent, statistical-physics-based model for the simulation of SMA. The underlying model is extended in view of the implementation in a non-affine micro-sphere formulation, enabling the capturing of polycrystalline behaviour and the simulation of three-dimensional boundary value problems. For the simulation of multi-cyclic SMA responses, and for the reflection of TRIP steel in general, plasticity effects should be considered in addition to phase-transformations. A coupling to plasticity is introduced, where each phase considered in each spatial direction of the polycrystal has individually evolving plastic strains. The interactions between dislocation densities, i.e. plastic strains, and evolving phase fronts are additionally accounted for by means of a so-called plasticity inheritance law.

Keywords: SMA, TRIP steel, phase-transformations, plasticity, polycrystal, finite element simulation

1. Introduction

For industrial applications, materials with advanced functional properties like TRIP steels and SMA offer significant advantages over standard engineering materials. As an example, technical components made of functional materials provide macroscopic mechanical properties like hardness, stiffness and ductility, tailored to a desired application. The macroscopic properties result from a complex microstructure, which leads to the demand for accurate constitutive models that facilitate not only the prediction of the material response but also allow for optimised structural designs. Due to materials considered in the work, the coupling of phase-transformations and plasticity involves several complex physical mechanisms to be captured in a numerically efficient yet physically sound manner.

2. Modelling of phase-transformations and coupling to plasticity

The material considered is assumed to consist of \( \nu \) phases, where the volume fraction of each phase \( \alpha \in \{1, \ldots, \nu\} \subset \mathbb{N} \) is defined as \( \xi^\alpha := \lim_{\nu \to 0} \left( \frac{\nu^\alpha}{\nu} \right). \) A Helmholtz free energy potential \( \psi^\alpha = \hat{\psi}^\alpha (\varepsilon^\text{dev}, \varepsilon^\text{vol}, \varepsilon^\text{pl}, \theta) \) is assigned to each individual phase \( \alpha \), where the potential takes the form

\[
\psi^\alpha = \psi^\alpha_{\text{dev}} + \psi^\alpha_{\text{vol}} + \psi^\alpha_{\text{therm}} + \psi^\alpha_{\text{chem}}.
\]

The elastic energy of each phase is additively decomposed into deviatoric and volumetric contributions, \( \psi^\alpha_{\text{dev}} = \hat{\psi}^\alpha_{\text{dev}} (\varepsilon^\text{dev}) \) and \( \psi^\alpha_{\text{vol}} = \hat{\psi}^\alpha_{\text{vol}} (\varepsilon^\text{vol}) \), respectively. The energy related to thermal expansion is accounted for by the contribution \( \psi^\alpha_{\text{therm}} = \hat{\psi}^\alpha_{\text{therm}} (\varepsilon^\text{dev}, \varepsilon^\text{vol}, \varepsilon^\text{pl}, \theta) \), and the temperature-dependent chemical energies take the form \( \psi^\alpha_{\text{chem}} = \hat{\psi}^\alpha_{\text{chem}} (\theta) \). In order to capture plasticity resulting from deviatoric deformation states only, the plastic strains \( \varepsilon^\text{pl} \) are, besides their influence in thermal expansion, accounted for in each phase solely within the deviatoric energy term but not in the volumetric energy contribution [5]. The model presented in this work involves the Helmholtz free energy contributions of the phases are specified as

\[
\begin{align*}
\rho_0 \psi^\alpha_{\text{dev}} &= \frac{1}{2} E^\alpha_{\text{dev}} (\varepsilon_{\text{dev}} - \varepsilon^\alpha_{\text{tr,dev}} - \varepsilon^\alpha_{\text{pl}})^2, \quad (2) \\
\rho_0 \psi^\alpha_{\text{vol}} &= \frac{1}{2} E^\alpha_{\text{vol}} (\varepsilon_{\text{vol}} - \varepsilon^\alpha_{\text{tr,vol}})^2, \quad (3) \\
\rho_0 \psi^\alpha_{\text{therm}} &= -\kappa^\alpha E^\alpha (\varepsilon_{\text{tr}} - \varepsilon^\alpha_{\text{tr}} - \varepsilon^\alpha_{\text{pl}}) [\theta - \theta_0], \quad (4) \\
\rho_0 \psi^\alpha_{\text{chem}} &= \rho_0 c^\alpha_p \left( 1 - \log \left( \frac{\theta}{\theta_0} \right) \right) - \rho_0 \lambda^\alpha_2 \left[ 1 - \frac{\theta}{\theta_0} \right]. \quad (5)
\end{align*}
\]

Figure 1: Gibbs potential contributions \( g \) of a 3-phase material.

The intersection curves of the elliptic paraboloids are minimised iteratively, yielding the Gibbs energy barriers that have to be overcome for the initiation of phase-transformation [4].

For each phase \( \alpha \) a volumetric-deviatoric-type split of total strains \( \varepsilon = \varepsilon_{\text{dev}} + \varepsilon_{\text{vol}} \) as well as transformation strains...
\( \varepsilon_{\text{vol}}^{\text{dev}} + \varepsilon_{\text{vol}}^{\text{tr}} \) is considered, cf. [4]. In line with experimental observations, the volumetric contribution \( \varepsilon_{\text{vol}}^{\text{dev}} \) of the total transformation strain \( \varepsilon_{\text{vol}}^{\text{tr}} \) is set to zero for SMA. In contrast, to consistently capture the transformation behaviour of TRIP steel, a non-zero volumetric transformation strain has to be introduced as TRIP steels show a significant change in volume during transformation from austenite to martensite. The evolution of volume fractions is derived using an approach from statistical physics, cf. [1, 2, 3, 4].

2.1. Extension to polycrystals – homogenisation via the microsphere approach

The scalar-valued model for the interaction of phase-transformations and plasticity is embedded in a non-affine microsphere framework, enabling the simulation of three-dimensional homogeneous load cases and to solve three-dimensional boundary value problems. In the small strain kinematic setting considered, the local deformation state \( \varepsilon := \nabla \varepsilon_i \) is split into volumetric and deviatoric parts in terms of \( \varepsilon = \varepsilon_{\text{vol}} + \varepsilon_{\text{dev}} \) with \( \varepsilon_{\text{vol}} = 1/3 \operatorname{tr}(\varepsilon) I \) and \( \varepsilon_{\text{dev}} = \varepsilon - 1/3 \operatorname{tr}(\varepsilon) I \), where \( I \) denotes the second-order identity tensor. Both tensorial quantities are then projected onto each spatial integration direction \( r_{i} \in \mathbb{U}^2 \) considered for the micro-sphere scheme.

In the context of the micro-sphere approach, the macroscopic stress tensor \( \sigma \) is obtained by integration over the unit sphere, resulting in the numerical approximation

\[
\sigma = \frac{1}{4\pi} \int_{\mathbb{U}^2} \frac{\partial \Phi}{\partial \varepsilon} \, d\alpha \approx \sum_{i=1}^{n} \frac{\partial \Phi}{\partial \varepsilon_i} \, \tilde{w}_i = \sum_{i=1}^{n} \sigma_i . \tag{6}
\]

2.2. Finite element implementation

As a representative finite element simulation we consider a plate of the size \( 4 \times 4 \times 1 \text{ mm} \) with a central hole of a \( 2 \text{ mm} \) diameter, cf. Fig. 3. We use a mesh of 768 hexahedral elements and apply Dirichlet boundary conditions to the \( y = -2 \) and \( y = 2 \) face of the body, where the \( y = -2 \) face is chosen to be spatially fixed such that \( u_x|_{y=-2} = u_y|_{y=-2} = u_z|_{y=-2} = 0 \).

References

Semi-active stabilization of smart structures subjected to impact excitation

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Abstract

In the work, a novel control method to stabilize vibrations of high structures is presented. The control is realized by changes of the stiffness parameters of the structural couplers. A seismic pulse excitation applied to the structure is submitted as a kinematic excitation. For such a representation the designed control law provides the best rate of the energy dissipation. Performance in different structural settings is studied by means of the stability analysis. Then, the efficiency of the proposed strategy is examined via numerical simulations. In terms of the assumed energy metric, the controlled structure outperforms its passively damped equivalent by over 50 percent.

Keywords: structural control, semi-active control, smart materials, smart buildings, stabilization

1. Introduction

Elimination of vibration is essential in the period of rapid technological development. Although theoretical basis is extensively elaborated, final solutions are not sufficiently implemented. Structures and technical devices are more complex nowadays and are subjected to extreme loads. It occurs in crash engineering, structures exposed to explosions, damages from seismic or paraseismic vibrations, etc. Classical methods that enhance the load carrying capacity do not increase sufficiently the resistance to incidental extreme loads. Active or semi-active techniques exhibit further possibilities in development. In the work, we design and examine a semi-active control method to reduce the vibration of high structures, i.e., buildings, chimneys, masts etc., subjected to short term excitations or abrupt load. Seismic pulse excitation applied to a bending structure will be submitted as a kinematic excitation. The proposed approach can also be addressed to structures subjected to the danger of damage of vital elements, for example, a guy rope in a mast.

Reduction of amplitudes of vibration of slender structures were considered in numerous publications. Examples of applications in automotive and aerospace industry, civil engineering structures, etc. are countless. Modern buildings are getting higher, and structures become more slender. Increasing height of skyscrapers is accompanied by increased flexibility and a lack of sufficient internal damping of vibrations. A new system can provide the most effective way of controlling structural response to environmental loads such as earthquake excitations [1, 2]. When the structural damping is insufficient, supplementary damping devices can be introduced. The auxiliary systems accomplish different strategies of energy dissipation. Indirect energy dissipation is the most popular reduction method of wind-induced vibrations but it can also be successfully applied to seismic technology. These methods utilize inertial effects and generally are called Tuned Mass Dampers (TMDs) [3, 4]. In many cases, insufficient space in buildings precludes the use of traditional TMD. Therefore, alternatives devices such as pendulums, inverted pendulums, slide-platform and rubber or hydrostatic bearings can be used. Review of the various auxiliary damping devices can be found in [5, 6].

In the work, we propose an efficient method for semi-active stabilization of transverse motion of beams induced by a kinematic impulse motion of the support. The stabilization is done by changes of the stiffness parameters of the structure’s couplers. We derive the strategy of the control of switched elastic connectors that mistune the motion of the structure and remove the peak energy in time intervals. The efficiency is examined by numerical experiments.

2. Mathematical model

Our goal is to reduce the physical model of high buildings to a relatively simple structure. Two coupled cantilever beams that describe high structures, i.e., masts, towers or high buildings were assumed. The transverse motion, perpendicular to the length direction, is composed of bending and shearing state. Especially skyscrapers constructed as cores surrounded by the walls of the enclosure exhibit significant shear. We opted for the Timoshenko model of beams since it allows to contribute high shear stiffness. For the sake of the stability analysis and control law design we will introduce a simplified equivalent of the considered system. Each of the beams will be represented as a simple oscillator (see Fig. 1). The parameters of the oscillators are computed such to mimic the dynamics of the first natural modes of the beams.

Figure 1: Cantilever beams system reduced into two coupled simple oscillators
### 3. Control law design

Introducing the state vector \( \bar{x} = [\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4]^T \) the dynamics of a simplified system is governed by

\[
\dot{\bar{x}} = \bar{A} \bar{x} + u \bar{B} \bar{x}.
\]

For such a representation we consider the energy function given by

\[
V = \frac{1}{2} \bar{x}^T Q \bar{x},
\]

where \( Q = \text{diag}(k_1, m_1, k_2, m_2) \). Its time derivative is of the form \( \dot{V} = \bar{x}^T Q \bar{A} \bar{x} + u \bar{x}^T Q \bar{B} \bar{x} \). Substitution of the system matrices yields

\[
\dot{V} = -c(\bar{x}_2 - \bar{x}_4)^2 - u(\bar{x}_1 - \bar{x}_3)(\bar{x}_2 - \bar{x}_4).
\]

We use the control \( u^* \) providing the best instantaneous energy dissipation, namely

\[
\forall t: \quad u^*(t) = \arg\min_{u \in [u_{\text{min}}, u_{\text{max}}]} \dot{V}(\bar{x}(t)).
\]

From (2) we conclude that

\[
u^*(t) = \begin{cases} u_{\text{max}} & \text{if } (\bar{x}_1(t) - \bar{x}_3(t))(\bar{x}_2(t) - \bar{x}_4(t)) > 0, \\ u_{\text{min}} & \text{otherwise}. \end{cases}
\]

Analysing the structure of \( \dot{V} \), we observe that the control (4) guarantees a permanent decrease of the energy, except for the time instants when \( \bar{x}_2 = \bar{x}_4 \). The asymptotically stability is assured only if the states remains desynchronized until the origin is reached. We can also observe that larger desynchronization provides better rate of decrease of the energy. All these facts motivate the analysis on the dynamics of the synchronization. Our goal is to answer two key questions: How the control given by (4) does impact on the system synchronization? How can we protect against the synchronization to keep a desired performance? The synchronization is studied by means of the analysis on the dynamics of the relative state

\[
\dot{\epsilon} = A_\epsilon \epsilon + u B_\epsilon \epsilon + F(\bar{x}).
\]

Here \( \epsilon_1 = \bar{x}_1 - \bar{x}_3 \) and \( \epsilon_2 = \bar{x}_2 - \bar{x}_4 \). We will examine the energy function corresponding to the relative dynamics \( V_\epsilon = \frac{1}{2} \epsilon^T Q_\epsilon \epsilon \), where \( Q_\epsilon = \text{diag}(k_1 + k_2, m_1 + m_2) \). It can be shown that

\[
\dot{V}_\epsilon = \begin{cases} \epsilon_2(\bar{x}_1 - \bar{x}_3)\epsilon_2 + & k_1 \left( \frac{m_2}{m_1} - 1 \right) \bar{x}_1 + k_2 \left( \frac{m_1}{m_2} - 1 \right) \bar{x}_3 \epsilon_2 + \\ - c \left( \frac{1}{m_1} + \frac{1}{m_2} \right) (m_1 + m_2) \epsilon_2^2 + & - u \left( \frac{1}{m_1} + \frac{1}{m_2} \right) (m_1 + m_2) \epsilon_1 \epsilon_2. \end{cases}
\]

From (6) we observe that the controller tends to synchronize the oscillators with the maximum rate of convergence. This is undesired effect, since, as we stated before, the best rates of decrease of the energy \( \dot{V} \) are provided when the states are desynchronized. The key in resolving this antagonistic issue lies within the structure of terms in the first and the second lines in (6). By taking \( k_1 \neq k_2 \) the first line term oscillates between positive and negative values. If, for instance, we assume \( k_2 > k_1 \) and the initial condition such that \( (\bar{x}_1(0) + \bar{x}_3(0))(\bar{x}_2(0) - \bar{x}_4(0)) > 0 \), then under \( k_2 < k_1 \) large enough, we have \( V_\epsilon > 0 \) for some time period. In that period the system energy \( \dot{V} \) decreases very quickly. The latter change of the sign of \( (\bar{x}_1(0) + \bar{x}_3(0))(\bar{x}_2(0) - \bar{x}_4(0)) \) does not result in significant lose of the performance. Analogous conclusion can be stated for the second line term when \( m_1 \neq m_2 \).

### 4. Numerical validation

Simulations are carried on under the following setting. Two couplers are symmetrically crossed (the crossing provides slower synchronization) and located between the beams at the ending section. We consider four variants: variant 1–both couplers are not controlled (referred as the passive case), variant 2 (variant 3)–only one of the couplers is under control, variant 4–both couplers are controlled. Both beams are given with identical parameters and are initially excited with identical velocity. Displacement of the beam ending point and the metric corresponding to the potential energy are compared in Fig. 2 and Fig. 3, respectively. Each of the controlled variants clearly outperforms the passive case (see also Tab. 1).

![Figure 2: Displacement of the ending point of the beam](image1)

![Figure 3: Evolution of the metric corresponding to the potential energy](image2)

**Table 1:** Comparison of the total energy metric (values are normalized to the variant 1)

<table>
<thead>
<tr>
<th>Variant</th>
<th>Metric Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>0.466</td>
</tr>
<tr>
<td>3</td>
<td>0.466</td>
</tr>
<tr>
<td>4</td>
<td>0.430</td>
</tr>
</tbody>
</table>

### References


Analytical modelling of a piezoelectric displacement amplifier with two pairs of flexure hinges

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Abstract

This article presents an analytical method for displacement and internal axial force calculations in a planar piezoelectric flexensional transducer with two pairs of flexure hinges. The actuator is composed of a piezoceramic rod and two rectilinear metal beams which are discretely attached to the rod by means of rigid links. Monolithic hinge lever mechanisms are applied to magnify the transverse displacement of the beam. That can be done due to the eccentricity of the beams with regard to the centrally located rod. In the model, each hinge is represented by a pin strengthened by a rotational spring. The analytical model is based on the Euler–Bernoulli beam theory and established on the basis of the principle of stationary value of total potential energy. The electro-mechanical constitutive equations for piezoelectric material are used the formulation of the energy functional. During numerical computations, the deflection and the internal axial force generated by an electric field application are determined by changing actuator properties such as the distance between the beams and the rod and the stiffness of the constricted hinges. The obtained results and final conclusions can be directly applied in the design process of the flexensional actuators.

Keywords: piezoelectric actuation, flexure hinge, flexensional transducer, geometrical nonlinearity

1. Introduction

Flexensional actuators, since their invention in 1990s, have been thoroughly used in many applications, especially as displacement transducers, shape controllers, precise micropositioners and in vibration control. Having different designs, their principle of operation applies electric field parallel or perpendicular to the polarisation direction in a piezoceramic rod or piezostack, respectively, to expand it in the longitudinal direction and convert this displacement into the transverse displacement of beams, which are mounted eccentrically to the centrally located rod. To amplify the device response monolithic hinge levers have been implemented [1]. The idea behind this solution is to reduce the bending stiffness at the hinges, while maintaining enough axial rigidity of the active parts of the actuator to finally obtain greater magnification of the out-of-plane displacement. Accepting the crucial role of flexensional piezoceramic-metal composite actuators in various applications, tailoring their electro-mechanical performance received attention of several authors for different types of transducers, which has been reported in [2]. In the paper the static behaviour was analysed of a flexensional actuator consisting of two rectilinear metal beams, mounted by means of two hinged links with an offset to a centrally located piezoceramic rod, in order to estimate the effectiveness of flexure hinges design on the out-of-plane displacement of such a device.

2. Analytical model

The considered actuator is doubly symmetric; the upper part of its simplified model is sketched in Fig. 1. Two flexure hinges of transducer are placed at its end as shown in Fig. 2. In the work the transversal displacement of beams and the axial force generated by electric field of vector $\mathbf{E}$, applied to the piezoceramic rod, are studied as functions of the offset distance of the metal beams with reference to the rod ($e^*$) and the flexural stiffness of the hinges represented in the model by linear rotational springs of known stiffnesses ($C, C_0$). Some details of mathematical modelling of such transducers for static and dynamic purposes were presented in [2,3].

![Figure 1: Model of flexensional transducer with constricted hinges](image1)

![Figure 2: Flexure hinge design](image2)

In a flexensional electro-mechanical transducer, depending on the direction of the electric field applied to the piezorod with regard to the polarisation vector $\mathbf{P}$, the beams can deflect inward or outward. The reduced linear piezoelectric constitutive equations have the following form:

$$
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
D_x
\end{bmatrix}
= 
\begin{bmatrix}
E_{31} - e_{31}n_n^e \\
e_{31}n_n^e \\
\xi_{33}\end{bmatrix}
\begin{bmatrix}
e_x \\
ie_x \\
w^e
\end{bmatrix}
= 
\begin{bmatrix}
E_{31} - e_{31}n_n^e \\
e_{31}n_n^e \\
\xi_{33}\end{bmatrix}
\begin{bmatrix}
e_x \\
ie_x \\
w^e
\end{bmatrix}
$$

(1)

The strain-displacement relation in beams according to the von Karman theory is as follows:

$$
\varepsilon(\xi) = \frac{dw(\xi)}{d\xi} + \frac{1}{2}\left[\frac{d^n w(\xi)}{d\xi^n}\right]^2 - \frac{2}{E_{31}}\frac{d^2w(\xi)}{d\xi^2},
$$

(2)

where: $\varepsilon_{31}$ denotes Young’s modulus of the piezo-material along the strain axis, $\varepsilon_{11}$ is the piezoelectric constant ($\varepsilon_{11} = d_{31}E_p$), $E_i$ is a homogeneous electric field ($E = V/h_0$), $V$ is the operating
voltage applied to the piezoelectric rod, \( D \) is the electric displacement, \( \varepsilon_{33} \) stands for the permittivity coefficient of the piezoelectric material and \( \phi(\xi) \) is the strain resulting from the axial displacement of the rod.

The problem was formulated on the basis of the principle of stationary value of total potential energy, thus the governing equations can be derived in the form:

\[
\begin{align*}
 w''(\xi) + s''w'(\xi) &= q(\xi), \quad (3) \\
 u_c''(\xi) &= 0, \\
 s^2 &= \lambda \left\{ u'(\xi) + \frac{1}{2} w'(\xi) \right\}, \\
 s_x^2 + f^2 - s^2 &= 0 \\
 \end{align*}
\]

(4) (5) (6)

where: \( q(\xi) \) is the distributed load of beams, \( s'', s_x, s' \) are the piezoelectric force resulting from the applied voltage, the axial force in the rod, the axial force in the beam, respectively, and \( \lambda \) is the beam's slenderness parameter.

After the solution of Eqns (3-5) under the imposed geometric and natural boundary conditions the relation is obtained between the axial displacements of the actuator components and the piezoelectric force. Numerical analysis of the problem makes possible to investigate the electromechanical performance of the actuator for its different geometric and physical parameters.

3. Results of numerical calculations

Figure 3 illustrates the relationship between piezoelectric force \( f \), generated by the electric field (up to 3.5 kV/mm) applied in the direction perpendicular to the piezorod long axis, and axial force \( s \) along the beam for a constant value of spring stiffness \( c = 10 \) and different values of \( \alpha \). While a beam of the actuator is subjected to a load of constant intensity \( q = -0.4 \) applied over its entire length, the beam is elongated with force \( s \), this value rises with growing values of stiffness \( c \).

The curves shown for different values of stiffness \( c \) manifest the applicability of piezo actuation for creating the desired beam deflection in both directions: towards or outdoors the transducer. Points \( A_i \) (\( i = 1, 2, 3, 4 \)) in Fig. 4 exhibit the level of lateral displacement originating from the external distributed load. Points \( C_i \) (\( i = 3, 4 \)) placed on the abscissa demonstrate the necessary magnitudes of force \( f \) for a complete reduction of beam deflection created by a load \( q \). At these points the beam changes its inward deflection to an outward deflection, or vice versa, depending on the direction of the piezoelectric force.

4. Conclusions

Generation of the axial force by electric field applied to the piezorod is strictly dependent on the rigidity of constricted hinges. The less rigid a support, the greater electric field necessary to be applied to induce a particular axial force. On the other hand, having a constant force, the less deflection of beams, the more rigid their support is. Taking into account such a duality, the scale of the actuation must be controlled due to the maximum electric field as the quantity which is characterized by each piezoelectric material. Hence, optimization is required during the design process of such actuators to balance both stiffness of the hinges and the offset distance.

References


Frequency domain model of active vibration absorber based on SMA spring

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Abstract

The paper presents a vibration reduction system with a controlled dynamic vibration absorber (DVA). In DVA shape memory alloy (SMA) springs are used. These springs serve to shape the characteristics of the dynamic vibration absorber (DVA). Therefore it is possible to adapt the resonant frequency of the absorber to actual operating conditions. The paper formulates a mathematical description of the 2-DOF system along with a viscoelastic model of the controllable SMA spring. The model was developed and experimental tests of the vibration reduction system with DVA were conducted. System vibration transmissibility functions were established. It was shown that the developed vibration absorber system may compensate for the changes in disturbance frequency to as great an extent as ±10%.

Keywords: actuator, shape memory alloy, model, vibration reduction system

1. Introduction

Shape Memory Alloys (SMA) are materials widely used to designing actuators for positioning systems. These actuators have many advantages. They are light, strong and silent, but they have many disadvantages too e.g. nonlinear characteristics.

Three effects present in SMA are distinguished in relation to phase transitions: one-way and two-way shape memory effect and pseudoelasticity [1]. Due to their unique properties such alloys are used in many fields of technology [2] and medicine. As mentioned above they are most often used as actuators or joining elements and as protheses, orthotics, implants or other elements implanted into the human body in the form of braces, screws, stents, clamps, etc. But in this paper we concentrate on the usage of the alloys in vibration control systems (VRS).

Because of their properties SMA are starting to be used in VRS. This paper presents a controlled dynamic vibration absorber in which springs made of SMA are used. Such springs modify the spring constant as a result of changes in temperature so they shape the characteristics of the dynamic vibration absorber (DVA).

The main purpose of VRS is minimisation of acceleration or displacement amplitudes [3]. In the cases where the dominant frequency is possible to distinguish, dynamic vibration absorbers (DVA), also called tuned mass dampers (TMD), are used; they are designed for a single frequency of disturbance. The effectiveness of vibration reduction is greatest when damping in the absorber system is minimal. In such a case the vibration reduction bandwidth is the narrowest. In a situation where the frequency of disturbance changes, then additional damping broadens the frequency scope of the absorber operation instead of its effectiveness. The use of controlled absorbers is an alternative method to compensate for the influence of the change in frequency of disturbance vibrations.

In this paper developed SMA springs with controllable stiffness are used in DVA. These springs serve to shape the characteristics of the DVA. The model of the spring and 2-DOF is formulated.

2. SMA spring

The spring used in the absorber was constructed of six bilaterally supported bars made of NiTi alloy with the composition (48Ni-46Ti-6Cu WT%). The spring was tested in laboratory. Results of tests were presented in the papers [4,5].

The formulated mathematical model of SMA spring constitutes a viscoelastic model with coefficients k_2 and c_2 depending on frequency and/or temperature. The parameters of the spring model (1), were experimentally determined for the SMA temperature, ranging from 25°C to 80°C.

\[ F_s = k_s(T,\omega)x + c_s(\omega)\dot{x}, \]  

where:

\[ k_s(T,\omega)=70952-213.01\omega-5.214\omega^2+1148.8T \]

\[ c_s(\omega)=1.91+17100 1/\omega \]

T – temperature of the SMA spring,  
\[ \omega \] – frequency of excitation.

Chosen characteristics of the real SMA spring and obtained from its model are presented in Fig. 1.

Figure 1: Hysteretic characteristic of SMA spring

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3. Active vibration absorber based on SMA spring

The mathematical model of DVA was formulated based on its calculation scheme presented in Fig. 2. A DVA in the form of mass \( m_2 = 12 \text{kg} \) was connected to the host mass \( m_1 = 25 \text{kg} \) by means of a SMA spring. This spring was described by means of a controllable spring element \( k_2(T, \omega) \) and a controllable damping element \( c_2(\omega) \). The host mass \( m_1 \) is supported by spring \( k_1 \) and damper \( c_1 \); mass \( m_1 \) is excited by means of kinematic excitation: \( z_w = A \sin(\omega t) \). The model was formulated in the form of a system of equation:

\[
Ax = Bu,
\]

where:

\[
A = \begin{bmatrix}
-k_1 + c_1 s^2 & k_2 + c_2 s \\
-k_2 s + c_2 s & k_1 + c_1 s^2
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
-k_2 s + c_2 s \\
0
\end{bmatrix},
\]

\[
x = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad u = \begin{bmatrix} z_w \\ 0 \end{bmatrix}.
\]

Figure 2: Calculation scheme of the DVA

Figure 3 presents the vibration transmissibility functions for the dynamic vibration absorber DVA as a function of frequency of the signal \( z_1 \). These characteristics were achieved for the absorber’s mass \( m_2 \). As can be seen, the resonant frequency increases along with an increase in temperature. Therefore, controlling the spring temperature we also control the resonant frequency of the vibration absorber. It means that we can adjust the vibration absorber to the frequency of disturbance \( z_w \) by changing the temperature of the SMA spring. Figure 4 presents selected characteristics for transmission of the disturbance \( z_w \) on the host mass \( m_1 \) for the selected temperatures of the system 25°C, 60°C, 80°C. It can be seen that the first resonance of the system changes a little and is practically independent of the temperature, whereas the antiresonance resulting from the work of the DVA changes its frequency along with modification of the resonant frequency of the vibration absorber.

For this DVA which uses spring with controllable stiffness nonlinear controller was developed. Main aim of the controller is tuning resonant frequency of DVA to frequency of disturbance \( z_w \). In the control system there was used the fact that resonance frequency can be estimated by natural frequency. Phase shift \( \varphi \) between displacement \( z_2 \) and displacement \( z_1 \) amounts -90°. For that reason it was decided that signal of feedback is to be cosine of phase shift angle.

This signal is estimated using upon formula (3). Error of estimation is inversely proportional to observation time \( T_o \), and is equal to zero for observation time \( T_o \) being multiple of forced vibration period. In Fig. 4 is shown (black line) transmissibility function of controlled vibration absorber. As we can see bandwidth of vibration reduction is wider than in cases of non-controlled DVA (coloured lines). In such case frequency of disturbance could change even ±10%.

\[
\cos(\varphi) = \frac{1}{T_o} \int_{-T_o}^{T_o} z_2(t) z_1^*(t) dt
\]

References

Multiscale homogenization of magneto-electric composites: how the ferroelectric polarization affects the product properties

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Abstract

The coupling between electric polarization and magnetization can find many applications in data storage devices or sensor technologies. Since the magneto-electric (ME) coupling in natural materials and in most of the synthetic single-phase materials generally vanishes at room temperature, two-phase composites are attractive candidates for the generation of large ME-coefficients at room temperature. The talk focuses on the theoretical framework and numerical two-scale homogenization of two-phase magneto-electric composites in consideration of ferroelectric pre-polarization and the comparison of the determined ME-coefficients to experimental measurements.

Keywords: multiscale homogenization, FE²-method, magneto-electric composites, effective properties, ferroelectric switching

1. Introduction

Materials with ferroelectric or ferromagnetic properties are used in many fields of modern technologies. Some of these devices could be improved by combining different ferroic characteristics in so called multiferroics. They can exhibit a coupling between polarization and magnetization and allow also for new applications, such as electrical magnetic-field sensors or electric-write/magnetic-read-memories, see [1] or [2]. However, natural and synthetic single-phase materials with magneto-electric properties mostly show this coupling far below room temperature, due to their crystallographic properties. Measured ME-coefficients are between $5 \cdot 10^{-12}$ s/m and $30 \cdot 10^{-12}$ s/m at temperatures between 4.2 K and 270 K, see for example [3]. An exception is BiFeO₃, which unfortunately features a very low coupling at room temperature. This disadvantage has been circumvented by manufacturing magneto-electric composites, which consist of magneto-mechanically and electro-mechanically coupled phases. The idea behind the development of such composites is to generate the desired magneto-electric effect as a strain-induced product property, see for instance [4, 5]. This product property is not present in each of the constituents, but is a result of their interaction. In the case of ME composites, we distinguish between the direct and converse ME effect. The direct effect characterizes magnetically induced polarization: an applied magnetic field yields a deformation of the magneto-mechanically coupled phase which is transferred to the electro-mechanically coupled phase. As a result, a strain-induced polarization in the electric phase is observed. On the other hand, the converse effect characterizes electrically activated magnetization: an applied electric field yields a deformation of the electro-mechanical phase which is then transferred to the magneto-mechanical phase. The result is thus strain-induced magnetization. Several experiments on ME composites showed remarkable ME coefficients, which are orders of magnitudes higher than those of single-phase materials, see [5]. The magnitude of the effective ME-coefficient depends on the ferroic properties of the individual phases as well as on the morphology of the microstructure. In order to regard both effects we derive a homogenization approach, which allows for the direct integration of ME composite microstructures in macroscopic simulations. This work is an extension of [6] to magneto-electro-mechanically coupled problems, see also [7, 8].

2. Theory of multiscale homogenization for ME composites

For the applied homogenization approach a magneto-electro-mechanically coupled boundary value problem is solved on two scales. Instead of defining a macroscopic material law, at each macroscopic integration point a representative volume element (RVE) is attached. Suitable macroscopic quantities, such as the strains, electric and magnetic fields, have to be localized on the microscale. In order to define the microscopic boundary value problem, energetically consistent boundary conditions have to be applied on the RVE. Based on the work of [9] we apply a generalized macro-homogeneity condition of the form

$$\sigma : \ddot{\varepsilon} - D : \varepsilon - B : H = \frac{1}{V} \int_{RVE} (\sigma : \dot{\varepsilon} - D : \varepsilon - B : H) \, dV, \quad (1)$$

which can be fulfilled using periodic boundary conditions on the RVE. In order to obtain the microscopic quantities such as the stresses, the dielectric displacement and the magnetic induction, the weak forms of the balance equations have to be solved on the microscale. Then, a homogenization step is performed, in which average values of the microscopic quantities are determined, with

$$\langle \bullet \rangle = \frac{1}{V} \int_{RVE} \langle \bullet \rangle \, dV. \quad (2)$$

These homogenized variables have to be transferred to the associated points on the macroscale, where finally, the macroscopic boundary value problem is solved. In order to obtain quadratic convergence on the macro-scale, we have to linearize the macroscopic weak forms consistently. This goes along with the determination of the linear increments of the con-

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performing and remanent strains of individual barium titanate unit cells can change the free energy, whereby the remanent polarizations are motivated by the switching criterion for the remanent polarization based on the ferroelectric phase will be modelled through a microscopic approach as in case of pure barium titanate. In order to account for ME composites, because it activates the electro-mechanical coupling effects.

This pre-polarization process is crucial in the manufacturing of barium-titanate/cobalt-ferrite composites. In this context, we focus on the heterogeneous morphology of the microstructure, inhomogeneous distributions of the microscopic electric fields and orientations, taken from Figure 1. Due to the heterogeneity of the matrix material, we assume a uniform unpolar character of the matrix material, we assume a uniform isotropic distribution of the barium titanate unit cells. In order to represent the initial orientation distributions of the tetragonal crystallographic unit cells is applied. In order to load the macroscopic bvp with attached RVE and distributions of the barium titanate unit cells. The heterogeneous microstructure causes an inhomogeneous distribution of the microscopic electric fields and orientations, taken from [8].

The constitutive quantities, which appear as

\[
\begin{bmatrix}
\Delta \sigma \\
\Delta \mathbf{D} \\
\Delta \mathbf{B}
\end{bmatrix} =
\begin{bmatrix}
\mathbf{C} & - \mathbf{e}^T & - \mathbf{q}^T \\
- \mathbf{e} & - \mathbf{e} & - \mathbf{q} \\
- \mathbf{q} & - \mathbf{q} & - \mathbf{p}
\end{bmatrix}
\begin{bmatrix}
\Delta \mathbf{E} \\
\Delta \mathbf{E}_T \\
\Delta \mathbf{H}
\end{bmatrix},
\]  

(2)

The macroscopic moduli contain the ME-coefficient

\[
\sigma = \frac{\partial \mathbf{B}}{\partial \mathbf{E}} = \left[ \frac{\partial \mathbf{D}}{\partial \mathbf{H}} \right]^T
\]

and is determined through the homogenization procedure [7, 8].

3. Numerical example and discussion

The goal of the presented simulation is to take a closer look at the ferroelectric pre-polarization process of the matrix material. This pre-polarization process is crucial in the manufacturing of ME composites, because it activates the electro-mechanical coupling of the ferroelectric matrix. After sintering of the composite, the spontaneous polarizations of the matrix are orientated isotropically as in case of pure barium titanate. In order to account for different polarization states, the non-linear hysteretic response of the ferroelectric phase will be modelled through a microscopic switching criterion for the remanent polarization based on the work of [10]. In the work the switching criterion is motivated by a change of the free energy, whereby the remanent polarizations and remanent strains of individual barium titanate unit cells can perform 90° or 180° switchings. A homogenization approach based on the orientation distributions of the tetragonal crystallographic unit cells is applied. In order to represent the initial unpolar character of the matrix material, we assume a uniform isotropic distribution of the barium titanate unit cells in the three dimensional space through the use of a triangular collocation on a geodesic dome.

In the numerical example the macroscopic body is loaded with alternating electric potentials and the resulting electric fields are localized on the microstructure, see Figure 1. Due to the heterogeneous morphology of the microstructure, inhomogeneous distributions of the microscopic electric fields and stresses are obtained. These quantities drive the ferroelectroelastic switching in the attached crystallites, which result in different local hysteresis loops across the RVE. The talk will address the influence of different pre-polarization states on the overall ME coupling of barium-titanate/cobalt-ferrite composites. In this context, we will also compare our results to experimental measurements [11]. Here, it will be shown that the assumption of perfect piezoelectric material response of the ferroelectric phase will generally lead to an overestimation of the obtainable ME-coefficient.

References

Finite displacement dynamic model of twin beams with controllable damper

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Abstract

Lateral vibrations of two parallel cantilever beams joined at their free ends by a viscoelastic member are studied experimentally and theoretically. A dynamic model of the structure is proposed. It fits the experimental data well and allows to estimate the shear modulus and damping coefficient of the member. The model can be useful for the development of semi-active control strategies for double-beam systems with controllable damping members.

Keywords: sandwich beam, double-beam system, shear deformation, magnetorheological elastomer, semi-active damping

1. Introduction

Systems of two parallel elastic beams can be found in various devices and structures – examples include aircraft wing spars, double-beam cranes, bridge spans, or linear guideways in ploters. Recently semi-active methods of vibration suppression for such systems have been developed. In Ref. [1] the controlled delamination of a two-layer beam is employed for the releasing of strain energy accumulated in the deformed structure. The research Ref. [2] deals with semi-active damping of two beams joined by elastomer composite with iron particles, whose stiffness and dissipative properties increase when it is exposed to magnetic field. A simple switching strategy allows to reduce vibrations more effectively than in the case of the elastomer permanently activated. A similar system is experimentally investigated in Ref. [3], however in this study an elastic hermetic container filled with granules and subjected to underpressure acts as the damping member.

The scope of the research is to provide a dynamic model for systems analyzed in two former papers.

2. Experiment

The scheme of the investigated system is depicted in Fig. 1. Two aluminum beams of length \( L = 700 \text{ mm} \), width \( b = 25 \text{ mm} \) and thickness \( d = 2 \text{ mm} \) are mounted in parallel in a clamped configuration and joined at their free ends by a damping member made of MS-polymer adhesive. When the beams are deflected, the member undergoes shear deformation, which is the main source of elastic and dissipative forces in the system. The mass of the member amounts to \( 2M = 18 \text{ g} \), length \( 2a = 33 \text{ mm} \), height \( 2h = 15 \text{ mm} \), and width equals to \( b \). In order to keep the gap between the beams constant over their lengths, two lightweight rolls were placed at distances of \( x_1 = 230 \text{ mm} \) and \( x_2 = 470 \text{ mm} \) from the support. Three laser displacement sensors were aimed at the system at distances 230, 460 and 695 mm. An initial deflection of the system was applied, and after releasing of the structure, the vanishing free vibrations were recorded.

Figure 1: Scheme of the analyzed system.

3. Model

Lateral vibrations in \( x-y \) plane are studied, and \( w_{1,2}(x,t) \) denote displacements of beams in direction \( y \). The beams are considered linear Bernoulli-Euler cantilevers, subjected to both internal and external viscous damping. Gravity acts perpendicularly to \( x-y \) plane, so its influence can be neglected.

The anti-buckling rolls are treated as linear springs of stiffness \( K_r = 20000 \text{ N m}^{-1} \), which generate transverse forces

\[
F_{r_{1,2}} = -K_r(w_{1,2}(x_{1,2}, t) - w_2(x_{1,2}, t)).
\]  

By assumption, the member is made of the Kelvin-Voigt material characterized by shear stress-strain relation \( \tau = G\dot{\varphi} + G^*\varphi \), where \( G \left[ \text{Pa} \right] \) is the Kirchhoff modulus and \( G^* \left[ \text{Pa s} \right] \) is the damping coefficient. The member is modelled as a two-link diagonal truss, exerting forces to both beams as depicted in Fig. 2. It is short enough to neglect its rotary inertia, but the mass is included.

Figure 2: Forces generated by the truss and acting on the beams.

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It is assumed that the end parts of both beams (i.e. over the damping element) remain almost straight, their slopes of deflection are approximately equal to \( \varphi \), and the value of \( \varphi \) is small, so \( \sin(\varphi) \approx \varphi \) and \( \cos(\varphi) \approx 1 \). Then the components of elastic forces in the assumed coordinates amount to

\[
\begin{align*}
F_{e, x}^f &= -\frac{Gb}{2}(\ddot{w} + 2a\varphi), \\
F_{e, y}^f &= -\frac{Gb}{2a}(\ddot{w} - 2a\varphi), \\
F_{e, z}^f &= \frac{Gb}{2}(\ddot{w} - 2a\varphi),
\end{align*}
\]

where

\[
\ddot{w} = w_1(L - a, t) - w_2(L - a, t),
\]

\[
\varphi = \frac{1}{2}\left(\frac{\partial w_1}{\partial x}(L - a, t) + \frac{\partial w_2}{\partial x}(L - a, t)\right)
\]

The components of dissipative forces are given by analogous formulas, in which \( G \) is replaced with \( G^* \), and \( \ddot{w}, \varphi \) - with their time derivatives.

The dynamics of the system is governed by two classical equations of lateral beam vibrations, with additional point forces generated by the springs and the truss. The following equation describes the motion of beam 1

\[
(pA + M\delta_{L-a})\frac{\partial^2 w_1}{\partial t^2} + EI\frac{\partial^4 w_1}{\partial x^4} + E^*\frac{\partial^2 w_1}{\partial x^2}\frac{\partial^2}{\partial t^2} + E^*\frac{\partial^2 w_1}{\partial x^2}\frac{\partial^2}{\partial t^2} + c\frac{\partial w_1}{\partial t} + \\
- F_{r, x, \delta x_1} - F_{r, x, \delta x_2} - P\frac{\partial^2 w_1}{\partial x^2} + \\
- F_{r, y, \delta L} - F_{r, y, \delta L, -2a} - F_{r, y, \delta L, -2a} - F_{d, y, \delta L} - F_{d, y, \delta L, -2a} = 0,
\]

where \( A = bd \) is the cross-sectional area of the beam, \( I = bd^3/12 \) - the second moment of an area, \( E^* \) denotes the internal damping coefficient of aluminum, and \( c \) is the external damping coefficient of air. The value \( P = F_{r, x, +} + F_{r, x, -} + F_{r, y, +} + F_{r, y, -} \) denotes the sum of forces generated by the truss and acting on the beam in direction \( x \). It is assumed that all axial forces are concentrated at the beam tip, because part of the beam over the truss is a short, so its slope is small.

A dynamic equation of beam 2 is analogous. The sign at the axial force changes to \( \cdot - \cdot \), because if one beam is being compressed, the other one is stretched. Obviously the \( \cdot - \cdot \) sign at transverse forces is replaced with \( \cdot + \cdot \). The points, where the forces generated by the truss act, are swapped.

4. Verification

A continuous problem was discretized using the Galerkin procedure based on the cantilever beam eigenfunctions. The number of base vectors was enough to provide a satisfactory accuracy of the approximate solution. The resultant set of ordinary equations was solved using the Fehlberg method.

Firstly, the stiffness and dissipative properties of the single aluminum beam were estimated basing on free vibrations induced in the first mode. The obtained values were \( EI = 1.01 \text{ N m}^2 \), \( E^* = 0.00026 \text{ N m}^2 \text{s}^{-2} \) and \( c = 0.01283 \text{ Pa s} \). Afterwards, the model of a double-beam system was fitted to the experimental data. This data was acquired from the free vibration trial, with an initial tip deflection of 49 mm, and lasting 6.3 seconds. The fitting quality was measured by Pearson correlation coefficient between empirical and theoretical responses, averaged over sensors \( S_1, S_2 \) and \( S_3 \). The values of \( G \) and \( G^* \) which maximize the quality criterion were chosen as estimators. A simple optimization technique based on the systematic search yielded to \( G \approx 365 \text{ kPa} \) and \( G^* \approx 0.683 \text{kPa s} \). The optimal value of mean correlation coefficient equal to 0.988 indicates that the model fits the experimental data well.

Figure 3 presents the comparison of experimental and theoretical time series of transverse displacements at the position of sensor \( S_3 \), and over the whole time of the trial. In Fig. 4 the deflection of the beam is shown over its entire length. The dots denote the actually measured displacement during the selected moments of the first cycle. The first cycle was chosen because of the highest deflection, which yielded to the strongest and easily observed S-shaped deformations of the beam.

5. Conclusions

A dynamic model of the system of twin cantilever beams connected at their free ends by an elastomer damping member is proposed. The model fits the experimental data well. The Kirchhoff modulus and shear damping coefficient of the elastomer are identified. The proposed model can be a basis for the development of optimal control strategies for double-beam systems with adaptive damping members. It may be also useful for establishing such geometrical and physical parameters of these systems supposed to provide the highest efficiency of vibration suppression.

References


Phase-field modelling of twinning and martensitic transformation at finite strain

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Abstract

We develop a micromechanical phase-field model that describes the transformation between the austenite and twinned martensites. The new model constrains the volume fractions of both parent and internally twinned phases such that the physically motivated bounds are not violated. As an application, we studied the twinned martensite and austenite-martensite interfaces in the cubic-to-orthorhombic transformation in a CuAlNi shape memory alloy and estimated the elastic part of the interfacial energy.

Keywords: phase-field modelling, martensitic phase transformation, shape memory alloys, interface structure

1. Introduction and description of the model

Shape memory alloys are materials with a complex response during deformation. The material can exist in different phases, usually called austenite (parent phase) and martensite (product phase). When being deformed, the material can undergo phase transformation and twinning, which is accompanied by storing energy in the interfaces and which often leads to formation of laminate microstructures. The microstructure is essentially multi-scaled, with characteristic dimension given by spacing of austenite and martensite plates and the twin spacing of two different variants of martensite.

We develop a finite-strain phase-field model of twinning and martensitic transformation in shape memory alloys. The model is based on the minimization of the total rate-potential which describes all processes in the material, namely it consists of the free energy, comprising the elastic and interfacial energy, and the dissipation potential. The interfacial energy describes the energy of diffuse interfaces of a chosen width (parameter of the model) which is a characteristic feature of phase-field modelling [4].

Compared to other phase-field models for twinning [1, 2] our model uses a different mixing of the corresponding transformation strains, so it could have been easily generalized to more variants of phases. Also it is important that our mixing is consistent with respect to the volume changes. Finally, the order parameter, that is in case of our model interpreted as the volume fraction, is restricted so that it does not exceed the physical range. This is implemented with the augmented Lagrangian method, see [7].

In order to model microstructure with the austenite and twinned martensite, we extended the model by introducing two hierarchical order parameters. The volume fraction of the austenite and the other differentiates between two variants of martensite are described. The free energies and dissipation potential are modified consistently. The resulting partial differential equations are solved using the finite element method.

As an application, we study the austenite-martensite interfaces in the cubic-to-orthorhombic transformation in a CuAlNi shape memory alloy. For this purpose we use anisotropic elastic energy with the elastic constants of single-crystalline austenite and martensite of CuAlNi taken from the literature.

Both presented problems are solved in a two-dimensional domain, however, three components of the unknown displacement field are considered. We choose a special 2D domain lying in the plane whose normal $\vec{n}$ is parallel with the vector $\vec{m} \times \vec{l}$, where $\vec{m}$ is the normal of the interface between austenite and twinned martensite obtained from the solution of habit plane equation and $\vec{l}$ is the normal of the interface between martensites from the twinning equation.

2. Shape of the austenite-martensite interface and the elastic microstrain energy factor

Using our model we find the shapes of the interface between austenite and twinned martensite for four different types of possible geometrical microstructures of CuAlNi. The problem is computed in the periodic domain $\Omega_{\text{per}}$ of the shape of parallellogram where the normal of lateral sides is equal to $\vec{l}$ and the normal of the top and bottom side is equal to $\vec{n}$.

For the order parameters the full periodicity is prescribed. For the displacement we prescribe a boundary condition with a periodic fluctuation, i.e. $\vec{u} = (F - I) \vec{X} + \vec{\psi}_B$, where the deformation gradient $F$ mixes the deformation gradients of austenite and both variants of martensite scaled by average volume fractions. The average volume fraction of austenite is the only prescribed value (one half is chosen in this problem), the average volume fraction of martensites are such that formation of a compatible interface between austenite and twinned martensite is enforced.

The four types of microstructures differ in the geometry and also in a size-independent energy factor $\Gamma^e$ defined by

$$\Gamma^e = \frac{1}{hA_B} \int_{\Omega_{\text{per}}} \psi_B \, dX,$$

where $\psi_B$ is the elastic bulk free energy, $h$ is the twin spacing and $A_B$ is the nominal area of the macroscopic austenite-martensite interfaces in the reference configuration.

Stupkiewicz et al. [5] studied the shapes and values of $\Gamma^e$ for sharp interface between austenite and twinned martensite. They solved a purely elastic problem looking for such interface between austenite and twinned martensite which minimizes the elastic strain energy. In our case we use the phase-field model for dynamical simulation of the evolution of the material and in a steady solution we obtain shapes of the produced interfaces, which are similar to their results. Also the values of $\Gamma^e$ com-

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puted by ourselves converge close to their values as the interface thickness goes to zero.

Figure 1: Shapes of interface between austenite and twinned martensite for four different types of microstructures. Austenite is depicted in green, one variant of martensite in red and the other in blue.

3. Full microstructure of the grain-like domain

In another problem we compute the full microstructure for two variants of martensite and austenite in a circular grain-like domain with the fixed boundary condition for the displacement with the average volume fraction of austenite equal to 0.4. The result for the circular domain with the diameter 600 nm is shown in Fig. 2. We study the dependence of the number of austenite-martensite interfaces on the diameter of the domain. By minimizing the energy accumulated on the rigid boundary, interfacial energy between the twins and interfacial energy between austenite and twinned martensite have been estimated by analytic formulae [3, 6]. That estimate is compared with the result obtained from the finite element computation in Fig. 3. The analytical result overestimates the result obtained from the phase-field model, however, the scaling law is correctly predicted.

![Analytical estimate comparison](image)

Figure 3: Dependency of number of austenite-martensite microstructure on the diameter, comparison of the analytical estimate and finite element computation

References


The influence of external excitation on the dynamics of milling process

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Abstract

Numerical analysis of the milling process with added external excitation is presented in this paper. The analysed model takes into account flexibility of the tool and the workpiece. The dynamics of the milling process is described by the discontinuous ordinary differential equation with time delay, which can cause process instability. The stability lobes diagram was determined numerically for selected system parameters. In order to reduce harmful vibrations the concept of the use of active piezo-elements is presented here. In addition, the work shows numerical results of chatter control in closed loop using a PD controller.

Keywords: chatter, milling, nonlinear vibrations, time delay, stability

1. Introduction

At present, the high speed machining, especially high speed milling, plays an important role in the manufacturing process. During the machining, at specific combination of cutting depth and spindle speed, the chatter may arise. This phenomenon is harmful, mainly due to heavy vibrations of the tool which cause low surface quality.

In order to overcome difficulties related to chatter vibrations several methods of their elimination or suppression are mentioned. The tools activated by piezoelectric elements [2], a change of dynamic properties of the tool [1] may serve as examples.

2. Model of milling

The models of milling process are non-smooth by nature because a cutting tool has several cutting teeth, which are in contact with a workpiece during certain time intervals. The rest of time, the cutting edge is not in contact with the workpiece. This causes discontinuities, which make difficulties in getting analytical solutions and numerical simulations as well. Therefore, a proper modelling of the milling process is very important from technical point of view.

Differential equations of dynamics of the milling process considered as a two degrees of freedom system take the form

\[
\ddot{x}_i(t) + 2\zeta_i\omega_i\dot{x}_i(t) + \omega_i^2x_i(t) + \gamma_{1i}r_i(t) = \frac{1}{m_1}\sum_{i=1}^{z} F_i(t)
\]

where:

- \(m_1, m_2\) - modal mass of the tool and workpiece,
- \(\zeta_1, \zeta_2\) - damping coefficient of the tool and workpiece,
- \(\omega_1, \omega_2\) - natural frequency of the tool and workpiece,
- \(\gamma_{1i}, \gamma_{2i}\) - nonlinear stiffness coefficient of the tool and workpiece.

The resultant cutting force is described by the equation

\[
F_i(t) = g_p(t)(-F_{zp}(t)\cos\phi_i(t) - F_{zp}(t)\sin\phi_i(t))
\]

The cutting force acting on i-th tooth (i=1,2,..., z) in the x direction depends on an angular tool position \(\phi_i\), and consists of a tangential \(F_{zp}\) and a normal \(F_{zp}\) force component. \(z\) means the number of tool teeth, and \(g_p\) defines when i-th tooth is active. The tangential and radial cutting force acting on the tool are proportional to the axial depth of cut \(b\) and chip width \(w_p\) according to the equations

\[
F_{zp}(t) = K_t^p w_p(t)^\gamma, \quad F_{zp}(t) = K_n^p w_p(t)^\gamma
\]

Where, \(K_t^p, K_n^p\) are specific cutting forces which depend on the cutting material properties. Typical relationship between \(K_t\) and \(K_n\) for classical materials is \(K_n = 0.36 K_t\). The coefficient \(\alpha\) also depends on the material, and is usually estimated from 0.75 to 1.

\[
w_p(t) = \left[ f + (x_i(t) - x_e(t)) - f_x(t(\lambda)) \sin\phi_i(t) \right]
\]

The chip width \(w_p(t)\) is a function of the feed \(f\), the present tool and workpiece vibrations \(x(t)\) and vibrations of the previous tooth \(x(t-\tau)\). Where, \(\tau = 60/zn\) is the tooth passing period, \(n\) means rotational speed of the tool, \(z\) is number of teeth. Additionally, in order to control vibrations during milling process, harmonic motion of the workpiece represented by \(f(t)\cos(\phi)\) is added. Where, \(f, \phi, \lambda\) mean the amplitude and frequency of the external excitation signal.

The step function \(g_p(t)\) is defined in order to check whether the tool is in cut or not:

\[
g_p(t) = \begin{cases} 1, & \varphi_{on} < \varphi_i < \varphi_{off} \\ 0, & \text{elsewhere} \end{cases}
\]

In down-milling process the entry \(\varphi_{on}\) and exit \(\varphi_{off}\) angles are defined as follows

\[
\varphi_{on} = \arccos\left(\frac{D - 2a}{D}\right), \quad \varphi_{off} = \pi.
\]

Where, \(D\) denotes the tool diameter and \(a\) is a radial depth of cut. In case of full immersion milling analysed here, the entry and the exit angles take the value

\[
\varphi_{on} = 0, \quad \varphi_{off} = \pi
\]

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3. Numerical simulation

Based on differential equations of motion (1) the numerical simulations were performed in the environment Matlab-Simulink using the Runge-Kutta method of fourth order with variable step of integration. The values of parameter used in the simulation are shown in Table 1. As a result of the numerical simulations a graph of stability region of milling process is obtained (Fig. 1). The greyscale indicate the value of vibrations amplitude of the workpiece. The unstable lobes presented on stability lobes diagram (SLD) in Fig. 1 are a bit different from classical ones known in literature. This is because our model has the cubic nonlinearity expressed by the coefficient $\gamma_1$ and $\gamma_2$.

Table 1: Parameters of milling model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_1$</td>
<td>0.1824 [kg]</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>865.43 [rad/s]</td>
</tr>
<tr>
<td>$\zeta_1$</td>
<td>0.0406 [-]</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>2e12 [N/m^3]</td>
</tr>
<tr>
<td>$m_2$</td>
<td>3.02 [kg]</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>318.93 [rad/s]</td>
</tr>
<tr>
<td>$\zeta_2$</td>
<td>0.0396 [-]</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>2e12 [N/m^3]</td>
</tr>
</tbody>
</table>

In order to eliminate self-excited vibration in milling process we have used the external displacement $f_1 \cos(\lambda t)$ that introduces the additional workpiece vibrations (active elimination). The numerical study is performed to evaluate the applicability of the proposed method considering a closed loop control with a PD controller. The PD controller is intended to minimize the displacement between tool and workpiece. Numerical simulations were carried out to select the parameters by which PD controller reduce vibration. Figure 2 shows the bifurcation diagram for $n=7000\text{rpm}$ without controller. The influence of parameters $P$ and $D$ of the controller are shown in Fig. 3. Choosing the parameters $P=0.0012$ and $D=0.0017$ a significant reduction of vibration in two areas was observed.

4. Conclusions

The application of an external force to control the milling process has been demonstrated. Numerical studies were carried out for the non-linear model of milling which includes discontinuities of the process. It was shown that added external excitation together with the closed loop PD control may lead to significant decrease of the vibration levels for selected parameters of the process.

References


Mechanism of bi-direction laser bending for micro systems
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Abstract
Laser-based non-contact micro-adjustment method is a way of going beyond limits of traditional mechanical techniques applied for precise alignment during assembly of miniature opto-electro-mechanical devices. This paper presents experimental and numerical investigation of thermal micro-bending mechanism, which allows for bi-directional deformation, i.e. either towards or away from the laser beam, dependent on the applied processing parameters. An experimentally validated numerical model explains behaviour of a stainless steel cantilever beam subject to the Nd:YAG laser pulse. With a constant pulse duration, the direction of bending depends on the laser beam power. The revealed mechanism of bending involves significant positive longitudinal plastic strain in certain material regions. The deformation results from a considerable temperature gradient across the width of the cantilever beam, with some contribution of the temperature gradient in the thickness direction. Application of the mechanism opens up new opportunities for the laser-based micro-adjustment technology.

Keywords: laser forming, laser bending, laser micro-adjustment, thermo-elastic-plastic deformation

1. Introduction
Laser-induced micro-deformations are widely applied in manufacturing processes of the electronic industry, e.g. in production of hard disk drives [3] and miniature electric relays [4]. The leading companies of the sector have patented numerous practical solutions based on local laser heating of material that allow precise, non-contact and fast positioning of parts and sub-assemblies, such as magnetic read/write heads, optical fibres, lenses and photodiodes [6]. Adjustment of critical dimensions with micrometer or milliradian accuracy is employed during assembly stages in mass-production [2].

The three fundamental mechanisms of laser-induced deformation identified to date are: the temperature gradient mechanism, the upsetting mechanism and the buckling mechanism. Activation of the pure upsetting mechanism results in homogenous plastic compressive strain distribution across material thickness, which yields material shortening and thickening in the heated region. It is applied for example in the laser micro-adjustment technology using two-bridge actuators [5].

Bending effect, i.e. the out-of-plane deformation, can be produced with the two other mechanisms. While the instability inherent to the thermal buckling mechanism is a limiting factor in its practical use, the temperature gradient mechanism has a drawback of producing bends always in one direction, i.e. towards the heat source (e.g. laser beam). The paper presents experimental and numerical investigations of a thermal micro-bending mechanism, which enables bi-directional deformation, i.e. either towards or away from the laser beam, dependent on the applied processing parameters.

2. Experiments
The samples of dimensions 50 x 4.05 x 0.55 mm made of 18-8 type stainless steel, clamped in the cantilever arrangement, were heated with a stationary Nd:YAG laser beam (Fig. 1). They were annealed prior to laser bending experiments in 400°C for a half an hour in order to reduce initial residual stresses and to increase coupling of laser radiation due to created oxide layer.

3. Numerical simulation
Sequentially coupled thermal and mechanical analysis using the finite element method was performed with the ABAQUS system. The power density distribution of the multimode laser beam was approximated by a top-hat model of constant intensity over the laser spot. Constant laser pulse duration 1.05 s was applied throughout the experiments with power levels ranging from 21 to 72 W. Fluctuations of the laser beam power are estimated as ±5% of the nominal value. Accuracy of pulse time duration was approximately ±0.05 s. Non-contact measurements of deformation were performed with a high-accuracy optical micrometer.

The laser beam was defocussed to obtain a spot of diameter 3.6 mm on the material surface. The spot was located on the longitudinal axis of symmetry of the specimen. Constant laser pulse duration 1.05 s was applied throughout the experiments with power levels ranging from 21 to 72 W. Fluctuations of the laser beam power are estimated as ±5% of the nominal value. Accuracy of pulse time duration was approximately ±0.05 s. Non-contact measurements of deformation were performed with a high-accuracy optical micrometer.

Figure 1: Schematic of the laser bending process. Definition of the positive angular deformation $\beta$.

Three-dimensional linear elements with 6 and 8 nodes were used: wedge elements DC3D6 and hexahedral DC3D8 for thermal problem, and compatible elements C3D6 and C3D8 for mechanical problem. Ten layers of elements were applied in the thickness direction of the specimen in order to accurately model the gradient of temperature and the bending effect.
Thermal dependence of material data as well as convective and radiative heat dissipation was respected in modelling. Material yield stress dependence on temperature was adopted from [1] using the 0.2% strain limit and the room temperature yield stress value 234 MPa.

Elastic-perfectly plastic isotropic material model was applied. Material hardening was neglected, as plastic deformation occurs mainly at high temperature, where strengthening is substantially reduced by dynamic recovery processes. The Huber-Mises-Hencky yield criterion was employed.

4. Results and conclusion

Figure 2 presents results of experimental measurements and numerical calculation of the angle of bend during laser heating and shortly afterwards.

Figure 2: Time runs of the bend angle in experiments and numerical simulation for the laser power 59 W.

Dependence of the final bend angle on the applied laser beam power is shown in Fig. 3.

Figure 3: Dependence of the final angle of bend on the applied laser beam power. Error bars represent sample standard deviation.

Some discrepancy between numerical results and experimental measurements, seen for the highest power level 72 W in Fig. 3, can be attributed mainly to the poor accuracy of the material yield stress value at temperatures above 1000°C. Taking into account limited accuracy of the available material data, the obtained simulation results are regarded as satisfactory validation of the numerical model.

Figure 3 reveals a new effect in laser micro-bending. Dependent on the applied laser beam power, with constant pulse duration, either positive or negative final angular deformation can be obtained. Mechanism of this deformation can be explained considering thermal gradients in the directions of axes 2 and 3, and the resulting distribution of longitudinal plastic strain component $\varepsilon_{11}^{pl}$ (PE11), as shown on one of the two symmetric halves of the model (Fig. 4).

Figure 4: Distribution of plastic strain component $\varepsilon_{11}^{pl}$ at the end of heating stage for laser power 59 W.

Plastic strain distribution results from a considerable temperature gradient across the width of the cantilever beam, i.e. in the direction of axis 2, with some contribution of the temperature gradient in the thickness direction (axis 3), at the highest temperature during thermal cycle. Final deformation of the beam is a result of a play between negative strain in the central region and positive strain close to the edges of the beam, both influenced by the temperature gradient in the thickness direction. Application of this mechanism opens up new opportunities for the laser-based micro-adjustment technology.

References


Soft Methods and Inverse Analysis in Mechanics of Structures and Materials
– a Session in Honor of Prof. Zenon Waszczyszyn in connection with His 80th Birthday
and in the recognition of important scientific achievements in Mechanics

organized by T. Burczyński and L. Ziemiański
Multiobjective optimization of electrothermal microactuators by means of Immune Game Theory MultiObjective Algorithm

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Abstract

The paper presents application of the IMGAMO (Immune Game Theory MultiObective Algorithm) in multiobjective optimization of electrothermal actuators. Different types of functionals are formulated, which depends on equivalent stresses, deflection of the actuators, force generated by the actuators and volume of the actuators. Values of the functionals are calculated on the basis of results obtained from numerical simulations. Boundary-value problem of electro-thermo-mechanical analysis is solved by means of Finite Element Method software. Numerical examples of multiobjective optimization of chevron-type electrothermal actuators are presented.

Keywords: multiobjective optimization, artificial immune system, game theory, finite element method, MEMS, microactuators, electrothermal actuators

1. Introduction

Electrothermal microactuators have proved to be very useful for generating force or displacement in MEMS. Comparing to the other types of microactuators (ex. electrostatic, piezoelectric) electrothermal microactuators can generate larger force per unit volume. Low electric potential used for control of such structures is another advantage of the electrothermal actuators. The deflection of the electrothermal actuator is produced when the electrical potential difference is applied to electrical pads. Due to the material properties - high electrical resistivity and thermal expansion, arms of the actuator can elongate. Mainly two types of the electrothermal actuators are used in MEMS: U-beam or V-beam. U-beam microactuator consist of beams (arms) which have different cross-section area. It produces different amount of Joule heating which causes bend of the actuator. For the V-beam actuator displacement is produced due to the pre-bending angle of beams (arms). Microactuators composed of greater amount of pairs of beams are named chevron type (Fig. 2).

The problems of optimal design of electrothermal actuators have been considered by many researches [3, 4]. In order to efficiently optimized such structures, proper optimization technique have to be applied, especially when more than one criterion is taken into account. Multiobjective optimization by means of evolutionary algorithms of U-beam actuators was solved by authors in [2]. The present work propose methodology of multiobjective optimization of actuators which is more effective, comparing to the previous work, if more then three criteria are considered. It is possible by application the in-house implementation of algorithm based on Artificial Immune System and the Game Theory. The proposed algorithm was tested on several mathematical benchmark problems showing its superiority comparing to other optimization techniques.

2. Formulation of the multiobjective optimization problem

The goal of the multiobjective optimization is to find vector of design variables (e.g.: geometry of the structure, material properties, boundary conditions) for which defined objective functions achieve extrema. Optimization task can be formulated as minimization or maximization problem. For the minimization problem vector of design variables \( \mathbf{x} = [x_1, x_2, ..., x_n] \) are searched which minimizes the vector of \( k \) objective functions.

\[
f(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), ..., f_k(\mathbf{x})]
\]  

(1)

Generally \( p \) equality constrains and \( m \) inequality constrains are imposed on the optimization problem.

\[
h_i(\mathbf{x}) = 0 \quad i = 1, 2, ..., p \quad g_i(\mathbf{x}) \geq 0 \quad i = 1, 2, ..., m
\]  

(2)

Thus in multiobjective optimization decision domain is \( n \)-dimensional and objective domain is \( k \) dimensional. Pareto concept requires the definitions of such terms as: dominated solutions, non-dominated solutions, Pareto-optimality and Pareto front. Considering two vectors \( \mathbf{x} \) and \( \mathbf{y} \) in the searching domain:

- solution \( \mathbf{x} \) strongly dominates \( \mathbf{y} \), if:
  \[
  \forall i \in \{1, 2, ..., k\} : f_i(\mathbf{x}) < f_i(\mathbf{y})
  \]  

(3)

- solution \( \mathbf{x} \) weakly dominates \( \mathbf{y} \), if:
  \[
  \forall i \in \{1, 2, ..., k\} : f_i(\mathbf{x}) \leq f_i(\mathbf{y}) \quad \land \quad \exists j \in \{1, 2, ..., k\} : f_j(\mathbf{x}) < f_j(\mathbf{y})
  \]  

(4)

- solution \( \mathbf{x} \) is neutral (incomparable) to the \( \mathbf{y} \), if:
  \[
  \exists i, j \in \{1, 2, ..., k\} : f_i(\mathbf{x}) < f_i(\mathbf{y}) \quad \land \quad f_j(\mathbf{y}) > f_j(\mathbf{x})
  \]  

(5)
Solution $x$ is optimal in the Pareto sense, if there are no other solutions which dominates $x$ in whole searching domain. The set of Pareto-optimal solutions creates Pareto front (Fig. 1). For the three criteria set of Pareto-optimal solutions creates surface, whereas for higher dimensions additional technique is needed for representation of the Pareto front.

Figure 1: Representation of the Pareto front for the bi-objective case

3. Multiobjective optimization algorithm

The metaphors of the game theory and immunology are used to solve the problems of multiobjective optimization using IMG-AMO (Immune Game Theory Multi-Objective Algorithm) [5]. Each player has its own objective (a payoff function in the Nash equilibrium). The strategy for a particular player is the optimum solution for this player’s problem remembering that other players also play their best strategies. The solution of the optimized problem consists of several parameters, each of which is assigned to one of the players. Each player optimizes only its parameters (its strategy) taking the rest of them as constant. The rest of the parameters are set by taking the best solutions the from other players. Solutions from all players should establish the solution of the problem. Then all players use the immune algorithm to optimize their objectives. Details of the algorithms were described in [5].

4. Formulation of the problem

Multiobjective optimization of the chevron-type electrothermal actuator is considered (Fig. 2).

Figure 2: Geometry of the chevron-type microactuator

The deflection or force generated by the microactuator can be calculated analytically only for the straight shape arms. For more complicated shapes and if the stress field should be examined, numerical model have to be created. In order to simulate numerically electrothermal actuators coupled electrical thermal-mechanical analysis have to be solved. Such problem is described by the appropriate partial differential equations [6]. The equations with arbitrary geometries and boundary conditions are solved by Finite Element Method. This problem is weakly coupled and it requires solving electrical, thermal and mechanical analysis separately. Coupling is carried out by transferring loads between the considered analysis and by using staggered procedures. Matrix equations of electrical, thermal and mechanical problem can be expressed as follows:

$$K_E V = I$$

$$K_T T = Q + Q_E$$

$$K_M U = F + F_T$$

(6)

where: where: $K_E$ is the electrical conductivity matrix, $K_T$ is the thermal conductivity matrix, $K_M$ is the stiffness matrix, $Q_E$ is the heat generation vector due to current flow, $F_T$ is the force due to thermal strain vector, $V$, $T$, $U$ are the nodal vector of voltage, temperature and displacements, respectively, $I$, $Q$, $F$ are the nodal vector of current, heat fluxes and applied forces, respectively. The thermal and mechanical problems are coupled through thermal strain loads $F_T$. Coupling between the electrical and thermal problems is done by heat generation due to the electrical flow $Q_E$. FEM software MSC.Mentat/Marc is adopted to solve the problem. The preparation of the geometry, finite element mesh, boundary conditions and performing the analysis of this coupled problem have been done automatically by means some additional codes (written in C++ and script languages implemented in preprocessor Mentat). After solving coupled analysis, the values of the functionals are calculated on the basis of output files generated by MSC.Marc. Following functionals are defined: minimization the volume of the structure, minimization of the maximal value of the equivalent stress, maximization the deflection of the actuator.

References


Influence of frequency range of surface waves on estimation of parameters of heterogeneous concrete using non-contact method

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Abstract

This work is focused on studies of influence of selection of the frequency range of dispersion curves of surface waves on efficiency of estimation of the bulk elastic wave velocities and thickness of layers of non-homogeneous concrete. The experimental data comes from non-contact ultrasonic method and estimation of material properties uses optimization procedures. As the model the Thomson-Haskel theory of propagation of surface waves in layered media is used. The model predicts dispersion relationships of pseudo Rayleigh waves. The experimental dispersion curves are obtained by using the Slant-Stack transform. Quality of estimation was assessed based on analysis of fitting of dispersion curves and evaluation of the estimation error of the thickness of the top layers and the velocity of transverse waves for the individual layers.

Keywords: surface waves, concrete, cover layer, inhomogeneity, dispersion, non-contact measurements, inversion, NDT

1. Introduction

For concrete building structures which are influenced by external environment and interactions with humidity, changes of temperature (particularly in cases of negative temperatures), chemical or biological factors, the degradation of material properties starting from the concrete cover and penetrating throughout structural elements is observed. The changes of properties of concrete cover may concern mechanical or structural characteristics and usually exhibit as decreased strength and stiffness as well as increase of porosity and permeability. The changes have a direct influence on accelerated degradation of structures (also their reinforcements) and finally lowering their durability. More and more interest among researchers developing NDT methods for concrete is focused on the application of non-contact ultrasonic methods. The efforts are concentrated on techniques useful for assessment of strength and durability parameters (e.g. structural parameters like porosity or permeability) and for identification of thickness of the layers of non-homogeneous concrete, [1,4].

This work is devoted to the problem of influence of selection of the frequency range of experimentally determined dispersion curves of surface waves on efficiency of estimation of the bulk elastic wave velocities and thickness of the layers of non-homogeneous concrete.

2. Materials and experimental setup

The studies are performed for model inhomogeneous materials - concrete composites consisting of two or three layers of homogeneous materials (Fig. 1). The top layers with thickness from a few millimeters to a few centimeters are made from concrete classes: C8, C12, C16 and bottom layer (deep layer) is made from concrete of type C20/25, i.e. with higher quality concrete. The thickness of the cover and deep layers are 10 or 20 mm and 100 mm, respectively. The thickness of the cover layer guarantees penetrability of the applied ultrasound. The measurements are made with help of the automated system for non-contact studies of surface waves with frequencies from 40 kHz to 250 kHz (Fig. 2). The gas matrix transducers (The Ultran Group) with main frequencies 50, 100, and 200 kHz are used. The signals of leaky waves are acquired for 37 points with the spatial step equal to 5 mm. The signals are processed off-line using Slant-Stack transformation, giving maps of velocity of surface wave as the functions of frequency (Fig. 3), [2,4]. From them the dispersion curves are extracted being the input data for the solution of inverse problems.

Figure 1: One of the samples of studied materials - two layer concrete of class C8 (cover) over C20

Figure 2: The automated experimental system for studies of surface waves (T- transmitter, R- receiver, SW-surface wave)
The identification procedure uses the Haskell model of surface wave propagation in multi-layered half space which relates dispersion characteristics of wave modes with mechanical parameters of layers and their thicknesses, [3]. The algorithm of solution of inverse problem concentrates on the fundamental wave mode and applies hybrid optimization method which combines local (bisection method) and global (pattern search method) extremum search. The optimization procedure runs number of times with different starting parameters and delivering solutions with various quality of fitting of the model predictions to experimental dispersion curves (error functions), [5]. The results of identification are compared with the known parameters of studied model materials (e.g. thickness of layer and velocities of shear waves determined directly), [2].

3. Results and conclusions

The parameters assumed as unknown were velocities of shear waves in the two layered systems and thickness of the cover layer. The other parameters: densities and velocities of longitudinal waves in the materials were assumed as known and the values determined from the independent direct tests. The results of identification for a studied concrete sample of class C8-C20 with ~1.5 cm thick cover layer and using two different frequency ranges of dispersion curves are visualized in Fig. 3. In the first column of Fig. 3 the input (experimental) dispersion curve is shown with red points illustrating the frequency range of dispersion curve loaded as input data for identification procedure. The second column shows comparison of the dispersion curves obtained as the result of solution of inverse problem - best fit (black line) with the experimental dispersion curve (red line). In the third column distributions of the shear wave velocity VS as the function of depth measured from the surface and determined for few inversions which guarantee the smallest errors, along with the averaged values and standard deviations are presented. The coordinates corresponding to the jump of the averaged shear wave velocity is assumed as the thickness of the cover layer evaluated from inversion. In the first considered case (first row in Fig. 3) the results for a wider frequency range of dispersion curve from 46kHz to 230kHz are illustrated. The averaged thickness of the cover layer obtained by inversion was ~14.6mm (std 6.5%) while the thickness measured by a ruler was ~14.9mm (relative differences ~2%). In the second case (second row in Fig. 3) the range of dispersion curve was from 41kHz to 125kHz and this led to the thickness of the cover layer about 20 mm (relative differences ~34%). More data for different concrete composites and frequencies of ultrasonic transducers were considered. The results show a significant role of frequency range of the experimental dispersion curve used as the input for inversion. The range influences both the values of identified thickness of the cover layer and the shear wave velocities of concrete layers.

4. References


Design of fuzzy logic controller for a unloading system in mechatronic device for gait reeducation

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Abstract

The article presents structural solution and a control system of a device responsible for patient unloading during therapeutic tasks related to the walk re-education process. The device is incorporated in a complex mechatronic system which constitutes a rehabilitation device. Unlike traditional devices of this type which often operate as classic hoists, the applied system is equipped with two independent motors. One of the motors is responsible for the dynamic compensation of the patient body weight and acts as the so-called “elastic series actuator” – a drive applied in walking machines and human-assisting robots. The other is responsible for the operation of the winding reel. Due to the need to ensure proper cooperation between the two engines, the selection of the optimal control system of the device constitutes an interesting engineering problem. The work presents a solution using an implemented algorithm based on fuzzy logic. The selection of parameters was conducted based on numerical studies in which the walk of a patient through a step was simulated.

Keywords: mechatronic device, gait reeducation, control system, fuzzy logic controller, unloading system.

1. Introduction

Mechatronic devices equipped with intelligent control systems are starting to take on tasks that used to be conducted by humans only. This may be exemplified by modern rehabilitation devices which take the place of physical therapists or at least significantly ease their work [1]. As far as rehabilitation equipment is concerned, the device supporting the process of walk re-education designed at the Department of Theoretical and Applied Mechanics is one of such solutions. It operates as a travelling crane in which the unloading system (a modified hoist) performs a movement following the moving patient while unloading the patient with a pre-set force value. The patient may thus move at the entire area, limited only by the structure of the crane. This type of solution allows to perform various types of exercises under the rehabilitator supervision, e.g. climbing stairs, squats or standing up. Another matter that is essential to the rehabilitation process is also to ensure the patient a possibility to move in the most natural manner. The unloading devices used until now have only allowed for a movement in a single direction. This limits the patient freedom of movement and the applied force largely hinders the natural moves of the rehabilitated person. As the experimental studies analysing the displacement of the centre of gravity of the unloaded person during walk with the use of the videogrametric method, the application of the follow-up movement in all directions (both in the sagittal and frontal planes of the patient) allows to walk in a near-natural manner.

2. Construction of the device

From a structural point of view, the device resembles a travelling crane in which the girder travels on tracks located in the top part of the support frame of the device. The structure is made of aluminium profiles. In one direction, the follow-up movement of the crane keeping up with the patient (horizontal plane movement) is conducted using a belt transmission and using a screw transmission in the other. The electrical part of the device is comprised of BLDC motors controlled by servo inverters operating in speed mode [7].

The operation of the unloading system should guarantee a rapid reaction to the changes of the unloading force values as well as to allow the patient to move along a track with elements of a different height. To allow the fulfillment of both criteria, two separately operating motors were installed in the device [4]. The first using a planetary gear, drives the reel of the hoist (Z1 drive), which allows to control the height of the sling. The second engine has been coupled with a screw drive which is responsible for the movement of the travelling element which is separated from the pulley with elastic elements (Z2 drive).

Figure 1: Scheme of unloading system

A cable is stretched on the pulley – one end the cable is fixed to a drum and the other to the orthopaedic harness in which the patient is located. The diagram of the developed drive was presented in Fig. 1.

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Moreover, the device is equipped with designed measurement systems: sensor of the rope deflection angle and the sensor for the measurement of the unloading force.

3. Control system

The control of the device is conducted in a real-time mode, using RT-DAC4/PCI cards connected to a control cabinet by means of a dedicated signal conditioning interface.

The algorithm responsible for the follow-up movement utilizes PD regulators operating in a feedback loop. The misalignment signal is equal to the deflection angle of the rope along a given direction [2].

In case of the patient movement along a non-uniform track (climbing stairs, inclined planes), the control unit of the unloading system also manages the operation of both the drives – Z1 and Z2.

Due to the fact that the control unit of the unloading system is a multi-input and multi-output system, the control algorithm (in opposition to the control of the drives responsible for the follow-up movement) has been based on fuzzy logic [3,5,6].

After formulating the control rules, the ranges of the membership function were optimized. The selection of these parameters was conducted using the developed and identified numerical model of the device. During the numerical studies, the patient walk over a 8 cm high obstacle was simulated. Genetic algorithm was selected as the optimization algorithm.

![Figure 2: Change of unloading force values as a function of time](image)

![Figure 3: Displacement of the pulley in the Z2 axis as a function of time](image)

The graphs above shows the values obtained from the numerical simulations: displacement of the pulley in the Z2 axis (Fig. 2) and the value of unloading force (Fig. 3).

Figure 4 shows the experimental research of the unloading system.

4. Conclusions

The article presents a control unit of the unloading system installed in a mechatronic device used for the re-education of walk. Ensuring the cooperation of both the motors incorporated in the drive resulted in the increase of the total misalignment of the unloading force but has not changed the value of the maximal misalignment (in relation to the previously developed control using a single drive). The force unloading the patient varies in the range of ±40 N in relation to the pre-set value. The proposed control system thus allows to conduct exercises in unloaded conditions, which require significant vertical movement of the patient. This is especially significant in training e.g. climbing the stairs – one of the basic daily functions of all persons.

References

On improving evolutionary algorithms applied to chosen problems of mechanics

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Abstract

Advances in development of several new concepts of acceleration techniques for the Evolutionary Algorithms (EA) are discussed in the paper. Though our approach is general, the target objective of this research is development and application of the improved EA to chosen problems of mechanics, including residual stress analysis in railroad rails and vehicle wheels, as well as the Physically Based Approximation of experimental and/or numerical data. Due to slow convergence of the standard EA, our current research is mostly focused on development of various new very efficient speed-up techniques proposed, including smoothing and balancing, adaptive step-by-step mesh refinement, as well as a posteriori solution error analysis and related techniques. In the paper we analyse efficiency of chosen speed-up techniques using several simple but demanding benchmark problems. Preliminary results obtained for numerical tests indicate a clear possibility of practical application of the improved EA to large non-linear constrained optimization problems.

Keywords: Evolutionary Algorithms, large non-linear constrained optimization problems, computation efficiency increase, residual stress analysis, physically based approximation

1. Introduction

In the paper we discuss a development of several new acceleration techniques for the Evolutionary Algorithms (EA) applied to chosen problems of computational mechanics. In particular, we take into account two important engineering applications, namely residual stress analysis in railroad rails and vehicle wheels [2, 4], and a wide class of problems resulting from the Physically Based Approximation (PBA) of experimental and/or numerical data [4]. Moreover, the improved EA may be also applied to a wide class of other engineering and scientific problems formulated in terms of large non-linear constrained optimization.

In contrast to a majority of deterministic methods, the EA may be successfully applied with similar efficiency to both the convex and non-convex problems [1]. However, the standard EA are generally rather slowly convergent methods. Therefore, the research is focused, first of all, on significant acceleration of the EA-based solution process.

2. Problem formulation

We consider a wide class of large non-linear constrained optimization problems, where a function is investigated, usually in a discrete form of its nodal values. In general, the considered optimization problems may be posed as follows:

Find a function \( u = u(x) \), \( x \in \mathbb{R}^N \), that yields the stationary point of a functional \( F(u) \), satisfying the equality \( A(u) = 0 \), and inequality constraints \( B(u) \leq 0 \). After discretization we find a vector \( u = \{u_i\} \) consisting of nodal values \( u_i \), \( i = 1, 2, ..., n \). These nodal values are defined on a mesh formed by arbitrarily distributed nodes. Here, \( N \) is the dimension of the physical space (1D, 2D or 3D), and \( n \) is a number of decision variables.

In the particular case of the PBA approach [4], the functional to be optimized and related constraints consist of the experimental, and theoretical parts. The experimental part is defined as the weighted averaged error resulting from discrepancies between the measured data and its approximation.

The theoretical part is based on a known theory (e.g. energy functional in mechanics), and/or on a heuristic principle (e.g. smoothness requirement).

3. Acceleration techniques for the EA

The EA are understood here as genetic algorithms with a real-value coding. The standard algorithm consists of three operators: selection, crossover and mutation [1]. Significant acceleration may be obtained in various ways, including appropriate hardware and algorithm improvements. So far we have proposed and preliminarily tested several new acceleration techniques based on simple concepts [3, 5], including smoothing and balancing, a posteriori solution error estimation and related techniques, as well as the use of a step-by-step mesh refinement. We present a brief overview of the proposed methods, as well as advances in their development. Some of these techniques are problem- (or class of problems) oriented, the other are of more general character.

3.1. Smoothing

Smoothing techniques are addressed to optimization problems where a smooth (at least in subdomains) function is investigated. Two different approaches for smoothing of raw EA results are considered. The first one uses an extra procedure based on the Moving Weighted Least Squares (MWLS) technique [3, 5]. The second approach is based on the mean solution curvature which is introduced into the fitness function evaluating individuals in a population [3].

3.2. A posteriori error analysis and related techniques

The a posteriori error analysis is based on the assumption that it is possible to generate reference solutions of sufficient quality for the error estimation. We have already proposed such a new technique for the reference solutions generation, based on the stochastic nature of the EA [5]. We have also proposed and tested improved mutation, crossover and selection operators taking advantage of information about the local and global solution errors. Other related techniques include solution...
averaging and cloning, generation of population of representatives, as well as non-standard distributed and parallel computations [5].

3.3. Adaptive step by step mesh refinement

The approach using step-by-step mesh refinement starts the analysis from a coarse mesh, where a solution is obtained much faster than in the fully dense mesh case. However, such solution usually is not precise enough. The precision of the solution is increased by inserting new nodes in the possible best locations. Such process is repeated until a sufficiently dense mesh is reached, and errors in all nodes are smaller than their admissible values assumed. In the iterative solution procedure applied the initial function values at inserted nodes are found by using an approximation built upon the coarse mesh nodal values. Furthermore, step-by-step mesh refinement may be combined with other already mentioned acceleration techniques [3, 5].

4. Sample numerical results

The efficiency of the EA speed-up techniques was evaluated using several benchmark problems, including residual stress analysis in selected elastic perfectly-plastic bodies under various cyclic loadings, including combined ones [3, 5]. We also investigated several PBA benchmark problems [3], including smoothing of beam deflections, and reconstruction of residual stresses. Both simulated pseudo-experimental data, as well as real experimental data were used. The main objective of the executed tests was to evaluate the possibility of practical application of the improved EA to optimization problems involving large number of decision variables. A formulation of one of the simplest PBA benchmarks is as follows:

Given free-supported beam displacements \( w^{exp} \), measured at points \( x_i \), \( i = 1, 2, 3, \ldots, m \) we seek nodal values \( w \), \( i = 1, 2, 3, \ldots, n \) of smoothed displacements \( w \).

Find the stationary point of the functional:

\[
\Phi(w) = \lambda \Phi^e(w) + (1-\lambda) \Phi^f(w), \quad \lambda \in [0, 1],
\]

where

\[
\Phi^e(w) = \frac{1}{m} \sum_{j=1}^{m} \left( (\pi_j - w^{exp}_j) / e_j \right)^2,
\]

\[
\Phi^f(w) = \int_0^L \kappa^2 \sqrt{w^{ref}_j} \, dx + \int_0^L \left( \frac{w_j}{w^{ref}_j} \right)^2 \, dx + \frac{1}{n} \sum_{i=1}^{n} \left( w_i / w^{ref}_i \right)^2,
\]

satisfying boundary conditions

\[
w(0) = w(L) = 0,
\]

admissible local error constraints

\[
|\pi_j - w^{exp}_j| \leq e_j,
\]

and admissible global error constraint

\[
\sqrt{\Phi^f(w)} \leq e_g.
\]

In the above formulation \( \pi_j \) is an approximation built upon sought values \( w_i \) at node \( x_i, e_i \) and \( e_g \) are admissible errors, \( L \) is the beam length, \( \kappa \) is the mean solution curvature at the point \( \pi_j \) and \( w^{ref}_j \) are reference values, \( \lambda \) is a scalar weighting factor determining a reasonable balance between the experiment and theory [4].

The results obtained so far indicate a possibility of practical application of the improved EA to non-linear optimization problems involving large number of decision variables and constraints. Future research will be focused on the application of the improved EA to real large complex engineering problems, including residual stress analysis in railroad rails and vehicle wheels, as well as the PBA of experimental data.

5. Final remarks

The results obtained so far indicate a possibility of practical application of the improved EA to non-linear optimization problems involving large number of decision variables and constraints. Future research will be focused on the application of the improved EA to real large complex engineering problems, including residual stress analysis in railroad rails and vehicle wheels, as well as the PBA of experimental data.

References


Event-driven approximate reasoning

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Abstract

The paper presents a concept of the application of event-driven architecture in expert system. The considered expert system is a dynamic expert system based on the multimodal statement networks. It is presented a concept of statement and multimodal statement networks. The expert system applies a notice board for the values of statements to be stored. It was assumed that the change of statement values presented on the notice boards could be described as an event. Thus it is proposed to use an event-driven architecture technique in the inference process in such expert system.

Keywords: artificial intelligence, event driven systems, intuitionistic network

1. Introduction

One of the main objectives of technical diagnostics is the assessment of technical condition of devices and machines. Due to the development of methods involving both the possibility to carry out measurements and also the processing of the existing data, the assessment of technical state can be carried out based on a great amount of data. Such a large number of input data leads to a large number of investigated diagnostic symptoms. In such cases, it is recommended to apply expert systems. Each of the expert systems consists of many typical modules. They include:
- user interface module,
- knowledge base module,
- database module,
- inference module.

Considering an expert systems for their operation, we distinguish two groups of such systems. The first group includes static systems. Static systems work in the so-called off-line mode and search for the solution in fixed environment. The reasoning process in this kind of systems is realized using fixed data that are collected from the technical object. The second group of expert systems includes dynamic systems. Dynamic systems work in the on-line mode. The reasoning process in this kind of systems is realized in limited time and with limited resources. Thus the reasoning process could be realized using different methods. One of the methods assumes that the data acquired from the technical object are frozen for some time interval and the reasoning process is realized like in static systems.

An interesting example of dynamic systems designed to support processes of machine monitoring are systems based on notice boards. A special class of this kind of expert systems are systems based of intuitionistic notice boards [2]. In this kind of systems the board consists of notices including statements. A notice consists of a statement whose content serves as a notice content. A statement is a declarative expression, resulting from observed facts, or representing an opinion, which can be either exclusively true, or exclusively false [2]. Results of the system could be presented as a colored rectangle on the notice board view [3].

The notice presented on the notice board consists of two elements: statement content and statement value. Statement content is a fixed element of notice and statement value is a variable element of notice.

An interesting solution which can be realized in dynamic expert systems could be event driven reasoning. In this solution the reasoning process will be started by different events placed on the notice boards.

2. Event driven systems

The event driven systems are systems based on an Event Driven Architecture (EDA) [5]. The event driven architecture is a programing paradigm determining the behavior of a computer program based on events [4]. An event is an occurrence or happening, which originates inside or outside a system, and is significant for, and consumed by, a system’s component [1]. Events are characterized by their types. We can distinguish two types of events [1]:
- Primitive events,
- Composite events.

Primitive events, are atomic and occur at one point in time. Composite events include several primitive events. In this case primitive events occur in a time range and have a specific pattern. In composite event we can distinguish an initiator and a terminator. An initiator is a primitive event, that start the pattern of composite event. A terminator is a primitive event that completes the composite event. The occurrence time can be defined as a time of the terminator or can be represented as a pair of times, one for the initiator event, and the other for the terminator event [1].

An event can be defined as a pair of two values:

\[ e = <h, b> \]  

where \( h \) is an event header and \( b \) is an event body. The event header is a vector that include information about the event such as:
- event name,
- occurrence time,
- occurrence number,
- event source,
• possible other elements that are included in the event specification.

The event body is a vector that include the details of the detected state change.

Event-driven architecture is used in many domains e.g. process management [6], programming applications or in managing and control systems. One of the possibility of application of event-driven architecture is to use it in reasoning process.

3. Event driven reasoning

One of the main elements of each expert system is the knowledge base. The accumulated knowledge is the basis for carrying out the inference process. The knowledge in expert systems can be described in different forms. One the most interesting form of knowledge representation useful for diagnostic systems are multimodal statement networks [7]. The value of statements that is used in the inference process in this kind of network in many cases are evaluated using data acquisition systems or based on values of other statements.

In dynamic expert systems the process of evaluating statement values is realised in the on-line mode. The calculated statement values are the basis for the process of inference, which is realized in limited time and with limited resource. These limits are tried to be reduced in different ways. One possibility of reduction is the use of such an event driven architecture technic. In expert systems based on the notice board, the statements value are written on it. According to the definition of the event, a change in the statement value on the notice board, can be regarded an event, occurring in a given system.

Let the change in statement value will be an event $e_i$. Then we can define a set of events $E$, describing the events that can occur in the system:

$$ e_i \in E $$

Each event can be described by a pair of vectors $h$ and $b$ in accordance with the formula (1). In this case the header of the event can be described as follows:

• event type,
• statement name,
• statement value timestamp,
• source of statement value,
• range of statement value where the event is omitted.

In the body of the event we can describe the following elements:

• old value,
• new value.

All events should be consumed by the system. So in the dynamic system a special function which could be run when the event occurs should be defined. The function may perform one of the following scenario:

• if the difference between old value and new value is to small (less than range of statement value defined in event header) the event should be omitted,
• the event could initialize the process of evaluation of other statements value, using some equations or other special prepared procedures,
• the event could initialize the inference process.

The application of EDA provides an opportunity for the inference process to a limited extent. We can imagine, that for the purposes of determining the value of the statement will be necessary to use an inference process. In this case, we can isolate a small statement network for which the inference process will be conducted. This will allow for a better use of available resources and reduce the time of the ongoing process of inference.

4. Conclusions

The application of event-driven architecture in dynamic expert systems is an interesting solution for the operation of dynamic expert systems. It allows for a better use of available resources and reducing the time needed to carry out the inference process. This is especially helpful in currently available multithreaded operating systems, where are available multiple processors and large memory resources. This approach also allows for the integration of expert systems with data acquisition systems. Changes in the values of conditioned data could call the appropriate automatic reaction of expert system and run an inference process.

References


On-line identification of elastic parameters in composite laminates using Lamb waves monitoring and Bayesian filtering

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Abstract

The paper presents a new approach to the problem of identification of elastic parameters of homogeneous, elastic and hexagonally orthotropic plates. The proposed solution is based on dispersion curves for Lamb waves propagating in free waveguides and Bayesian inference for sequential estimation of elastic parameters. We solve the problem considering the unknown elastic parameters state variables of a stationary dynamic system and formulating the sequential identification problem as a Bayesian state estimation problem. We solve the problem using particle filtering and show results in case of elastic parameters estimation for a thin orthotropic plate.

Keywords: Lamb waves, dispersion curves, thin orthotropic plate, finite element method, Bayesian state estimation, particle filtering

1. Introduction

Currently guided Lamb waves are often used for non-destructive identification of elastic constants of materials. In general, the identification procedures are based on minimization of the discrepancy between experimental and numerical or analytical dispersion curves. Thus, they are unable to characterize reconstruction uncertainty in a systematic manner. In this context Bayesian methods can be useful offering a systematic approach to uncertainty quantification [1]. Bayesian methods are also sequential, solving identification problems recursively. Recently, Słoński in his paper was applied particle filter in the problem of identification of elastic parameters of aluminum thin plates [4].

In this work an application of particle filter for sequential stochastic identification of elastic parameters of thin plates using Lamb waves monitoring is proposed. The procedure is based on the comparison of numerical and experimental dispersion curves. The identification results are then presented in the form of a posterior probability density distribution over elastic parameters and the posterior describes the uncertainty. The proposed procedure is verified on an example of pseudo-experimental dispersion curves computed for a thin orthotropic plate.

2. Identification algorithm

We formulate the sequential identification problem as a Bayesian state estimation problem. The elastic parameters are assumed not to change in time, so they are regarded time-independent state variables, see [4] for details. The main goal of Bayesian state estimation is sequential inference of the posterior distribution \( p(x_{k+1}|Y_{1:k+1}) \) starting from a prior distribution \( p(x_1|Y_1) \). The inference is performed recursively in two steps: prediction step and update (correction) step. In the first step the prediction of state variables distribution \( p(x_{k+1}|Y_k) \) before applying new measurements is done. This distribution is computed using the sum rule of probability and integrating out the state variables as

\[
p(x_{k+1}|Y_{1:k}) = \int p(x_{k+1}|x_k)p(x_k|Y_{1:k}) dx_k.
\]  

(1)

Then the new measurements \( y_{k+1} \) are used to update the prior to obtain the posterior distribution \( p(x_{k+1}|Y_{1:k+1}) \) applying the Bayes’ rule

\[
p(x_{k+1}|Y_{1:k+1}) = \frac{p(y_{k+1}|x_{k+1})p(x_{k+1}|Y_{1:k})}{p(y_{k+1}|Y_{1:k})},
\]

(2)

where the denominator in (2) is computed from

\[
p(y_{k+1}|Y_{1:k}) = \int p(y_{k+1}|x_{k+1})p(x_{k+1}|Y_{1:k}) dx_{k+1}.
\]

(3)

The update step in Eq. (2) can be also written in the recursive form that is more useful for obtaining particle filter algorithm. Using Bayes’ rule we can rewrite Eq. (2) as

\[
p(x_{k+1}|Y_{1:k+1}) = p(x_k|Y_{1:k}) \frac{p(y_{k+1}|x_{k+1})p(x_{k+1}|Y_{1:k})}{p(y_{k+1}|Y_{1:k})}.
\]

(4)

The Bayesian state estimation described above gives the posterior distribution over the states. It does not give however, the way to find the solution efficiently using both equations (1) and (2). In addition, the exact inference is intractable and an approximate method has to be applied. In this work a particle filter (PF) algorithm is used. It is based on sequential Monte Carlo sampling and described in [3, 4].

3. Numerical experiments

The effectiveness of the proposed method is assessed performing numerical exercises for an orthotropic plate. The properties of the plate (Young’s moduli, Poisson’s ratios and mass density) and the plate thickness, applied in the experiments, are presented in Fig. 1.

Having defined the plate parameters, a pseudo-experimental fundamental antisymmetric dispersion curves \( \lambda_0 \) were computed using numerical approach described in [5]. In this approach it is assumed that the dynamic problem is formulated as a plain strain problem and solved by numerical simulations via commercial finite element code Abaqus in a few series of modal analyzes. The numerical model in Abaqus for a 30mm by 1.2mm plate segment has 3600 square CPE4 elements with characteristic length \( l_e = 0.1 \text{mm} \). These dispersion curves were approximated using basis functions and corresponding parameters found with the
least square method. The parameters were treated as the observed variables $y$. Fig. 1 presents fundamental dispersion curve $A_0$ for the orthotropic plate and its approximation using 5 basis functions proposed in [2].

![Figure 1: Fundamental dispersion curve for orthotropic plate with thickness $h=1.2mm$ and its approximation using 5 basis functions](image)

Initial and uncertain knowledge about Young’s modulus $E_1$ is represented by a prior distribution $p(x_0)$. We applied a Gaussian prior probability density distribution $p(x_0) = N(\mu_0, \sigma_0^2)$, with mean value $\mu_0 = 130.0$ GPa and standard deviation $\sigma_0 = 1.3$ GPa (coefficient of variation (CoV) was 1%). Fig. 2 shows the prior.

![Figure 2: Prior and posterior distributions for Young's modulus](image)

The approximate posterior distribution of Young’s modulus given pseudo-experimental dispersion curves $P_N(x_k|y_k)$ in the $k$-th step was computed using the particle filter-based identification procedure described above. In experiments, we applied $N = 2000$ particles to obtain the approximate posterior distribution and the number of steps in the sequential identification was set to $K=500$. The posterior has mean value $\mu_{\text{post}} = 131.0$ GPa and standard deviation $\sigma_{\text{post}} = 0.19$ GPa (CoV is 0.15%). Fig. 3 shows the sequential nature of the elastic constant identification process by plotting the evolution of the mean value of the posterior distribution and the corresponding plot for the one-standard deviation error bars as a function of the step number. There is also shown a solid horizontal line representing the reference Young’s modulus value (131.0 GPa) applied in numerical experiments. From the plot, it may be observed that the estimation process converged to the reference value quite rapidly.

![Figure 3: Plot of evolution of mean value of the posterior](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean value (GPa)</td>
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<td>131.0</td>
</tr>
<tr>
<td>Standard deviation (GPa)</td>
<td>1.3</td>
<td>0.19</td>
</tr>
<tr>
<td>COV (%)</td>
<td>1.0</td>
<td>0.15</td>
</tr>
</tbody>
</table>

4. Final conclusions

This paper presents an application of Bayesian methods and particle filter for reconstruction of elastic parameters of plates. The proposed procedure is based on the comparison of experimental and numerical dispersion curves from guided Lamb waves monitoring. Taking into account the assumed experimental errors and considering propagation of errors in the sequential estimation, the uncertainty in the identified value of Young’s modulus $E_1$ is less than 0.5%. More results for other elastic parameters will be presented during the conference.

References


Identification of material parameters in thin elastic plates: 
basic problems of neural networks and Lamb waves applications

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Abstract
The paper deals with numerical problems corresponding to the formulation of Hybrid Computational Systems (HCS). The systems are completed of two complementary parts: i) Part A is related to the analysis of direct (simulation) problems, ii) Part B corresponds to reverse (identification) problems. Lamb waves technique is applied in Part A. The inverse problem is analyzed by Artificial Neural Networks (ANNs) trained ‘off line’. The novelty of the presented paper is the separation of Parts A and B by means of a set of internal parameters. Numerical problems related to the proposed approach are discussed and illustrated by the identification of parameters for isotropic and composite plates.

Keywords: hybrid computational systems, Lamb waves, artificial neural networks, identification of thin plate parameters

1. Introduction
A new research of science and technology, called Structure Health Monitoring (HSM), has been increasingly developing for some time [1]. An important role in SHM plays systems, which on the base monitoring or measurements can reflect the actual state (health) of structures. This permits control of the structure and warning against failure or dangerous events. Non-destructive methods of structure examination and ‘on line’ methods of information transmission are especially valuable for aspects of SHM.

It was proved in the analyses of many engineering problems, that the Artificial Neural Networks (ANNs) can be efficiently applied in the analysis of reverse (identification) problems [2]. This feature is complementary to direct (simulation) features of other basic numerical methods, e.g. FEM or a number of experimental laboratory tests or measurements on real structures.

From the point of view of SHM especially interesting is formulation of Hybrid Computational Systems (HCS), composed of three components, i.e. data from tests, their processing and neural identification. Thus, HCS can play the role of an original non-destructive method. This is a part of a research grant *.

Basic components of HCS are experimental data related to propagation of ultrasonic waves in solid media. A special kind of these waves, called Lamb Waves (LWs), was selected as a good tool for the evaluation of material properties and detection of various defects, cf. references in extensive literature, e.g. see [6].

2. Development of HCS at RGNN and SemNN
At the turning of 1995/96, an informal Research Group on Applications of NN in Civil Engineering (called RGNN for short) was organized at the Institute on Computer Methods in Civil Engineering of Cracow UT. The standing Seminar (SemNN) was also organized as accompanying part of the RGNN. This activity attracted participants from eight Polish TUs; see references in [3].

During of more than fifteen years of the RGNN and SemNN activity, a great attention was focused to the analysis of inverse problems since just in such topics the ANNs occurred to be a new, numerically efficient tool. The corresponding research and engineering applications needed formulation of different HCSs. They were developing by RGNN participants, especially from Cracow, Rzeszów and Zielona Góra TUs., see [3]. It is worth mentioning PhD theses by B. Miller, J. Kaliszuk and P. Nazarko as well as dissertations written for position of Assoc. Prof. by W. Lakota, E. Pabisek and B. Miller.

The most fruitful was the time 2013-15 when the grant (*) was under development. One of the main topics of the grant concerned the research activity on a new HCS, discussed in the presented paper. In this field, the cooperation with L. Ambroziszki and P. Pačko from AGH Krakow was unavailable.

3. Modification of the Four Essential Steps algorithm
A flow-chart algorithm, discussed in [1], is a scheme of a general algorithm for the identification of damage in thin plates, see, Figure 1. This algorithm was modified in [4] to have a full correspondense to the proposed new HCS. The flow-chart was divided into two Parts A and B. The Part A is completed of three Essential Steps, related to the direct analysis. The Part B contains an extended Essential Step IV.

4. Novelties of the proposed HCS
4.1. HCS for identification of isotropic plates
It was assumed that identified plates are homogenous, of constant thickness \( d = 2h \) and density \( \rho \) made of elastic material with two mechanical parameters \( E \) and \( \nu \).

The main attribute of the problem is the Lamb dispersion curve (LDC). It is a dependent variable \( k(f)\) : where: \( k \) – wave number measured in \([m^{-1}]\), \( f \) [MHz ] – frequency of vibrations, \( \text{spar} \) – selected set of independent variables.

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Figure 1: Four essential steps for identification of plate parameters applying LCS and ANN

The main idea of the separation lies in the possibility to “learn” both Part A and B in the ‘off line’ mode. Then, the internal parameter set $\alpha$ is introduced in order to transmit information from the direct to the reverse part of HCS.

The separation of Parts A and B leads to “numerical on line” approach. Instead of the classical approach, minimizing the “distance” between the experimental and Lamb DC $k_{\text{exp}}^\text{exp}$, only one substitution approach is applied. This means that the values of the internal parameters $\{\alpha_j\}$, computed as outputs of Part A are inputs to the ‘off line’ trained ANN in Part B.

In such a way, the commonly used classical approach, which needs the application of different numerical methods, e.g. FEM BEM FDM and other special methods, can be eliminated.

Other novelties in [4] correspond to the introduction of approximate dimensionless LDC equations.

The proposed approach was examined by some case studies, using experimental tests, as well as pseudo-experimental and noisy data.

4.2. Identification problems for composite plates

The isotropic material discussed above has two independent material parameters $E$ and $\nu$. In the grant (*) we started from the hexagonal orthotropy with five independent stiffness parameters $C_{ij}$. Such a model of material well corresponds to the uniaxially reinforced composite lamina.

It was proved in paper [5], which is a generalization of [4], that the separation of Parts A and B in a corresponding HCS is also possible for anisotropic materials.

One of the main problem are very different and comparatively small values of stiffness parameters. For instance, the stiffness corresponding to tension of composite reinforcement $C_{11} \approx 130$ MPa vs. $C_{44} \approx 4.0$ MPa for shear. Moreover, such low values of parameters are measured with about 20% accuracy, see [6].

Another important question is the sensitivity of stiffness to variations of parameters. This feature strongly influences generation of patterns for the ANNs trained ‘off line’.

Unlike the isotropic material, where only one LDC can be used for the mode $A_0$, see [4], in case of the above mentioned composite material we have to apply two modes $A_0$ and $B_0$, see Figure 2. The uniqueness of the stiffness evaluation demands three independent guided directions, e.g. with inclinations from the direction of reinforcements which are $\alpha = 0^\circ, 90^\circ$ and $45^\circ$.

Such problems are discussed in [5], and now they are under development in grant (*)

References

Damage detection and evaluation in GFRP strip based on elastic wave propagation and support vector machines classification

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Abstract

An idea of damage detection and evaluation in glass fiber reinforced polymer (GFRP) was proposed in the paper. The approach uses the phenomenon of elastic wave propagation and multi-class classification using Support Vector Machines (SVM). The set of laboratory experiments was performed in GFRP strip with bonded piezoceramics transducers and two various types of damage were analyzed. The damage caused by the influence of high temperature was considered in 4 cases differing in the area of damaged surface, while the impact damage was consider in 5 cases. The proposed approach may be an efficient tool for monitoring of composite structures or their members especially for aircraft industry but also for a civil infrastructure. The results presented in the paper proved usefulness of proposed approach.

Keywords: damage detection, glass fiber reinforced polymer, elastic wave propagation, support vector machines

1. Introduction

The dynamic development of sciences related to the design and manufacturing of composite materials influenced their prevalence in many areas of human activity, even where the priority is the safety of structure users. Hence there is a need to design, develop and improve the diagnostics systems oriented on application in composite structures, taking into account the its complexity [1]. It should be emphasized that the inhomogeneity of composite materials in general impedes efficient detection and identification of damages. In addition, developing a system to diagnose the state of composite structures should reflect the diversity of damage occurring in such structures. The most frequent defects are delamination, thermal and chemical damage of the composite matrix.

Among the developed non-destructive techniques (NDT) for damage detection and identification the methods based on the phenomenon of elastic wave propagation are widely used, also in the composites structures [2]. The use of elastic wave propagation phenomenon allows to monitor large structures, the hidden and inaccessible areas or even operating machines.

2. Laboratory experiments

2.1. Model and laboratory equipment

The laboratory specimen was a GFRP strip with permanently bonded piezotransducers (Fig. 1). The strip dimensions were equal to 600 x 25 x 1.4 mm. The "pitch-catch" method was used. The function wave generator TTi TG1010 was used to involve the elastic waves. Next, the signal was amplified by linear amplifier EPA-104 (Piezo System, Inc.) and introduced directly to the piezoceramic element. Structure response at control point no. 2 was then amplified by charge amplifier and signal conditioner (422E12, PCB Piezotronics, Inc.). Next, the received wave was recorded by a digital oscilloscope LeCroy.

2.2. Damage cases

Two various damage types were considered. The first damage was caused by the influence of high temperatures. It was realized by placing a defined area of the sample in a stream of hot air (240°C) for approx. 10 minutes. Four sizes of faulted areas were analyzed. Second damage was caused by the point impact. In this case, 5 sizes of defect was considered. The rates of damages were chosen in a such way that the failures were invisible on the revers side of the specimen.

3. Signal data processing

As an excitation signal modulated 4 cycles of sin wave impulse was used. The excitation was induced with operating frequency equal to 72 kHz.

Fig. 2: Laboratory specimen and setup

An example of registered signal was shown on Fig. 2. The section of the signal which was analyzed defines vertical dashed lines. It was not possible to separate wave reflected form the damage. Damage presence cause rather changes in the wave propagated across the discontinuity.
For each analyzed state 225 response signals were recorded. Due to the fact that measurements were not fully-repeatable and the recorded signals were noised averaging of samples were performed. Two ways of averaging were used: n following samples, \( n \) random samples. This led to a reduction in the number of patterns.

As a signal parameters time-of-flight (ToF), maximum amplitude (A) and energy of signal (E) were extracted.

### 4. Damage detection and evaluation

Three cases of inverse problem were considered. In order to identify specimen state Support Vector Machines Binary Decision Tree (SVM BDT) [3] was utilized. As the input vectors various combination of signal parameters ToF, A, E were used. SVMs with radial basis kernel functions were used.

For thermal damage 4-level classification was performed. 5 states of specimen were analyzed: undamaged (unD) and 4 sizes of damage D1, D2, D3 and D4. At first stage of BDT a structure can be classified as undamaged (unD) or damaged (D1, D2, D3, D4). Next the damage patterns (D1, D2, D3, D4) can be split to related to the specimen with smallest damage D1 and the others (D2, D3, D4). In the last step, the specimen with two largest (D3, D4) damages were separated.

As a measure of the accuracy of the classification correct rate (CR) parameter were used. CR was calculated as a the ratio of the number of correctly classified patterns to the total number of classified patterns.

The results of thermal damage classification are shown in Fig. 3 where classification correct rates were calculated for classification based on 3-element input vector.

A satisfactory classification effectiveness was achieved for levels 1-3 as well for 2- and 3-elements input vector while for level 4 it was necessary to use energy parameter E.

Moreover, for levels 1-3 averaging of 3 samples allows to achieve over 95 % accuracy of classification. In case level 4, such accuracy require averaging of 7 samples.

In case of impact damage, state D4 from previous task was assumed as initial undamaged state. Besides 5 damaged cases (D1 to D5) were considered. The SVM classifiers were used to perform 5-level classification.

Results in form of CR parameter are shown in Tab. 1.

<table>
<thead>
<tr>
<th>Level</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR</td>
<td>0.71</td>
<td>0.77</td>
<td>0.77</td>
<td>0.83</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.81</td>
<td>0.83</td>
<td>0.82</td>
<td>0.83</td>
<td>0.84</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>0.82</td>
<td>0.91</td>
<td>0.94</td>
<td>0.97</td>
<td>0.97</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>0.69</td>
<td>0.69</td>
<td>0.68</td>
<td>0.67</td>
<td>0.69</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>0.64</td>
<td>0.64</td>
<td>0.70</td>
<td>0.79</td>
<td>0.78</td>
<td>0.82</td>
</tr>
</tbody>
</table>

It can be stated here that all levels of classification were performed with over 80 % efficiency with the expection of level 4 where CR did not achieve 70 %.

In the third case of inverse problem 3 states of specimen were considered: undamaged (unD), thermal damage (D I) and thermal and impact damages simultaneously (D II). At first level patterns related to undamaged and damaged and were separated. Then cases with one damage were split to those connected with 2 damages. Results of 2-level classification are collected in Tab. 2.

<table>
<thead>
<tr>
<th>Level</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>CR</td>
<td>0.95</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>0.65</td>
<td>0.67</td>
<td>0.67</td>
<td>0.70</td>
<td>0.72</td>
<td>0.76</td>
</tr>
</tbody>
</table>

Damaged state was detected almost perfectly while separation pattern related with one damage and two damages was performed with 75 % accuracy.

### 5. Conclusions

The performed tests proved that damage detection and evaluation is possible with the use of elastic wave propagation phenomenon. Patterns classification using SVMs BDT exhibit to be efficient tool in damage detection and evaluation. The presented approach can be the part of the automated system of damage detection and evaluation.

### References


MS23

Theoretical, Computational and Experimental Mechanics for Coupled Field Problems and Multiphase Materials

organized by C.S. Drapaca, S. Hartmann, J. Leszczyński, S. Sivaloganathan and W.Sumelka
Derivation and numerical solution of fractional Euler-Bernoulli beam equation

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Abstract

In the paper a new formulation of the Euler-Bernoulli beam equation, based on the fractional calculus, is proposed. The fractional Euler-Bernoulli beam equation is derived using a variational approach. Such formulation leads to an equation containing the left and right fractional Caputo derivative, simultaneously. The obtained equation is solved numerically. Finally, an illustrative example is shown.

fractional differential equation, numerical solution, Euler-Bernoulli beam equation, Caputo derivatives

1. Introduction

Fractional differential equations have recently played a very important role in various fields of science. It is caused largely by the fact that the fractional derivatives are non-local operators and depend on the past values of a function (the left derivative) or the future values of a function (the right derivative) [2].

In this manuscript, a procedure is proposed for constructing the fractional Euler-Bernoulli beam equation utilising the fractional Riesz-Caputo derivatives. In this paper we propose a new formulation of the Euler-Bernoulli beam equation based on the fractional variational calculus and then we present a numerical scheme for the solution of such equation.

2. Fractional operators

Here, we only recall necessary definitions of fractional operators (see [2]). The left and right Caputo derivatives of order \(1 < \alpha \leq 2\) are defined as follows

\[
C_{D+}^\alpha u(x) := \left\{ \begin{array}{ll}
\frac{1}{\Gamma(\alpha)} \int_{0}^{x} \frac{u(\tau)}{(x-\tau)^{1-\alpha}} d\tau & \text{for } 1 < \alpha < 2 \\
\frac{D^2 u(x)}{D^2} & \text{for } \alpha = 2
\end{array} \right.
\] (1)

\[
C_{D-}^\alpha u(x) := \left\{ \begin{array}{ll}
\frac{1}{\Gamma(\alpha)} \int_{x}^{L} \frac{u(\tau)}{(\tau-x)^{1-\alpha}} d\tau & \text{for } 1 < \alpha < 2 \\
\frac{D^2 u(x)}{D^2} & \text{for } \alpha = 2
\end{array} \right.
\] (2)

where \(D^2\) is operator of the second order derivative and operators \(I_{D+}^\alpha\) and \(I_{D-}^\alpha\) are respectively the left and right fractional Riemann-Liouville integrals of order \(\alpha > 0\) defined by

\[
I_{D+}^\alpha u(x) := \frac{1}{\Gamma(\alpha)} \int_{0}^{x} \frac{u(\tau)}{(x-\tau)^{1-\alpha}} d\tau \quad (x > 0)
\] (3)

\[
I_{D-}^\alpha u(x) := \frac{1}{\Gamma(\alpha)} \int_{x}^{L} \frac{u(\tau)}{(\tau-x)^{1-\alpha}} d\tau \quad \text{for } (x < L)
\] (4)

where \(\Gamma\) is the Euler Gamma function.

3. Mathematical model and numerical solution

In classical mechanics, the minimization of the potential energy for a supported beam of length \(L\) with a downward transverse load per unit length \(f(x)\), requires that the functional

\[
V = \int_{0}^{L} F(x, u, u’) dx
\]

\[
= \int_{0}^{L} \left[ \frac{1}{2} EI (u’(x))^2 - f(x) u(x) \right] dx
\] (5)

be minimized, where \(EI\) is the bending stiffness, which is constant, and \(u(x)\) is the static deflection of the beam.

The individual terms \(\frac{1}{2} EI (u’(x))^2\) and \(f(x) u(x)\) represent potential (strain) energy due to bending and potential energy due to the lateral deflection respectively. The boundary conditions for the supported beam look as follows

\[
u(0) = u'(0) = u(L) = u'(L) = 0
\] (6)

The corresponding Euler-Lagrange equation for the considered problem has the form

\[
\frac{\partial F(x, u, u’)}{\partial u} + \frac{d^2}{dx^2} \left( \frac{\partial F(x, u, u’)}{\partial u’} \right) = 0
\] (7)

which leads to the Euler-Bernoulli beam equation

\[
EI \frac{d^4 u(x)}{dx^4} - f(x) = 0
\] (8)

In this manuscript, a procedure is proposed for constructing the fractional Euler-Bernoulli beam equation utilising the fractional variational calculus. We start from the functional (5) and we replace the second order derivative by the left Caputo derivative (1) in the following way

\[
V_{frac} = \int_{0}^{L} F_{frac}(x, u, C_{D+}^\alpha u) dx
\]

\[
= \int_{0}^{L} \left[ \frac{1}{2} EI \left( c^{\alpha-2} C_{D+}^\alpha u(x) \right)^2 - f(x) u(x) \right] dx
\] (9)
Next, using results presented in [3] we get the corresponding form of the fractional Euler-Lagrange equation for the problem (9), which has the following form
\[
\partial F_{free} (x, u, C D^{\alpha}_{0+} u) + C D^{\alpha}_{L-} \left( \partial F_{free} (x, u, C D^{\alpha}_{0+} u) \right) = 0
\]
and leads to the fractional Euler-Bernoulli beam equation
\[
\ell^{2(\alpha - 2)} E I C D^{\alpha}_{L-} C D^{\alpha}_{0+} u (x) - f (x) = 0
\]
where \( \ell \) is a length scale [4].

3.1. Numerical scheme

In this section a numerical scheme for Eq. (11) is presented. We introduce the homogeneous grid of \( n + 1 \) nodes with the constant time step \( \Delta x = L/n; 0 = x_0 < x_1 < \ldots < x_i < \ldots < x_n = L \), and \( x_i = i \Delta x, i = 0, 1, \ldots, n \).

In our previous work [1] we determined the discrete form of the composition of the left and right fractional Caputo derivative of order \( \alpha \in (1, 2] \). On the basis of these results, we present the final discrete form of the problem (11) and (6)
\[
\begin{align*}
  u_0 &= 0 \\
  u_1 &= 0 \\
  \sum_{j=1}^{n+1} v (n - i, n - j) \sum_{k=-1}^{j+1} v (j, k) u_k &= f (x_i) / \beta
\end{align*}
\]
where \( \beta = \ell^{2(\alpha - 2)} E I \), and coefficients \( v \) look as follows
\[
v (i, j) = \frac{\Delta x^{-2\alpha}}{1 - (-1)^{i+j}}
\]
\[
\begin{array}{ll}
  (i - 1)^{3-\alpha} & \text{for } j = -1 \\
  -(i + \alpha - 3) i^{2-\alpha} & \text{for } j = 0 \wedge i = 1 \\
  2a - 3 & \text{for } j = 0 \wedge i = 1 \\
  i^{3-\alpha} - 4 (i - 1)^{3-\alpha} & \text{for } j = 0 \wedge i = 1 \\
  (i - 2)^{3-\alpha} & \text{for } j = 1 \wedge i = 2 \\
  +2 (i + \alpha - 3) i^{2-\alpha} & \text{for } j = 1 \wedge i = 2 \\
  2 - \alpha & \text{for } j = 1 \wedge i = 1 \\
  6 - (\alpha - 1) 2^{\alpha} & \text{for } j = 1 \wedge i = 2 \\
  -2 i^{2-\alpha} & \text{for } j = 1 \wedge i = 2 \\
  -2 i^{2-\alpha} & \text{for } j = 1 \wedge i = 2 \\
  -2 (i - 1)^{3-\alpha} & \text{for } j = 1 \wedge i = 2 \\
  -6 (i - 2)^{3-\alpha} & \text{for } j = 1 \wedge i = 2 \\
  +i (i - 3)^{3-\alpha} & \text{for } j = 1 \wedge i = 2 \\
  -i (i + \alpha - 3) i^{2-\alpha} & \text{for } j = 1 \wedge i = 2 \\
  (i - j + 2)^{3-\alpha} & \text{for } j = 2, \ldots, i - 2 \\
  -4 (i - j + 1)^{3-\alpha} & \text{for } j = 2, \ldots, i - 2 \\
  +6 (i - j)^{3-\alpha} & \text{for } j = 2, \ldots, i - 2 \\
  -(i - j - 1)^{3-\alpha} & \text{for } j = 2, \ldots, i - 2 \\
  -4 i^{2-\alpha} & \text{for } j = i - 1 \\
  3^{2-\alpha} - 4 - 2^{2-\alpha} + 6 & \text{for } j = i - 1 \\
  2^{2-\alpha} - 4 & \text{for } j = i + 1
\end{array}
\]

One can observe that for the calculation of values \( u_0, u_1, \ldots, u_n \) we need to solve the system of \( n + 1 \) linear equations.

3.2. Example of computations

On the basis of the numerical scheme presented in the previous section we implemented an algorithm in Maple and carried out computational simulations for different values of parameters \( \alpha \) and \( \ell \). We applied the LU decomposition method in order to solve the system (12). In all presented examples we assumed \( L = 1, f^* (x) = -1 \) (where \( f^* (x) \) denotes \( f (x)/EI \)), and \( \Delta x = 0.002 \). Numerical results are presented in Fig. 1 and 2.

Figure 1: The comparison of the static deflections of the beam for the length scale \( \ell = L/2 \) and the order of fractional derivatives \( \alpha = 1.9 \) and the length scale \( \ell \in \{ L/2, L/4, L/6, L/8, L/10 \} \)

Figure 2: The comparison of the static deflections of the beam for the order of fractional derivatives \( \alpha = 1.9 \) and the length scale \( \ell \in \{ L/2, L/4, L/6, L/8, L/10 \} \)

4. Conclusions

In the paper a new formulation of the Euler-Bernoulli beam equation using fractional calculus is presented. The obtained description is non-local. The influence of length scale \( \ell \) and of the order of fractional derivatives \( \alpha \) are illustrated by the analysis of the solution of the problem.

References


High-temperature deformation of polysynthetically twinned crystals of TiAl – numerical modeling of yield point

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Abstract

In the contribution, a crystal plasticity-based model is presented which captures the yield stress of polysynthetically twinned crystals of TiAl from room temperature up to elevated temperatures around 900°C. In this temperature range, the temperature dependence of material parameters as well as the coupling between thermal and mechanical quantities have to be considered. While the latter is known from standard thermodynamic arguments, it is not straightforward to sufficiently capture the temperature dependent behavior of polysynthetically twinned crystals in the parameters of the constitutive model. A respective set of temperature-dependent model parameters is selected to consider the influence of different microstructural lengths within polysynthetically twinned crystals and to account for the typical yield stress temperature anomaly, observed in intermetallic compounds. The presented model captures the plastic anisotropy of a single polysynthetically twinned crystal as a function of temperature including the temperature anomaly and the microstructural strengthening by different phase boundaries. The simulation results are in good agreement with the experiments in literature.

Keywords: thermomechanical modeling, crystal plasticity, TiAl, polysynthetically twinned crystal, Hall-Petch relation

1. Introduction

In the context of high-temperature lightweight construction, TiAl alloys are well-known for their superior thermomechanical properties [1]. To make best use of their advantages over, e.g., Nickel-based alloys, a reliable prediction of their yield strength especially at preferred operating temperatures is crucial. A respective model has to consider several restrictions and interrelations arising from the complex micromechanics of the here studied fully-lamellar TiAl alloys.

2. Micromechanics of polysynthetically twinned crystals

In mesoscale, TiAl alloys with a fully-lamellar microstructure are composed of the so-called colonies of parallelly aligned lamellae. This lamellar compound is characterized by various phase boundaries between the occurring phases which cause a complex micromechanical behavior. To investigate the micromechanical interactions in the lamellar compound, a single set of parallel aligned lamellae, the so-called polysynthetically twinned crystal, is best suited. As a basic component of the microstructure, the behavior of a polysynthetically twinned crystal strongly influences the mechanical behavior of the alloy. A schematic drawing of a polysynthetically twinned crystal is given in Fig. 1.

2.1. Lattice structure

Polysynthetically twinned crystals exhibit a strong orientation relation between the hexagonal lattice of the α\textsubscript{2}-phase and the tetragonal lattice of the γ-phase which is the cause for the strict parallel alignment of the lamellae. In this orientation relation, the γ-phase may occur in six different rotation variants forming the so-called domains which are separated by grain boundary-like interfaces.

2.2. Plastic anisotropy

The interfaces between lamellae with spacing λ\textsubscript{L} and domain boundaries with a mean diameter λ\textsubscript{D} act as strong barriers for dislocation motion. As a result, polysynthetically twinned crystals exhibit a strong plastic anisotropy, i.e., the yield stress is varying with the angle between loading direction and lamellar plane. Under load angles near 90° and 0° the yield stress is 5 to 7 times higher than at intermediate load angles around 45°.

2.3. Yield stress temperature anomaly

Besides strengthening by phase boundaries, the yield stress of polysynthetically twinned crystals – and therefore their critical resolved shear stresses – exhibits a distinct temperature dependence. As observed in other intermetallic compounds, the yield stress of polysynthetically twinned crystals rises with
temperature up to a peak usually located between 500°C and 800°C. After this peak, the yield stress decreases rapidly.

3. **Model**

A modeling approach for the plastic behavior is based on crystal plasticity. The elastic deformation is modeled in terms of the Helmholtz free energy density according to the hyperelastic framework. The thermomechanical coupling is conducted as shown in [2].

3.1. **Plastic anisotropy**

Anisotropic flow behavior of crystalline structures is well described by crystal plasticity. In the case of polysynthetically twinned crystals, single phases can be modeled using crystal plasticity but the effect of the lamellar and domain boundaries on the yield stress is not considered intrinsically in this formulation. Therefore strengthening by these boundaries is introduced into the model via the critical resolved shear stresses using a Hall-Petch type relation. Following the ideas in [3], the different slip and twinning mechanisms are classified into three morphological classes so that only three critical resolved shear stresses have to be determined in function of microstructural length.

3.2. **Temperature anomaly**

The temperature dependence of the basic material parameters e.g. Young’s modulus is taken from literature. The temperature yield stress anomaly, however, is incorporated into the critical resolved shear stresses of the crystal plasticity model. To model the anomaly in the critical resolved shear stresses as a continuous function of temperature, a sinusoidal power law is used. The fitting parameters in the respective power law are determined from experiments with polysynthetically twinned crystals [4] and α2 single crystals [5].

4. **Results**

The simulations are carried out with a RVE of a polysynthetically twinned crystal subjected to periodic boundary conditions. Comparison of simulation results to literature shows a good agreement for the investigated range of temperatures and microstructural parameters [6].

4.1. **Plastic anisotropy**

As shown in Fig. 2, the model captures the plastic anisotropy in good correlation with the experimental results of several authors for different lamellar thicknesses \( \lambda_l \) and domain sizes \( \lambda_D \).

4.2. **Temperature anomaly**

The temperature dependence of the simulated yield stress reflects the characteristics of experimental curves for the three distinctive load angles 0°, 45° and 90° very well (cf. [6]). The peaks of the anomalous yield stress curves correspond well to experimental findings.

4.3. **Hall-Petch strengthening**

A strong influence of the microstructural lengths on the yield stress is captured by the model for a wide range of \( \lambda_l \) and \( \lambda_D \). The temperature dependence of the respective Hall-Petch parameters is captured in the range of experimental scattering (cf. [6]).

![Figure 2: Room temperature yield stress of polysynthetically twinned crystals as function of load angle under uniaxial compression. Comparison with experiments.](image)

5. **Conclusions**

The presented model may be used to predict the yield stress of polysynthetically twinned TiAl crystals in a wide temperature range and for a wide range of microstructural parameters. Although formulated for a polysynthetically twinned crystal, the model represents a good starting point for thermomechanical analyses of realistic TiAl microstructures under operating conditions.

**References**


Theory and Numerics of Monolithic and Partitioned Thermo-Mechanical Coupling

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Abstract

The mechanical behavior of materials is mainly influenced by the applied load and thermal agencies. In this case, the constitutive equations, which determine the stress state, depend on the temperature as well, and the temperature distribution is influenced by the deformation and by internal dissipation. This is modeled by the balance of linear momentum, the heat equation and by a set of constitutive equations of rate-type. In this presentation, we show the numerical treatment of this coupled system within the context of the method of vertical lines. For the spatial discretization, we draw on h- or p-version finite elements, for the time discretization the resulting system of differential-algebraic equations is solved using high-order, diagonal-implicit Runge-Kutta methods. One particular more general method turns out to be the Backward-Euler method, which is commonly applied. The resulting coupled system of these implicit schemes yield in every point in time a system of non-linear equations, which can be computed by various schemes, for example, the Newton-Raphson method and its variants or by a non-linear Block-Gauss-Seidel scheme (BGS). Within the latter approach partitioned solution methods can be treated, their convergence is investigated. In this context, Aitken-relaxation and Quasi-Newton methods are applied. Numerical examples are considered to study the theoretical and numerical approach and compare the monolithic and the partitioned approach.

Keywords: thermo-mechanical coupling, finite elements, time integration, monolithic approach, partitioned approach

1. Introduction

Thermo-mechanical coupling has been treated for a long time, see, for example, [1, 2, 7, 8, 14], and the literature cited therein. There are, however, different approaches coupled in a greater or less extent. We propose a canonical approach which is based on the vertical method of lines. This method states that the entire discretization of a boundary-value problem is carried out within two steps. In the first step, the spatial discretization is carried out, in the second step the time-discretization is performed.

Constitutive models of rate-type are applied, where internal variables evolve in order to describe the hardening behavior of the material. Commonly, these are systems of ordinary differential equations of the first order. They determine the stress state due to the balance of linear momentum. In the latter we assume quasi-static processes, so the wave propagation is not considered. This partial differential equation is discretized using finite elements, where one can apply low-order linear or quadratic (h-elements), mixed, or even high-order finite elements (p-version finite elements). Within the latter approach the integrated Legendre polynomials are applied. [4]. The result of the discretization is a system of non-linear equations. The same approach is applied to the non-stationary, non-linear heat equation. The result of the discretization is a system of ordinary differential equations,

\[ g_n(t,u,\Theta,q) = 0, \]
\[ q(t) = r_0(t,u,\Theta,q), \]
\[ C_{\Theta}(t,u,\Theta) \dot{\Theta}(t) = r_\Theta(t,u,\Theta,q), \]

where \( u \) defines the unknown displacement degrees of freedom, \( q \) all internal variables evaluated at all Gauss-points of the entire structure, \( \Theta \) the unknown temperature degrees of freedom, and \( t \) the time. \( g_n \) represents the discretized principle of virtual displacements, \( r_0 \) the evolution equations of the internal variables, \( r_\Theta \) the heat equation, and \( C_{\Theta} \) the heat capacity matrix, which can depend on the temperature and deformation as well. This system has to be closed by consistent initial conditions, \( u(t_0) = u_0 \), \( q(t_0) = q_0 \), and \( \Theta(t_0) = \Theta_0 \).

2. Time-Discretization

As shown in [5], or in a more generalized way in [10], high order time-integration schemes can be applied to the system of differential-algebraic system (2) (DAE-system), where accordingly, diagonally-implicit Runge-Kutta methods (DIRK) turn out to be a very similar algorithmic structure as existing Backward-Euler based implementations. Thus, the presentation is restricted to a Backward-Euler method. However, it can easily be extended to DIRK-methods. The application of the integration step yields

\[ \Theta_{n+1} = \Theta_n - \frac{\Delta t}{C_m} q_n, \]
\[ Q_n = Q_{n+1} - \frac{\Delta t}{C_m} q_n, \]
\[ u_{n+1} = u_n + \Delta t \left( \frac{C_m}{C_m} \Theta_{n+1} - \Theta_n \right), \]

where \( C_m \) is the mass matrix. The above steps are valid for the non-stationary, non-linear heat equation. The resulting system of differential-algebraic equations is solved using high-order, diagonal-implicit Runge-Kutta methods. One particular more general method turns out to be the Backward-Euler method, which is commonly applied. The resulting coupled system of these implicit schemes yield in every point in time a system of non-linear equations, which can be computed by various schemes, for example, the Newton-Raphson method and its variants or by a non-linear Block-Gauss-Seidel scheme (BGS). Within the latter approach partitioned solution methods can be treated, their convergence is investigated. In this context, Aitken-relaxation and Quasi-Newton methods are applied. Numerical examples are considered to study the theoretical and numerical approach and compare the monolithic and the partitioned approach.

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representing a system of non-linear equations

\[ \begin{align*}
G_n(u, \Theta, q) &= 0 \\
G_{n+1}(u, \Theta, q) &= 0 \\
L_{n+1}(u, \Theta, q) &= 0
\end{align*} \]  

(7)

which has to be solved at each point in time (here, \( t_{n+1} \)). Here, we have omitted the time \( t_{n+1} \) and the index \( n + 1 \) for brevity.

3. Solution Procedures

The system of non-linear equations (7) has to be solved, where we assume two possibilities. The first is a monolithic approach, where in a single program the entire system (7) is solved using the Multilevel-Newton algorithm (MLNA). According to [9] the classical two-level procedure, where the internal variable computation is carried out on Gauss-point level, can be applied. On global level the discretized equilibrium conditions and the heat equations are solved, whereas on Gauss-point level the internal variables are computed.

Alternatively, one can apply two finite element programs with two different meshes. In this case the system (7) is solved using the BGS-method, where Eqn.\((7)_2\) is computed within one program, and Eqns.\((7)_{1,3}\) are calculated with the MLNA in another FE-program. This is the so-called partitioned approach. If the rate of iterations is one, it represents the classical staggered scheme. There are several issues to be addressed in the case of a partitioned approach. Firstly, the solution of each field has to be interpolated and projected onto the Gauss-points of the other field. Thus, a nearest neighbor search algorithm has to be applied. [11]. Secondly, the BGS-iterations have to be accelerated if the strength of the coupling requires. Acceleration schemes of the paper show what has been developed in the context of fluid-structure interaction. These are the Aitken-relaxation, see [11], and the Quasi-Newton method proposed in [3]. In the context of volumetric coupling of thermo-elasticity this is studied in [6].

First, we compare both schemes, i.e. the monolithic and the partitioned approach. Subsequently, the partitioned approach and both their acceleration techniques as well as the influence of the interpolation are studied. The constitutive models applied are proposed in [15] and the temperature-dependent quantities are based on the inquiry in [13].

References


Computer simulation of the effective viscosity in Brinkman’s filtration equation using the Trefftz method

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Abstract

The paper presents determination of the effective viscosity in Brinkman’s equation by a numerical simulation of an imaginary physical experiment with a viscometer. The model of a porous medium and the applied method of solution are very simple. In the idealized problem we consider axial flow through an infinite array of cylindrical rods. Assuming that the flow in such a porous medium is described by the Brinkman filtration equation, effective viscosity can be calculated as a function of porosity. In the paper, a relation between the volume fraction porosity and the effective viscosity is given for triangular and square arrays of rods.

Keywords: porous material, effective viscosity, Brinkman’s equation, Trefftz method

1. Introduction

For a slow incompressible fluid flow in porous media the Darcy’s equation is usually applied in the form:

\[ q = -\frac{k}{\mu} \nabla P \]  

(1)

where \( q \) is the macroscopic velocity, \( P \) is the pressure, \( \mu \) is the dynamic viscosity of the fluid, \( k \) is the permeability of the porous medium.

For a porous medium with a high porosity in the presence of a free fluid region, or for a wall-bounded porous medium, a more proper filtration equation is the Brinkman equation in the following form [2]:

\[ \nabla P = \frac{\mu}{k} q + \mu \nabla^2 q \]  

(2)

where \( \tilde{\mu} \) is the effective viscosity of the porous medium. As a matter of fact, this is the Stokes equation for creeping flow with the Darcy resistant term. In contrast to Darcy’s equation, Eqn (2) includes the term related to the viscous transfer of momentum. Validity or theoretical justification of the Brinkman equation was presented in papers [3,5,9,13,14,15,17]. In the original paper [2] Brinkman did not consider the filtration problem but the viscous force on a dense swarm of particles due to fluid flow. The proposed equation is used as the filtration equation and Brinkman’s paper has been cited over 800 times according to the ISI Web of Science database. Almost all authors using the Brinkman equation have supposed that the effective viscosity \( \tilde{\mu} \) is equal to the viscosity \( \mu \) of a pure fluid.

The number of papers, where the effective viscosity has been distinguished from the viscosity of pure fluids, is very limited. Brinkman [2] suggested a possible use of Einstein’s formula for a dilute suspension:

\[ \frac{\tilde{\mu}}{\mu} = 1 + 2.5\varphi \]  

(3)

where \( \varphi \) is the volume fraction of the skeleton of a porous medium (porosity is equal to 1 – \( \varphi \)). This formula suggests that the effective viscosity of the porous medium is greater than the viscosity of the pure fluid. Beyond any doubt, this is the case for a suspension. However, a rigid porous medium, whose skeleton is fixed, is not necessarily the same case. For the porous medium in a form of fixed spheres, Lundgren [9] obtained \( \tilde{\mu}/\mu \) as a function of the volume fraction. The value of \( \tilde{\mu}/\mu \) rises slightly above 1 as the volume fraction increases from zero. This function attains its maximum near \( \varphi = 0.2 \) and decreases rapidly when \( \varphi > 0.3 \). Koplic et al. [6] calculated the energy dissipation in extensional flow around an isolated stationary sphere, and determined the effective viscosity as

\[ \frac{\tilde{\mu}}{\mu} = 1 - 0.5\varphi. \]  

(4)

The value is less than for a pure fluid.

On the other hand, Kim & Russel [5], Martys et al. [10] and Givler & Altobelli [4] obtained the effective viscosity greater than the viscosity of pure fluids.

In 1967 Bevers & Joseph published a paper about the boundary condition between the porous and free fluid regions [1]. The authors investigated unidirectional flow in a channel adjacent to a porous medium, and the study indicated that the volume flux in the channel was enhanced by the presence of the porous medium. They proposed a boundary condition which allows to take into account this case. Taylor [16] also observed that the Beavers-Joseph boundary condition can be deduced as a consequence of what is now commonly called the Brinkman equation, although Taylor did not use this label. This idea was developed in detail by Neale & Nader [12]. They showed that the problem of flow in a channel bounded by a thick porous wall yields the same solution with the Brinkman equation as with the Darcy equation together with the Beavers-Joseph condition, provided that the constant \( \alpha \) in this condition is identified as \( \sqrt{\tilde{\mu}/\mu} \). Experimental or computational data for the constant \( \alpha \) can be used for determination of the effective viscosity. In experiments by Beavers & Joseph, this constant for a certain material was less than unity but for another material exceeded unity. The same computational results were presented in papers [7,8].

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The purpose of the paper is to determine the effective viscosity as a function of porosity by an imaginary physical experiment which is realized by a numerical simulation. For this reason, an idealized porous material is considered. Such a material is formed by parallel cylindrical rods arranged in a square or a triangular array. For a pure fluid, one method of the fluid viscosity measurement is to analyse shear flow in a viscometer (the concentric-cylinder viscometer). A similar flow is simulated numerically in the paper. The porous region is placed between two parallel plates (Fig. 1). One plate is fixed whereas the other moves with a constant velocity. The determination of the effective viscosity is possible by calculation of the shear stress on the moveable plate. The microstructural shear flow problem is solved by means of the Trefftz method. Velocity of the flow between the parallel plates is approximated by a linear combination of special purpose Trefftz functions [11]. Numerical results are presented in Fig. 2. According to our calculations, in all the considered cases the effective viscosity in the Brinkman equation is less than the pure fluid viscosity.

References


Determination of Elastic Material Parameters in Higher-order Continua Based on Size-Dependent Bending Behavior of Epoxy and SU-8

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Abstract

In the context of the static theory of elasticity for isotropic and nonsimple materials, different approaches of generalized continua are presented in order to model size-dependent deformation behavior that is experimentally verified. The modified Strain gradient-, micropolar-, couple stress- and surface theories are used and constitutive relations are derived following a cascade of principles of rational mechanics. Analytical solutions in the context of Euler-Bernoulli beam theory, as well as numerical implementations in terms of finite element variational formulations of higher-order theories are presented and solved with the help of the open-source Finite Element code FEniCS. Atomic Force Microscopy (AFM) investigations of the engineering polymer materials SU-8 and epoxy are performed and higher-order material parameters, i.e. material length scale parameters are obtained by a method of least squares from analytical and experimental data.

Keywords: couple stress theory of elasticity, size effect, variational formulation, atomic force microscopy, length scale parameter

1. Introduction

A quantitative understanding of size effects in microstructures is of great importance during the modeling phase of Micro- and Nanoelectromechanical Systems (MEMS/NEMS), either in a physically reasonable manner or as a technique of homogenization concerning materials with intrinsic nano-, micro- or even macro-structures. As far as size effects in elasticity are concerned an increase in bending rigidity of micro-beams is observed in [1, 2, 3]. For the reason that conventional theories, based on the BOLTZMANN continuum, are not able to predict size effects, higher-order continua are considered in the work. Elastic moduli for the materials epoxy and SU-8 are measured in micro-scale using an AFM technique and evaluated with analytical and numerical approaches in order to identify the additional parameters of the models.

2. Higher-order continua

Recent concepts of higher-order continua allow to separate micromorphic and strain gradient theories as extensions of the conventional BOLTZMANN continuum. Additionally, a theory of material surfaces is taken into account, based on the idea that the elastic behavior of the surface of a solid may be separated from its volume. With the help of specific restrictions, the micromorphic continuum is simplified to the micropolar-, and to the Couple Stress continuum (CS). The same procedure is applied to strain gradient theories, which are transferred to the Modified Strain Gradient theory (MSG) and the CS theory as well.

2.1. Modified strain gradient theory

Based on the third formulation of the linear strain energy density \( u^{3RD} \) of isotropic and nonsimple materials of the gradient type, postulated by MINDLIN [4], and assuming that the anti-symmetric part of the gradient of rotation \( \chi^S_{ij} \) does not influence the energy, the linear strain energy \( U \) of the MSG theory reads [5]:

\[
U = \tilde{U} \left( \varepsilon_{ij}, \varepsilon_{mm}, t, \eta_{ijk}, \chi_{ij}^S \right) = \int \left( \frac{1}{2} \lambda \varepsilon_{ii} \varepsilon_{kk} + \mu \varepsilon_{ij} \varepsilon_{ij} + \mu \varepsilon_{ij} \varepsilon_{ij} + \eta_{ijk} \eta_{ijk} + \chi_{ij}^S \chi_{ij}^S \right) \mathrm{d}V,
\]

(1)

where \( \lambda \) and \( \mu \) are LAMÉ’s constants, \( \varepsilon_{ij} \) the small strain tensor, \( \eta_{ijk} \) the deviatoric part of the symmetric part of the second gradient of displacement, and \( \chi_{ij}^S \) the symmetric part of the gradient of rotation. Using EULER-BERNOUILLI (EB) assumptions for the vector of displacement and applying the principle of virtual displacements to the strain energy and the work done by the external force \( F \), a differential equation of the sixth order is generated and solved with proper boundary conditions for a cantilever beam loaded at one end (cf. Fig. 1).

![Coordinate system of the EB beam](https://via.placeholder.com/150)

Figure 1: Coordinate system of the EB beam (\( L/T/W \) denote length, thickness and width).

Assuming POISSON’s ratio equal to zero the customised solution for the elastic modulus \( E^* \) of the present model, divided by the conventional solution reads:

\[
E^* = E \frac{L^3}{3} \left[ C_1 L^3 + C_2 L^2 + C_3 L + C_4 + \frac{C_5}{L} + C_6 e^{-\frac{L^3}{S}} \right].
\]

(2)

where \( E \) denotes the macroscopic elastic modulus (from conventional continuum), and \( S \) and \( K \) are substitutions, which depend on \( \ell \) and the geometry. The coefficients \( C_1, \ldots, 6 \) are adapted to classical and higher-order boundary conditions.
2.2. Couple stress theory

If the additional and intrinsic degree of freedom, i.e., rotation, is completely assigned to the macroscopic deformation and no influence of strain gradients is assumed, then the model is called pseudo- or indeterminate COSserat model. The linear strain energy \( U \) of isotropic and simple materials and the customised analytical solution for the elastic modulus \( E^* \) of the present model, divided by the conventional solution, reads:

\[
U = \int_V \left( \mu \varepsilon_{ij} \varepsilon_{ij} + \mu^2 \chi_{ij}^2 \chi_{ij} \right) dV, \quad E^* = E \left( 1 + \frac{\beta^2}{\gamma^2} \right).
\]  

(3)

The latter formula is derived using the EB assumptions and applying the principle of virtual displacements to the strain energy and the work done by the external force \( F \). Through this means, a differential equation of the fourth order is generated and solved with proper boundary conditions for a cantilever beam (cf. Fig. 1).

2.3. Surface elasticity (SE)

The surface of a solid typically exhibits characteristics that may differ from the properties of its volume. The continuum mechanical description of a surface stress tensor is based on [6, 7, 8]. Starting from the most general case of an arbitrarily curved and smooth surface, the concept of co- and contravariant components of tensor representation will be used. For linearly elastic and isotropic materials without residual surface stresses, the surface stress tensor \( \tau^\alpha \beta \) and surface strain tensor \( e^\alpha \beta \) are:

\[
\tau^\alpha \beta = \lambda^\alpha \varepsilon \delta^\alpha \beta + 2 \mu^\alpha \varepsilon^\alpha \beta, \quad e^\alpha \beta = \frac{1}{2} \left( u^\alpha_{,\beta} + u^\beta_{,\alpha} \right),
\]

(4)

where \( u_{,\alpha} \) is the displacement vector of the surface and \( (\lambda^\alpha, \mu^\alpha) \) are Lamé’s constants of the surface. A surface strain energy and an analytical solution for the elastic modulus \( E^* \) of the present model (assuming rectangular EB beams with flat surfaces), divided by the conventional solution, are represented as follows:

\[
U = \int_V \left( \mu \varepsilon_{ij} \varepsilon_{ij} + \mu^2 \chi_{ij}^2 \chi_{ij} \right) dV + \int_{\partial V} p \varepsilon_{ij} \varepsilon_{ij} dA, \quad E^* = E \left( 6 \frac{\ell}{T} + 2 \frac{w}{W} \right).
\]

(5)

3. Finite element approach

A variational formulation of the CS theory is derived as:

\[
\int_V \left( \sigma_{ij} \delta e_{ij} + \mu_{ij} \delta \chi_{ij}^2 \right) dV - \int_{\partial V} p \delta u_i dA + \int_{S} \frac{1}{2} \left[ \mu_{ij} \delta \varphi_{ij} \right] n_i dS = 0,
\]

(6)

starting from the balance of linear momentum and angular momentum, and using GAUSS theorem in its general form, including jump terms to account for jumps of the couple stress vector \( \varphi \) at the element’s interfaces \( S \). Equation 6 has been implemented in FEniCS, providing a mesh that consists of second order tetrahedral elements to account for the second-order derivatives of \( u \). For spatial discretization the GALERKIN method is applied and the system matrix is solved based on the method of GAUSS elimination. A common DIRICHLET boundary condition on the surface at \( x = 0 \) and a load application on the surface at \( x = L \) make it possible to model a cantilever. A deviation of less than 5% of deflections in comparison to CS analytics is achieved for a minimum number of elements.

4. AFM experiments on epoxy and SU-8

Static bending tests are performed on freestanding microbeam structures. A load of \( 0.5 < F < 250 \) \( \mu \text{N} \) is applied using an off-axis laser-reflective Atomic Force Microscope (AFM) and deflections of \( 40 \text{ nm} < w < 10.0 \) \( \mu \text{m} \) are recorded. Assuming rectangular cross-sections of the specimens, the classical relation between the AFM measures \( (F/w) \) and the elastic modulus from the measurement \( E^* \) is used:

\[
E^* = \frac{4L^3 F}{W^4 w^3}.
\]

(7)

5. Results and discussion

The results of the bending experiments show an increase of nearly twice the value of conventional elastic modulus (cf., Fig. 2). The corresponding additional material coefficients were fitted to the aforementioned analytical models and finite element formulation (cf., Table 1) in good accordance. If the additional material coefficients are set equal to zero, then the aforementioned theories reduce to the classical continuum theory, without the presence of a size effect \( (E^*/E = 1) \).

![Figure 2: Mean values of experiments and smoothed FE results.](image)

Table 1: Results

<table>
<thead>
<tr>
<th>Material</th>
<th>Theory</th>
<th>Parameter I</th>
<th>Parameter II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Epoxy</td>
<td>CS</td>
<td>( E = 3.93 ) GPa</td>
<td>( \ell = 7.75 ) ( \mu \text{m} )</td>
</tr>
<tr>
<td></td>
<td>MSG</td>
<td>( E = 4.14 ) GPa</td>
<td>( \ell = 1.39 ) ( \mu \text{m} )</td>
</tr>
<tr>
<td></td>
<td>SE</td>
<td>( E = 4.36 ) GPa</td>
<td>( E^S = 14.1 \text{ kN/m}^{-1} )</td>
</tr>
<tr>
<td></td>
<td>FEM</td>
<td>( E = 4.20 ) GPa</td>
<td>( \ell = 2.5 ) ( \mu \text{m} )</td>
</tr>
<tr>
<td>SU-8</td>
<td>CS</td>
<td>( E = 4.13 ) GPa</td>
<td>( \ell = 2.5 ) ( \mu \text{m} )</td>
</tr>
<tr>
<td></td>
<td>MSG</td>
<td>( E = 4.14 ) GPa</td>
<td>( \ell = 1.39 ) ( \mu \text{m} )</td>
</tr>
<tr>
<td></td>
<td>SE</td>
<td>( E = 4.36 ) GPa</td>
<td>( E^S = 3.95 \text{ kN/m}^{-1} )</td>
</tr>
<tr>
<td></td>
<td>FEM</td>
<td>( E = 4.20 ) GPa</td>
<td>( \ell = 2.5 ) ( \mu \text{m} )</td>
</tr>
</tbody>
</table>

References

Generalized Fractional Calculus of Variations and its Applications

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Abstract

A review is made of the recent generalized fractional calculus of variations. In the problems under consideration classical integrals are substituted by generalized fractional integrals, and the Lagrangians depend not only on classical derivatives but also on generalized fractional operators. Choosing particular operators and kernels, some results are obtained available in the literature of fractional variational calculus and mathematical physics.

Keywords: fractional derivatives, fractional integrals, Lagrangian, Euler–Lagrange equation, dissipative equation

1. Introduction

A classical problem of calculus of variations is formulated as follows: minimize (or maximize) the functional

\[ J(x) = \int_a^b L(t, x(t), x'(t)) \, dt \quad (1) \]

on \( D = \{ x \in C^1([a, b]) : x(a) = x_a, x(b) = x_b \} \), where \( L : [a, b] \times \mathbb{R}^n \to \mathbb{R} \) is twice continuously differentiable. In mechanics, function \( L \) is called the Lagrangian; functional \( J \) is called the action. If \( x \) gives a (local) minimum (or maximum) to \( J \) on \( D \), then

\[ \frac{d}{dt} \partial_t L \left( t, x(t), x'(t) \right) = \partial_x L \left( t, x(t), x'(t) \right) \quad (2) \]

holds for all \( t \in [a, b] \), where we are using the notation \( \partial_i F \) for the partial derivative of a function \( F \) with respect to its \( i \)-th argument. This is the Euler–Lagrange equation. In mechanics, if Lagrangian \( L \) does not depend explicitly on \( t \), then the energy, \( E(x) := -L(x, x') + \frac{1}{2} m \mathbf{v} \mathbf{v} \cdot x' \), is constant along physical trajectories \( x \) (that is, along the solutions of the Euler–Lagrange equations). In real systems friction corrupts conservation of energy, so the usefulness of variational principles is lost: “forces of a frictional nature are outside the realm of variational principles”[5]. For conservative systems, variational methods are equivalent to the original method used by Newton. However, while Newton’s equations allow nonconservative forces, the later techniques of Lagrangian and Hamiltonian mechanics have no direct way to dealing with them. Let us recall the classical problem of linear friction:

\[ m \frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} \frac{\partial V}{\partial x} = 0, \quad \gamma > 0. \quad (3) \]

In 1931 Bauer proved that it is impossible to use a variational principle to derive a single linear dissipative equation of motion with constant coefficients like (3). The proof of Bauer’s theorem relies on the tacit assumption that all derivatives are of integer order. If a Lagrangian is constructed using fractional (non-integer order) derivatives, then the resulting equation of motion can be nonconservative. This was first proved by F. Riewe in 1996/97 [8, 9], marking the beginning of the Fractional Calculus of Variations (FCV). Further information on FCV is included in [1, 4, 6, 7].

2. Fractional Operators

This section presents definitions of fractional operators. Let \( \Delta := \{(t, \tau) \in \mathbb{R}^2 : a \leq \tau < t \leq b \} \). (4)

Let us consider a function \( k \) defined almost everywhere on \( \Delta \) with values in \( \mathbb{R} \). For any function \( f \) defined almost everywhere on \( (a, b) \) with value in \( \mathbb{R} \), the generalized fractional integral operator \( K_P \) is defined for almost all \( t \in (a, b) \) by

\[ K_P[f](t) = \lambda \int_a^t k(t, \tau)f(\tau)d\tau + \mu \int_t^b k(\tau, t)f(\tau)d\tau, \quad (5) \]

with \( P = (a, t, b, \lambda, \mu), \lambda, \mu \in \mathbb{R} \).

The generalized fractional derivative of Riemann–Liouville type, denoted by \( A_P \), is defined by

\[ A_P = \frac{d}{dt} \circ K_P. \quad (6) \]

The general kernel differential operator of Caputo type, denoted by \( B_P \), is given by

\[ B_P = K_P \circ \frac{d}{dt}. \quad (7) \]

In particular, for suitably chosen kernels \( k(t, \tau) \) and sets \( P \), kernel operators \( K, A \) and \( B \) reduce to the fractional integrals and derivatives of the Riemann–Liouville, Caputo or Hadamard type [3].

3. Variational Problem

Let \( R = (a, b, 1, 0) \), \( P = (a, t, b, \lambda, \mu) \) and

\[ A(y_a, y_b) := \left\{ y \in C^1([a, b]; \mathbb{R}) : y(a) = y_a, y(b) = y_b, \right\} \quad (8) \]

\[ K_P[y], B_P[y] \in C([a, b]; \mathbb{R}) \]
We consider the problem of finding a function \( y \) that gives minimum value to the functional

\[
\mathcal{I} : \mathcal{A}(y_a, y_b) \mapsto \mathbb{R}
\]

\[
y \mapsto K_R \int F(y, K_P[y], y, B_P[y], t) \, dt
\]

where \( K_R \) and \( K_P \) are generalized fractional integrals with kernels \( k(x, t) \) and \( h(t, \tau) \), respectively, being elements of \( L^n(\Delta; \mathbb{R}) \), \( B_P = K_P \circ \frac{d}{dt} \) and \( F \) is a Lagrangian of class \( C^1 \):

\[
F : \mathbb{R}^4 \times [a, b] \rightarrow \mathbb{R}
\]

\[
(x_1, x_2, x_3, x_4, t) \mapsto F(x_1, x_2, x_3, x_4, t)
\]

If \( y \in \mathcal{A}(y_a, y_b) \) is a minimizer of functional (9), then, under appropriated assumptions, \( y \) satisfies the generalized Euler–Lagrange equation

\[
k(b, t) \cdot \partial_t \psi^*(y)(t) - \frac{d}{dt} \left( \partial_t F(y)(t) \cdot k(b, t) \right)
\]

\[
- A_P \cdot k[b, \tau] \cdot \partial_t \psi^*(y)(\tau)(t) + K_P \cdot k[b, \tau] \cdot \partial_t \psi^*(y)(\tau)(t)
\]

\[
= 0, \quad t \in (a, b)
\]

where \( \psi^*(y) = (y(t), K_P[y](t), y(t), B_P[y](t), t) \) and \( P^* = \{a, t, b, \mu, \lambda\} \).

4. Examples

If the Lagrangian of functional (9) does not depend on generalized fractional operators \( B \) and \( K \), then the following result is obtained: if \( y \in C^1([a, b]; \mathbb{R}) \) is a solution to the problem of extremizing

\[
\mathcal{I}(y) = \int_a^b F(y(t), \dot{y}(t), t) \, dt
\]

then

\[
\partial_t F(y(t), \dot{y}(t), t) - \frac{d}{dt} \partial_t F(y(t), \dot{y}(t), t)
\]

\[
= \frac{1}{k(b, t)} \left( \frac{d}{dt} k(b, t) \right) \partial_t F(y(t), \dot{y}(t), t).
\]

We recognize the generalized dissipative parameter \( \delta(t) = \frac{1}{k(b, t)} \frac{d}{dt} k(b, t) \) on the right hand side of (13).

Let us consider kernel \( k^*([b, t] = \exp(\alpha(b - t)) \) and the Lagrangian

\[
L(y(t), \dot{y}(t), t) = \frac{1}{2} m \dot{y}^2(t) - V(y(t)),
\]

where \( V(y) \) is the potential energy and \( m \) stands for mass. The Euler–Lagrange equation (13) gives the following second-order ordinary differential equation:

\[
\ddot{y}(t) - \alpha \dot{y}(t) = -\frac{1}{m} V'(y(t)).
\]

Equation (15) coincides with (14) of [2], a modification of Hamilton’s principle.

The next example is the Caldirola–Kanai oscillator. Let us consider

\[
L(y(t), \dot{y}(t), t) = m \left( \frac{\dot{y}^2(t)}{2} - \omega^2 y^2(t) \right)
\]

and \( k(b, t) = \exp(\sin \beta \gamma t) \). Using our generalized Euler–Lagrange equation (13) with kernel \( k(b, t) \) to Lagrangian (16), we obtain

\[
\ddot{y}(t) + \beta \gamma \cos \beta \gamma \dot{y}(t) + \omega^2 y(t) = 0.
\]

Assume that the Lagrangian of functional (9) does not depend on \( K \) and let \( R = \{a, b, b, 1, 0\}, P = \{a, t, b, \mu, \lambda\}, k(x, t) \equiv 1, h^\alpha(t - \tau) = \tau^{1-\alpha}(1 - \tau)^{-\alpha}, \alpha \in (0, 1). \) Then the functional (12) takes the form

\[
\mathcal{I}(y) = \int_a^b F(y(t), \dot{y}(t), t) + \lambda_1 D^\alpha_0[y](t), \dot{y}(t) dt,
\]

\[
- \lambda V(y(t)) dt.
\]

In this case the Euler–Lagrange equations (11) leads to

\[
\ddot{y} - \gamma \dot{y} = -V'(y) - \gamma y.
\]

(20)

(for more details we refer the reader to [4]).

The result presented by F. Riewe in [8, 9] are obtained by choosing \( R = \{a, b, b, 0, 1\}, P = \{a, t, b, 1, 0\}, k(x, t) \equiv 1, h^\alpha(t - \tau) = \tau^{1-\alpha}(1 - \tau)^{-\alpha}, \alpha = 1/2 \) and assuming that the Lagrangian of functional (9) does not depend on \( K \).

Finally, it is worth noting that taking different kernels and parameter sets many other results known from the fractional variational calculus and mathematical physics can be obtained as corollaries of (11) (see [7]).

References


Simulation of extrusion processes in porous media

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Abstract

During the last decade computer methods became an effective tool to study fluid mechanical processes occurring in biomechanical systems. One of the most important concerns production of a droplet with the medicine against cancer. Such droplets come into being through nanotubes as pores and contact between solid and liquid porous filter (sieve plate), mass diffusion and/or viscous flow during extrusion of a lipid suspension through a polycarbonate porous by nanotube filter. The paper deals with the description of the coupled elastodiffusive model of that problem and computer simulations of droplet extrusion through such a filter.

Keywords: extrusion processes, porous media, lipid solution

1. Introduction

In recent times computing methods dealing with flow problems became very important in extrusion processes leading to produce a peculiar form of a biological based medium against a cancer. Formulation of solvent drops in sieve plates is very important phenomenon in solvent extraction. Small drupelets are usually desired in order to ensure a large interfacial area available for mass transfer to maximize the process efficiency. Formation of very small drops is, however, a common problem frequently encountered in the industrial extraction equipment. Very small droplets (50-400nm) are difficult to separate from the continuous phase and their presence can adversely affect the process. Formation of droplets can be affected by changes in flow conditions and in the mechanical structure of the sieve plates. Drop formation in sieve plates is, altogether, a complex phenomenon which depends on the flow velocity in the tubes, the material properties of the tube, the size and structure of the tube, the distance between the holes, and the alignment of the tubes on the sieve plate.

The extrusion process deals with the elastic properties of the porous filter (sieve plate), mass diffusion and/or viscous flow through nanotubes as pores and contact between solid and liquid phases during dynamical interactions.

2. Physical model of extrusion of lipid suspension

The main condition necessary to describe those complex problems is continuity of the layer stress and the pressure of the lipid drops across their interface. The surface tension of the drop has to be greater than its volume pressure to keep the drop stable along the plate channel and outside the plate.

These groups of mechanical description of the complex problem have to be taken into account:

• general elastodiffusive equations in the stress-diffusion form,
• elastodiffusive equations of the plate,
• flow equations of viscous fluid through elastic channels.

During extrusion processes a very slow flow occurs through capillary nanotubes. That phenomenon can be quite well described within a continual model of diffusion process in a continuous elastic body [8]. This way we can simulate elastodiffusive extrusion process assuming continuity conditions between pressure of the suspension drop and the stress state of the elastic solid across the interface between phases. Since we are directly interested in determination of the pressure-stress problem the most suitable to do that are the Beltrami-Mitchell stress equations containing the diffusion terms coupled with the diffusion evolution equations [1,2,3,4,5,8]:

$$\frac{\partial^2 \sigma_{ij}}{\partial t^2} + 2(\lambda + \mu) \frac{\partial \sigma_{kk,ij}}{\partial t} + \left(\frac{\lambda}{C_2^2} + \frac{1}{C_1^2}\right) \frac{\lambda \delta_{ij}}{3\lambda + 2\mu} C_{kk} +$$

$$+ J_{ij} + \alpha_c E \left( \frac{1}{1 - \nu} \nabla^2 c \delta_{ij} + \frac{1}{1 + \nu} c \delta_{ij} \right) = 0,$$

$$\frac{D_c}{\partial t} - D_c \frac{A_p}{S} \left( \frac{K_{w_r}}{\tau} \right) C_{kk} = 0,$$

where:

$$\lambda = \frac{E \nu}{(1 + \nu)(1 - 2\nu)} \quad \mu = \frac{E}{2(1 + \nu)} \quad c_1^2 = \frac{\lambda + 2\mu}{\rho} \quad c_2^2 = \frac{\mu}{\rho}$$

λ, μ denote Lame’s constants, E is Young’s modulus, ν is Poisson’s ratio, c is the mass concentration, α is the diffusion expansion coefficient [2], D is the diffusion coefficient, ε = Au/S is the porosity of density of tubes in the surface of the filter [4].

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$K \approx 1$ or $K = \left(1 - \frac{a}{r}\right)^2$.

$a$ is the drop radius, $r$ is the tube (pore) radius, $t$ informs how the diffusion distance is bigger of the filter (plate) thickness, $\alpha$ to be estimated [4]. The boundary conditions are applied in the form:

\begin{align*}
\sigma_{kk} &= -q, \quad c = c_p \quad \text{at the upper surface}, \\
\sigma_{kk} &= 0, \quad c = c_r \quad \text{at the lower surface},
\end{align*}

and initial conditions:

\begin{align*}
\sigma_{kk}(x_1,0) &= 0, \quad c(x_1,0) = 0,
\end{align*}

where $q$ denotes the pressure.

### 3. Numerical simulations

Let us consider the problem of extrusion of big liposome structure (lipid suspension) of elements $\phi = 10^{-5} \text{m}$ through polycarbonate filter $\phi = 10^{-7} \text{m}$. After 6 or 7 cycles of extrusion one arrive at a target diameter of drops i.e. $\phi = 10^{-7} \text{m}$. However, the first cycle is the most effective reducing the diameter of the drop to $\phi = 2 \times 10^{-7} \text{m}$ [6,7,9,10].

The problem is physically very complicated. Therefore, certain simplifications are necessary to do. They have concerned boundary conditions and material characteristics and were made basing on a little simplified mathematical model of viscous fluid flow through a nanotube. The main goal of the investigations was to find relations among basic system parameters and granulation effectiveness, i.e. relation between pressure of the suspension and depth of drops penetration into the filter tube.

The computer simulations were done with the help of Finite Element Method. Since the distances among the nanotubes and their length strongly suggest that the filter forms rather a collection of the separate tubes than the material where mass diffusion occurs in. Therefore, the simulations were done assuming the flow through a single nanotube (Fig. 1).

![Figure 1: Initial geometry of extrusion](image1.png)

![Figure 2: Liposome deformation as a function of filter material properties (I - III) and the external normalized pressure 62,5%, 94,0%, 100% (a, b, c)](image2.png)

These parameters were applied in numerical simulations of extrusion process presented in Fig. 2.

### References


Hyperelastic structural-mechanical model of filled rubber

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Abstract

The microstructure of filled rubber (heterogeneous filler network in rubber matrix) is modelled by a volume filled with thousands of rigid spherical inclusions. The inclusions can be grouped into secondary structures – fractal clusters and micropellets (dense globules of poorly mixed filler). Inclusions are connected by damageable links representing the mechanical behaviour of the elastomer in the gaps between filler particles. The mechanical response of the links depends on the initial gap between the pairs of inclusions. The formation of the interfacial polymer layer (10 nm thick) is taken into account. The volume is subjected to stepwise deformation. After each loading step, the equilibrium position of inclusions investigated, provided that the energy of deformed links is minimal. Hysteresis losses under cyclic loading conditions are modelled by breaking links. The influence of microstructure (filler fraction, cluster/random filler distribution, amount of micropellets) and properties of interfacial layers, including the local damage criteria, on the macroscopic characteristics of filled elastomers are discussed in the context of the proposed model.

Keywords: filled rubber, structural-mechanical model, interfacial interactions, hysteresis

1. Introduction

Reinforcement of elastomers by active fillers, in particular carbon black (CB) improves the mechanical and strength characteristics of rubbers. Elastic properties inherent in unfilled vulcanizates remain unchanged.

The CB is represented by primary indivisible aggregates of the order of 0.1…0.2 μm. However, the filler is added to the polymer in a pelletized form (grains of size 1–3 mm). During the process of mixing the composite ingredients these grains are separated and distributed throughout the matrix. Related with the mixing parameters and elastomer viscosity, part of the filler is left in the composite in the form of bulk inclusions – micropellets. At the same time CB aggregates in the polymer matrix form secondary structures – fractal clusters. At some volume fraction, the filler in the matrix forms a continuous network of clusters.

Experimental studies have revealed that the surface of active filler in a rubber is surrounded by polymer layer of a reduced mobility [1]. The thickness of this layer is about 2 nm, its properties are similar to the polymer in a glassy-like state. According to some hypotheses [2], the mobility of molecules increases gradually outward from the surface and at a distance of about 10 nm passes into the matrix.

In the work a structural-mechanical model for filled elastomers is proposed, taking into account peculiar microstructural features of the material. Inclusions are connected by hyperelastic links. Mechanical properties of these links depend on the size of the gaps between inclusions and the characteristics of hypothetical interfacial layers.

2. Description of the structural-mechanical model

The structure of the composite is represented by a system of rigid spherical inclusions. Modelling of interfacial interactions incorporates the links (Fig.1), which, in fact, are rods pivotally connected to the centres of inclusions, subjected to tension compression only.

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the gap - the Gent approach is used [3] (if the hydrostatic stress in the centre of the gap \( > 5/6E \), then the material, i.e. the corresponding link, is broken). The initial gaps, i.e. initial lengths of the links, are vary from 2 to 250 nm. The size of inclusions is assumed 80 nm.

The microstructure is modelled by a cube non-uniformly filled with spheres of equal sizes. The minimum acceptable space between the spheres is 2 nm. The spheres are assembled into fractal clusters or micropellets (spherical regions densely packed with inclusions). Comprehensive analysis of fractal structure and the synthesis algorithms are given in the work [4]. Apart from the clustered structure, the case of random filler distribution is also considered (Fig. 2). The filler volume fraction \( \phi \) in the investigated structures is 0.13 and 0.20.

Due to a constant volume, the structure deforms by the step-wise displacement of inclusions. After each deformation step, the coordinates of inclusions are refined due to means of forces that occur in links by minimizing the local elastic energy of links. The minimization is performed by the Nelder–Mead method. Achieving the optimal state of all inclusions, the configuration of links is specified: the link is broken at a maximum elongation. The part of the material in the gap with the broken link begins to respond to compressive forces only not resisting in other cases. If the links break, the procedure to search the system equilibrium is repeated. When a new optimal state of a structure is found, the structural-mechanical characteristics of the system are determined, so the next step of loading is performed.

3. Results and discussion

Two uniaxial stretching – compression cycles with an increasing amplitude (elongation ratio \( \lambda \) equal to 2 and 4) were applied to each structure. Example of a four – times stretched structure (initial structure given in Fig. 2b) is shown in Fig. 3.

At a certain local elongation of the gap between inclusions the corresponding link was broken. The typical behaviour of the rupture of the links is shown in Fig. 4a. While \( \lambda \) is less than the maximum value (unloading or repeating loading), the number of broken links remains unchanged. Thus the rupture of links leads to the hysteresis of the stress-strain curve (Fig. 4b), a typical pattern of the filled rubbers.

Simulations indicate that the presence of specific layers has a profound effect on the pair interaction of inclusions in the matrix even at a big gap (up to 250 nm) between inclusions. Based on the structural-mechanical model, it can be concluded that the absence of interfacial layers of variable stiffness does not lead to the reinforcement effect characteristic of filled vulcanizates.

It is shown that part of the polymer in the gaps between close inclusions (\( \delta_0 < 20 \) nm) does not practically respond to deformation – such inclusions move in groups, contribution of stress in these gaps to the total stress-strain state is rather low. Part of the polymer in the long links (when the interphase layers do not intersect, i. e. \( \delta_0 > 20 \) nm) bears the main load. The most deformable portion of the matrix in the models is the polymer in the gaps of \( \delta_0 = 80...120 \) nm.

References

Numerical simulations of mechanical properties of alumina foams based on computer tomography

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Abstract

The aim of the paper is to apply the results of microtomography of alumina foam to create the numerical model and perform the numerical simulations of compression tests. The geometric characteristics of real foam samples are estimated from tomographic and scanning electron microscopy images. The performance of the reconstructed models is compared to experimental values of elastic moduli and strength in the uniaxial compression test.

Keywords: compression test, alumina open-cell foam, Young’s modulus, compressive strength of alumina foams

1. Introduction

The mechanical properties and numerical model of real alumina foam, which is produced in the gelcasting process are studied. The finite element models of alumina open-cell foams are developed. Elastic properties and compressive strength are predicted and compared with experimental results. The numerical parameters which are needed to build the unit cell model are based on the data obtained from microtomography images of real foam. Alumina foams are studied also with the use of periodic models based on the crystallographic systems: simple cubic (sc), body centered cubic (bcc) or face centered cubic (fcc). Each lattice point is replaced by a bubble, whose radius corresponds to the mean value of cell radius distribution of alumina foam. In these models the interconnection radius between two bubbles is equal to the mean value of distribution of window radius for a real porous material. Using the procedures described in Ref.[4] the analysis of the microtomography images shows that the alumina foams are composed of approximately spherical cells interconnected by circular windows Ref. [6].

The porosity, cell radius distribution, and window radius distribution of an individual foam sample is computed and the geometry of a unit cell is proposed. Dimension of a finite element corresponds to the dimension of a single voxel and is equal to 16µm. In all numerical calculations the cube-shaped sample of the foam with dimensions of 400×400×400 voxels was considered. This assumption gives a representative volume element (RVE) of size 4×4×4 mm. To assure the better convergence of the numerical simulations the microtomography images of alumina foams were rendered by ScanIP software, Fig. 1.

The material of skeleton of the alumina foam is assumed to be isotropic and linearly elastic. The bottom surface of the sample is fully constrained and the top surface of the sample is moved parallel to the z-axis. As a result of numerical simulation of compression test of alumina foam for different values of porosity, the Young’s modulus and the strength of such foams are estimated. The comparison of experimental data Ref. [6] with numerical and analytical predictions Ref. [3] of Young modulus for Al₂O₃ ceramic foams of different porosity is presented. The analytical estimation shows good correlation with the results of experiment and simulation for the range of porosity (84%-90%).

Figure 1: The example of rendered alumina 86% porosity foam used in simulations.

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2. The quasi-brittle material model for Al₂O₃ foams

The model describing behaviour of considered alumina foam is defined in elastic range by Hooke’s law and inelastic range by the associated plasticity theory. The limit surface is defined by Burzyński paraboloid yield condition, see Refs. [1, 2, 8, 9]. The additive decomposition of strain tensor into elastic and inelastic part is given

\[ \varepsilon = \varepsilon^e + \varepsilon^{in} \]  

The Burzyński paraboloid yield condition is given in a form

\[ F = \frac{1}{2k} \left\{ 3(1 - k) \sigma_m + \sqrt{9(k - 1)^2 \sigma_m^2 + 4k^2\sigma_T^2} \right\} + \sigma_T^2 = 0 \]  

where \( k = \sigma_C^e / \sigma_T^e \), \( \sigma_C^e \) and \( \sigma_T^e \) are the initial yield stress in uniaxial compression and tension test, respectively, and \( \sigma_m \) is the mean stress \( \sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3 \), while \( q = \sqrt{3/2} : \! : \! \overline{s} \) and \( \overline{s} \) is the deviatoric stress tensor.

In addition the scalar damage parameter \( d \) is introduced.

\[ \sigma = (1 - d)D^e : (\varepsilon - \varepsilon^{in}) \]  

The inelastic part of strain rate is given by classical associated flow rule where \( F \) represents limit surface given in Eqn (2)

\[ \dot{\varepsilon}^{in} = \frac{1}{\partial F/\partial \sigma} \]  

In Eqn (5) parameter \( d \) describes the damage in skeleton material. The evolution of the damage parameter \( d \) is described by function \( \eta(\varepsilon^{in}) \), where \( \varepsilon^{in} \) is the equivalent inelastic strain, cf Ref. [5]. In the calculations the function \( \eta \) is linear function of \( \varepsilon^{in} \) with the limits:

\[ \eta(0) = 0, \eta(\varepsilon^{in}_0) = 0.9 \]  

The system of equations describing the deformation process of the open-cell foam is solved by algorithm using the return mapping procedure. The proposed algorithm is implemented in commercial FEM software ABAQUS/STANDARD with developed own UMAT subroutine, Ref. [5].

The ceramic foam is assumed isotropic. The bottom surface of the sample is fully constrained and the top surface of this sample is moved parallel to the z-axis.

Material data for \( \text{Al}_2\text{O}_3 \) are as follows: Young’s modulus \( E = 370 \text{ GPa} \), Poisson ratio \( \nu = 0.22 \), the initial compression yield stress \( \sigma_{Yc}^e = 2400 \text{ MPa} \), the initial tensile yield stress \( \sigma_{Yt}^e = 105 \text{ MPa} \), density \( \rho = 3.92 \text{g/cm}^3 \).

3. Summary and Conclusions

The model outlined in Section 2 is used to predict the response of alumina foam in the uniaxial compression. The calculations is terminated after the expected initial load maximum representing the strength of the foam skeleton. The material is quasi-brittle with the stress-strain response of the \( \text{Al}_2\text{O}_3 \) ceramic measured in a tests. The initial part of the response is nearly linear with Young’s modulus \( E \) in a good agreement with the measured values. An example of the comparison of normalized Young’s modulus with the experimental data (Ref. [7], [10]) and numerical predictions for the periodic models (sc, bcc and fcc) are presented in Fig. 2.

References


Influence of heat conduction on instabilities in large strain thermoplasticity

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Abstract

The paper deals with a thermomechanical model of elastic-plastic materials undergoing large deformations. The coupling between thermal and mechanical fields involves not only thermal expansion, but also so-called plastic heating and the dependence of yield strength on temperature which leads to thermal softening of material. In the paper the attention is focused on numerical simulations of loss of stability caused by geometrical and thermal effects. The regularizing influence of heat conduction on simulation results is investigated. Numerical tests are performed using finite element routines prepared with AceGen package and some aspects of this implementation are discussed as well.

Keywords: thermoplasticity, large deformations, softening, regularization

1. Introduction

When instabilities result in localized deformation modes an important difficulty occurs in mathematical modelling and numerical simulations: the considered boundary value problem can lose well-posedness and numerical simulation results are sensitive to discretization density in a pathological manner. Regularization (also called localization limiter) is then necessary. In the isothermal case the regularization can be provided by a non-local or rate-dependent enhancement of the constitutive description, in the thermomechanical model the coupling and in particular heat conduction can provide some localization limiting properties.

The aim of the paper is to verify, by numerical experiment using a fairly general model of thermoplasticity, the extent the thermomechanical model the coupling and in particular heat conduction can provide some localization limiting properties.

2. Thermoplasticity model

The analysis of fully coupled thermoplasticity models in the thermodynamic framework can be found in the papers of Simo, Miehe, Ortiz or Ristinmaa and their co-workers, e.g. [5, 6, 8].

The classical definitions of the deformation gradient \( F \) and its determinant \( J \) are adopted. The multiplicative decomposition of the deformation gradient into its mechanical and thermal part is considered in the form

\[
F = F^e F^p, \quad F^p = (J^0)^{1/3} I
\]

where the mechanical part of the deformation gradient is decomposed into elastic and plastic factors, the thermal part is assumed to be purely volumetric and \( J^0 = \exp(3\alpha_T(T - T_0)) \), where \( T \) denotes the absolute temperature, \( T_0 \) is the reference temperature and \( \alpha_T \) is the coefficient of linear thermal expansion.

The state of a material is expressed by the Helmholtz free energy potential which is assumed in a decoupled form:

\[
\psi(b^e, T, \gamma) = \psi^e(b^e) + \psi^\theta(T) + \Psi^\gamma(\gamma)
\]

where the first part \( \psi^e(b^e) \) is related to the elastic response and includes thermo-mechanical coupling responsible for thermal expansion, \( \psi^\theta(T) \) is a purely thermal part and, finally, the term \( \Psi^\gamma(\gamma) \) denotes the potential of isotropic strain hardening in plasticity (the specific definitions are omitted here). In Eqn (2) \( b^e = F^e(F^p)^T \) denotes the elastic left Cauchy-Green tensor and \( \gamma \) is a plastic strain measure.

To complete the constitutive description of the problem the yield condition which governs the plastic regime is defined

\[
F_p(\gamma, T) = f(\gamma) - \sqrt{2/3} \sigma_{yy}(\gamma, T) \leq 0
\]

where \( f(\gamma) \) is a stress measure and \( \sigma_{yy}(\gamma, T) \) denotes the yield stress which is assumed in a form including strain hardening and thermal softening.

Moreover, the constitutive assumption for heat conduction is the classical Fourier law for isotropic materials \( q = -k\nabla T \), which is formulated for the Kirchhoff heat flux vector \( q \) (\( k \) is the heat conduction coefficient).

The two governing equations are written in spatial description. The first one is the balance of linear momentum formulated in local form as follows

\[
J \nabla(\tau/J) + \rho_0 B = 0
\]

In Eqn (4) \( \nabla(\cdot) \) is the divergence computed with respect to Eulerian coordinates, \( \rho_0 \) is a reference density and \( B \) is a given acceleration field. The balance of linear momentum is completed with proper boundary conditions for displacements or tractions.

The second governing equation is the energy balance which is expressed in the following temperature form [6]

\[
\epsilon^T = J \nabla(-q/J) + \mathcal{R}
\]

where \( \mathcal{R} \) is a heat source density. In the presented model it includes heating due to plastic dissipation which is computed as the plastic work reduced with constant Taylor-Quinney coefficient [7]. Equation (5) is also completed with proper boundary conditions for temperature or heat flux.
3. Simulation tool

The finite element implementation is based on weak forms of the governing equations. The numerical simulations are performed using the symbolic-numerical packages AceGen (code generator) and AceFEM (finite element programme) in the Wolfram Mathematica environment [3]. The application of such software requires a specific approach and the development of appropriate algorithms based on potentials. This involves the formulation of a potential for the energy balance equation (5).

In order to solve the set of nonlinear equations using the finite element method a standard Newton-Raphson incremental-iterative procedure is adopted. The subroutine for the computation of finite element contribution to the global residual vector \( \mathbf{R} \), and to the tangent matrix \( \mathbf{K}_{el} \) is generated with AceGen. The important advantage of the package is automatic differentiation (AD) which allows the researcher to compute the contribution to the consistent tangent matrix automatically on the basis of proper constitutive relations. Due to this feature of the applied software, a monolithic solution scheme is used for coupled problems.

4. Simulation results

In the paper two benchmarks are considered: necking of a circular bar in tension and shear banding in a plate in tension. Here the latter one is briefly discussed. Plain strain conditions are simulated and symmetry of the plate is exploited. The deformation in the length direction is imposed and in the width direction is free. Three discretizations of the specimen are considered: coarse (called mesh 1), medium (mesh 2) and fine (mesh 3), which consist of 800, 3200 and 11480 elements, respectively. If the thermomechanical coupling involving thermal softening is adopted then strain localization is observed at the end of elongation process in the form of shear bands (see Fig. 1). Depending on the value of the heat conduction coefficient \( k \) the widths of bands differ significantly. The differences are also noticed in the diagram presenting the sum of reactions versus the load multiplier in Fig. 2. It can be observed that the larger the heat conduction coefficient is, the closer the results for each discretization are.

Figure 1: Deformed mesh 3 with distribution of \( \gamma \) for \( k = 0 \), \( k = 100 \), \( k = 400 \) at the end of elongation process

Figure 2: Sum of reactions \( \Sigma R \) vs displacement factor \( \lambda \) for the adiabatic case and for \( k = 400 \), and different discretizations

5. Conclusions

It has been shown that for significant heat conduction the problem of thermal softening is regularized, but when one approaches the adiabatic case \( k = 0 \) the results are mesh sensitive (the problem is ill-posed). It is therefore advisable to upgrade the constitutive model with non-locality, for instance in a gradient form [2]. But then, considering the presence of heat conduction, the question arises, how the two regularizing influences of for instance plastic strain gradients and heat conduction will interact.

References

Numerical simulations of auxetic metallic foam fabrication process

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Abstract

The subject of the study are metallic open-cell foams. In particular, the foam of Cu skeleton is considered. To simulate the deformation processes of such a material the finite element program ABAQUS is used. The tomogram reconstructing the 3D virtual volume of a real foam structure with the use of computed tomography is applied to formulate the finite element model of the convex open-cell foam cube of the edge of 800 voxels, 1 voxel = 2.52 10⁻⁶ m, created with application of ABAQUS/CAE. The initial cube of convex open-cell skeleton is subjected to three-axial compression applied as uniform displacements normal to the surface of cube faces in order to simulate numerically auxetic foam fabrication process. Repeating the process of three-axial compression for different displacement boundary conditions the optimal values of the face displacements can be obtained with respect to the required structure of the auxetic foam skeleton.

Keywords: auxetic foam, micro-tomography, tomograms, negative Poisson’s ratio, numerical simulation, metallic foam, open-cell foam, foam fabrication

1. Introduction

Depending on manufacturing method of metallic foams the resulting skeleton consists of convex or concave shape cells. The materials with convex cell structure reveal positive value of Poisson’s ratio, that is if a sample is stretching, then its cross-section is getting thinner. The complex structure of the foam related with reentrant cells produce the opposite effect during stretching of a sample, i.e. its cross-section is increasing. Then the negative Poisson’s ratio is observed and such foams become auxetic.

The aim of the present study is to tackle the following problems:

- how to produce tomograms, i.e. 3D virtual volume reconstructions of real foam structure with use of computed tomography [1],
- elaboration of the numerical simulations methodology of auxetic foam fabrication process with use of the tomograms.

The motivation for such an approach can be found in [2], where the following statement was formulated: „Ideally, in an attempt to reduce laboratory expense, one would like to make predictions of a new material’s behaviour by numerical simulations, with the primary goal being to accelerate the trial and error laboratory development of new high performance materials.” – from Preface, p. v. Accordingly, numerical simulation of fabrication process of auxetic foam is developed.

2. Description of the procedure

The subject of the study are metallic open-cell foams. In particular, the foam of Cu skeleton is considered. To simulate the deformation processes of such a material the finite element program ABAQUS is used. Finite element discretization is derived from real foam specimen with the use of computed tomography images implementing the procedures described in [3], [4], [5]. The initial cube of convex open-cell skeleton is subjected to three-axial compression applied as uniform displacements normal to the surface of cube faces, Fig. 1. In all numerical calculations the skeleton material is assumed as isotropic Cu – Young’s modulus: 126 GPa, Poisson’s ratio: 0.3 and yield limit: 20 MPa. The computational methods and procedures applied in the analysis of the micro tomography observations and numerical simulations of deformation process are presented in detail. An example is displayed in Fig. 1, where

Figure 1: a) Initial configuration of the skeleton of the convex open-cell Cu foam of 94% porosity, b) finite element model with the scheme of displacement boundary conditions.
the initial configuration of the skeleton of the convex open-cell Cu foam of 94% porosity is presented and the finite element model of the foam cube with the edge of 800 voxels is depicted. The dimension of a finite element corresponds to the dimension of a single voxel and is equal to $2.52 \times 10^{-6}$ m. The finite element mesh is obtained with use of ABAQUS/CAE.

3. Presentation of the results

Presentation of the results of numerical simulation is given in Figure 2.

![Figure 2](image1)

Figure 2: a) The resulting structure of the virtual auxetic foam with the estimated Poisson’s ratio: - 0.3, b) the picture of real Cu skeleton with reentrant cells obtained in [6]

4. Conclusions

The results of the above analysis can be applied for the prediction and optimisation of manufacturing requirements. Repeating the process of three-axial compression for different displacement boundary conditions the optimal values of the face displacements can be obtained with respect to the required structure of the auxetic foam skeleton. Analysis of the sequence of calculated deformation processes of the virtual cube leads to the required displacements of the faces of real Cu specimen and enables fabrication of auxetic foam without any damage or crushing of the skeleton ribs, what is quite frequently observed in real manufacturing processes. The proposed procedure of virtual experiments can be applied for convex cell foams of diverse skeleton materials and provides the possibility of omitting the expensive trial and error experiments leading very often to the total damage of real skeleton of the investigated metallic foam.

References


Volume fraction and finite-specimen size effects on a limited-permeable inclined crack in 2D magnetoelectroelastic media using distributed dislocation method

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Abstract

The distributed dislocation method (DDM) is applied to understand the effects of volume fraction and finite size of a specimen on a limited-permeable inclined crack in 2-D magnetoelectroelastic (MEE) media. The Gauss-Chebychev quadrature method is used to solve the simultaneous singular integral equations which are obtained after distributed dislocation modelling of the problem. The study is carried for different volume fractions, inclination angles and crack-face boundary conditions. The results of intensity factors obtained using DDM are validated with the results of X-FEM under impermeable crack-face boundary conditions on a finite specimen. The obtained results show the significant effects of volume fraction, finite size of a specimen and crack face boundary conditions on the generalized intensity factors (IFs).

Keywords: distributed dislocation method, magnetoelectroelastic, Gauss-Chebychev quadrature, limited-permeable

1. Introduction

Magnetoelectroelastic ceramics are extensively used as a key component in smart structures. Due to the fatigue and aging cracks develop in them. Hence, it is essential to analyze the fracture behaviour of such materials.

To study the fracture parameters in 2-D cracked finite MEE media, various computational techniques extended and applied to these problems such as BEM, FEM, X-FEM [1], etc. So far majority of the problems have been studied under impermeable crack-face boundary conditions. The study of MEE media subjected to limited-permeable conditions, which are also considered realistic crack-face boundary conditions, is limited[4] because it involves significant computational cost due to its non-linearity nature.

Since last two decades, Distributed Dislocation Method (DDM) has been applied to analyze the crack problems in an infinite domain of isotropic and anisotropic materials [3]. Using the principle of continuous distribution of dislocation Han and Dhanasekar [2] demonstrated this method to model curved cracks in solids of arbitrarily shaped finite geometries.

Due to the author’s knowledge this method has not been so far explored for studying the inclined crack in 2-D finite MEE media subjected to different volume fractions and crack face boundary conditions. Therefore, DDM is applied here to study the inclined crack in 2-D finite MEE media under limited-permeable crack-face boundary conditions and for different values of volume fraction.

2. Crack-face boundary conditions for MEE media

2.1. Limited-permeable crack-face conditions

\[
\sigma_{ij} n_j = 0 \quad \text{and} \quad \phi^+ = \phi^-,
\]

\[
D_x^+ = D_x^- = D_y^+ = D_y^- = -\kappa_c \frac{\Delta \phi(x_1)}{\Delta u_2(x_1)}.
\]

\[
B_2^+ = B_2^- = B_y^+ = B_y^- = -\mu_c \frac{\Delta \phi(x_1)}{\Delta u_2(x_1)}.
\]

\[ (2) \]

\(\kappa_c \) and \(\mu_c \) are the permittivity and permeability of the medium between crack faces. \(\Delta \phi(x_1) \) & \(\Delta \phi(x_1) \) are the electrical & magnetic potential jump, and \(\Delta u_2(x_1) \) is the crack opening displacement.

3. Formulation of the problem

A problem of finite specimen is considered by cut-out a rectangular specimen of size 2\(W\) x 2H from the infinite MEE domain as shown in Fig.1. The values of parameters which have been used for the analysis are also represented in Fig.1. A local coordinate system (\(\xi, \tau\)) is considered for convenience to handle arbitrary oriented crack situated in the specimen.

\(\sigma_{yy} = 10MPa,\) 
\(D_y = 0.01C/m^2,\) 
\(B_y = 0.1NA^{-1}m^{-1}\) 
\(c = 1mm, W/c = 20,\) 
\(H/W = 1, \alpha = \frac{\pi}{6}\)

Impermeable \(\kappa_c = 0, \mu_c = 0\)
Limited-permeable
\(\kappa_c = 8.85 \times 10^{-12},\) 
\(\mu_c = 1.256 \times 10^{-6}\)
Permeable
\(\kappa_c = 20000x8.85 \times 10^{-12},\) 
\(\mu_c = 20000x1.256 \times 10^{-6}\)

Material Constants: BaTiO3-CoFeO2 [1]

Normalized IFs
\(K^I = \frac{K_{II}}{\sigma_{yy} \sqrt{\pi \gamma}}, K^H = \frac{K_{III}}{\sigma_{yy} \sqrt{\pi \gamma}}, K^V = \frac{K_{IV}}{b_y \sqrt{\pi \gamma}}\)

Figure 1: Geometrical configuration and applied parameters

The crack and boundaries of the specimen are modelled as a continuous distribution of dislocations with generalized Burgers vectors \(b_h = [b_1 b_2 b_3 b_4 b_5 b_6]^T\) where \(b_h(h = 1 \text{ to } 3), b_4 \text{ and } b_5\)

*Author gratefully acknowledges NIT Uttarakhand, Srinagar(Garhwal), India for providing the research facility during the course of this work.
$b_i^\tau$ are mechanical displacement jump, an electrical potential jump and magnetic potential jump in the plane, respectively at points $(x_i, x_\tau_i)$. It is equivalent to a multiple cracks problem in the infinite solid with prescribed loads on the actual cracks faces and traction/induction free conditions on the boundary of the specimen [2, 3]. Expressing the cracks lines as $z_i^\tau = z_i^\tau_0 + \xi z_i^\tau_1$, where $z_i^\tau_0 = x_i + p_x \alpha_i^\tau$ and $z_i^\tau_1 = \cos(\alpha_i^\tau) + p_x \sin(\alpha_i^\tau)$, the dislocation $b_i^\tau$ becomes $b_i^\tau(\xi)$. Enforcing the applied surface traction (charge) conditions on each crack faces, a system of singular integral equations for dislocation density $b_i^\tau(\xi)$ is obtained as

$$
\frac{1}{2\pi} \int_{c_i} b_i^\tau(t, \xi) d\xi + \frac{1}{\pi} \sum_{i=1}^n \int_{c_i} K_i(t, \xi_j) b_i^\tau(\xi_j) d\xi_j = -f(t_i)_{[\xi_i]} < c_{\xi_i} \leq 5 \tag{3}
$$

where $K_i(t, \xi_j) = \text{im} \left( B(z_i^\tau(\xi_j) + t_i \xi_j^\tau - z_i^\tau(\xi_i) - \xi_i^\tau)^{-1} B^\tau \right)$, is a kernel function of the singular integral equations and are Holder-continuous along $c_{\xi_i} \leq \xi_i < c_i$. The eigenvalues $p_k$, matrices $B$ and $L$ depend upon the material constants, and are taken as defined in [1, 4].

The boundary conditions on inclined crack:

$$
t_n = -\sigma_{x\xi} \sin \alpha - \sigma_{x\tau} \cos \alpha, \tag{4}
$$

and on the boundaries of specimen:

$$
t_n = 0. \tag{5}
$$

For single valued displacements, electric potential and magnetic induction around a closed contour surrounding the cracks, the following conditions must also be satisfied

$$
\int_{c_i} b_i^\tau(\xi) d\xi = 0 \text{ for } i = 1 \text{ to } 5. \tag{6}
$$

The solution of the above system of singular integral equations is obtained using Gauss-Chebyshev quadrature method [3]. Once the generalized dislocation densities have been obtained, the intensity factors (IFs) at the crack tips ($\pm \epsilon$) are given by

$$
[K_{II} \ K_{III} \ K_{IV} \ K_{V}]^T = \begin{bmatrix}
\pi \\
\pi \alpha \\
\pi \alpha \\
\pi \alpha
\end{bmatrix}
\begin{bmatrix}
b_1(\pm \epsilon) b_2(\pm \epsilon) b_3(\pm \epsilon) b_4(\pm \epsilon) b_5(\pm \epsilon)
\end{bmatrix}^T. \tag{7}
$$

4. Numerical solution

The normalized IFs obtained using DDM are validated with X-FEM subjected to impermeable crack-face boundary conditions. In particular, here $W/c = 10$ considered for the analysis. Figure 2 justifies the good agreement between XFEM and DDM results.

![Figure 2: Validation of normalized IFs w.r.t inclination angle obtained using DDM with XFEM](image)

The variation of normalized IFs w.r.t $H/W$ for different crack-face boundary conditions are plotted in Fig. 3. All the normalized IFs decrease with the increase in $H/W$ ratio and remains almost constant for $H/W \geq 2$. The higher values at low aspect ratio shows the finite specimen effects on generalized IFs. Further, it shows that $K_{III}$ and $K_{IV}$ are independent of the crack-face boundary conditions whereas $K_{II}$ and $K_{V}$ are significantly dependent on it. The values of $K_{II}$ and $K_{IV}$ for limited-permeable crack-face conditions lie in between the values obtained under permeable and impermeable conditions.

![Figure 3: Variation of normalized IFs w.r.t H/W for different crack-face conditions](image)

Figure 4 shows the variations of normalized IFs w.r.t $c/W$ for different volume fractions under limited-permeable crack-face conditions. No effect is observed on $K_{I}$ and $K_{III}$ whereas $K_{II}$ and $K_{IV}$ significantly dependent on the volume fraction. $K_{IV}$ increases w.r.t volume fraction while $K_{II}$ shows the reverse behaviour. This is mainly due to the composition of BaTiO$_3$ and CoFeO$_3$ w.r.t volume fraction.

![Figure 4: Variation of normalized IFs w.r.t c/W for different volume fraction values](image)

5. Conclusion

The conclusions of the study are as follows:

1. The effect of finite specimen is observed on generalized normalized IFs. A specimen having $H/W = 2$ and $W/c = 20$ may be considered as an infinite domain.

2. Crack-face boundary conditions and volume fraction effects only $K_{II}$ and $K_{IV}$ whereas $K_{I}$ and $K_{III}$ independent of these.

3. The DDM is an efficient technique to analyze the fracture problems in 2-D finite MEE media.

References


Modelling the solidification of a liquid flowing in a narrow pipe using XFEM

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Abstract

The paper considers the solidification of an incompressible Newtonian fluid flowing through a circular cross-section microchannel subjected to the specified wall temperatures. It is assumed that the flow is laminar, driven by the pressure drop along the channel. Thermal properties of the phases are implicitly discontinuous across the interface. The location and topology of the interface change over time, the rate is dependent on the jump in the heat flux. The initially complex problem is reduced to the system that consists of the heat equation and a condition for the evolution of the interface. Two length scales are introduced into the equations and extended finite element method with the level set method are applied to solve the problem.

Keywords: extended finite element method, Stefan problem, level set method

1. Introduction

The freezing of water flowing through a pipe is a classical problem motivated by the damage due to ice expansion, Ref. [10]. Recently increased attention has been paid to the micro and nanoscales problems. The focus on the development of micro-devices leads to the new techniques. The phase change valve is an example of that technique since it is suitable for use in micro-channels, Ref. [1]. This form of valves uses an external cooling system to provide us with a result of the constant pressure drop Δp and the actual shape of the interface h(x, t). The initial temperature of the liquid is T₀.

The components of the velocity field u in the liquid phase are constructed assuming laminar flow and incompressible Newtonian fluid

\[ u_1(x, r, t) = \frac{2Q(t)}{\pi h(x, t)^2} \left(1 - \left(\frac{r}{h(x, t)}\right)^2 \right) \]

The derivation details are presented in [5].

The flux in the pipe is calculated using

\[ Q(t) = \frac{\pi \Delta p}{8m \int_0^{h(x, t)} \frac{1}{r} dx} \]

where h(x, t) is the distance from a point (x, 0) to the interface measured in r direction and m is a viscosity parameter.

Figure 1: Configuration of the narrow pipe problem
The evaluation of the level set function \( \phi \) into consideration the external, internal and Stefan conditions. The problem needs to be solved for discrete time steps, taking where

\[
\tilde{q}(\mathbf{x}, t) = k_S \nabla T(\mathbf{x}, t) \big|_{\Gamma^S} \cdot \mathbf{n}_I - k_L \nabla T(\mathbf{x}, t) \big|_{\Gamma^L} \cdot \mathbf{n}_I
\]

where \( \Gamma^S \) and \( \Gamma^L \) denote the solid and the liquid sides of the interface \( \Gamma_I \), respectively. The outward normal vector \( \mathbf{n}_I \) at the interface \( \Gamma_I \), pointing into the liquid phase, is defined.

Using length scales \( L_x = L \) and \( L_r = R \), we can define the following scaling for the space

\[
\hat{x} = \frac{x}{L_x}, \quad \hat{r} = \frac{r}{L_r}.
\]

The scaling (8) is introduced to Eqs (1)-(7) that results in non-dimensional formulation with respect to space.

3. Solution procedure

The finite element formulation of the problem is represented by the system of ordinary differential equations

\[
C^e \frac{dT}{dt} + BT + AT = f - q \quad t \in J.
\]

The problem needs to be solved for discrete time steps, taking into consideration the external, internal and Stefan conditions.

In this analysis, a fully implicit time approximation scheme is applied. When used in the system (9), the scheme gives

\[
\frac{1}{\Delta t} C^t = C^{t-1} + BT^n + AT^n = f - q
\]

where \( n = 1, 2, \ldots, N \) defines time step. The derivation details can be found in Ref. [6].

The extended finite element method is capable of describing discontinuities in the solution field by adding the enrichment term to the standard finite element approximation. In the paper, the abs-enrichment function proposed in Ref. [4] is used

\[
\Psi(x) = |N(x)||\phi| - |N(x)||\phi|
\]

where \( N(x) \) is the vector of the shape function and \( \phi \) is the signed distance function with respect to the interface.

The level set method is utilized to determine the front position. In the method moving interface \( \Gamma_I(t) \) is represented by the zero level set of a higher-dimensional surface \( \phi(t) \) such that

\[
\Gamma_I(t) = \{ x : \phi(x, t) = 0 \}.
\]

The evaluation of the level set function \( \phi \) in time under a velocity field \( \mathbf{u}_s \) is governed by the level set equation, which is a linear advection equation in the form

\[
\frac{\partial \phi(x, t)}{\partial t} + \mathbf{u}_s \cdot \nabla \phi(x, t) = 0 \quad \text{in} \quad \Omega \times J.
\]

In this study the penalty method is utilized in order to apply the internal condition (2).

4. Numerical results

The results are compared with these of the solution presented in Ref. [5]. The front positions at \( t=9.9 \) s and \( t=20.5 \) s in non-dimensional coordinates are shown in Fig. 2. The mesh of 150×50 rectangular elements and the time step \( \Delta t = 0.0125 \) s were used in the computations.

References


Topology optimization of a two-phase core of a sandwich panel

Tomasz Strek¹, Hubert Jopek²

¹ In the considered cases, the effective Poisson’s ratio was assumed dependent of the control variable \( r = r(x) \). Thus, objective functions for the optimization problem of the two-phase composite may be introduced:

\[
V_{\text{eff}}(r) = -\frac{E_{\text{transverse}}(r)}{E_{\text{longitudinal}}(r)}.
\]

Similarly, effective Young’s modulus as the ratio of an average longitudinal stress to average longitudinal strain

\[
E_{\text{eff}} = \frac{\overline{\sigma}_{\text{longitudinal}}}{\overline{\epsilon}_{\text{longitudinal}}}.
\]

The average stress and the average strain are defined as

\[
\overline{\sigma} = \frac{1}{S} \int \delta(x) dS, \quad \overline{\epsilon} = \frac{1}{S} \int \epsilon(x) dS
\]

where \( \delta \) and \( \epsilon \) are stated in a given direction (longitudinal or transverse), \( S \) is the volume of the considered composite.

3. Numerical results

In the considered cases, the effective Poisson’s ratio was assumed dependent of the control variable \( r = r(x) \). Thus, objective functions for the optimization problem of the two-phase composite may be introduced:

\[
V_{\text{eff}}(r) = -\frac{E_{\text{transverse}}(r)}{E_{\text{longitudinal}}(r)}.
\]

A control (design) variable is a generalized material density satisfying the following constraints [1]:

\[
0 < r(x) < 1 \quad \text{for} \quad x \in A_2
\]

The second is the integral inequality:

\[
0 < \int_{A_1} r(x) dA < A_1 A_2
\]

where \( A_1 \) is a fraction of the domain to use for the distribution of the second material, \( A_1 \) and \( A_2 \) are areas of the considered domain as shown in Figure 1.

For the purpose of the analysis of properties of two-phase sandwich plate parameters such as Young’s modulus, Poisson’s ratio and density were expressed by means of shape interpolation scheme SIMP (Solid Isotropic Material with Penalization) [1,4,7].

**Keywords:** auxetics, two-phase core, topology optimization, method of moving asymptotes (MMA)

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*This work was supported by grants of the Ministry of Science and Higher Education in Poland: 02/21/DS PB/3463/2015 and 02/21/DSPB/3465/2015.
The aforementioned parameters fulfill the formulas:

\[ E(r) = E_1 + (E_2 - E_1)r^p, \quad E_1 < E_2 \]  
\[ \nu(r) = \nu_1 + (\nu_2 - \nu_1)r^p, \quad \nu_1 < \nu_2 \]  
\[ \rho(r) = \rho_1 + (\rho_2 - \rho_1)r^p, \quad \rho_1 < \rho_2 \]

where penalization parameter is \( p > 1 \), \( E_i \) and \( E_2 \) are Young’s moduli, \( \nu_1 \) and \( \nu_2 \) are Poisson’s ratios, \( \rho_1 \) and \( \rho_2 \) are densities for the first and the second material, respectively.

Table 1. Materials properties

<table>
<thead>
<tr>
<th>Material 1</th>
<th>Material 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young modulus [Pa]</td>
<td>( E_1 = 10^7 )</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>( \nu_1 = 0.1 )</td>
</tr>
</tbody>
</table>

The boundary conditions (BC) used in calculation (see Figure 2) are as follows: right boundary: \( x = L \), \( y \in [0, L + h] \) - free BC; left boundary (symmetry): \( x = 0 \), \( y \in [0, L + h] \) - roller BC: \( \mathbf{n} \cdot \mathbf{u} = 0 \); bottom boundary (symmetry): \( y = 0 \), \( x \in [0, L] \) - roller BC: \( \mathbf{n} \cdot \mathbf{u} = 0 \); top boundary: \( y = L + h \), \( x \in [0, L] \) - roller BC: \( \mathbf{n} \cdot \mathbf{u} = 0 \); boundary load: \( \mathbf{F} \cdot \mathbf{n} = F L \mathbf{F} n \cdot \mathbf{u} = F L \). The method of moving asymptotes MMA [9,10] was used to find optimal distribution of two positive materials (see Table 1), and to achieve an auxetic composite (see Figure 3-4).
Anisotropic fractional non-local model

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Abstract

The description of anisotropic scale effect in the framework of fractional continuum mechanics is under consideration. The results are generalising the previous author’s formulation. The overall concept bases on the fractional deformation gradient which is non-local, as a consequence of fractional derivative definition.

Keywords: anisotropy, non-local models, fractional calculus, continuum mechanics

1. Introduction

Reliable modelling of heterogeneous materials, ranging from macro- to micro- and nanoscale of observation, in terms of continuum mechanics concept, needs non-local formulations [20]. The first articles in this area were released in the 1960s, and dates back to the scientists, such as Toupin [25], Mindlin [19], Eringen [8], Dillon [6], Dafalias [5], Bažant [4], Maugin [18], Aifantis [2], Fleck and Hutchinson [9]. These non-local models introduce a characteristic length concept, the parameter which is inherent to the inner material structure [10, 13, 17].

The regular motion of the material body \( B \rightarrow \mathcal{S} \) is a \( C^1 \) actual configuration of \( B \) in \( \mathcal{S} \), at time \( t \). We define the fractional deformation gradient and its inverse as follows (\( \alpha \in (0, 1) \))

\[
\mathbf{F}(\mathbf{X}, t) = \mathbf{F}^{\alpha-1}\mathbf{D}^{\alpha}\phi(\mathbf{X}, t), \quad (3)
\]

and

\[
\mathbf{F}(\mathbf{x}, t) = \mathbf{F}^{\alpha-1}\mathbf{D}^{\alpha}\varphi(\mathbf{x}, t), \quad (4)
\]

where \( D^{\alpha} \) is a fractional differential operator of Riesz-Caputo type, and \( \ell_X \) and \( \ell_\epsilon \) are length scales in \( B \) and \( \mathcal{S} \), respectively. For example the matrix representation of object \( \mathbf{F} \) is

\[
\mathbf{F} = \begin{bmatrix}
\ell \varphi^{\alpha_{11}} D^{\alpha_{11}} \varphi_1 \\
\ell \varphi^{\alpha_{12}} D^{\alpha_{12}} \varphi_1 \\
\ell \varphi^{\alpha_{13}} D^{\alpha_{13}} \varphi_1 \\
\ell \varphi^{\alpha_{21}} D^{\alpha_{21}} \varphi_2 \\
\ell \varphi^{\alpha_{22}} D^{\alpha_{22}} \varphi_2 \\
\ell \varphi^{\alpha_{23}} D^{\alpha_{23}} \varphi_2 \\
\ell \varphi^{\alpha_{31}} D^{\alpha_{31}} \varphi_3 \\
\ell \varphi^{\alpha_{32}} D^{\alpha_{32}} \varphi_3 \\
\ell \varphi^{\alpha_{33}} D^{\alpha_{33}} \varphi_3
\end{bmatrix}.
\]

In the following the kinematics bases on the introduced fractional deformation gradient and appropriate definition for stresses is necessary [24].

It should be emphasised that the anisotropic non-locality introduces a set material parameters: orders of fractional continua (\( \alpha_{ij} \)) and length scales (\( \ell_{ij} \)). They control the way in which the information from the surrounding influences particular point of interest. This result by far enhances modelling approach and allows closely mimic the experimental observations.

2. Fractional continua

The regular motion of the material body \( B \) can be written as

\[
x = \phi(\mathbf{X}, t), \quad (1)
\]

while its inverse is

\[
\mathbf{X} = \varphi(\mathbf{x}, t), \quad (2)
\]

thus \( \phi : B \rightarrow \mathcal{S} \) is a \( C^1 \) actual configuration of \( B \) in \( \mathcal{S} \), at time \( t \). We define the fractional deformation gradient and its inverse as follows (\( \alpha \in (0, 1) \))
4. Conclusions

The anisotropy of non-locality makes the modelling more reliable - by far allows easier mimic the experimental observations.

References


MS24
Thin-Walled Structures
organized by K. Magnucki, R. Mania and W. Pietraszkiewicz
Thermo-mechanical stresses in a brake disc

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Abstract

The aim of the study was to investigate the thermal and mechanical stresses occurring in disc brake during a single braking. Based on the finite element method, computational model was developed to simulate the process of frictional heating of the brake disc. The resulting temperature distributions were used to determine the associated quasi-static thermal stress. Thermal stress and stresses resulting from the occurrence of centrifugal forces, friction and pressure of the brake pad were combined on the basis of the superposition. The process of a single braking, from specified speed to a stop was analyzed. The calculations allowed to formulate general conclusions about the interactions of mechanical and thermal stresses and the stress state in the brake disc during braking.

Keywords: disc brake, frictional heating, thermo-mechanical stresses, finite element method

1. Introduction

In the operation of disc brake systems cases are known when large temperature gradients occurring in the friction pair elements lead to impaired braking action, or even to the destruction of its elements. On the contact surface of the rotating brake disc occurs short and intense heating, having the of a thermal shock nature. Traces of local overheating can appear, on the border of which thermal cracking grows rapidly. These phenomena can be observed even at a relatively low average temperature of the process and the mechanical loads which may cause under normal conditions only elastic deformation. Analysis of thermal stresses in the brake discs has been the subject of previous work [1,2] and this study makes an attempt was made to investigate the interactions of thermal and mechanical loads during braking.

2. Problem formulation

Heat flux density, which heats the brake disc in the contact zone with the brake pad, can be written as:

\[ q(r,t) = \gamma f \omega(t) r p_0 \]

where: \( f \) – coefficient of friction, \( \omega(t) \) – angular velocity, \( r \) – radial coordinate, \( p_0 \) – contact pressure. The heat partition ratio \( \gamma \) used in this expression is defined as follows [1]:

\[ \gamma = \frac{\sqrt{K_c \rho c_p}}{\sqrt{K_c \rho c_p} + \sqrt{K_r \rho c_p}} \]

Consider a single braking process from the initial angular speed \( \omega_0 = 88.464 \text{s}^{-1} \), to standstill during \( t_s = 3.96 \text{s} \). It is assumed that the pad is made of FMK-11 and the disc is cast-iron. The contact pressure is constant and equal \( p_0 = 1.47 \text{MPa} \). The initial temperature and the ambient temperature is equal to \( T_0 = T_a = 20 \degree \text{C} \), friction coefficient is constant and independent of temperature changes (\( f = 0.5 \)). It was also assumed that the material properties, working conditions are not temperature dependent. This calculation does not take into account the convective heat transfer (\( h = 0 \)) due to its negligible influence on the process of heating during the single braking. [3] (Fig. 1a).

![Figure 1: Scheme of the boundary conditions of heat conduction problem (a) and thermoelasticity problem (b)](image)

The solution of unsteady heat conduction problems led to the knowledge of temperature fields in the brake disc for the individual time steps of the braking process [4]. These results were used next as boundary conditions in the formulation of the thermoelasticity problem (Fig. 1b). In this step, the calculation takes into account the pressure, the friction force according to Coulomb’s law and centrifugal forces. Because of to the symmetry of the problem, only half of the disc thickness was considered, assuming no displacement nodes at plane \( z = -\delta \) in the \( z \) direction (\( u_z = 0 \)).

At the inner cylindrical surface of the disc (\( r = r_d \)) a lack of displacement in the circumferential direction (\( u_\theta = 0 \)) was established. This method of clamping gives the case of the floating brake disc, which can freely expand in the radial direction.
All calculations were accomplished using MSC.Nastran software and the authors' software used to prepare the boundary conditions and to automate the process of preparation of computational problems for the individual time steps. The developed computational models were tested using a finite element mesh with different densities. Finally, the disc region was divided into 86040 ‘CHEXA8’ type elements and 102960 nodes.

3. The results of calculations and conclusions

The results of the calculation of the temperature evolution and the equivalent Huber-Mises stress on the surface of the brake disk \((z = 0)\) for the selected radial coordinate \((r = r_d, r_p, r_m, R_d)\) during braking are shown in Fig. 2. The plot shows the temperature (the dotted line), the stresses resulting from the influence of mass inertial forces \((1)\), thrust of the brake pads \((2)\) and thermal stresses \((3)\). Resulting stresses \((4)\) where obtained on the basis of the superposition of all components mentioned above.

Analysis of the results of calculations indicate thermal load as the main determinants of the state of stress in the brake disc during braking. Centrifugal forces significantly affect the stress state at the beginning of the braking process near the inner regions of the disc (Fig. 2a, b). Taking into account the operation of the brake pads in the form of the friction and pressure in contact area, the maximum values of total stresses are lower. A brake pad softens sharp changes in the thermal stresses on the surface of the brake disc.

References


Hydroelastic stability of thin-walled structures interacting with liquids

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Abstract

The paper presents a 3D formulation of the spectral problem and finite element algorithm for its numerical implementation, designed to investigate natural vibrations and stability of pre-stressed shells and plates interacting with a quiescent or flowing fluid.

Keywords: hydroelasticity, stability, natural vibrations, shells, plates, FEM

1. Introduction

It is known that the interaction of elastic shells or plates with a liquid causes vibrations of the system [3]. In the context of this problem, along with the investigation into the dynamic response of the system it is expedient to determine critical flow velocities above which the structure loses stability. The advanced commercial software packages, which employ the resolving Navier–Stokes equations for describing liquid behavior, allow researchers to perform an unsteady two-way coupled fluid-structure interaction analysis. However, such a formulation of the problem explicitly lacks the stability criterion. This generates a need for solving a resource-intensive coupled problem at different flow velocities and to analyze the stability/instability of the mode on the basis of the energy criteria or for tracing the displacements at different points. Clearly, any preliminary knowledge of the critical velocity, for example, of its upper or lower boundary will essentially reduce computational costs.

In the paper we consider a simplified model, whose the numerical implementation is based on the finite-element model developed for the analysis of natural vibrations and hydroelastic stability of noncircular cylindrical shells and plates interacting with the internal fluid flow. Here we proceed from the supposition that the calculated values of the critical velocity are the lower bounds, which can be used as the initial approximations in the solution of the unsteady problem.

2. Mathematical statement of the problem

In the case of small perturbations the basic equations describing the non-vortex dynamics of compressible liquids are expressed in terms of the perturbation velocity potential \( \phi \) in the shell-linked coordinate system \((x, y, z)\). At the interface between the elastic structure and liquid \( S_0 \) we impose the impermeability condition. The application of the Bubnov–Galarkin method yields the following matrix relation [2]

\[
(K_f + A_f^*) \phi + M_f \ddot{\phi} - C_f \dot{\phi} - C_f \dot{\omega} - A_f \omega = 0
\]

where \( \omega \) is the normal component of the shell displacement vector. The expressions describing the deformation of elastic structure are derived using the relations of the non-linear theory of thin shells (plates), which relies on the Kirchhoff–Love hypothesis. For the mathematical statement of the problem we employ the principle of virtual displacements, taking into account the work done by the inertial forces and the hydrodynamic pressure, which is calculated at the wetted surface by the Bernoulli equation \( p = -\rho_f (\partial \phi / \partial t + \mathbf{U} \cdot \nabla \phi) \).

Linearization of the stress-strain relations with respect to the equilibrium state contiguous to the initial state allows us to formulate the hypothesis for pre-stressed non-deformed state, in its context the variational equation can be written as [1]

\[
\int_{S_f} \delta \varepsilon^T \mathbf{D} \delta \varepsilon dS + \int_V \rho \delta \dot{\varepsilon}^T \delta \varepsilon dV - \int_{S_f} \delta \varepsilon^T \mathbf{P} \delta S + \int_{S_f} \delta \varepsilon^T \sigma_0 \delta S = 0, \quad \mathbf{P} = \{0, 0, p, 0, 0, 0\}^T
\]

Here \( \varepsilon, \dot{\varepsilon} \) and \( \mathbf{P} \) are the vectors of generalized strains, displacements and pressure acting on the wetted surface; \( \mathbf{D} \) is the matrix of elastic constants of isotropic material; \( p \) and \( \rho_f \) are the density of the shell material and the fluid density; \( \mathbf{e} \) is the strain vector; \( \mathbf{U} \) is the fluid flow velocity; \( \sigma_0 \) is the matrix of the initial forces and moments. The implementation of the standard FEM procedures leads to the matrix equation

\[
(K_s + K_g) \mathbf{d} + M_s \ddot{\mathbf{d}} + \rho_f C_s \dot{\mathbf{d}} + \rho_f A_s \phi = 0
\]

In the case when the structure is exposed to high environmental temperatures the dependence of the material properties (elasticity modulus, Poisson’s ratio, coefficient of thermal expansion, thermal conductivity \( k \)) on temperature is expressed as a polynomial

\[
P(T) = P_0 \left( P_{-1} T^{-1} + P_1 T + P_2 T^2 + P_3 T^3 \right)
\]

Here \( P_i \) are the coefficients of temperature \( T \), which are unique for each particular material. The distribution of temperature through the shell thickness is determined by solving numerically a quasi-linear equation of steady-state thermal conductivity with corresponding boundary conditions

\[
\frac{d}{dz} \left[ k(z,T) \frac{dT}{dz} \right] = 0, \quad T|_{z=-h/2} = T_i, \quad T|_{z=h/2} = T_u
\]

where \( h \) is the thickness of structure. The forces and moments generated by the applied mechanical or temperature loads \( P_0 \), are used to form the geometric stiffness matrix \( K_g \). They are obtained by solving the static problem \( K_d \mathbf{d} = P_0 \).

To investigate the behavior of loaded shells or planes interacting with a quiescent or flowing liquid it is necessary to solve a

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coupled system of two equations (1) and (3) taking into account the existence of pre-stressed non-deformed state caused by the impact of the static force factors on the elastic structure

\[
(K + A) \{ \ddot{d} \phi \}^T + M \{ \dot{d} \phi \}^T + C \{ d \phi \}^T = 0 \quad (5)
\]

\[
K = \begin{bmatrix} K_s + K_y \\ 0 \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} -A_s \\ A_s \end{bmatrix}, \quad C = \begin{bmatrix} 0 & \rho_f A_s \\ -C_f & -C_f \end{bmatrix}
\]

Here \( K, M, C \) and \( A \) are the stiffness, mass, damping and hydrodynamic stiffness matrices.

Assuming that the perturbed motion of the shell and fluid is harmonic the system Eqn. (5) can be reduced through the algebraic manipulation to a standard eigenvalue problem

\[
\left[ \begin{array}{cc} 0 & I \\ -M^{-1} (K + A) M^{-1} C \end{array} \right] \{ q, f, q, f \} = \omega^2 \{ q, f, q, f \}
\]

Here \( I \) is the unit matrix, \( q \) and \( f \) are some coordinate functions. \( \omega \) is the characteristic quantity. The complex eigenvalues \( \omega \) were calculated using the ARPACK procedure, which is based on the implicitly restarted Arnoldi method. Simulation of shells with arbitrary cross-sections was performed by assuming that the curvilinear surface is approximated to a sufficient accuracy by a set of plane quadrilateral elements. For discretization of the fluid domain we used an 8-node finite element in the form of a prism.

3. Numerical results

In various practical applications the thin-walled structures are subjected not only to mechanical but also to temperature loads. The developed finite-element algorithm was used in a series of numerical experiments, which allowed to estimate the influence of both factors on the eigenfrequencies and vibration modes and also on the mechanical and hydroelastic stability of thin-walled cylindrical shells with arbitrary cross-section and thin plates interacting with a quiescent or flowing fluid. The analysis was performed under the assumption that the external surface of the structure is subject to uniformly distributed pressure and heating and the internal surface is at temperature \( T_i = 300 \) K.

3.1. Circular and elliptical cylindrical shells

The influence of kinematic boundary conditions prescribed at the structure ends was investigated for circular cylindrical shells. It was shown that the mechanical and temperature loads exert a destabilizing effect and reduce the vibration eigenfrequencies down to the values responsible for the onset of divergence instability. It was found out that shells clamped at both ends (hereinafter denoted by \( C \)) are more sensitive to heating, since they lose stability at essentially lower values of temperature. By contrast, for cantilevered shells (\( F \)) a rise of temperature is not so critical and leads to considerably less reduction of eigenfrequencies. In the case of hinged support of the right-hand end, the structure with such type of boundary conditions demonstrates permanent stability against temperature loads. This can be explained by the fact that the temperatures, which may possibly lead to the loss of stability, are higher than the melting point of the material under consideration.

For cylindrical shells with a elliptical cross-section containing a quiescent liquid the ratio of semi-axes of the ellipse, \( R_u \) and \( R_y \), is defined by the parameter \( \beta = R_u / R_y \) (\( \beta = 1 \) corresponds to a circular configuration). An increase in the level of liquid, both in the vertical and horizontal shells, leads to a decrease in the vibration eigenfrequencies of the system. It was found that the value of the critical pressure or temperature is practically independent on the fluid level. This is true only when the geometrical dimensions of the system are such that the hydrostatic load caused by the mass forces of the liquid is small. The results of investigation into the effect of liquid level, geometrical parameters and kinematic boundary conditions on the eigenfrequencies and vibration modes of elliptical cylindrical shells with a quiescent fluid under the action of different force factors are discussed in greater detail in [1].

Similar investigations were carried out for clamped-clamped shells interacting with a flowing liquid. In this case, an increase in the flow velocity leads to the onset of the divergence instability. It was established that the external hydrostatic pressure, heating of the external lateral surface of the shell, an increase in the shell length or in the level of the liquid in the shell are the factors that reduce the critical velocities, at which the loss of stability takes place. Conversely, the internal pressure, longitudinal tensile loads and increase in the shell thickness produce a stabilizing effect.

3.2. Rectangular plates

We also investigated the hydrodynamic stability of rectangular plate placed on top of the channel with rigid walls conveying liquid with the velocity \( U \). The analysis of possible combinations of kinematic boundary conditions prescribed at the plate ends revealed the existence of different types of instability. Some typical results are given in Table 1.

| Table 1: Dimensionless critical velocities and instability type |
|-----------------|-----------------|
| CCCC | CCCF | CCFF | FFCC |
| 3.7243 | 11.2002 | 1.8570 | 8.2761 |

It was found that the influence of the liquid layer on the eigenfrequencies and critical velocities responsible for the loss of stability is inessential in the case when the height of the channel exceeds the length of the longest side of the plate.

References


New structures shaped with the SADSF method and their properties in elastic state

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Abstract

The work presents a series of original thin-walled structures shaped with the use of statically admissible discontinuous stress fields (the SADSF method) assuming the condition of equalized effort in limit state. These are particular structures, build of plane elements, which are suitable for carrying torsion moment load despite the fact that all surfaces of the structures are accessible from outside. In the result of solving the problem, determination was made of approximate shapes and dimensions of the structures, and, first of all, 3D layout and systems of connections between their component elements. The work is complemented with results of numerical investigations (FEM), which confirm good properties of these structures also in the elastic range of load.

Keywords: thin-walled structures, limit analysis, shape and topology optimization, computer methods

1. Problem formulation and solution

A typical formulation of the shaping problem, and the solution found with the use of the SADSF method, is presented visually in Fig. 1, in which one of new solutions is shown. The set of data included only the material of the sought-after structure and the boundary conditions (Fig. 1a): the stress loads \( p \) applied to the borders \( S_0 \) (here they are reduced to two pairs of forces applied to extreme membranes) and geometry of the given borders (dimensions: \( c, L, h \) and \( \delta \)).

![Figure 1: Illustration of problem formulation and solution](image)

The solution obtained in this way has many faults, also significant ones, resulting mainly from properties of the SADSF method. It is an approximate method, operating on fields that satisfy only a part of the problem conditions, it assumes a rigid-plastic model of material and the criterion of equalized effort; it doesn’t take into account stability problems, not even considers stages of elastic work. However, this criticism loses its sharpness when we put the method faults together with the nature of problems to be solved. Note that:

1. The obtained statically admissible limit stress field (Fig. 1b) defines the sought-after structure in a complete way: determines its approximate shape and dimensions, indicates the places in which welds must be laid, and, first of all, determines layout and system of connections between component elements in space.

The layout of elements resulting from the solution guarantees that, although all surfaces of the shell are accessible from outside, it still exhibits high torsion strength. If any of the elements was removed, or deprived of ability to carry load that results from the field, then the load carrying capacity of the shell would decrease even several dozen times. With such a great scale of possible changes, approximation inaccuracies or other faults of the method seem not to be so important.

2. The problems similar to that illustrated in Fig. 1 are encountered in initial phase of design of any construction. At that moment, there are given only boundary conditions, and is little known about the structure itself.

2. Direct formulation and application version

In comparison to other methods that also undertake problems with unknown 3D layout of material, including the topology optimization methods [2] developed based on the concept of finite elements (FEM), the SADSF method shows uniqueness, which results from its two unique features:

1. It doesn’t make use of iterative corrections, thanks to which one can deal with the problems in which small changes in boundary conditions (or in constructional details) may cause large and far-reaching changes in the solution. Such a properties are characteristic for a majority of thin-walled structures.

2. In the direct approach, typical problems of the SADSF method (such as that in Fig. 1) lead to extremely difficult, non-linear problems with unknown discretization, containing singularities. For an arbitrarily assumed discretization, the solution usually does not exist.
Despite these facts, the SADSF method has a property that cannot be found among other methods: it makes it possible to build, within its framework, a relatively simple application version, in which these difficulties are circumvented. It consists in connecting – like building blocks in various allowable combinations – known particular solutions available for the designer in the form of a library.

Typical problems, such as those presented in this work, can be solved with the application version aided with the software developed by one of the authors [4]. The software does not impose any significant limitations on complexity of the designed structures.

3. Further example solutions

The assumed criterion does not lead to unique solutions, and gives quite a lot of freedom for creating variants. In Fig. 2, there are presented four other solutions, defined on complex fields, which are obtained by assembling various particular systems of stress discontinuity lines.

![Figure 2: Examples of contours obtained from other solutions](image)

These solutions, in the software denoted with symbols $hp$ and $s20$, are used in structures shown in Figs. 2a and 2b, respectively. The fields type $hp$ were also used in the structures from Figs. 2c and 2d, where they were assembled in different configurations. The complex field depicted in Fig. 1b is composed of 16 library fields type $d2hp$, and, in this unique case, there is shown the network of stress discontinuity lines. In the remaining cases, only contours of component fields are shown. To visualize 3D layout of the fields, the drawing of one of them is blank, the others are shaded.

It is worth noting that the presented spatial fields could be perceived as segments of more complex fields, and can be multiplied. Moreover, each of the fields was obtained by assembling, in different combinations, only one library field.

As it can be seen, the contours of complex fields define the structure of torsion-loaded, thin-walled beams, of rather specific construction (as far as layout and system of connections between component elements is concerned). The existence of such structures was suggested by other solutions described in [4] that were also obtained with the use of $hp$-type fields.

4. Properties of designed structures at the stage of elastic work. Domain of method application.

The elastic stage of work is not considered in the SADSF method, however, in the structures shaped with this method one finds generally well-equalized effort level, or equalized at least along free borders. Such a property was found (among other things) through FEM analyses of several dozen solutions [5] and in experimental investigations. No negative example was encountered, so far. Then, these are the structures which can be accepted at once, or subjected to further corrections.

However, as a primary area of the SADSF method application we recommend the initial phase of design, all the more so because this phase is deprived of engineering methods, and dominated by intuitive methods of design. On the other hand, it is known that this stage decides on most important load-carrying properties of the future structure. Especially in the case of thin-walled structures, it is very easy to make gross errors which result in low load-carrying capacity, despite the fact that, in many cases, small constructional changes could significantly heighten this capacity.

Application of the SADSF method makes it possible to design structures of good or very good strength properties. In this respect, the SADSF method is regarded as a complementary one to the FEM.

5. The state of fundamental research

Investigations on the SADSF method were initiated by Professor W. Szczepiński in 1968 [1]. It was the subject of research of many teams of scientists, mainly from the Institute of Fundamental Technological Problems of the Polish Academy of Sciences (the IPPT PAN). However, because for over 20 years one could not find a complete mathematical description of the method and develop algorithms, the investigations were abandoned. Today, such barriers do not exist any longer [3, 4]. However, fundamental research on the method are still possible when one applies numerical methods. This is because the functions on which the method conditions are based are, in most cases, so complex that cannot be analytically transformed, so they must be left in the form of algorithms. The most important thing is not to formulate the set of conditions and determine parameters of the field that satisfies the assumed boundary conditions, but to find such a system of stress discontinuity lines for which there exists a solution for this field. When discretization is not known, even the dimension of the system of equations and inequalities remains unknown, not to mention the possibility of making up the equations.

Continuation of this research has also a practical aspect. It may serve to find (among other things) new solutions which supplement the libraries of the method applications.

References

Procedure for Automatic Solution of Free Vibration Problem of Timoshenko Beam with Attachments

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Abstract

The procedure for an automatic solution of a free vibration problem of Timoshenko beam was created, with attachments like: translational and rotational springs, concentrated mass including the moment of inertia, linear oscillator and additional supports. The exact solution of a free vibration problem of the beam without attachments is considered in the formulation of a procedure. The Lagrange multiplier formalism was used to formulate and solve the vibration problem of a complex system. The proposed procedure can be used for a free vibration analysis of complex systems consisting of a beam with any number of discrete elements.

Keywords: Timoshenko beam, free vibration problem, attachments, Lagrange multiplier formalism, discrete elements

1. Introduction

The problem of free vibration of Timoshenko beams with attachments has been widely investigated and a broad overview of works is given in [2,3]. In the paper the procedure for automatic formulation of solution of free vibration problem of Timoshenko beam with attachments was presented. The Lagrange multiplier method presented in [2] and the solution of the free vibration problem of Timoshenko beam shown in [2,3] were used to formulate the solution of the problem of free vibrations of the considered types of complex systems.

2. The formulation and solution of the problem

The considered system consists of a Timoshenko beam with attachments like: translational and rotational springs, concentrated mass, linear oscillator and additional supports against the beam translation or rotation. The motion of the beam without the additional elements can be described as [1]:

\[
\frac{\partial^2 y(x,t)}{\partial t^2} + \rho \frac{\partial^2 y(x,t)}{\partial x^2} - \rho l (1 + \frac{E}{k G z}) \frac{\partial^2 y(x,t)}{\partial x^4} = 0,
\]

(1)

\[
\frac{\partial^2 \psi(x,t)}{\partial t^2} + \rho \frac{\partial^2 \psi(x,t)}{\partial x^2} - \rho l (1 + \frac{E}{k G z}) \frac{\partial^2 \psi(x,t)}{\partial x^4} = 0,
\]

(2)

where \(y(x,t)\) is the total deflection of the beam at point \(x\), \(\psi(x,t)\) is the angle of rotation due to bending, \(\rho(x)\) is the mass per unit length, \(\rho l\) is the mass moment of inertia per unit length about the neutral axis passing through the center and \(\rho\) is the mass density, \(E\) is the modulus of elasticity, \(l(x)\) is the area moment of inertia about the neutral axis, \(G\) is the shear modulus, \(A(x)\) is the cross-sectional area and \(k\) is the numerical factor depending on the shape of the cross section.

Considering the case of a free beam not connected with any additional elements the boundary conditions can be formulated as:

\[
\psi(x,t)|_{x=0} = 0, \quad \frac{1}{L} \int_0^L y'(x,t) \, dx - \frac{1}{L} \int_0^L \psi(x,t) \, dx|_{x=0} = 0,
\]

(3, 4)

\[
\psi(x,t)|_{x=L} = 0, \quad \frac{1}{L} \int_0^L y'(x,t) \, dx - \frac{1}{L} \int_0^L \psi(x,t) \, dx|_{x=L} = 0.
\]

(5, 6)

Solving the formulated boundary problem the total deflection \(y\) and rotation \(\psi\) may be expressed:

\[
y(x,t) = \sum_{i=1}^n Y_i(x) \xi_i(t), \quad \psi(x,t) = \sum_{i=1}^n \Psi_i(x) \xi_i(t),
\]

(7, 8)

where \(Y_i(x)\) denotes the \(i\)-th transverse vibration mode and \(\Psi_i(x)\) the \(i\)-th rotational vibration mode according to the chosen boundary conditions (3-6). The detailed solution was presented in [3].

With respect to the Timoshenko beam theory and considering the derived transverse \(Y_i(x)\) and rotational \(\Psi_i(x)\) modes the kinetic and potential energies of the beam can be expressed in the form:

\[
T_i(t) = \frac{1}{2} \sum_{i=1}^n M_i \ddot{\xi}_i^2, \quad U_i(t) = \frac{1}{2} \sum_{i=1}^n K_i \dot{\xi}_i^2,
\]

(9, 10)

where:

\[
M_i = \int_0^l Y_i(x) \rho A(x) dx + \int_0^l \Psi_i(x) \rho l(x) dx,
\]

\[
K_i = \int_0^l E I(x) \ddot{Y}_i(x) dx + \int_0^l k G A(x) \dot{\Psi}_i(x) dx.
\]

(11)

Using the expressions (9) and (10), according to the procedure described in the paper [2] the kinetic and potential energies including all elements of the system before their connection can be formulated:

\[
T = \frac{1}{2} \sum_{i=1}^n M_i \ddot{\xi}_i^2 + \frac{1}{2} m z_0^2 + \frac{1}{2} M z_1^2 + \frac{1}{2} J \dot{\phi}_1^2,
\]

(12)

\[
V = \frac{1}{2} \sum_{i=1}^n K_i \ddot{\xi}_i^2 + \frac{1}{2} K z_0^2 + \frac{1}{2} C \dot{\phi}_1^2 + \frac{1}{2} K M(z - z_1)^2,
\]

(13)

where \(K\) and \(K_M\) are the linear translational spring stiffnesses, \(m\) and \(M\) are the masses, \(C\) is the linear rotational spring stiffness, \(J\) is the rotary inertia and \(z, z_0, z_1, \phi_1, \phi_2\) are the motion coordinates of the additional elements.
The additional elements are connected to the beam at points $x_i (k=1,2,...,7)$ described by constraints:

$$
\begin{align*}
&f_1 = \psi(x_i) - z_i = 0, \quad f_2 = \psi(x_i) - z_2 = 0, \\
&f_3 = \psi(x_i) - \phi_3 = 0, \quad f_4 = \psi(x_i) - \phi_4 = 0, \\
&f_5 = \psi(x_i) - z_3 = 0, \quad f_6 = \psi(x_i) = 0, \quad f_7 = \psi(x_i) = 0.
\end{align*}
$$

(14)

The Lagrangian for the combined system may be written as:

$$
L = T - V + \sum_{i=1}^{10} \lambda_i f_i,
$$

(15)

where $\lambda_i$ is the Lagrange multiplier and $R$ is the number of the attachments in the system. Using Lagrange equations it holds

$$
M_\xi \ddot{\xi}_i + K_\xi \xi_i = \sum_{i=1}^{10} \lambda_i b_i,
$$

(16)

where

$$
b_i = \left\{ \begin{array}{c}
\dot{Y}(x_i) \
\psi_i(x_i)
\end{array} \right\}
$$

(17)

Assuming a simple harmonic motion:

$$
x_i = A e^{i\omega t}, \quad z_i = Z e^{i\omega t}, \quad \xi_i = Z e^{i\omega t}, \quad \lambda_i = \lambda_i e^{i\omega t}
$$

(18)

for $i = 1,2,5,6$.

The system of equations (16) may be solved for the $A_i, Z_1, \phi_k$ and $Z$ in terms of the $A_i$:

$$
A_i = \sum_{i=1}^{10} \lambda_i b_i, \quad Z_1 = A_i / K_1,
$$

(19)

$$
Z_2 = A_2, \quad \Phi_1 = A_i / C_1, \quad \Phi_2 = A_i / (J_0 \omega^2),
$$

(20)

$$
Z_2 = A_5, \quad \Phi_1 = A_i / (C_0 \omega^2), \quad Z = A_i / (C_0 \omega^2)
$$

Substitution of Eqs. (19) into Eqs. (14) gives

$$
CA = 0,
$$

(21)

where $A = [A_1, A_2, A_3, A_4, A_5, A_6, A_7]^T$, and $C$ is a symmetric square matrix of size equal to the number of attachments in the model. In the present case $C$ has the form

$$
C = \begin{bmatrix}
C_{11} + e_1 & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} & C_{17} \\
C_{21} & C_{22} + e_2 & C_{23} & C_{24} & C_{25} & C_{26} & C_{27} \\
C_{31} & C_{32} & C_{33} + e_3 & C_{34} & C_{35} & C_{36} & C_{37} \\
C_{41} & C_{42} & C_{43} + e_4 & C_{44} & C_{45} & C_{46} & C_{47} \\
C_{51} & C_{52} & C_{53} & C_{54} + e_5 & C_{55} & C_{56} & C_{57} \\
C_{61} & C_{62} & C_{63} & C_{64} & C_{65} + e_6 & C_{66} & C_{67} \\
C_{71} & C_{72} & C_{73} & C_{74} & C_{75} & C_{76} + e_7 & C_{77}
\end{bmatrix},
$$

(22)

where the following coefficients are introduced:

$$
C_{kj} = \sum_{i=1}^{10} b_{ki} b_{ji} K_{i-\omega^2 M_j},
$$

(22a)

$$
e_1 = \frac{1}{K_1}, \quad e_2 = \frac{1}{M_0 \omega^2}, \quad e_3 = \frac{1}{C_1},
$$

(22b-d)

$$
e_4 = -\frac{1}{J_0 \omega^2}, \quad e_5 = \frac{1}{K_0 \omega^2 - \frac{1}{M_0 \omega^2}}, \quad e_6 = 0, \quad e_7 = 0.
$$

(22e-h)

For a non-trivial solution of the system of equations (20) the determinant of the matrix $C$ is zero

$$
\det C = 0
$$

(23)

that is the equation for computation of the eigenfrequency $\omega$ of the combined system. In this equation the coefficients $C_{kk}$ characterize the beam as the base system element and the coefficients $e_k$ characterize the additional elements attached to the beam.

A number of attachments determines the size of matrix. A new set of equations of the form (20), according to the number of additional elements (attachments) connected with the beam to create the complex system, can be determined. Each matrix element depends on the coefficient $C_{kk}$ and the type of attachment characterized by the coefficient $e_k$. Thus, having defined the matrix $C$ the natural frequencies of the considered continuous-discrete system can be determined.

Additionally, after calculation of the frequency values of the complex system under consideration the mode shape corresponding to each determined frequency $\omega_k$ can be computed as presented in [3].

3. Conclusions

The procedure for automatic formulation of solution of free vibration problem of complex systems was worked out. The procedure makes it possible to obtain the solution of a free vibration problem of arbitrary complex system consisting of uniform Timoshenko beam with any number of attachments like: translational and rotational springs, concentrated mass, linear oscillator and additional support elements. While defining system parameters the solution results can be obtained numerically on the basis of a mathematical model. The formulation and solution of the problem was based on the Lagrange multiplier formalism. The important feature of the proposed procedure is that the mathematical model is created using the analytical solutions.

The proposed procedure can be easily improved for other continuous-discrete systems consisting of continuous element connected with any number of discrete elements.

References


Stability FEM analysis of steel members restrained by sheeting accounting for imperfections

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Abstract

In the paper the stability analysis is presented of steel hot-rolled beams or welded girders restrained by trapezoidal cladding. In order to determine the stability behaviour a nonlinear FEM model oriented to engineering structures with imperfections is developed. Attention is paid to modelling the connection between steel beams and cladding where trapezoidal sheeting is not only an element which transmits the external loads to main steel elements but also a structural member stiffening the beam. In the proposed FEM model stiffening effect of trapezoidal sheeting is modelled by elastic springs which stiffness is based on experimental data [4]. The post-buckling analysis is carried out employing shell elements and introducing initial geometric imperfections employing the method presented in [5]. It is demonstrated that in the case of lateral-torsional buckling an important role is played by rotational stiffness of the connection and by stiffness of the element and cladding. Therefore the most important factor in a numerical model is proper calibration of elastic spring stiffness. Moreover, an unstable post-buckling behaviour and high sensitivity to imperfections is presented by means of several examples.

Keywords: stability analyses, steel beams, trapezoidal cladding, initial geometric imperfections

1. Introduction

It is well known that cooperation between steel girders and sheeting provides in civil engineering several benefits. However, the problem of this cooperation is still the subject of theoretical and experimental studies presented in numerous papers. The first significant laboratory tests concerned the principles of the cooperation of trapezoidal sheets and steel columns and beams made of hot rolled or welded plate girders have been published already in the 60s and 70 by Errera, Pincus, Fisher [3]. The research conducted by Garncarek [4] was focused on experimental studies too. Their research provided the data to extend the theoretical background concerning the influence of trapezoidal sheeting on torsional/flexural stiffness of the beam, not included in the standard [2] previously.

In the paper an advanced numerical model is developed taking into account a wide range of steel hot rolled beams and welded girders cooperating with trapezoidal steel sheeting with various dimensions. The nonlinear analysis is carried out taking into account initial geometrical imperfections according to the method proposed by Garstecki and Rzeszut in [5].

2. Numerical examples

Let us consider trapezoidal sheets with the height of wave T130 and T153 with thickness 0.75 mm and 1.0 mm and double span welded plate girders with height 650mm. Figure 1 shows the numerical model (3D) where I-beam and trapezoidal sheet are created as a "shell deformable" elements (2D). Axial connectors are inserted in every wave connecting trapezoidal sheet through trough at a distance of 3 cm from the edge of the top flange of I-beam. The contact between the upper surface of the top flange of the cross-section and the trough of the trapezoidal sheet has been defined using surface to surface "Hard" contact. The beam flange is assumed penetrating surface "master" and the surface of trough of the sheet plays the role of penetrated surface "slave". The material properties assigning elastic-plastic properties are adopted to the sheet and beam as well. The applied boundary conditions reflect the conditions of a real construction. The sheet is connected to the purlins by means of special connectors which connected middle surfaces of trapezoidal sheet and I beams. Between the sheet and the beams a frictionless contact was defined. The edges of the sheet that are parallel to the purlins are free, whereas two other edges are simply supported.

Nonlinear numerical analysis is carried out for different initial geometrical imperfection pattern employing ABAQUS software. The imperfections are modelled in form of a displacement vector with the dimension and physical meaning adequate to the displacement vector in FEM model of the structure. The obtained results are presented in Fig. 2.

3. Conclusions

By means of several examples the interaction of sheeting with steel girders was studied and the applied numerical model was validated. It was found that the most important role in structural behaviour is played by appropriate modelling of connectors and proper calibration of their elastic springs stiffness. Moreover, it was also presented that structure under consideration demonstrates high sensitivity to imperfections and unstable post buckling behaviour.
Figure 1: Numerical model: a) boundary conditions, b) constructional details, c) connector

Figure 2: Equilibrium path of the girder-trapezoidal sheet structural system for different initial geometrical imperfections

4. References


Influence of rib configuration on stability of thin walled steel beams

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Abstract

Effectiveness of various rib configurations is studied. Linear stability analysis of a large number of beams for variable orthogonal and diagonal rib configurations and web slenderess ratios was conducted. The type of buckling modes was analysed and the lowest eigenvalues for global and local modes were found for each beam. It was observed that diagonal ribs provide better resistance against local and global instability. The latter effect can be assigned to increase torsional stiffness provided by diagonal ribs this effect is studied numerically in the paper. Several examples are solved aiming at quantitative evaluation of torsional stiffness of beams with diagonal ribs and their stability response in the form to be used by designers.

Keywords: thin walled beams, stiffening ribs in beams, diagonal ribs, stability of beams, interactive stability

1. Introduction

Modern methods of welding and corrosion protection make it possible to design steel girders with high slenderess ratios. As an effect steel consumption can be reduced and thus reduced is the amount of energy and environmental pollution connected with steel production. This belongs to principles of sustainable development which usually involves additional costs. We face an opposite relation because slender girders are usually less expensive in production, transport and erection. These aspects are worth noting, because they will determine permanent trend to design more and more slender structural elements. Slender elements are susceptible to loss of stability, therefore this failure mechanism must be considered carefully in design. Stability of thin walled beams and columns has been broadly discussed in the literature. Currently the research is focussed on local and interactive instability, elastic-plastic effects and sensitivity to imperfections. Shear resistance of webs considered as thin panels stiffened by ribs was studied experimentally and numerically in [5, 6]. Elastic-plastic instability of plated members in channel sections was investigated in [3] using numerical and experimental approach. The theory and applications of sensitivity analysis oriented to thin walled beams, columns and frames are presented in [7, 8]. Influence of head plates on critical bending moments in beams was evaluated in [2]. Rigid head plates constrain the warping, therefore the bimoment stress can appear in support regions and result in essential increase of critical load in global buckling. To protect a thin walled girder against local instability ribs with proper configuration and spacing must be introduced. Classical rib configurations used by design engineers are the orthogonal ones. In [4] a diagonal rib configuration was proposed. The theoretical model was created, where the actual diagonal ribs were substituted by continuously distributed fibres similarly inclined to the longitudinal axis of the beam as the inclination of discrete ribs. It was demonstrated that diagonal ribs provide a higher torsional stiffness and can also ensure higher resistance against global instability. Total manufacturing cost of girders with orthogonal and diagonal ribs was evaluated in [1] using the actual unit costs of steal, cutting, welding and corrosion protection.

2. Problem description

Stability of a large number of girders with I section for various orthogonal and diagonal rib configurations was analysed using FEM with shell elements. Based on the linear stability formulation the lowest eigenvalues associated with global and local forms of instability were computed. Comparing the diagonal rib configurations with the orthogonal counterparts of a similar cost it appeared that the diagonal ribs provide higher critical loads both in global and local instability.

Figure 1: Variation of eigenvalue depending on the web slenderess ratio. Upper plots: diagonal ribs inclined 45°, 54° and 61°. Lower plots: orthogonal ribs with variable length of longitudinal ribs k=a/L = 0, 0.2, 0.4, 0.6 and 1.0 (a=length of horizontal rib in m, G indicates global instability, L – local type)

Results of a linear stability analysis shown in Fig. 1 refer to girders with the dimensions: length of span $L=7.5$ m, height of web $h_w=0.74$ m, thickness of webs $t_w=3-6$ mm, flanges
180x10 mm and thickness of ribs 10 mm. There are two pin supports at the ends of the beam and a vertical point load applied at the upper flange in the mid-span point. The ends of beams are restrained against torsion. Note that girders with orthogonal ribs demonstrated a global form of buckling for the whole analysed range of slenderness ratios and eigenvalues are low. This response could not be improved by the modification of orthogonal ribs. Contrarily, diagonal web configurations demonstrated different forms of buckling, depending on the web slenderness ratio. A low slenderness ratio equal 123, generated global form of instability, medium ratio=148 gave rise to interaction of global and local forms, whereas for high ratios 185 and 247 the lowest eigenvalue was associated only with local forms. Diagonal ribs proved more effective than, the orthogonal ones for the whole domain of slenderness ratios.

In the following a smaller set of girders with diagonal and orthogonal ribs were subjected to nonlinear stability analysis accounting for initial local geometric imperfections with variable intensity. The exemplary plots are shown in Fig. 2.

Figure 2: Nonlinear response of a girder with diagonal ribs for various intensity of initial local imperfections

In the analysis shown in Fig. 2 the recommendations of EC 3 [9] were followed. The Abaqus system with shell finite elements was used. The elastic-plastic model of steel with minimal hardening of isotropic type was introduced (Fig. 3). The following material parameters were set: Young’s modulus $E=210\text{GPa}$, Poisson’s ratio $\nu=0.3$ and yield stress $f_y=235\text{MPa}$. Riks method was employed in the analysis.

An analogous analysis was carried out for girders of the same dimensions and initial imperfections but with orthogonal ribs. In comparison with Fig. 2 the plots demonstrated lower limit load proportionality factors and greater sensitivity to imperfections.

The next part of the study was devoted to quantitative evaluation of the increase of the torsional stiffness of girders obtained by introduction of diagonal ribs. Several examples were solved numerically. Torsion of cantilever and simply supported beams was examined for variable ribs configurations and dimensions. Three to four times higher torsional stiffness of girders with diagonal ribs was observed in comparison with orthogonal ribs.

References
A consistent plate theory for compressible hyperelastic materials

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Abstract

The paper presents a finite-strain plate theory for compressible hyperelastic materials, consistent with the principle of stationary three-dimensional (3-D) potential energy. Based on the 3-D nonlinear elasticity and by a series expansion, we deduce a vector plate equation with three unknowns. The success relies on deriving exact recursion relations for the coefficients from the 3-D system. The plate equation can be recast into a weak form, which naturally provides some practical boundary conditions on the edge. As an example, the pure bending problem of a hyperelastic block is considered, and the plate theory gives second-order correct results comparing with the exact solution. The advantages of the present plate theory include consistency, high accuracy, without high-order stress resultants and preserving local force-balance structure in all three directions.

Keywords: plate theory, nonlinear elasticity, finite strain

1. Introduction

Plate theory is a classical research field, which has attracted extensive research interest since the 19th century. Early attempts on plate theories relied on a priori hypotheses, mostly motivated by engineering intuition. The well-known plate theories include Kirchhoff-Love theory, Mindlin-Reissner theory, von Kármán plate theory and the third-order shear deformable plate theory [6]. Although widely used and accepted in the research community, they may fail to provide good results for relatively thick plates and/or for finite-strain problems. In recent decades, some consistent mathematical methods, such as asymptotic method [1] and method of formal scaling [5], have been employed to derive or justify the above theories.

Recently some consistent approach based on 2-D variational (or virtual work) principle has been adopted to derive plate theories [4]. In particular, significant progress has been put forward by Steigmann [7, 8], who constructed an a priori 2-D energy to incorporate both stretching and bending effects. However, his plate theory is mainly restricted to the traction-free case.

The work intends to deduce a consistent finite-strain plate theory from the the 3-D differential formulation or the 3-D weak formulation, with no special restrictions on applied loads.

2. The 3-D formulation

We consider a homogeneous thin plate of constant thickness composed of a compressible hyperelastic material. A material point in the reference configuration $\kappa = \Omega \times [0, 2h]$ is denoted by $X = (x, Z)$. The deformed position in the current configuration $\kappa_t$ is denoted by $x$. Then, the deformation gradient is given by

$$ F = \frac{\partial x}{\partial X} = \frac{\partial x}{\partial r} + \frac{\partial x}{\partial Z} \otimes k = \nabla x + \frac{\partial x}{\partial Z} \otimes k, $$

(1)

where $\nabla$ is the in-plane two-dimensional gradient, and $k$ is the unit normal to the reference bottom surface $\partial \Omega$. Suppose the strain energy function of the hyperelastic material is $\Phi(\mathbf{F})$, so the nominal stress tensor is readily defined as $S(\mathbf{F}) = \partial \Phi / \partial \mathbf{F}$.

For the case of dead-loading and in the absence of body forces, the 3-D potential energy $E$ is given by

$$ E = \int_{\Omega} \int_{0}^{2h} \Phi(\mathbf{F}) dZ dr - \bar{V}, $$

(2)

where $\bar{V}$ is the load potential from the applied tractions. The associated 3-D weak formulation $\delta E = 0$ can be easily obtained and is equivalent to the following 3-D differential system

$$ \text{Div} \mathbf{S} = 0, \quad \text{in} \quad \Omega \times [0, 2h], $$

$$ S^T \mathbf{k} \mid_{Z=0} = -\mathbf{q}^-, \quad S^T \mathbf{k} \mid_{Z=2h} = \mathbf{q}^+, \quad \text{in} \quad \Omega, $$

$$ x = \mathbf{b}(s, Z), \quad \text{on} \quad \partial \Omega_0 \times [0, 2h], $$

$$ S^T \mathbf{N} = \mathbf{q}(s, Z), \quad \text{on} \quad \partial \Omega_b \times [0, 2h], $$

(3)

where $\mathbf{q}^-, \mathbf{q}$ and $\mathbf{b}$ are the prescribed data on the 3-D boundaries, and $\mathbf{N}$ is the unit outward normal to the edge.

3. The 2-D plate theory

3.1. Derivation of the vector plate equation

First, we adopt a series expansion of the deformed position from the bottom surface

$$ x(r, Z) = \sum_{j=0}^{4} \frac{1}{j!} Z^j \mathbf{x}^{(j)}(r) + Z^5 \mathbf{x}^{(5)}(r, Z^*), $$

(4)

where $0 \leq Z \leq 2h$ and $\mathbf{x}^{(j)}(r) = \frac{\partial x}{\partial Z} \mid_{Z=0}$. Similarly, $\mathbf{F}$ and $\mathbf{S}$ can be expanded in series like (4), and all the coefficients in their expansion can be expressed in terms of $\mathbf{x}^{(j)}$ by (1) and the definition of $\mathbf{S}$.

Substituting the expansions into the field equation (3)1, we obtain a sequence of equations. The first three can be utilized to derive the recursion relations for $\mathbf{x}^{(j)}$ ($j = 2, 3, 4$) in terms of $\mathbf{x}^{(1)}$ and $\mathbf{x}^{(0)}$. Furthermore, the bottom traction condition (3)2 serves to express $\mathbf{x}^{(1)}$ by $\mathbf{x}^{(0)}$. Together with all these recursion
relations, the top traction condition (3), furnishes the 2-D plate equation
\[ \nabla \cdot \mathbf{S} = -q, \quad \mathbf{q} = \frac{q^+ + q^-}{2h}, \quad \mathbf{S} = \frac{1}{2h} \int_0^{2h} \mathbf{S} \text{d}Z, \quad (5) \]
where \( \mathbf{S} \) is the averaged stress and is truncated with an \( O(h^3) \) error. Therefore, the system is a fourth-order differential equation for the vector \( x^{(0)} \), and once \( x^{(0)} \) is solved the high-order coefficients \( x^{(j)} (j = 1, \ldots, 4) \) can be recovered by the recursion relations.

3.2. Boundary conditions

We intend to propose some appropriate 2-D boundary conditions for the plate equation (5) from the 3-D ones in (3). First, on the boundary \( \partial \Omega_0 \) (refer to the third line of (3)), we adopt the following two conditions
\[ x^{(0)} = b^{(0)}(s), \quad \bar{x} = \bar{b}, \quad (6) \]
where a bar over a quantity is defined in the same way as \( \mathbf{S} \) in (5) and \( b^{(0)} = b|_{x=0} \). Second, on the boundary \( \partial \Omega_q \) (refer to the fourth line of (3)), we adopt
\[ S^T \mathbf{N} = \frac{1}{2h} \int_0^{2h} q \text{d}Z = q_0, \]
\[ \int_0^{2h} (Z - h) S^T \mathbf{N} \text{d}Z = \int_0^{2h} (Z - h) q \text{d}Z = m_0(s), \quad (7) \]
where \( q_0 \) is the averaged traction and \( m_0(s) \) is the moment vector (including bending moment and twisting moment).

It can be checked that the 2-D plate equation (5) and boundary conditions (6, 7) are consistent with the 3-D weak formulation, i.e., they ensure each term in the variation \( \delta E \) to be either zero or \( O(h^3) \). The associated 2-D weak formulation for (5) can also be derived and it naturally leads to suitable boundary conditions in various practical cases, such as a clamped edge, a pinned edge and a clamped saddle edge.

4. An example

In this section, we consider pure bending of a rectangular block for Hill’s class of compressible materials with [3]
\[ \Phi(F) = \frac{1}{2} \lambda (\text{tr} F^m)^2 + \mu \text{tr} [(F^m)^2], \quad (8) \]
where \( E^m \) and \( m \) are Lamé constants and \( m \) is a real parameter. Coordinates \( (X, Y, Z) \) and \( (\theta, \gamma, r) \) are used respectively for the reference and current states (see Figure 1). Then, the bending of the block is described by
\[ \theta = \alpha X/L_1, \quad r = r(Z), \quad \gamma = \lambda_2 Y, \quad -L_1 \leq X \leq L_1, \quad 0 \leq Z \leq 2h, \quad -L_2 \leq Y \leq L_2, \quad (9) \]
where \( \alpha \) is the bending angle, and without loss of generality we set \( \lambda_2 = 1 \) and \( L_1 = 1 \). For the purpose of illustration, we restrict ourselves to \( m = 1 \).

The exact solution is given by [9]
\[ r^*(Z) = -\frac{e^{\alpha Z} (1 - 2\nu) + e^{2\nu} - e^{2h - \alpha Z} + 1}{\alpha (1 + e^{2h}) (\nu - 1)}, \quad (10) \]
where \( h = \alpha h \) and \( \nu \) is the Poisson’s ratio. By our plate theory, we first expand the unknown \( r(Z) \) (as refer to (4))
\[ r(Z) = r_0 + r_1 Z + \frac{r_2}{2} Z^2 + \frac{r_3}{6} Z^3 + \frac{r_4}{24} Z^4 + \cdots. \quad (11) \]
Finally, the plate equation (5) furnishes an algebraic equation for \( r_0 \), from which we obtain
\[ r_0 = \frac{3 - 3h^2 + 2\lambda h^2}{\alpha (3 + 2\lambda h^2)}. \quad (12) \]
By means of Taylor expansions for (10, 12) and recursion relations, the plate theory yields \( O(h^3) \)-correct results for \( r_i \) \( (i = 0, 1, 2, 3) \) with all bending angles. As a result, all strains and stresses are correct up to \( O(h^3) \). 

5. Conclusion

In the paper, a finite-strain plate theory is developed with no special restrictions on loadings or the order of deformations. The advantages of the present plate theory include consistency with 3-D weak formulation, preserving local force-balance structure in all three directions, without high-order stress resultants, and high accuracy for a finite bending problem.

References

Global postbuckling analysis of functionally graded box section cantilever

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Abstract

The abstract presents some results concerning FEM geometrically nonlinear stability analysis of a thin walled box section, made of Functionally Graded Material (FGM). The analysis for a compressive force with loading imperfection was performed using shell finite elements. For FGM cross section consisting of ceramic and metal constituents the power law distribution of material properties was assumed through the shell thickness. The influence of the load application method on the critical buckling force was studied. The global postbuckling analysis for a functionally graded box section for different values of power law index was performed. The results computed for middle and neutral shell base surface were compared.

Keywords: nonlinear stability analysis, postbuckling analysis, box section, neutral surface, functionally graded materials,

1. Introduction

Functionally graded materials (FGMs) are microscopically inhomogeneous composites characterized by continuous gradation of material properties. In the paper, the box section with internal surface made of metal and external surface of ceramic is analysed. Large displacement and postbuckling response of FGM square plates under mechanical loading was studied in Ref. [7]. Local stability analysis of isotropic box section struts is presented in Ref. [4]. The influence of initial geometric imperfections, shape and duration of pulse loading on local dynamic buckling response of box section column was investigated in Ref. [5]. In the paper a global postbuckling analysis was performed for thin-walled FGM box section beams with loading imperfection. Other papers concerned with buckling analysis of FGM shells may be found in the review article [6]. In the calculations presented here, the nonlinear 6-parameter shell theory with drilling degree of freedom (see Ref. [2]) is used which kinematic model has the character of Cosserat medium. Application of this theory to the geometrically nonlinear analysis of FGM shells was presented in Ref. [3]. In Ref. [3] the shell base surface was assumed as geometrical middle surface. Reference [1] showed that taking the reference surface as physical neutral surface yields similar set of equations as for isotropic shells. Moreover, this approach is reasonable in engineering applications because it is easier and more effective computationally. To evaluate this method in nonlinear stability analysis, numerical results obtained for a neutral shell base surface were compared with the reference results for a mid-surface based formulation.

2. Method

In functionally graded shells the volume fraction of constituent materials varies through the shell thickness h. Here, the power law distribution is assumed, so the material properties P are described by the functions of the thickness coordinate z

\[ P(z) = (P_c - P_m) \left( \frac{z}{h} + \frac{1}{2} \right)^n + P_m, \]

where the subscripts c and m refer to ceramic and metal constituents. The constitutive relation for the middle shell base surface was presented in Ref. [3].

The physical neutral surface for FGM shells is shifted from geometric mid-surface by

\[ z_0 = \int_0^{h/2} 2E(z)dz / \int_0^{h/2} E(z)dz = h / 2 (n + 1) \left( E_c - E_m \right) \]

toward ceramic rich surface. There are no stretching-bending components in the constitutive relation for neutral surface approach. Therefore, the governing equations are of a simple form as for homogenous isotropic shell theory.

The geometrically nonlinear analysis with loading imperfection was performed by the Authors’ FEM program coded in Fortran. In calculations the 16-node C⁶ Lagrangian type shell finite element with 6 DOFs in each node and fully integrated matrices was used. The postbuckling equilibrium path was obtained removing imperfection force for an arbitrary chosen level of load. The calculated global buckling load for limiting cases of metal and ceramic box section was compared with a theoretical value obtained for cantilever beam solution

\[ F_{crit} = \frac{\pi^2 E_I}{4L^2} \]

3. Results

As an example box section cantilever beam was analysed, see Fig. 1. The geometric dimensions were assumed as follows: L=6, a=b=0.3, h=0.008 [m]. The following values of Young’s moduli \( E_c = 1.51 \times 10^6 \) [Pa], \( E_m = 0.7 \times 10^9 \) [Pa] and Poisson’s ratio \( v = 0.3 \) were used in calculations. The regular discretization \((2+2+2+2+40\times40\) of box section beam is presented in Fig. 1. The micropolar constants (Ref. [3]) were assumed as \( N=0.707 \), \( \rho/\rho_0=0.001 \) and reference compression force \( F_{ref}=4P_0=1000 \) [N].

Firstly, the influence of load application method on critical buckling force for FGM box section was evaluated. Uniformly distributed nodal forces and point loads P at box corners were taken into account. Additionally, the influence of reinforcement of the cantilever end (Fig. 1) by a triple increment of its thickness was studied. The loading imperfection forces \( q=0.04P \) at box corners were assumed to get the postbuckling response.
The calculated equilibrium paths for exponent $n=2$ (Eqn (1)) in the region close to bifurcation point were compared in Fig. 2.

Next, the global postbuckling analysis for FGM cantilever beam was performed. The point loads at box corners and no reinforcement at cantilever end were assumed. The influence of power law index $n$ on nonlinear equilibrium curves is shown in Fig. 3. The equilibrium paths are presented only for neutral surface formulation, because the choice of reference surface has small impact on the results. The buckling forces 6889 and 14860 [N] calculated for limiting cases of metal and ceramic cross sections are in good agreement with theoretical values 6909 and 14903 [N] obtained from Eqn (3).

Table 1: Influence of exponent $n$ and choice of shell base surface on value of buckling force

<table>
<thead>
<tr>
<th>Surface</th>
<th>Metal</th>
<th>$n=2.0$</th>
<th>$n=1$</th>
<th>$n=0.5$</th>
<th>Ceramic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Middle</td>
<td>6.889</td>
<td>9.599</td>
<td>10.927</td>
<td>12.245</td>
<td>14.860</td>
</tr>
</tbody>
</table>

4. Conclusions

In the postbuckling analysis of FGM box section cantilever it was observed that the uniformly distributed load triggered local deformation under load. Reinforcement of the box section end is an effective method to eliminate this form of deformation. However, in the nonlinear stability analysis of FGM cantilever beam the point loads are applied at box corners, because this method is simpler and also almost free from local deformation form under load.

The nonlinear analysis shows that the character of response curves of FGM box section beam has similar character to the paths of isotropic shells, lying between curves of metal and ceramic box section. Global buckling of cantilever causes bending and increased compression in a single wall of the box section. The consequence of flexion is premature local buckling of this and the adjacent walls. As a result of interaction of global and local buckling, the bifurcation point in FEM solution for a shell box section is unstable. This is contrary to a postbuckling curve of beam solution. The critical forces calculated for a middle reference surface are greater than that obtained for a neutral surface approach.

References

A layer-wise user element for the analysis of photovoltaic modules

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Abstract

Laminated glasses and photovoltaic modules are usually composed of relatively stiff skin layers and a compliant core layer, which contains the solar cells and their encapsulation. If the stiffnesses of the layers differ considerably, one cannot apply first-order shear deformation plate theories to these laminated plates. Therefore, a layer-wise theory for laminates has been developed [2]. In order to obtain numerical solutions based on this layer-wise theory, this contribution introduces a user element, which is a quadrilateral serendipity element with quadratic shape functions. The governing equations of the layer-wise theory are presented since they serve as starting point to derive the principle of virtual work and the element stiffness relation. Finally, the element has nine degrees of freedom including two components of the in-plane displacement vector of the laminate, two components of the relative in-plane displacement vector of the skin layers, the deflection, two components of the cross-section rotation vector of the laminate, and two components of the relative rotation vector of the skin layers. The Abaqus subroutine “User Element” is utilised to implement the element. To verify the numerical results, a closed-form analytical solution based on the layer-wise theory is derived.

Keywords: layer-wise theory, user element, laminates, photovoltaics

1. Introduction

Crystalline photovoltaic modules are usually composed from front and back glass or polymer layers and a core layer with solar cells embedded in a polymeric encapsulant. Materials like ethylene vinyl acetate (EVA) and polyvinyl butyral (PVB) are used to encapsulate the solar cells [4].

Figure 1: Crystalline solar module [4]

The photovoltaic modules feature high differences in material properties of the layers. Let \( G_T \) be the shear modulus of the skin layers and \( G_C \) the shear modulus of the polymeric core layer. The ratio of the shear moduli \( \mu = G_C / G_T \) for materials used in photovoltaic panels is in the range between \( 10^{-3} \) and \( 10^{-2} \), depending on the type of polymer and the temperature [4]. For the comparison, classical sandwich panels are composed from materials with \( \mu \) in the range of \( 10^{-2} \) and \( 10^{-1} \). Besides, the core layer of a classical sandwich structure is much thicker than the skin layers, whereas in photovoltaic applications the core layer is much thinner than the skin layers.

Recently, layer-wise theories have been developed and applied in order to analyse laminates. Within the layer-wise theory (LWT), balance and constitutive equations are derived for individual layers. With constitutive assumptions for interaction forces and compatibility conditions, a model for the layered system is derived. For laminates with core layer from soft polymers, a LWT is presented in [2] for plates. Based on this LWT, this contribution presents a user element. First of all, the LWT is introduced. Then, the principle of virtual work is derived and the element is implemented with an Abaqus subroutine. In order to verify the results, a closed-form solution is derived.

2. Layer-wise theory

Figure 2 shows a rectangular plate that is composed of a top layer (index T), a core layer (index C), and a bottom layer (index B). The coordinate system with the orthonormal basis \( \mathbf{e}_1, \mathbf{e}_2, \mathbf{n} \) and the corresponding coordinates \( x_1, x_2, z \) is applied. The origin of the \( z \) coordinate is located in the middle of the core layer, i.e. \(-h_B - h_C/2 \leq z \leq h_C/2 + h_T \). \( h_T, h_C, \) and \( h_B \) denote the thicknesses of the top, core, and bottom layer respectively.

Figure 2: Photovoltaic module subjected to uniform surface load

The LWT takes a transverse load \( q = q_n \) and a tangential load \( s = s_n \mathbf{e}_n \) at the top layer into account [2]. Thereby, Greek letters used for indices take the values 1 or 2 and the Einstein summation convention is applied if indices appear twice in one term. Bold lower-case letters represent vectors. Since there is a tangential loading, also the membrane forces are considered. For each of the three layers, equilibrium conditions for the membrane
forces, the shear forces, and the moments exist. The constitutive and kinematic equations are also referred to each layer individually. Linear elastic behaviour is assumed and only laminates which are symmetric with respect to the midplane are considered such that the membrane and bending states are decoupled.

3. User element

In general, the principle of virtual work (PVW) can be formulated as follows:

\[ \delta W_m = \delta W_{ea} \]  

(1)

To deduce the PVW, the equilibrium conditions according to [2] are multiplied with the corresponding virtual degrees of freedom (DOFs). Afterwards, all equilibrium conditions are added, the integral over the surface of the plate is determined, and the two-dimensional Gauss’s theorem is applied to obtain the PVW. In order to simplify the equations, one introduces new DOFs:

\[ u = \frac{1}{2} (u_x + u_b), \quad u_e = \frac{1}{2} (u_x - u_b) \]  

(2a)

\[ \varphi = \frac{1}{2} (\varphi_x + \varphi_b), \quad \varphi_e = \frac{1}{2} (\varphi_x - \varphi_b) \]  

(2b)

\[ u = u_{ex}, e_e \] is the average in-plane displacement vector, while \[ u_e = u_{ex}, e_e \] is the relative in-plane displacement vector. In analogy, \[ \varphi = \varphi_{ex}, e_e \] represents the average cross-section rotation vector and \[ \varphi_e = \varphi_{ex}, e_e \] is the relative cross-section rotation vector. For the user element, the well-known quadrilateral two-dimensional serendipity element with eight nodes and quadratic shape functions is chosen. Then, the usual procedure to derive the element stiffness relation from the PVW is applied, e.g. [3]. In the end, the user element has nine DOFs: two components \[ u_{ex} \] of the average in-plane displacement vector of the laminate, two components \[ u_{ex}, e_e \] of the relative in-plane displacement vector of the skin layers, the deflection \[ w \] (which is assumed to be equal for all three layers), two components \[ \varphi_{ex}, e_e \] of the average cross-section rotation vector of the laminate, and two components \[ \varphi_{ex}, e_e \] of the relative rotation vector of the skin layers.

The user element models all three layers with the Mindlin plate theory. In order to avoid shear-locking effects, selective integration is applied, cf. [3]. The element is implemented with the Abaqus subroutine “User Element”.

4. Closed-form solution

If the plate is symmetric and loaded only with the transverse load \( q \), the boundary value problem can be formulated as a system of three differential equations [2]:

\[ \Delta \Delta \tilde{w} = \frac{q}{D} \]  

(3a)

\[ \Delta \Psi - \tau^2 \Psi = 0 \]  

(3b)

\[ \Delta \Phi - \frac{\rho^2}{D} \Delta \Phi = - \frac{Hq}{\rho^2 D} \]  

(3c)

\( w, \Psi, \) and \( \Phi \) are three scalar functions, and \( \Delta (\ldots) = (\ldots)_{11} + (\ldots)_{22} \) represents the Laplace operator. \( D \) is the bending stiffness of the laminate, \( \tau \) is a constant, \( \rho \) is a shear rigidity parameter of the laminate, and \( H \) is an average thickness.

Equations (3a) and (3b) are well-known from the FSDT [1]. Eqn (3c) provides a correction for the LWT [2]. In order to formulate the closed-form solution for a rectangular plate, a series approach is utilised, cf. e.g. [5]. We assume that the edges \( x_1 = 0 \) and \( x_1 = x_1 \) of the plate in Fig. 2 are simply supported. For the edges \( x_2 = \pm \frac{1}{2} l_2 \), arbitrary boundary conditions can be applied. The following series solutions fulfil the differential equations and the boundary conditions along the simply supported edges automatically:

\[ w(x_1, x_2) = \sum_{m=1,3, \ldots}^{\infty} w_m + Y_m (x_2) \sin (\lambda_m x_1) \]  

(4a)

\[ \Psi(x_1, x_2) = \sum_{m=1,3, \ldots}^{\infty} Z_m (x_2) \cos (\lambda_m x_1) \]  

(4b)

\[ \Phi(x_1, x_2) = \sum_{m=1,3, \ldots}^{\infty} \left[ p_m + R_m (x_2) \right] \sin (\lambda_m x_1) \]  

(4c)

with \( \lambda_m = \frac{\pi m}{l_1} \).

The functions \( Y_m(x_2), Z_m(x_2), \) and \( R_m(x_2) \) contain constants which are determined by the boundary conditions along the edges \( x_2 = \pm \frac{1}{2} l_2 \).

In order to verify the user element with the closed-form solution, a laminate, which is symmetric with respect to the midplane, with freely supported edges \( x_2 = \pm \frac{1}{2} l_2 \) is chosen as example. The layers are isotropic, and \( \mu \approx 10^{-4} \) holds true. Figure 3 shows the deflection and a shear force along the path \( x_2 = 0 \) mm as two chosen examples. The results of the user element approach the results of the closed-form solution with good precision. So finally, the user element has been verified successfully.

![Figure 3: a) Deflection \( w(x_1, 0) \), b) Shear force \( Q_2(x_1, 0) \)](image)

References


On the plates and shells with initial surface stresses

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Abstract

Recently the interest grows to the development of the theory of surface elasticity with respect to nanotechnologies. Nanostructured materials demonstrate promising properties different from those of bulk materials, in general. In particular, nanosized specimens exhibit a size-effect of dependence of apparent (effective) material properties such as Young’s modulus on specimen size. One of possible explanation of these phenomena is consideration of surface-related phenomena and their influence on the effective properties at the macroscale. In fact, surface elasticity may dramatically change effective (apparent) properties of nano- and microstructure materials.

The aim of the lecture is to discuss the effective properties of thin-walled structures in detail, tangential and bending stiffness parameters taking into account surface/interfacial initial stresses. Here we consider the effective properties of plates and shells considering various models of surface elasticity and analyzed difference between models. Mostly we consider the Gurtin-Murdoch model of surface elasticity. We show that the surface elasticity results in a positive size-effect of is stiffening of nano-sized bodies compared to their bulk counterparts. Special attention is paid to residual surface stresses. Unlike to surface elastic moduli there are no restrictions for values of residual stresses. In particular, we show that the compressive residual stresses may lead to negative size-effect of decreasing of an effective stiffness for nano-sized specimens. Making use of the variational technique we discuss the influence of initial/residual stresses on the stiffness parameters.

Keywords: surface stresses, plates and shells, nonlinear elasticity, initial stresses

1. Introduction

The mostly used model of surface elasticity for elastic solids under large deformations was proposed by Gurtin and Murdoch [1]. From the physical point of view the model describes a nonlinear elastic solid with an elastic membrane attached on its surface. The stress resultant tensor acting in the membrane can be considered a surface stress tensor. Thus, for the surface elasticity model, in addition to the three-dimensional constitutive equations, the two-dimensional constitutive relations are also required. Recently the Gurtin-Murdoch model found many applications in micro- and nanomechanics [2, 3, 4]. In particular, the surface elasticity is used for the explanation of the size-effect observed at the nanoscale. The Gurtin-Murdoch model is also applied for extension of models of plates and shells to the nanoscale, see for example [5, 6], where it is shown that the surface elasticity results stiffening of nano-sized bodies in comparison with their bulk counterparts. Such behaviour is often called the positive size-effect.

In the lecture we intend to consider residual/intial surface stresses and the effective properties of thin-walled structures that is tangential and bending stiffness parameters considering these surface phenomena. Unlike to surface elastic moduli there are no restrictions for values of residual stresses. In particular, we show that the compressive residual stresses may lead to negative size-effect that is to decreasing of an effective stiffness for nano-sized specimens. Using the variational technique [7] we discuss the influence of initial/residual stresses on the stiffness parameters.

2. Basic equations of surface elasticity

The Lagrangian equilibrium equations and the boundary conditions take the following form:

\[
\nabla_s \cdot P + \rho f = 0, \quad (\mathbf{n} \cdot P - \nabla_s \cdot S)_{|_{\Gamma_s}} = t, \\
\mathbf{u}_{|_{\Gamma_s}} = \mathbf{u}_0, \quad \mathbf{n} \cdot P_{|_{\Gamma_f}} = t.
\]  

(1)

Here \( P \) is the first Piola-Kirchhoff stress tensor, \( \nabla_s \) the Lagrangian three-dimensional (3D) nabla operator, \( \nabla_s \) the surface (2D) nabla operator, \( S \) the surface stress tensor of the first Piola-Kirchhoff type acting on the surfaces \( \Omega_s \), \( \mathbf{u} \) the displacement vector, \( f \) and \( t \) the body force and surface loads vectors, respectively, and \( \rho \) the mass density. We assume that the displacements are given on the part \( \Omega_s \) of the body surface, while on \( \Omega_f \) the surface stresses \( S \) are absent. Equation (1) is the so-called generalized Young-Laplace equation describing the surface tension in solids.

For a bulk material we use the standard constitutive relations of the nonlinear elasticity

\[
P = \frac{\partial W}{\partial F} \quad W = W(F), \quad F = \nabla_s \mathbf{x},
\]  

(2)

where \( W \) is the strain energy density, \( F \) the deformation gradient, and \( \mathbf{x} \) the position vector in the actual configuration. The surface stress tensor \( S \) is similar to the membrane stress resultants tensor

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and expressed with the use of the surface strain energy density $U$
\[ S = \frac{\partial U}{\partial F_s} \quad U = U(F_s), \quad F_s = \nabla_s x|_{\Omega_s}, \]
(3)
where $F_s$ is the surface deformation gradient.
For solids with initial/residual surface stresses we introduce the initial surface energy and initial surface stresses as follows
\[ U_0 = U(F_s^0), \quad S_0 = \frac{\partial U}{\partial F_s^0}. \]
(4)
Tensor $F_s^0$ describes the initial deformation from stress-free state to the chosen initial configuration. It can be considered as the given parametric tensor in the constitutive equations. The strain energies can be considered functions of the right Cauchy–Green strain tensor and its surface analogues
\[ W = W(C), \quad U = U(F_s^0 \cdot C_s \cdot F_s^0)^{1/2}, \]
(5)
where $C = F \cdot F^T$ and $C_s = F_s \cdot F_s^T$. For isotropic materials $W$ and $U$ are functions of the principal invariants
\[ W = W(I_1, I_2, I_3), \quad U = U(J_1, J_2), \]
(6)
where $I_1 = \text{tr} C$, $I_2 = \frac{1}{2} \left[ \text{tr}^2 C - \text{tr} C^2 \right]$, $I_3 = \text{det} C$, $J_1 = \text{tr} F_s^0 \cdot C_s \cdot F_s^0$, $J_2 = \text{tr} \left( F_s^0 \cdot C_s \cdot F_s^0 \right)^{1/2}$.

3. Transitions to the plates and shells equations
In the following we use the exact nonlinear resultant theory of shells [8, 9]. Within this theory the non-linear 3D equations of solids can be reduced to exact 2D equations. In particular, for the nonlinear elastic shell-like body without surface stresses, i.e. when $S = 0$, the through-the-thickness integration technique gives the following relations between $P$ and resultant tensors of the first Piola-Kirchhoff type:
\[ T = \int G \cdot P \, d\zeta, \quad M = - \int G \cdot P \times z \, d\zeta, \]
(7)
where $\zeta \in [-h/2, h/2]$ is the normal coordinate, $h$ is the shell thickness, $z$ is the base reference deviation and $G$ the geometrical tensor (shifter) defined by [8, 9]. The through-the-thickness integration technique was extended in [5, 6] for the case of surface stresses. In particular, it was shown that the stress resultants for the shell with surface stresses can be represented as a sum of two terms
\[ T^r = T + T_s, \quad M^r = M + M_s, \]
(8)
where $T$ and $M$ are the classical stress and couple stress resultant tensors given by (7), while $T_s$ and $M_s$ are the resultant tensors induced by surface stresses.
Here we extended the results of [5, 6] considering initial surface stresses, that is when $S_0 \neq 0$. Let us note that unlike to the linear model of surface elasticity where surface elastic moduli form positive surface energy density, there are no mathematical restrictions for values of residual stresses. As a result, the linear Gurtin-Murdoch model lead to stiffening of materials, the bending stiffness and tangential stiffness of a plate with surface stresses are both greater than in the case of shell without surface stresses. The influence of initial surface stresses is more complex.
In particular, we show that the compressive residual stresses may lead to negative size-effect that is to decreasing of an effective stiffness for nano-sized specimens. This can be also the reason of the experimentally observed self-buckling of shells of nanosized thickness. Using Rayleigh and Courant’s variational principles [7, 10] we consider also the influence of initial surface stresses on the stiffness of the shells and solids with surface stresses.

References

The Effect of the Impulsive Dynamic Loads on the Barrel Type Helicoidal Bars

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Abstract

In the study, dynamic analysis of linear viscoelastic barrel helices having elliptical cross section is examined and the effects of the triangular and rectangular impulsive dynamic loads are investigated. Analysis based on the Timoshenko beam theory is employed in the Laplace space using the mixed finite element method. Two nodded curved element is used. The exact nodal curvatures are approximated over the elements by linear shape functions that are used for interpolating field variables. It is assumed that, the results are transformed back to time domain numerically by the Modified Durbin’s transformation algorithm. Numerical results for the effect of the impulsive dynamic loads on the barrel type helicoidal bar are presented as benchmark examples for the literature.

Keywords: barrel type helicoidal bar, mixed finite element, linear viscoelastic material, the Laplace space

1. Introduction

The viscoelastic behaviours of straight bars was examined extensively by many authors [1,3,6,8,10], however, few studies exist in literature that investigates the viscoelastic behavior of helicoidal bars [5,9]. In this study, a mixed finite element formulation based on the Timoshenko beam theory is used for dynamic analysis of a cantilevered linear viscoelastic barrel type helicoidal bar subjected to external dynamic loads. The solutions are carried out in Laplace transform space. Viscoelastic material behavior is simulated a by standard model [4,7]. The cantilevered linear viscoelastic barrel helicoidal bar having elliptical cross-section is handled to investigate the effect of the impulsive dynamic loads on the barrel type helicoidal bar are presented as benchmark examples for the literature.

2. Formulation in the Laplace space

2.1. Helix geometry

In the Cartesian coordinates, the geometrical properties of the helix are \( x = R(\phi)\cos \phi \), \( y = R(\phi)\sin \phi \), \( z = p(\phi)\phi \), \( p(\phi) = R(\phi)\tan \alpha \) where \( \alpha \) denotes the pinch angle, \( R(\phi) \) and \( p(\phi) \) signify the centerline radius and the step for unit infinitesimal arc length \( ds = \sqrt{R^2(\phi) + p^2(\phi)}d\phi = c(\phi)d\phi \). In the case of barrel helix, the radius at any point on the helix geometry is \( R(\phi) = R_{\text{avg}} + (R_{\text{max}} - R_{\text{avg}})(1 - \phi/2\pi)^2 \), where \( n \) is the number of active turns, \( R_{\text{avg}} \) and \( R_{\text{max}} \) are the radii of the helix.

2.2. Field equations and functional

The necessary field equations for the dynamic analysis in the Laplace space are given in the Frenet coordinate system.

\[
\begin{align*}
-T_n - \Phi + \rho A z^2 \psi &= 0 \\
M_n - t \times \Phi - m + \rho f z^2 &= \Omega = 0
\end{align*}
\]

(1)

\[
\begin{align*}
\Phi - t \times \Omega - C_T &= 0 \\
\Omega - \Phi = 0
\end{align*}
\]

(2)

where the Laplace transformed variables are denoted by the over bars, comma as a subscript under the variable designates the differentiation with respect to \( s \), \( z \) is the Laplace transformation parameter. \( \vec{u}(\vec{x}, \vec{z}, \vec{\tau}) \) is the displacement vector, \( \vec{\Omega}(\vec{\Omega}_n, \vec{\Omega}_r, \vec{\Omega}_s) \) is the rotation vector, \( \vec{f}(\vec{f}_n, \vec{f}_r, \vec{f}_s) \) is the force vector, \( \vec{M}(\vec{M}_n, \vec{M}_r, \vec{M}_s) \) is the moment vector in the Laplace space, \( \rho \) is the density of material, \( A \) is the area of the cross section, \( l(l_s, l_f, l_r) \) is the moment of inertia of the cross section, \( \vec{q} \) and \( \vec{m} \) are the distributed external force and moment vectors in the Laplace space, \( \vec{C}_n \) and \( \vec{C}_r \) are the compliance matrices in the Laplace space. The functional of the structural problem in the Laplace space is

\[
\begin{align*}
1(\vec{y}) &= \left[ -[\vec{u}, \vec{T}, \vec{f}, \vec{M}] + [\vec{x} \times \vec{O}, \vec{T}] - \bar{\Omega} - \bar{z}[\bar{C}, \bar{\Omega}] - \bar{f}[\bar{C}, \bar{T}, \bar{\Omega}] + \bar{z}[\bar{C}, \bar{f}] - \bar{z}[\bar{C}, \bar{M}] + \bar{z}[\bar{C}, \bar{M}] + \bar{z}[\bar{C}, \bar{M}] \right]
\end{align*}
\]

(3)

The detailed formulation of the functional is documented in [5].

2.3. Mixed finite element formulation

The two-nodded curvilinear elements based on Timoshenko beam theory are generated in Laplace space. Using the subscripts \( i,j \) to represent the node numbers of the bar element, the linear shape functions \( \phi_1 = (\phi - \phi_i)/\Delta \phi \) and \( \phi_2 = (\phi - \phi_j)/\Delta \phi \) are employed in the finite element formulation, where \( \Delta \phi = (\phi - \phi_i) \). The non-cylindrical helix geometry

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is introduced into the mixed finite element formulation by considering the variable nodal curvatures of a two noded curved element. Each node has 12 degrees of freedom namely, $\pi, \Omega, T, M$.

2.4. The viscoelastic model and modified Durbin's algorithm

The viscoelastic material properties are implemented into the formulation through the use of the correspondence principle [2]. The analysis is carried out in the Laplace space and the results are transformed back to time space numerically by using the modified Durbin's algorithm [4,7]. It is assumed that, the linear viscoelastic material exhibits standard type distortional behavior while having elastic Poisson's ratio. In that case, $\sigma = \nu$ and the form of complex shear modulus $G$ are $G = G_e (1 + \beta \tau_r^2) / (1 + \tau_r^2)$ where the modulus of shear is $G_e$, $\beta = G_t / G > 1$ and $\tau_r$ is the retardation time. $G$ and $G_t$ are the equilibrium value and the instantaneous value of relaxation function associated with shear modulus [5].

3. Numerical examples and discussion

A viscoelastic cantilever barrel type helicoidal bar is solved. The helix geometry has $n=5.5$ number of active turns, the height of the bar is $H=6$ m and the minimum radius of helix to maximum radius of helix ratio $R_{\text{max}} / R_{\text{min}} = 0.5$ where $R_{\text{max}} = 3$ m. The major and minor radii of elliptical cross section $a = 25$ cm and $b = 15$ cm, respectively. The material parameters are $G = 80$ GPa, Poisson’s ratio $\nu = 0.3$, $\beta = 3$, $\tau_r^2 = 0.0005s$, $0.005s$, $0.01s$, $0.1s$ and the material density $\rho = 7850$ kg/m$^3$. The bar is subjected to an external dynamic load $P = P(t)$ acting from the tip of it. The quasi-static and dynamic responses of the bar are determined within $0 \leq t \leq 80s$ by considering two different time histories of the point load $P$, namely, rectangular and triangle impulsive types with $t_{\text{load}} = 30s$ having an intensity of $P = 1$ kN and $P = 2$ kN, respectively. The parameters which are used in the analysis for the modified Durbin's algorithm are $N = 2^{10}$ and $aT = 6$. The vertical displacement $u_z$ at the tip of the barrel type bar and shear force $T_z$ at the fixed end are determined using 100 finite elements for two different impulsive loads. In Figures 1-4, the time histories of $u_z$ and $T_z$ are presented to investigate the influences of the viscoelastic parameter $\tau_r^2$, which is used to describe the standard type of distortional behavior, on the dynamic response of the bar. It is observed that, due the values of $\tau_r^2$ increase, $u_z$ and $T_z$ dissipate, the amplitudes of vibration decrease and approaches to the quasi-static case. This situation is more pronounced on the dynamic behavior of the barrel type helix under the triangle impulsive loading.

References


Abstract

A quadrilateral flat shell finite element for the analysis of buildings and construction parts representing thin-walled reinforced concrete structures is proposed. This finite element in the shape of a plane convex quadrilateral describes the behavior of concrete and reinforcement. The axes of the reinforcement rods can be rotated on arbitrary angle around the middle surface normal that allows modeling foundation plates, floor slabs and walls of complex geometry and implement this approach in the software used to analysis of real structures.

Keywords: structural mechanics, finite element method, deformation theory of plasticity, reinforced concrete, principle of virtual work.

1. Introduction

There are many publications today on behaviour analysis of thin-walled concrete structures. In many studies based on various versions of the theory of plasticity, the bounding surface of concrete is replaced by the yield surface, and the process of cracking is modeled descending branch of the $\sigma - \varepsilon$ diagram [1]. Other studies simulates a crack formation directly [3].

The paper presents an approach in which the concrete and reinforcements are implemented in the same flat shell finite element that allows us to consider the construction of complex geometric shapes with distorted meshes, and axes of reinforcing rods can be arbitrary focused concerning axes of local coordinate system of an proposed finite element.

We take into account the shear stiffness of reinforcement as well as tension-compression stiffness. In many cases, this avoids the geometric instability, since the stretched zone of concrete does not work on tension, and if the shear stiffness of reinforcement is ignored, zero rows and columns may appear in the tangent stiffness matrix of finite element. In addition, we obtain a condition

$$\frac{\xi}{\varepsilon_{\text{min}}} = 1 - \left(\frac{1 - h_s}{h_s}\right) \left(1 - \alpha \right) \frac{E_s}{E},$$

where $\xi$ sets the extension of the descending branch in stretched concrete (Fig. 3), $\alpha$ – the residual durability of the cracked concrete, $h_s$ is a height of compressed zone, $\mu_c$ – reinforcement ratio for stretched rebar, $E$ – deformation modulus for concrete, $E_s$ – Young’s modulus for steel (see also Fig. 1, Fig. 3). Expression (1) restricts decreasing intensity of descending branch in $\sigma$-$\varepsilon$ diagram for stretched zone of concrete. For instance, in the case of stretched bar the violation of (1) results in loss of stability.

2. Mechanical models for concrete and steel

The behaviour of concrete is presented by relations of the deformation theory of plasticity with elements of degradation simulating the cracking process. We use an asymmetrical $\sigma - \varepsilon$ diagram describing the tensile zone as well as the compressed one. The main relations for active loading are formulated in terms of residual strains, which allows modeling the switch from tension to compression and inversely. Pade approximation of bilinear $\sigma - \varepsilon$ diagram as well as Euro-International Concrete Committee (ECC) diagram for compressive zone together with three-linear diagram for tensile zone (Fig. 3) is considered for concrete.

The deformation theory of plasticity in terms of residual strains is applied for reinforcements too. We use symmetrical bilinear $\sigma - \varepsilon$ diagram as well as its exponential approximation [4] for steel. The elastic unloading are taking into account for both: concrete and reinforcements. We assume the small displacements and rotation angles. Article [4] presents the details. Figure 1 depicts the general view of proposed finite element.

Here, $z_s - z_d$ are the distances between corresponding reinforcement layers $s_l - s_d$ and the middle surface. We use a principle of virtual work for obtaining of element stiffness matrix, tangent matrix and right hand side vector:

$$\int_\Omega [\mathbf{D} \mathbf{e}^T \sigma + \mathbf{D} \mathbf{t}^T \tau] d\Omega +$$

$$+ \sum_s \int_\Omega \frac{A_s}{h_s} \left[ \sigma \delta e_s + m \left( \tau_{xy} \delta y_s + \tau_{xz} \delta z_s \right) \right] d\Omega - \delta \mathbf{d}_{\text{ext}} = 0,$$

where $\mathbf{e}^T = (\varepsilon_x, \varepsilon_y, \gamma_{xy})$, $\mathbf{t}^T = (\tau_{xx}, \tau_{yy}, \tau_{xy})$ are components of the strain tensor in concrete; $\mathbf{D} = (\sigma_{xx}, \sigma_{yy}, \tau_{xx}) - \text{com-
ponents of stress tensor in concrete; \( \varepsilon_x, \gamma_{xy}, \gamma_{xx} \) – correspondingly the axial and shear strains in reinforcement layer \( s \) \( (s = s_1, s = s_2, \ldots) \); \( \sigma_x, \tau_{xy}, \tau_{xx} \) – respectively axial and shear stresses; \( n \) – direction vector, which is orthogonal against axes \( s \) and \( x \); \( A_x, b_x \) – cross-section area and step between rods in reinforcement layer \( s \). \( m_s \) – reduction multiplier taking into account that not all cross-section area takes part in shear. For circular cross-section \( m_s = 0.66 \). The first integral in (2) describes virtual work of concrete, second – virtual work of reinforcement layers. We assume that all rods of a given layer are equal, lay on equal distance one from other and step between rods is essentially less than dimensions of finite element. Therefore we can replace the sum of virtual works for each rod belonging to \( s \) layer by integral in area of finite element \( \Omega \), as is done in (2). The last term in (2) denotes virtual work of external load.

We use a Mindlin-Reissner shell theory and mixed interpolation of tensorial components to avoid a shear locking [2]. The kinematic coupling conditions of concrete and reinforcements of layer \( s \) with assumption of no slip between concrete and rods are:

\[
\begin{align*}
\varepsilon_x &= \varepsilon_x \cos \varphi^2 + \varepsilon_z \sin \varphi^2 + \frac{1}{2} \gamma_{xy} \sin 2\varphi \\
\gamma_{xy} &= \sin 2(\varepsilon_x - \varepsilon_z) + (\cos \varphi^2 - \sin \varphi^2) \gamma_{xy} \\
\gamma_{xz} &= \gamma_{xz} \cos \varphi + \gamma_{yz} \sin \varphi
\end{align*}
\]

where \( \varphi \) is an angle between local axis \( Ox \) and axis of rods in layer \( s \). We use poly-linear shape functions and trapezoid integration method over thickness and Gauss integration method over domain.

3. Numerical results

The transverse bending of square plate subjected to 16 concentrated forces (Fig. 2, specimens 825 – 827 from [5]) with plan dimension \( 2 \times 2 \) m and thickness \( h = 12.2 \) cm is considered. We analyse quarter part of plate with taking into account symmetry conditions. Remaining parameters are: \( z_{xx} = \pm 5.4 \) cm, \( z_{xy} = \pm 5.05 \) cm, \( A_x = A_y = 0.407 \) cm\(^2\), \( h_{xx} = h_{yx} = 10 \) cm, \( \sigma_x = 26.5 \) MPa, \( \sigma_y = 1.3 \) MPa, \( \sigma_z = 408 \) MPa, \( E = 30000 \) MPa, \( E = 201000 \) MPa.

![Figure 2: Comparison of numerical results with experiment [5]](image)

Here, \( z_{xx}, z_{xy} \) are distances between middle surface and upper (+) and lower (-) reinforcement layers in the directions \( Ox \) and \( Oy \). \( A_x, A_y \) – cross-reference areas of reinforcement rods in the directions \( Ox \) and \( Oy \). \( h_{xx}, h_{xy} \) – steps between rods in the directions \( Ox \) and \( Oy \). \( \sigma_x, \sigma_y \) – compressive and tensile strength of concrete (Fig. 3), \( \sigma_t \) – yield stress of steel, \( E \) – initial deformation modulus for concrete, \( E_s \) – Young’s modulus for steel.

![Figure 3: \( \sigma-\varepsilon \) diagram for concrete, \( E_s = 0.0035 \), \( \varepsilon_t = 1.41 \varepsilon_s, \sigma_s = 0.85 \sigma_c \)](image)

Experimental curves demonstrate scattering of experimental results. Curve \( a \) corresponds to case when \( m_s = 0 \) (reinforcement works only on tension-compression), \( \xi = 40, \alpha = 0.5, \beta = 0 \); curves \( b, c, d \) – \( m_s = 0.66 \) (reinforcement works on tension-compression and shear), \( \xi = 50 \) for \( h_c \) and \( \xi = 20 \) for curve \( d \), \( \alpha = 0.05, \beta = 0 \). We take bilinear \( \sigma-\varepsilon \) diagram for steel (curves \( a, b, d \)) and exponential approximation of bilinear diagram for curve \( c \). The taking into account of shear stiffness in reinforcement makes a design model stiffer (curves \( b, d \) in comparison with curve \( a \)). On the other hand, it is a good natural regularisation of numerical solution when descending branch of \( \sigma-\varepsilon \) diagram for concrete is appeared. When this stiffness is ignored (\( m_s = 0 \), curve \( a \)) we cannot reduce \( a \), because numerical solution lacks a stability due to small tangent stiffness of concrete in stretched zone. With \( m_s = 0.66 \) (curves \( b, c, d \)) we can essentially reduce \( a \).

While using exponential approximation of bilinear diagram for steel (curve \( c \)) design model becomes less stiff. The behaviour of curve \( d \) is close to snap (red arrow in Fig. 2) because extension of descending branch (\( \xi = 20 \)) is close to \( (\cos \alpha = 12) \), that confirms the condition (1).

This test demonstrates a satisfactory ability of proposed approach to predict a load-carrying capacity and behaviour of thin-walled reinforced concrete structures – fragments of floor slabs. In addition, proposed approach properly works with skew-angular finite elements [4].

References


Buckling and optimal design of a cylindrical shell stiffened by annular plates

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Abstract

Buckling of a circular cylindrical shell stiffened by annular plates under the uniform external pressure is studied. In order to estimate the axisymmetric pre-buckling state and obtain the solution of a buckling problem asymptotic methods are applied. The simple approximate formula for the calculation of the critical pressure is obtained. The optimal parameters corresponding to the maximum critical pressure for the stiffened shell with given mass are evaluated.

Keywords: stiffened cylindrical shell, buckling, optimal design, asymptotic methods

1. Introduction

In Refs. [1], [3] and in almost all publications of ring-stiffened shell buckling the rings are modeled as circular beams. However, wide rings should be considered as annular plates. Buckling of an annular plate attached to the edge of a cylindrical shell was studied in Ref. [2].

2. Pre-buckling state

Consider the axisymmetric deformation of a cylindrical shell joined with identical annular plates under the uniform external pressure \( p \) (see Fig. 1).

![Figure 1: Cylindrical shell stiffened by annular plates](image)

We choose the radius \( R \) of the cylindrical shell as the characteristic length. Then non-dimensional equations describing the axisymmetric deformation of the cylindrical shell can be written as

\[
\begin{align*}
\frac{dT_1^{(k)}}{ds} &= 0, \quad \frac{dQ^{(k)}}{ds} + T_2^{(k)} + \frac{\sigma p}{Eh} = 0, \\
Q^{(k)} &= -\frac{dM^{(k)}}{ds}, \quad M^{(k)} = \frac{Eh^2}{12} \frac{d^2w^{(k)}}{ds^2}, \\
T_1^{(k)} &= \frac{du^{(k)}}{ds} - \nu w^{(k)}, \quad T_2^{(k)} = \nu \frac{dw^{(k)}}{ds} - w^{(k)},
\end{align*}
\]

where \( s \in [0, l] \) is the axial coordinate, \( l \) is the shell length, \( T_1^{(k)}, T_2^{(k)}, Q^{(k)}, M^{(k)} \) are the stress-resultants and stress-couple, \( u^{(k)}, w^{(k)} \) are the components of a displacement for \( s \in [s_{k-1}, s_k] \), \( k = 1, 2, \ldots, n, n = n_s + 1, n_s \) is a number of annular plates, \( s_k, k = 1, 2, \ldots, n_s \) are the coordinates of stiffened parallels, \( s_0 = 0, s_n = l, \sigma = 1 - \nu^2, \nu \) is Poisson’s ratio, \( E \) is Young’s modulus, \( h \) is the shell thickness.

We get the equations for an axisymmetric membrane deformation taking \( M^{(k)} = 0 \) in Eqn (1).

The non-dimensional equations for a tangential axisymmetric plate deformation are

\[
(rT_{1p})' - T_{2p} = 0, \quad rT_{1p} = ru_p' + \nu u_p, \quad rT_{2p} = u_p + \nu ru_p'.
\]

Here \( (\cdot)' \) denotes the derivative with respect to the radial coordinate, \( r \in [1, r_1], r_1 \) is the outer radius of the plate, \( T_{1p} \) and \( T_{2p} \) are the radial and hoop stress-resultants, \( u_p \) and \( v_p \) are tangential components of the displacement.

The transverse bending stiffness of a plate is not taken into account because it is much smaller than the tangential plate stiffness. It is assumed that the outer plate edge is free, the shell edges are freely supported and that the plate and the shell are made of same material. Then solutions of Eqn (1) and Eqn (2) satisfy following boundary conditions

\[
\begin{align*}
T_{1p}(s) &= 0, \quad T_{1}^{(1)} = u_1^{(1)} = M^{(1)} = 0, \quad s = s_0, \\
T_{1}^{(k)} = u_1^{(k)} = M^{(k)} = 0, \quad s = s_k, \quad T_{1}^{(k+1)} = T_{1}^{(k+1)}, \\
w_1^{(k)} &= w_1^{(k+1)} = w_1^{(k+1)}, \quad \delta^{(k)} = \delta^{(k+1)}, \quad M_1^{(k)} = M_1^{(k+1)}, \\
hQ_1^{(k)} &= hQ_1^{(k+1)} - aT_{1p}(1), \quad s = s_k
\end{align*}
\]

where \( a \) is the dimensionless plate thickness, \( k = 1, 2, \ldots, n_s \).

Let us seek a solution of Eqn (1) as the sum of the membrane state and the edge effect integrals. For example,

\[
w_1^{(k)} = \frac{p}{Eh} + \sum_{j=1}^{4} D_j^{(k)} f_j^{(k)}(s).
\]

Here the first term is a solution of the membrane equation, \( D_j^{(k)} \) are the arbitrary constants. The edge effect functions \( f_1, f_2 \) and \( f_3, f_4 \) are localized in small neighborhoods of the parallels \( s_{k-1} \) and \( s_k \) correspondingly.

The general solution of Eqn (2)

\[
u = C_1x + C_2/x, \quad T_{1p, 2p} = \gamma C_1 + \delta C_2/x^2,
\]

where \( \gamma = 1 + \nu, \delta = 1 - \nu \), depends on two arbitrary constants \( C_1 \) and \( C_2 \).
Substituting solutions (3) and (4) into boundary conditions we obtain arbitrary constants and pre-buckling stress-resultants $T_{1p}$ and $T_{2p}$:

$$T_{1p} = \frac{\sigma_p r^2 (r^2 - r_1^2)}{E h A_p r^2}, \quad T_{2p} = -\frac{\sigma_p r^2 (r_1^2 + r^2)}{E h A_p r^2},$$  

(5)

where $A_p = \gamma (\gamma r_1^2 + \delta) + (3 \sigma) \gamma/4 \alpha (r_1^2 - 1) h^{-3/2} / 2$.

3. Buckling of an annular plate

To analyzed buckling of a cylindrical shell stiffened by identical annular plates under the external pressure $p$ we use the same asymptotic approach as for the solution of Eqn (1). We seek the solution of buckling shell equations as a sum of a membrane state and edge effect integrals. In the first approximation we get the eigenvalue problem for buckling of annular plate. The non-dimensional buckling plate equation can be written as

$$\frac{d^4 w_p}{dr^4} + \frac{2}{r} \frac{d^3 w_p}{dr^3} - 2 m^2 + 2 \beta_1 \frac{d^2 w_p}{dr^2} + \frac{m^2 (m^2 - 4 + \beta_2)}{r^2} w_p = 0,$$

(6)

where $m$ is the circumferential wave number, $w_p$ is the normal deflection,

$$\beta = \frac{12 T_{1p}(1)}{a^2}, \quad t_k = \frac{r^2 T_{1p}(r)}{T_{1p}(1)} \quad k = 1, 2.$$

(7)

We assume that $a \leq h$. Then the boundary conditions on inner edge of the plate are

$$w_p(1) = w'_p(1) = 0$$

(8)

The outer edge of the plate is free.

The approximate solution of the eigenvalue problem for Eqn (6) in the case $b = r_2 - 1 < 1$ was obtained in Ref. [2]. The least positive eigenvalue $\beta_p$ may be found as

$$\beta_p = \frac{1}{b} \min (m_0^2 \beta_0^2(m_0)),$$

(9)

where $m_0 = bm$ and $\beta_0$ is the least positive root of equation $F \sinh \gamma \sin \alpha + G \cosh \gamma \cos \alpha + H = 0$.

$$H = \beta_0^2 (2 \nu - \delta^2), \quad F = (\beta_0^2 + \delta^2) \sqrt{\beta_0^2 - 1},$$

$$\alpha = m_0 \sqrt{\beta_0^2 - 1}, \quad \gamma = m_0 \sqrt{\beta_0^2 - 1}.$$  

(11)

It follows from Eqn (5) and Eqn (7) that the critical pressure

$$p_p = \frac{E h a^2 A_p}{12 \sigma (r_1^2 - 1)} \beta_p.$$  

(12)

The pressure $p_p$ causes the buckling of the second type, for which many small pits appear on the surface of the plate (see Ref. [2]).

4. Opimal parameters for the stiffened cylindrical shell

Let us compare the critical pressures $p_0$ and $p_p$ for the non-stiffened and the ring-stiffened cylindrical shells of equal mass with simply supported edges. We assume that rings are identical and have rectangular cross-sections with the thickness $a$ and the width $b = ka$. Then masses of the non-stiffened and the ring-stiffened shells are $M_0 = 2 \pi R^2 p h_0 a$ and $M_s = 2 \pi R^2 \rho (ht + n_1 ka^2)$ correspondingly, where $h_0$ and $h$ are the thicknesses of the shells, $\rho$ is the mass density. It follows from the equality $M_0 = M_s$, that $d = h_1 / h_0 < 1$. The parameters of the stiffened shell, for which the function $f_c = p_c / p_0$ attains its maximum $f_c^*$ are called the optimal parameters.

To find the critical pressures $p_0$ we use the Southwell-Papkovich formula

$$p_0 = \frac{4 \pi E h_0^3}{6 v^2 (1 - v^2) r^3}.$$  

(13)

If the parameter $k$ is sufficiently small we can model the ring as the circular beam. For this case in assumption on the uniform rings arrangement the approximate formula

$$f_c = f_c^* = n d^2 / l^2$$

was obtained in Ref. [1]. Here $d_*$ is the root of the equation

$$d^3 - \frac{B}{\eta_c A^2} (d - 1)^2 = 0,$$

(15)

satisfying the inequality $0 < d_* < 1$.

$$\eta_c = n^4 / 3 - 1, \quad B = \frac{\sigma n k}{\eta_c^2 h_0^2}, \quad A = n k h_0.$$  

(16)

The relative critical pressure $f_c^*(k)$ corresponds to the buckling of the first type, which is similar to the buckling of the non-stiffened shell. If the parameter $k$ grows, the critical pressure $f_c^* (k)$ increases while $k < k_*$. At $k = k_*$ the buckling of the first type switched on the buckling of the second type. Therefore if $k > k_*$ one can find the critical pressure from Eqn (12). We denote $f_c^* (k) = p_p / p_0$ for the plate of the thickness $a = a_*$. and the width $b = b_*$. The function $f_c^* (k)$ decreases as $k$ increases. Hence the relative critical pressure

$$f_c^* (k) = \min (f_c^*(k), f_c^*(k))$$

(17)

attains its maximum value at $k = k_*$, and $b_* = k_0 a_*$ is the optimal ring width. For $l = 4$, $h_0 = 0.01$, $n_0 = 6$, $\nu = 0.3$ the function $f_c^* (k)$ is plotted in Fig. 2.

Figure 2: Dependence of relative critical pressure $f_c^* (k)$ on the parameter $k = b / a$.

The parameter $k_* = 32.4$ is the root of the equation $f_c^* (k) = f_c^* (k)$. For the shell under consideration $a_* = 0.0037$, $b_* = 0.12$, $d_* = 0.93$, $f_c^* = 5.9$, and the critical external pressure of the ring-stiffened cylindrical shell with optimal parameters is almost 6 times higher, than the critical pressure for the non-stiffened shell of same mass.

References


Numerical analysis of thin-walled cold-formed steel column-base over-roofing solutions

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Abstract

In an increasingly developing world, in which sustainability has become not only an option but more a demand, over-roofing of existing buildings, together with over-cladding forms an optimum complete retrofitting solution to which the construction sector tends every day, instead of the counterpart solution of demolishing the old structures and building new ones. Henceforth, the lightness, reversibility and time consuming features of the materials used in building these over-roofing systems made the engineers turn to steel-based solutions. In this context, the paper summarizes three types of steel-based over-roofing structural solutions, i.e. a) hot-rolled profiles; b) rectangular hollow sections and; c) cold-formed profiles. The investigation is mainly focused on details concerning the connection of over-roofing to the existing structure. Further on, details of a parametric study on these column-base connections for cold-formed steel solution are presented, in both semi-rigid and rigid solutions.

Keywords: over-roofing, light steel structure, column-base connections, numerical modelling

1. Introduction

In the case of existing buildings, nearly most of all construction activities are related to maintenance, refurbishment and conversions. Weighing the increasing necessity for a sustainable construction, the percentage of adaptation works in comparison to the newly built ones tends to surpass the second, especially on the long term foreseeable future, in the developed parts of the world [5]. Over-roofing is the general term used to describe the installation of a new roof on an existing building, creating new habitable spaces. However, over-roofing frequently forms part of a more complex modernization scheme which includes over-cladding also, proving the entire concept presented above as a complete retrofitting solution.

2. Current situation of building stock in Romania

Referring to current building stock in Europe, there are about 196 millions of dwellings corresponding to about 160 millions of buildings, while new buildings represent yearly about 1.5% of the building stock [2]. However, the measured rate of retrofitting today is still much lower than it is foreseen on the long-term future ambitions.

In Romania, the existing building stock presents large retrofitting problems. According to The National Institute of Statistics on March 2002, 4,234,173 households were built in the urban area, from which 3,021,122 households in blocks of flats. From these, 81,964 units (blocks) are made out of large precast reinforced concrete panels [3]. These large prefabricated blocks were built in three main periods, i.e.: (a) 1962-1975; (b) 1975-1982; (c) 1982-1989 [4].

The renovation of buildings using over-roofing solutions requires a good understanding of the building physics requirements in terms of structural performance. Although in Romania, in the last decade, a lot of standards and guidelines concerning the structural, technological and energy efficiency for these interventions have been elaborated, some aspects regarding urban planning and exterior aesthetics of new-added storeys have been ignored while applied in reality. Therefore, in the present context, there is a large variety of geometries, volumes and aesthetics for the over-roofing and over-claddings even in the boundaries of the same neighbourhood, degrading the overall visual image. In this context, steel structures are often the only solution to such complex renovation problems, as it combines the benefits of lightweight components, flexibility in planning, long span capabilities, robustness and durability, with economy and speed of construction on site.

3. Over-roofing solutions based on intensive use of steel

The design of the structures subjected to over-roofing is composed of two stages: (i) verification of the current state of the original structure and (ii) verification of the structure after intervention (including the new-added steel structure). The state of the original structure and the refurbished newly added one was done through 3D analyses using ETABS computer code, by using shell finite elements (see Fig. 1).

Figure 1: 3D view of for common block typology T744R
The building was checked according to the actual standards, proving to be resistant to all loading combinations. These verifications also showed that, by removing the top thermal insulation layers from the terrace, the adding of a supplementary over-roofing floor would not affect the capacity of existing concrete structure.

Three types of solutions for an over-roofing based on intensive use of steel were chosen for numerical studies: (a) hot-rolled profiles (IPE beams and HEB columns); (b) RHS for columns and IPE beams; (c) cold-formed steel profiles. Therefore, pinned/semi-rigid and rigid column-based connections were further investigated, for all 3 cases, by using numerical analyses, in order to determine the optimum solution for connecting these over-roofing systems. However, the paper is focused on numerical and parametric details of the cold-formed steel case scenario (see Fig. 2).

4. Numerical analysis

In order to prove the efficiency of the column-base connections chosen for the study, numerical simulations were performed using FE software ABAQUS 6.7 [1]. Semi-rigid and rigid base connections using cold-formed steel profiles (CF) will be detailed in the following, as presented in Fig. 2.

![Figure 2: Cold-formed steel column-base connections](image1)

Elasto-plastic material models were used for all elements. SHELL elements of S4R type, with 4 nodes, reduced integration, 6 DOF per node were used for modelling the cold-formed steel sections, while for all other sections, BRICK elements of C3D8R type, with 8 nodes, reduced integration, 6 DOF per node, were used. In order to obtain accurate results a standard implicit solver was used. Initial convergence problems were issued by a preliminary step of preload and the actual load of 200 mm displacement was imposed through a Riks step.

The following results were obtained: (1) the first connection was modelled as semi-rigid but presents mostly a pinned behaviour. The maximum stresses were reported in the upper flanges of the section on the direction of loading, but also around the holes of the webs (see Fig. 3a); (2) for the rigid solution, the plastic stresses achieve maximum values both in the flanges and the webs; a web crippling phenomenon occurs (see Fig. 3b).

![Figure 3: Distribution of stresses in the based connections](image2)

The force-displacement curves obtained for each case scenario are presented in Figure 4.

![Figure 4: Force-displacement curves for the semi-rigid and rigid cold-formed steel base connections](image3)

The above force-displacement curves show, as expected, the semi-rigid solution shows smaller loading capacities than the rigid one, but the intention was to reduce the local effects on the existing structure. The stress level on the existing concrete floors and the additional walls are small, thus do not introduce supplementary stresses on the existing structure.

The scope of the numerical program is a better understanding of the behaviour of these types of column-base connections and to find the optimum solution for this kind of over-roofing systems, by limiting the stresses to the existing concrete panel building. Based on this numerical study several types of these connections will be experimentally investigated.

5. Conclusions

The study presented in the paper shows two possibilities of connecting steel-intensive over-roofing to the existing concrete buildings. Numerical analysis shows the light steel-intensive solutions are ideal systems for over-roofing the existing large precast concrete panel buildings due to their lightness, reversibility and clean sites; also, they can adapt to existing structural systems, so several structural typologies can be regarded. Numerical analysis shows that the cold-formed connections withstands a considerable amount of loading, for this kind of a four-class section. However, the main concern remains to minimise the structural intervention on the existing concrete structure. Experimental tests will be performed in order to validate the proposed solutions.

References

Numerical modal analysis of three-blade helicopter rotor model elaborated with the Finite Element Method

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Abstract

In the paper modal analysis of the helicopter rotor model with three composite blades was performed. The simulations were done using the finite element method. A number of simulations for different models of structure were performed. The numerical model with the hub modelled as a rigid body was also examined. In addition, the laboratory test stand was built in order to perform experimental verification. The presented results of numerical simulations are the first phase of research on the dynamics of helicopter blades.

Keywords: modal analysis, helicopter rotor, FEM, composite, blade, Abaqus

1. Introduction

Modal analysis of the structures is a widely used technique in practice. As a result of modal analysis, the natural frequencies and corresponding modes of free vibration and damping coefficients is obtained. In the carried out tests most classical and non-classical effects observed in beam-like structures are taken into account and examined e.g. in [1-3].

The aim of the current study was to examine numerically the natural frequencies and modes of three blades helicopter’s rotor model. The simulations were performed using a commercial package Abaqus. At the next step, the obtained results will be compared with the experimental results.

2. Research object

The object of analysis was three blade helicopter rotor. The blades of rotor was made of glass-epoxy unidirectional composite prepreg (R-glass material). Configuration of composite plies was as follows: [45/45/90]. The geometrical dimensions of each blade were: the length – 350 mm, the width – 34 mm, beam thickness – 1.8 mm (each piezoelectric layer had a thickness of 0.3 mm). The rotor head/hub was made of Polyamide 6 (PA 6) - TECAMID 6. The drive shaft was made of steel 18G2A. The handle of the beams also was made of this steel as well as aluminium alloy (A1). In numerical simulations the following data provided by the manufacturer of the composite were used: tensile moduli $E_1=56$ GPa, $E_2=16$ GPa, shear moduli $G_{12}=4.05$ GPa, $G_{13}=G_{23}=4$ GPa and Poisson’s ratio $\nu_{12}=0.4$. Mechanical properties of other materials are as follows: steel 18G2A: $E=210$ GPa, $G=80$ GPa, $\nu=0.3$; Polyamide 6 (PA 6): $E=1.6$ GPa, $G=0.6$ GPa, $\nu=0.42$; aluminium alloy (A1): $E=69$ GPa, $G=29.5$ GPa, $\nu=0.33$.

3. Numerical model of three blades helicopter’s rotor

The FE model of the composite blades was made using continuum shell finite elements (denote as SC8R) with reduced integration. This Abaqus enables modelling of the composite elements as a set of orthotropic layers in plane-stress state. Individual layers of the laminate was made according to Layup-ply technique. The numerical model of other elements was made using the C3D20RE and C3D10 type solid elements. All elements used a reduced integration method. The mechanical boundary conditions of the numerical model were realized by restraining the nodes located on foundation all the translational degrees of freedom and modelling this way the beam’s restraint. The combination of all elements was realized by defining interactions as “TIE”, what resulted in linking the degrees of freedom of nodes in contact on the appropriate surfaces of the model. The 3D model of the system is presented in Fig. 1.

Figure 1: The 3D model of the three blade helicopter rotor

4. Modal analysis and results

In order to check the dynamic behaviour of the three blades helicopter’s rotor a numerical modal analyses were performed. In the finite element method simulations the Lanczos algorithm was used in order to determine the natural frequencies and corresponding modes of free vibrations of the system. The results of modal analysis are shown in Table 1. The obtained results were compared with the natural frequency of a cantilever beam. The determined first flapwise bending mode (a), chordwise bending mode (b) and torsion mode (c) are shown in Fig. 2. The numerical model with the hub modelled as a rigid body was also examined.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Whole rotor</th>
<th>Cantilever beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>First flapwise bending mode</td>
<td>14.6</td>
<td>4.4</td>
</tr>
<tr>
<td>Second flapwise bending mode</td>
<td>90.5</td>
<td>27.7</td>
</tr>
<tr>
<td>First chordwise bending mode</td>
<td>113.0</td>
<td>76.0</td>
</tr>
<tr>
<td>First torsion mode</td>
<td>118.3</td>
<td>101.0</td>
</tr>
</tbody>
</table>

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All material data and specimen dimensions used for the numerical tests correspond to the actual beam installed in Laboratory of Dynamics and Strength of Materials at Lublin University of Technology to be used later for experimental validation. The experimental model of helicopter’s rotor is presented in Fig. 3.

5. Conclusion

Finite element model of three blade helicopter rotor was performed. Numerical model FEM was built in the commercial system Abaqus. The natural frequencies and corresponding modes of free vibration was determined. The obtained results were compared with the cantilever beam. The natural frequencies of the rotor and the cantilever are very different in terms of quantity. The numerical model with the hub modelled as a rigid body was also examined. Next, the results obtained by finite element method will be compared with experimental measurements.

References


Some features of vibrations of compound shells of revolution

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Abstract

Based on the assumptions of the Kirchhoff–Love shell model, numerical–analytical technique is developed for designing the lower frequencies of compound coaxial shells of revolution with various geometrical shapes. The technique employs variable separation method, incremental search and orthogonal–sweep methods with solving Cauchy problems by the fifth–order Runge–Kutta scheme.

The vibration features of compound systems are studied in comparison with vibrations of their constituent members. Using as an example the dependence \( \omega(k) \) very along a meridian, and (\( j=1,2,...,J \)) in other cases, it is shown that vibrations of the system as a whole may have qualitative distinctions in comparison with vibrations of individual shells (\( \omega \) is the minimum value of natural frequency, \( k \) is the wave-making mode in the circumferential direction).

Keywords: thin–walled systems, shells of revolution, natural frequencies, classical theory, numerical analytical technique, vibration features

1. Introduction

A great many modern structures are made in the form of thin–walled systems composed of coaxial joined shells of revolution with various geometrical shapes (reservoirs of various destinations, protective coatings of nuclear reactors, frames of rockets and underwater vehicles, etc.).

Studying dynamical characteristics of such complex objects make it possible to trace resonance regimes of their operation and to prevent emergency under the action of real service loads.

The overwhelming majority of studies on this subject are referred to shell systems of zero Gaussian curvature in the form of various combinations of coaxial cylinders, cones, and circular plates (e.g., Ref. [1, 4]). Many papers are focused also on the analysis of natural frequencies of systems with spherical members of nonzero but constant curvature (e.g., Ref. [3, 5]). At the same time, in accordance with elevated requirements to strength and reliability of modern technical equipment, the structures are built up with complex geometrical shape containing, in particular, torus-elliptic members.

The present work addresses vibration features of compound systems with complicated geometry.

2. Problem statement and solving method

Consider a system of \( J \) shells of revolution with different geometry, which are coaxial with respect to the \( \theta \)–axis and describe in the coordinate system \( \alpha, \theta (\omega=\{a_1, \ldots, a_2, \ldots, a_J \}) \) \( (j=1,2,...,J) \) very along a meridian, and \( \theta \) is the central angle of the section \( z=\text{const} \). The general view of its generatrix–meridian changing along the axis is presented in Fig.1.

The shells may be one-layered or composed of several layers with conditions of perfect contact at interfaces. It is assumed that their materials work in the range of elastic deformations and may be both isotropic and orthotropic.

3. Analysis of natural frequencies of compound shells

Let us consider the system of three joined members (Fig. 2) including the cone (length \( l_1 \), radiuses \( r_1 \) and \( R_1 \)), cylinder (length \( l_2 \), radius \( R_2 \)), and torus with elliptic cross-section \( (a \text{ and } b \text{ are semi-axes, } r_2 \text{ is distance between the ellipse center and axis of revolution, } \phi_2 \text{ is the half-angle at the pole}) \). All shells are of orthotropic material of the same thickness \( h \).

The particular case, when the third member is made in the form of a spherical isotropic ring \( (a=b=R_1, r_2=0) \), was considered in Ref. [5]. It is used to test the technique developed.

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Figure 1: General view of the generatrix-meridian of the shell system

The system ends may be under arbitrary homogeneous physically consistent boundary conditions. The equilibrium conditions of static characteristics and continuity conditions of kinematical characteristics at contact lines of adjacent shells are formulated in the general coordinate system \( r\theta z \).

Small undamped vibrations of such system are considered within the framework of the known assumptions of the Kirchhoff–Love theory. In order to solve associated two-dimensional eigenvalue problems, the numerical–analytical technique making use the Fourier variable separation method, incremental search method (\( \Delta \lambda \) – method), and the orthogonal sweep method in combination with the solving Cauchy’s problems by the fifth–order Runge–Kutta scheme (Merson’s modification) was developed in Ref. [2].
The distinctions in the values of the lower frequencies for input data:
\[ R=1 \text{ m}, r_1/R=0.4, l_1/R=2.5, b/R=0.01, l_2/R=1.2, h/R=0.01, \phi_0=0^\circ, 30^\circ, 45^\circ, \]
\[ E=69.58 \text{ GPa}, \mu=0.31, \rho=2700 \text{ kg/m}^3. \]

obtained in the present work and in Ref. [5] with the mixed series and finite element methods do not exceed 1%.

The fact that the results obtained coincide with high accuracy is evidence in favour of efficiency of the technique proposed as applied to the class of compound shells of revolution being considered.

Figure 2: The generatrix of a cone-cylinder-torus ellipse system

Vibration features of the compound system are analyzed using as an example the conditional dependence \( \omega(k) \), which is traditional for shells of revolution (\( \omega \) is the minimum value of natural frequency, \( k \) is the wave-making mode in the circumferential direction). This dependence for components of the system separately (Fig. 3, a: cone (CON), cylinder (CYL), torus-ellipse (TEL)) and for the system as a whole (Fig. 3, b: CON-CYL-TEL) is compared in the form of frequency parameter \( f=f(k) \) \( (f=10^{-3} \omega/2\pi, \text{Hz}) \) for input data:
\[ b=R, \ a/b=0.5, \ l_1/R=l_2/R=1.2, \ r_1/R=1.4, \ E_\alpha=4.4E_\theta, \ E_\theta=1.3E_\theta, \ G_{\alpha\theta}=0.37E_\theta, \mu=0.18, \rho=\rho_0. \]

As seen in Fig. 3, the dependence \( f=f(k) \) for individual members of the system is of a usual view being the nonmonotonic function with one minimum. This dependence for the system as a whole reveals several local minima at \( k=1, k=3, k=6 \), due to simultaneous work of joined shells as a single whole.

Figure 3: Values of the frequency parameter \( f=f(k) \) for the shell components separately (a) and for the system as a whole (b)

4. Conclusions

Using the assumptions of the Kirchhoff–Love classical shell theory, the numerical analytical technique is developed for designing the natural vibration frequencies of compound coaxial shells of revolution with the elements of nonzero Gaussian curvature. The technique employs the Fourier variable separation method, incremental search (\( \Delta(\lambda) \)–method), and orthogonal-sweep methods with the Cauchy problems being solved by the fifth-order Runge–Kutta scheme. The technique was tested inductively comparing with the results obtained by other authors.

Considering as an example the conditional dependence \( \omega(k) \), which is traditional for shells of revolution, it is shown that vibrations of the shell system, as a single object, may have qualitatively dissimilar character in comparison with vibrations of its constituent parts (\( \omega \) is the minimum value of natural frequency, \( k \) is the wave-making mode in the circumferential direction).

References

Stabilization of a magneto-elastic Mindlin-Timoshenko plate model

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Abstract

In the paper we concern stabilization of a model for the magneto-elastic interactions of an elastic field and a magnetic field in an electrically conducting Mindlin-Timoshenko plate. A particular question arises, what kind of mechanical damping will render the model uniformly exponentially stable. We show that the inclusion of nonlinear locally distributed damping in the interior of the plate furnishes uniform exponential stability when the damping term exhibits linear behaviour near the origin.

Keywords: Mindlin-Timoshenko, magneto-elastic, stabilization, locally distributed damping, observability estimate

1. Introduction and statement of the problem

Interactive models in elasticity have been extensively studied during the past few decades with applications in smart material technology. Such models include hybrid systems, e.g. plate-beam models, fluid-structure models including structural acoustic models, thermo-elastic models and magneto-elastic models. At the same time there has been a strong tendency in recent years to use plate models based on Mindlin-Timoshenko theory Ref. [7] rather than Euler-Bernoulli, Kirchoff or von Kármán models. With the inclusion of shear effects over and above displacement and rotary inertia effects, these models provide for thicker plates and higher frequencies of the flexural motions of the plate.

A model for the magneto-elastic interactions of an elastic field and a magnetic field in an electrically conducting Mindlin-Timoshenko plate, was first introduced in Ref. [4] using the classical system of magneto-elasticity Ref. [1] as a starting point for the magneto-elastic Mindlin-Timoshenko plate model achieved in Ref. [4] by incorporating nonlinear locally supported damping in the interior of the plate. It turns out that uniform exponential stability can be achieved provided that the damping term exhibits linear behaviour near the origin. The route to uniform polynomial stability of the model subject to fully nonlinear growth conditions on the damping term, is briefly outlined.

2. The model, hypotheses and notations

Following the procedure outlined in Section 1, the magneto-elastic waves effected by the dynamic interaction of the elastic field \( U := [\psi, \phi, w] \) and the magnetic field \( h = [h^x, h^y, h^z] \) in a two-dimensional plate with interior \( \Omega \), are governed by the ensuing interactive system:

\[
U_{tt} - \nabla \cdot S(U) - (\nabla \times h) \times H_0 + p(x, U_t) + Q = 0
\]

\[
h_t + \nabla \times (\nabla \times h) - \nabla \times (U_t \times H_0) = 0
\]

\[
\nabla \cdot h = 0
\]

On \( \partial \Omega \) :

\[
U = 0, \quad h \cdot n = 0, \quad n \times (\nabla \times h) = 0
\]

Initial Conditions:

\[
U|_{t=0} = U_0, U_t|_{t=0} = U_1, h|_{t=0} = h_0
\]

Here \( \Omega \) is an open, bounded, simply connected domain in \( \mathbb{R}^2 \) with smooth \( C^2 \) boundary \( \partial \Omega \). The variables \( \psi, \phi \) denote the shear angles of plate filaments and \( w \) represents the transversal displacement of the mid-plane of the plate. \( H_0 = [H_0^x, H_0^y, 0] \) denotes the uniform magnetic field. By incorporating the density \( \rho \) and the plate thickness \( b \) in the usual inner product on \( \mathbb{R}^3 \), these parameters have been taken as unity - the magnetic permeability \( \mu \) and the electrical conductivity \( \sigma \) are also taken as unity.

\( S \) and \( Q \) denote, respectively, the stress tensor and shear force vector given by

\[
S(U) = D(\psi_x + \nu \phi_x) D(\frac{1-\nu}{2}(\psi_y + \phi_y)) K(\psi + w_x)
D(\frac{1-\nu}{2}(\psi_y + \phi_y)) D(\phi_y + \nu \psi_y) K(\phi + w_y)
\]

\[
Q(U) = K[\psi + w_x, \phi + w_y, 0]
\]

with the coefficients \( K \) and \( D \) respectively shear the modulus and the flexural rigidity.

The damping term \( p \) is given by

\[
p(x, U_t) = [p^x(x, \psi_t), p^y(x, \phi_t), p^z(x, w_t)]
\]

Thus damping is applied in all three PDEs of the equations of the Mindlin-Timoshenko system. However, the damping term is active only in a neighbourhood \( \omega \) in \( \Omega \) of a portion of the boundary where geometric conditions hold, i.e.

\[
\Gamma(x_0) = \{ x \in \partial \Omega; (x - x_0) \cdot n \geq 0 \} \subset \partial \Omega
\]
The locally distributed damping term $p$ satisfies the following conditions:

(a) $p : \Omega \times \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is continuous, $p(x, s) \cdot s \geq 0 \forall s \in \mathbb{R}^3, x \in \Omega$

(b) $p$ is continuously differentiable in $\Omega \times \mathbb{R}^3$

(c) If $p = [p^1, p^2, p^3]$, then

$$\sum_{i,k=1}^3 u^i \frac{\partial p^k}{\partial x^k} u^k \geq 0 \forall u = [u^1, u^2, u^3], s = [s^1, s^2, s^3] \in \mathbb{R}^3$$

(d) There exist constants $k_j, j = 0, 1, 2, 3$, and a number $p \geq 0$ such that the following growth conditions hold:

$$|p(x, s)|^s \leq k_0 a(x) |s|^2 \quad \text{if } |s| \leq 1$$

$$|p(x, s)|^s \leq k_1 a(x)|s| \quad \text{if } |s| \geq 1$$

where $a : \Omega \rightarrow \mathbb{R}^3$ is a member of $L^\infty(\Omega)$ satisfying $a(x) \geq \alpha_0 > 0$, in $\omega, \omega_0$ a constant

Thus the nonlinear damping term behaves linearly near the origin whereas far from the origin it may be strongly nonlinear.

The following spaces will be used in the sequel:

$$H^m(\Omega), m \geq 1, 1 \leq d \leq 3$$

are Sobolev spaces with norms $\| \cdot \|_m$

$$H_0 := \{ f : f \in (L^2(\Omega))^3; \nabla \cdot f = 0 \text{ in } \Omega, f \cdot n = 0 \text{ on } \partial \Omega \}$$

$$\mathcal{H} := \left( H_0^1(\Omega) \right)^2 \times \left( L^2(\Omega) \right)^3 \times H_0$$

$$Z := \{ f : f \in (H^{2}(\Omega))^3 \cap H_0, n \times (\nabla \times f) = 0 \text{ on } \partial \Omega \}$$

$$\mathcal{X} := \left( H^2(\Omega) \cap H^1_0(\Omega) \right)^3 \times \left( H^1_0(\Omega) \right)^3 \times Z$$

3. Main result

The main ingredient of the paper is the uniform exponential stabilization of $Pr(P)$. First we show unique solvability of the model by considering $Pr(P)$ as an abstract evolution problem of the form

$$\dot{U} + AU = F(U), t > 0$$

$$U(0) = U_0$$

where $U = [U, U_1, h], A$ is a linear operator that describes the coupled dynamics of the conductive plate, with domain $\mathcal{D}(A) := \mathcal{X} \subset \mathcal{H}$ and $F(U) = [0, -p(U_1), 0]$. We then construct a $C_0$ semigroup of contractions for the homogeneous evolution problem whence the existence of a solution of the non-homogeneous problem, global weak or strong, depending on the regularity of the initial data, can be established.

Our approach in accomplishing exponential stability consists of direct PDE estimates comprising intricate energy equalities and inequalities for the full nonlinear model. These furnish an observability estimate for the energy $E(t)$ of the system. Here the fact that $N = 2, N$ the dimension of $\Omega$, plays a crucial role. Application of the classical semigroup property (see p. 408 of Ref. [6]) now furnishes uniform exponential decay of the energy of the system. Hence we obtain

**Theorem** Assume that $C_0 K < 2D$ holds, with $K$ and $D$ respectively the shear modulus and the flexural rigidity in $Pr(P)$, and $C_0$ Poincaré’s constant. If $[U_0, U_1, h_0] \in \mathcal{D}(A), p > 0$ ($p$ the number in the growth conditions on $p$ ), then the global strong solution of $Pr(P)$ satisfies

$$E(t) \leq CE(0) \exp(-\gamma t) \forall t \geq 0$$

with $C$ and $\gamma$ positive constants with $C$ dependent on $p > 0, \|U_0\|_2, \|U_1\|_2, \|h_0\|_2, \|a(x)\|_{L^\infty}$ and $|\Omega|$. When $[U_0, U_1, h_0] \in \mathcal{H}, p = 0$ the result holds for global weak solutions and $C$ is independent of the initial data.

It should be pointed out that the "regional constraint" $C_0 K < 2D$ is a mathematical condition, used together with Poincaré’s inequality, to obtain absorption of lower order terms, arising from the use of Mindlin-Timoshenko theory, in an energy estimate. The restriction can be circumvented by using a classical compactness-uniqueness argument. However, the constants introduced in the estimates are then “uncontrollable”.

**Remark.** In the event of fully nonlinear localized damping, i.e., when the damping term does not behave linearly near the origin, polynomial decay of the energy $E(t)$, at a rate of order $\frac{1}{t^2}$, with $\delta$ dependent on the growth of $p$, can be obtained applying Nakao’s lemma Ref. [8].
Numerical investigation of bending and buckling of seven-layer sandwich plates

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Abstract

The paper is focused on the numerical analysis of a seven-layer plate made of steel sheets. The main corrugated core is placed between two faces being sandwich plates, whose core is also corrugated. The directions of both corrugations are perpendicular to each other. Two load cases are considered: the transverse pressure and the axial compression. The first load case allows to investigate the bending stiffness of the plate and the second one gives the possibility to determine the buckling resistance of the plate. The influence of the length-to-width ratio as well as the direction of corrugation on the stiffness of the plate is analysed. A high buckling resistance of the proposed structure is pointed out.

Keywords: sandwich structure, rectangular plate, corrugated core, bending

1. Introduction

The bending stiffness of a plate can be increased by designing it as a sandwich structure. In such a structure most of the load is carried by the faces of a high stiffness. Further increase of the stiffness can be achieved by making the faces itself as a sandwich plates. This way a seven-layer plate is obtained consisting of the main core and two sandwich faces in which two faces and the core can be distinguished. Such structures have been analysed in terms of stability in conference proceedings [5].

The core in the present investigation has the form of a trapezoidal plate. Analytical description of the stiffness of such core can be found in [2, 4]. Numerical investigations of the stiffness of the sandwich plates with a corrugated core is presented in [1]. Experimental investigations and FE modelling of the impact response of multi-layered structures with corrugated cores are presented in [3].

The goal of the paper is to investigate the influence of the direction of corrugation and the dimensions of a plate on its stiffness. Buckling resistance of a square plate is also analysed. All calculations are made with the use of the finite element method (FEM).

2. Dimensions and FE model of the plate

The plate is composed of flat metal sheets of the thickness t = 0.8 mm and two types of corrugated sheets of the same thickness. Dimensions of the corrugations are shown in Fig. 1. Two types of load were investigated as shown in Fig. 2. The uniform transverse pressure allows to investigate the bending stiffness of the plate whereas the axial compression gives the possibility to determine the buckling resistance of the plate. In both load cases the model is simply supported at all edges.

As a base structure for bending analysis the plate of the dimensions a = 1196 mm and b = 1200 mm has been chosen. Two families of plates have been created by changing the ratios a/b and b/a as follows: 2/3, 1/2, 1/3. This way the influence of dimensions as well as the direction of corrugation on the stiffness of the plate can be investigated.

For buckling analysis the dimensions of the plate are: a = 4048 mm and b = 4000 mm. Two simulations have been conducted, one for each direction: x and y. This way the influence of the direction of corrugation on the buckling resistance can be determined.

The finite element model was prepared with the use of shell elements (see Fig. 1c). The mid-surface of each layer of the plate was modelled and the bonding conditions between the layers have been imposed. Since the problem is symmetrical with respect to two symmetry planes only a quarter of the plate was considered. The mechanical properties of the material are: Young’s modulus $E = 200000$ MPa, Poisson’s ratio $\nu = 0.3$.

![Figure 1: Dimensions of the plate (a) and (b); FE model (c)](image_url)
3. Results of analyses

3.1. Pressure conditions

In order to determine the influence of the dimensions of the plate and the direction of corrugation on the stiffness of the plate the linear static analysis was performed. The uniformly distributed pressure \( p = 0.1 \) MPa was applied to the upper face of the plate.

Since the main core and the core of the faces is corrugated the stiffness of the plate is influenced strongly by the shear effects in these layers. The deformation due to shear stresses is visible in Fig. 3 in which the sections of the square plate made in the midlength in both directions are shown.

![Figure 2: Loads applied in the analyses](image)

The deformation is smaller for the main core which is stiffened with three-layer faces. The core in the faces deforms more since from the outer side it is stiffened with a thin sheet only. The sheet follows the deformation of the core and forms regular waves on both sides of the seven-layer plate.

Since the corrugation is different in both cores another static analysis was performed to determine the influence of the direction of corrugation on the stiffness of the plate. The results are shown in Fig. 4. On the horizontal axis the width to length ratio is shown. The vertical axis corresponds to the stiffness of the plate – the applied load has been divided by the maximum deflection.

Both curves presented in the plot overlap except the point corresponding to the smallest ratio \( a/b \) or \( b/a \). In this case the plate in which the main core is corrugated in the longitudinal direction is about 15% stiffer than the plate with transversely corrugated main core.

3.2. Axial compression

A presented seven-layer structure is susceptible to the local buckling. In order to avoid this phenomenon large dimensions of the model were proposed to perform the buckling analysis, namely \( 4000 \times 4048 \) mm. For these dimensions the plate buckled in a global way: one half-wave in both directions. The buckling load is almost the same for two cases considered. For the compression in the \( x \) directions it has the magnitude of 2985 kN and in the \( y \) direction – 2929 kN. The buckling may be considered elastic since the stresses under the critical load are below 375 MPa.

4. Conclusions

In the paper seven-layer sandwich plates were investigated. Based on the results of bending analysis it is seen that the direction of corrugation of both cores (the main one and the one in the faces) do not influence the bending stiffness of the plate. Small differences of the stiffness are observed for plates in which the width-to-length ratio is small. The deformation of the outside sheets due to shear stresses (Fig. 3b) suggests that the increment of the thickness of these sheets may increase the stiffness of the seven-layer plate.

Based on the results of buckling analysis it is seen that the buckling resistance of the presented plate is the same in both directions. Moreover, the resistance is very high even for a thin sheets used in the model.

References

Tolerance modelling of dynamics of microstructured functionally graded plates

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Abstract

Microstructured functionally graded plates are considered. The size of the microstructure is of an order of the plate thickness. To take into account the effect of the microstructure on dynamic behaviour of these plates the tolerance modelling is applied. Using this method model equations with smooth functional coefficients involving terms dependent of the microstructure size are derived.

Keywords: functionally graded plates, effect of the microstructure size, thin plates, medium thickness plates, tolerance modelling, dynamics

1. Introduction

In the note plates with constant thickness \( d \) made of two materials are analysed. On the microlevel plate components are tolerance-periodic distributed along one direction \( x=x_1 \), cf. Figure 1. But averaged plate properties are slowly-varying along this direction, normal to interfaces between materials. Hence, such plates consist of many small elements on the microstructure parameter \( l \), cf. \([8,2,5]\); material properties are tolerance-periodic functions in \( x \) and independent of \( y, z \).

These plates can be investigated using various kinematic assumptions. Here, two various model equations are presented – one based on the thin plates theory relations and other – based on the medium thickness plates theory. Because the governing equations have highly oscillating, tolerance-periodic, non-continuous functional coefficients in \( x \), they are not relevant to analyse various problems of these plates. To investigate thermomechanical problems of FGM-type structures (also for plates) modelling methods for periodic structures can be used, cf. \([7,8]\), but the effect of the microstructure size is usually neglected in these model equations.

The aim of this note is to present averaged equations of the tolerance modelling, cf. \([8,2]\), involving terms describing the effect of the microstructure size. Similar tolerance model for thin transversally graded plates under the condition \( d<<l \) is shown in \([8,2,5]\).

2. Fundamental relations

Let us denote \( x=(x_1, x_2), \quad x=x_1, \quad z=z_1 \); and \( \Omega = \{ (x,z) : -2 \leq z \leq 2, x \in \Pi \} \) as the region of undeformed plate, with midplane \( \Pi \) and the constant plate thickness \( d \). Let \( \Delta = [-1/2/1] \times \{0\} \) be the “basic cell” on \( \Omega \), with cell length \( l \). Hence, such plates consist of many small elements on the microstructure parameter \( l \), and is called the microstructure parameter.

Denote by \( \partial_0 \) derivatives of \( x_0 \), and also \( \partial_{i,j}=\partial_{x_i} \partial_{x_j} \). Properties of the plate material, as mass density \( \rho \) and elastic moduli \( a_{ijkl} \), are assumed to be tolerance-periodic functions in \( x \), even functions in \( z \), and independent of \( y \). Denote also \( c_{ijkl}=a_{ijkl}-a_{ikjl} \), \( (\alpha_{ij})^{-1} \).

Hence, a mass density per unit area \( \mu \), a rotational inertia \( \vartheta \) and stiffnesses \( b_{ij}, d_{ij} \) of the plate are defined:

\[
\mu = \int_{-d/2}^{d/2} dz_2 \vartheta = \int_{-d/2}^{d/2} dz_2 \int_{-d/2}^{d/2} dz_1 \int_{-d/2}^{d/2} dz_3 b_{ijkl} = \int_{-d/2}^{d/2} dz_1 \int_{-d/2}^{d/2} dz_3 d_{ijkl} = \int_{-d/2}^{d/2} dz_1 \int_{-d/2}^{d/2} dz_3 \int_{-d/2}^{d/2} dz_2, \quad (1)
\]

being tolerance-periodic functions in \( x \). Let \( \mu_i \) be plate displacements, \( p \) - loads normal to the midplane.

Using the kinematic assumptions of the thin plates theory, dynamics of the functionally graded plates is described by:

\[
\partial_{x_1} (b_{ij} \partial_{x_1} u_{ij}) + \mu_{ij} - \partial_{x_1} (\vartheta \partial_{x_1} \theta_{ij}) \partial_{x_1} = p, \quad (2)
\]

for the plate deflection \( u_{ij} \).

From the kinematic assumptions of the medium thickness plates theory the system of three equations is derived:

\[
\partial_{x_1} (b_{ijkl} \partial_{x_1} \theta_{ij}) - d_{ijkl} (\mu \partial_{x_1} \mu_{ij} - \theta_{ij}) \partial_{x_1} = 0, \quad \partial_{x_1} (d_{ijkl} (\vartheta \partial_{x_1} \theta_{ij} - \vartheta_{ij}) \partial_{x_1} = -p, \quad (3)
\]

for the plate deflection \( u \) and rotations \( \theta_{ij}, \alpha=1.2 \).

The governing equations (2) and (3) have highly oscillating, tolerance-periodic, non-continuous functional coefficients in \( x \). To derive model equations with smooth continuous coefficients the tolerance modelling is applied.

3. Tolerance modelling

Basic concepts and assumptions of the tolerance modelling are presented in \([8,2,3,4]\). They are: an averaging operator, a slowly-varying function, a tolerance-periodic function, a highly oscillating function, a fluctuation shape function. Here, only...
two of them are mentioned. Let \( \Delta(x) = x + \Delta \), \( \Pi_\alpha = \{x \in \Pi : \Delta(x) \subset \Pi \} \), be a cell at \( x \in \Pi_\alpha \). The averaging operator for an arbitrary integrable function \( f \) is defined by:

\[
\langle f \rangle(x) = \int_{\Pi_\alpha} f(y) dy.
\]

For the tolerance-periodic in \( x \) function \( f \) its averaged value by (4) is a slowly-varying function in \( x \). The important concept of this method is also the fluctuation shape function, which is assumed in the form of a saw-like function of \( x \), cf. [3,4].

Using the above concepts the fundamental assumptions of the tolerance modelling for the considered plates can be formulated. The main assumption is the micro-macro decomposition of basic unknown fields in the problem under consideration. Below, there two independent assumptions are introduced, leading to various tolerance models of functionally graded plates.

The micro-macro decomposition for the thin plates, in which it is assumed that the plate midplane displacements \( u_i \) (\( i = 1,2,3 \)) can be decomposed as for periodic plates, cf. [6]:

\[
u_i(x,z,t) = \bar{u}_i(x,t) + w_i(x,t), \quad u_i(x,z,t) = -[\ddot{\bar{u}}_i(x,t) + h(x)\vartheta_i(x,z,t)],
\]

with new unknowns: macrodeflection \( w \) and fluctuation variables \( \vartheta \) (\( \alpha = 1,2 \)), being slowly-varying functions in \( x \), and the known fluctuation shape function \( h \), assumed in the form of a saw-like function of \( x \).

The micro-macro decomposition for the medium thickness plates, in which displacements of the plate midplane \( u_i \) (\( i = 1,2,3 \)) are assumed as for periodic plates, cf. [1]:

\[
u_i(x,z,t) = \bar{u}_i(x,t) + w_i(x,t), \quad u_i(x,z,t) = -[\ddot{\bar{u}}_i(x,t) + g(x)\vartheta_i(x,z,t)],
\]

with new unknowns: macrodeflection \( w \), fluctuation variables \( \vartheta \) (\( \alpha = 1,2 \)), and fluctuation variables \( \vartheta_a \) (\( \alpha = 1,2 \)), being slowly-varying functions in \( x \), and the known fluctuation shape function \( g \), which has the form of a saw-like function of \( x \).

The second assumption of the tolerance modelling is the tolerance averaging approximation in which it is assumed that terms of an order of tolerance parameter \( \delta \) are negligibly small in the modelling.

The above concepts and assumptions can be applied in various tolerance modelling procedures, cf. [8,2]. Here, the procedure from [3] is used, which can be divided on four steps. The first of them is to substitute micro-macro decompositions given in Eqn (5) or Eqn (6) to Eqn (2) or Eqn (3), respectively. In the second step these equations are averaged by using the averaging operator (4). In the third step the problem to find the fluctuation variables is formulated. To find these unknowns the orthogonal method is used, i.e. the governing equations (2) or (3) are multiplied by the fluctuation shape function and then averaged by formula (4). The fourth step is to substitute micro-macro decompositions (4) or (5) into obtained equations.

4. Tolerance models equations

From the above tolerance modelling procedure governing equations of two various tolerance models are derived.

4.1. Tolerance model equations of the thin functionally graded plates

The tolerance modelling replaces thin plates equations (2) by the following governing equations:

\[
\begin{align}
& \ddot{\bar{u}}_{a12} + \bar{a}_{12} \ddot{w} + \bar{b}_{12} \ddot{\vartheta}_a + \ddot{\vartheta}_a \bar{g} + \ddot{\vartheta}_a \bar{v}_a = -\vartheta_a \bar{g} \bar{v}_a - \bar{b}_{12} \ddot{\bar{u}}_a \bar{h}^2 \bar{v}_a, \\
& \bar{a}_{12} \ddot{w} + \bar{b}_{12} \ddot{\vartheta}_a + \ddot{\vartheta}_a \bar{g} + \ddot{\vartheta}_a \bar{v}_a = -\vartheta_a \bar{g} \bar{v}_a - \bar{b}_{12} \ddot{\bar{u}}_a \bar{h}^2 \bar{v}_a.
\end{align}
\]

Equations (7) are differential equations – one for macrodeflection \( w \) and two for fluctuation variables \( \vartheta_a \).

4.2. Tolerance model equations of the medium thickness functionally graded plates

The tolerance modelling leads from medium thickness plate Eqs. (3) to the following governing equations:

\[
\begin{align}
& \ddot{\bar{u}}_{a12} + \bar{a}_{12} \ddot{w} + \bar{b}_{12} \ddot{\vartheta}_a + \ddot{\vartheta}_a \bar{g} + \ddot{\vartheta}_a \bar{v}_a = -\vartheta_a \bar{g} \bar{v}_a - \bar{b}_{12} \ddot{\bar{u}}_a \bar{h}^2 \bar{v}_a, \\
& \ddot{\bar{u}}_{a12} + \bar{a}_{12} \ddot{w} + \bar{b}_{12} \ddot{\vartheta}_a + \ddot{\vartheta}_a \bar{g} + \ddot{\vartheta}_a \bar{v}_a = -\vartheta_a \bar{g} \bar{v}_a - \bar{b}_{12} \ddot{\bar{u}}_a \bar{h}^2 \bar{v}_a.
\end{align}
\]

It is a system of differential equations – two for macrorotations \( \vartheta_a \), one for macrodeflection \( w \) and two equations for fluctuation variables \( \vartheta_a \).

In the above equations the effect of the microstructure size is described by the underlined terms involving the microstructure parameter \( l \). Equations (7) or (8) have physical sense for unknowns \( w \), \( \vartheta_a \), \( \vartheta_a \), being slowly-varying functions in \( x \), for every \( t \). It is a certain a posteriori criterion of physical reliability for these models. Boundary conditions have to be formulated only for macrofunctions: macrodeflection \( w \) and macrorotations \( \vartheta_a \) on all edges. However, boundary conditions for fluctuation variables \( \vartheta_a \) and \( \vartheta_a \) can be defined only on edges \( x = 0, L \).

5. Remarks

The tolerance modelling used for governing equations of thin or medium thickness functionally graded plates with a microstructure makes it possible to pass from the equations with tolerance-periodic, non-continuous functional coefficients to equations with averaged, slowly-varying functional coefficients, taking into account the effect of the microstructure size.

References

Bending of thin rectangular polyethylene plate with ferrofluid in inhomogeneous magnetic field

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Abstract

The subject of the study is a rectangular plate made of polyethylene (PE). The plate consists of three layers: two external faces and internal core. The core of a cellular structure is filled with ferrofluid. Cells in internal layer prevent from the ferrofluid flow in the middle plane of a plate. The plate load is generated by inhomogeneous magnetic field changing in space. The magnetic field acts perpendicularly to the plate and along a middle plane, generated by the system of Helmholtz and Golay coils. These coils are based of modern MRI tomographs (Magnetic Resonance Imaging). Bending of the plate is caused by magnetic field acting on ferrofluid in porous cells. Bending of the plate is approximated by a bicubic spline function presented by normalized B-spline functions. Efficiency of proposed algorithms is proved comparing the test results with numerical simulations performed by FEM using the Autodesk Simulation v.14 program. Numerical algorithm determines bending of the plate under a current load induced by the variation of magnetic field.

Keywords: Rectangular plate, ferrofluid, Helmholtz coil, Golay coil, collocation method, B-spline functions

1. Introduction

Smart materials detect a wide spectrum of applications from industry to everyday object such as toys and sports equipment. The production and properties of smart materials is described in [3]. Ferrofluids are type of smart materials, their application in the form of smart plates was presented in [4,5], where dynamic equilibrium paths for the plate introduced into magnetic field was determined. Bending control of the plates is presented in [7,8]. In the paper bending of rectangular PE plate is described, using ferrofluid in cells as a smart material under magnetic field generated by coils used in MRI tomographs [1].

2. Rectangular plate under magnetic field

A rectangular plate model is based on a classical Kirchhoff-Love hypothesis.

![Plate with acting load and coordinate system.](image)

Bending of plate shown in Fig. 1 describes Eqn (1)

\[
\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} - \frac{1}{D} \left( q + n_x \frac{\partial^2 w}{\partial x^2} + n_y \frac{\partial^2 w}{\partial y^2} \right).
\]

where

\[
D = \frac{EH^3}{12(1-\nu^2)},
\]

- $E$ - Young's modulus,
- $\nu$ - Poisson's ratio,
- $h$ - thickness of the plate,
- $w = w(x,y)$ - bending deflection of the plate

with boundary conditions chosen from the following cases:

- clamped edge: \(w = 0, \frac{\partial w}{\partial n} = 0\),
- simply supported edge: \(w = 0, \frac{\partial^2 w}{\partial n^2} = 0\),
- free edge: \(\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial s^2} = 0, \frac{\partial^3 w}{\partial n \partial s} = 0\),

where \(n \in \{x, y\}\). A plate is made of PE, the cells in an internal layer are filled with ferrofluid.

2.1. Magnetic field generation

The magnetic field that acts on ferrofluid in plate is generated by two subsystems of coils- Helmholtz coil (HC) and Golay coil (GC). A Helmholtz coil consists of two loops of wire of radius \(r\), distance between loop equals \(r\). Golay coil is a saddle coil with the arcs of 120°. Both systems generate inhomogeneous magnetic field \(H\) that varies in space with a relative error.
 wherein $H_0$ is magnetic field in centre of coil system. Inhomogeneous $\eta$ of Helmholtz and Golay coil are shown in Fig. 2 and Fig. 3 respectively.

\[
\eta = 100 \left( \frac{H - H_0}{H_0} \right) \% \tag{2}
\]

Figure 2: Inhomogeneous of magnetic field for the Helmholtz coil.

\[
\left[ n_x, n_y, q \right] = \frac{1}{2} \mu_0 \chi (1 - \chi) \mathcal{Y} H^2 \tag{3}
\]

where $\mu_0$ - vacuum permeability,  
$\chi$ - magnetic susceptibility,  
$H$ - magnetic field.

Figure 3: Inhomogeneous of magnetic field for the Golay coil.

Magnetic field load acting on a ferrofluid in plate is

\[
w(x, y) = \sum_{i=-1}^{N+1} \sum_{j=-1}^{M+1} \psi_{i,j}(x) \tilde{B}_j(y), \tag{4}
\]

with unknown coefficients $\psi_{i,j}$.

Necessary approximations of partial derivatives appearing in equation of bending of the plate and in boundary conditions are obtained given, that a bicubic spline function is a polynomial spline function with respect to each of independent variables separately [5,6].

A system of algebraic equations for coefficient $\psi_{i,j}$ takes the form

\[
\sum_{r=-2}^{2} \sum_{s=-2}^{2} \psi_{i+r,j+s} q_{i+r,j+s} = b_{i,j}, (i = 1, \ldots, N - 1; j = 1, \ldots, M - 1). \tag{5}
\]

The rest of the equations can be obtained for boundary conditions using derivative schemes [5].

Figure 4: Scheme of bending of the plate with ferrofluid in magnetic field.

The efficiency of these algorithms is proved comparing the test results with numerical simulations performed by FEM using the Autodesk Simulation v.14 program. Bending of example plate obtained with proposed algorithm is presented in Fig. 4.

4. Conclusions

In the paper a model of the plate with ferrofluid in cells was introduced. Magnetic field load was generated by Helmholtz and Golay coils used in MRI tomographs. Solution of a plate at bending was obtained by a collocation method with the use bicubic spline functions. Prepared numerical algorithm determines bending of the plate under current load induced by means of changing the magnetic field.

References


Global and local elastic-plastic stability of FML columns of open and closed cross-section

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Abstract

The columns are built of rectangular flat plates. The material is GLARE 3 [5] with an even number of glass reinforced layers, the outer layers are always aluminum. The overall laminate is symmetric. The dimensions of structures are chosen in such a way that the stability loss occurs in the elastic-plastic range for aluminum layers. The problem of inelastic buckling is investigated using the analytical-numerical method prepared for the analysis of elastic stability of multi-layered thin-walled columns. The relationships between stresses and strains for a component elastic-plastic layer are derived on the basis of the J2-deformation theory of plasticity and/or J2-flow theory (incremental theory). On the other hand the same relations are written for an orthotropic elastic layer. Comparing the appropriate coefficients in both relations the instantaneous “conventional” parameters of orthotropy can be found out. Buckling stresses and buckling modes for the considered structures are estimated and presented in figures.

Keywords: buckling stress, elastic-plastic range, FML plates, deformation and incremental theories of plasticity

1. Introduction

A prismatic thin-walled composite structure built of plates connected along longitudinal edges was considered. The structure is simply supported at its ends. In order to account for all modes of global, local and coupled buckling, a plate model of a thin-walled structure was applied.

2. Method of solution

In the study, the classical laminate plate theory (CLPT) is employed to obtain the governing equilibrium equations of thin FML structures. The concept of postbuckling paths involves general asymptotic theory of stability. Among all versions of the general nonlinear theory, Koiter’s theory [3] of conservative systems is the most popular, owing to its general character and development. After Byskov and Hutchinson [1] formulated it in a convenient way. The theory is based on asymptotic expansions of the postbuckling path and is capable of considering nearly simultaneous buckling modes and is formulated only for elastic structures.

Differential equations of equilibrium were obtained from the variational method, taking into account: Lagrange’s description, full Green’s strain tensor for thin-walled plates and the second Piola-Kirchhoff’s stress tensor. The study is based on a numerical method of transition matrix using Godunov’s orthogonalization [4].

It is assumed that the material the structure is governed by Hooke’s law.

The consideration of displacements and load components in the middle surface of walls within the first order approximation, and led to precise geometrical relationships analysis of all possible buckling modes. The most important advantage of this method is that it allows to describe a complete range of behavior of thin-walled structures from all global (flexural, flexural–torsional, lateral, distortional buckling and their combinations) to the local stability, including a mixed buckling mode [4].

In order to obtain the equilibrium equations of individual plates for the asymptotic analytical-numerical method, the nonlinear theory of composite plates (CLPT) was modified.

In the paper the problem of stability in the elastic-plastic range of thin-walled FML structures is investigated using the method for the analysis of elastic stability of thin-walled composite structures. The relations between the stresses and strains for a component elastic-plastic plate are derived on the basis of deformation and incremental theory of plasticity for Ramberg-Osgood formula. On the other hand the same relations are formulated for an elastic composite plate. Comparing the appropriate coefficients in both relations the instantaneous conventional parameters of ‘elastic composite’ for layers corresponding each other of FML structures can be found out. Thus the problem of inelastic stability of FML structures can be investigated similarly to the problem of elastic composite structures.

The problem is solved numerically. For a given geometrical parameters, material constants of each FML layer and for the assumed number of half-waves the elastic buckling stress for the considered FML structure is calculated. If this value is greater than the proportional limit σ0 in the longitudinal direction the values of stresses σ in each plasticized layer are found out for a considered value of uniaxial strain ε in a whole structure and the next corresponding “conventional parameters of orthotropy” K11-K 33 (eq,1) of each considered layer are also found. Applying these parameters in each plasticized layer in the elastic solution for elastic composite structures a new value of the buckling stress is calculated. Further a method of secants is used to obtain the value of buckling elastic-plastic stress of the whole structure with the accuracy 0.01%. The proposed method allows to consider the transition of buckling mode together with the increase of loading as distinct from the usual assumption that the elastic-plastic buckling mode is analogical to the elastic one.

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Table 1: Material data [4]

<table>
<thead>
<tr>
<th></th>
<th>Elastic properties</th>
<th>Plastic properties (N=1.8- see eq.3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Young’s modulus</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>Aluminium layer</td>
<td>$E_a= 700 \text{ GPa}$</td>
<td>$\nu_{ax}=0.3$</td>
</tr>
<tr>
<td>Prepreg layer</td>
<td>$E_r=30.75 \text{ GPa}$</td>
<td>$\nu_{rr}=0.144 = \nu_{21}$</td>
</tr>
</tbody>
</table>

The solutions given here are valid in the cases of the uniform compression of the FML structure. Others types of loadings need further investigation.

Elastic range Inelastic range

$$\sigma_x = K_{11} \varepsilon_x + K_{12} \varepsilon_y, \quad \sigma_y = A_{11} \varepsilon_x + A_{12} \varepsilon_y, \quad \tau_{xy} = K_{33} \gamma_{xy}, \quad \tau_{xy} = A_{33} \gamma_{xy},$$

(1)

The coefficients $A_{11}$-$A_{33}$ (eq.1) determined on the basis of the $J_2$- deformation or $J_2$- flow theory of plasticity [2] depend on Young’s modulus, secant and tangent moduli for the considered material characteristics in the inelastic range.

The elastic-plastic stress-strain behaviour of an aluminum layer is described by a Ramberg-Osgood representation of a following type (see Table 1):

$$\varepsilon = \frac{(E - E_Y) \varepsilon}{1 + \left(\frac{(E - E_Y) \varepsilon}{\sigma_Y}\right)^N}, \quad \text{for } \sigma_0 \leq \sigma_Y$$

(2)

3. Calculation results

Columns under investigation are built of alternate aluminum sheets and unidirectional high-strength glass fiber layers. The outer layers are always aluminum so that the number of glass fiber layers is always one less. A total number of layers in the considered material equals 13 and a total wall thickness of a column is 4.3 mm.

For $m=1$ the values of global buckling stress estimated on the basis of both theories of plasticity nearly coincide equal to 240 MPa. Local buckling stresses and the corresponding buckling modes differ related to the applied theory: of plasticity $m=16$, $\sigma_{cr}=224.83$MPa (deformation theory of plasticity); $m=18$, $\sigma_{cr}=266.13$MPa (incremental theory of plasticity) or $m=15$, 232.83MPa (elastic theory).

![Figure 1: Plots of buckling stress versus number of half-waves $m$ for a column of a square cross-section (130x130mm). Length $l=2000$mm.](image1)

![Figure 2: Plots of of buckling stress versus number of half-waves $m$ for a column of a top hat cross-section with dimensions: 130x65x15mm. Length of a column equals $l=2000$mm. S denotes symmetric and A antisymmetric conditions imposed along symmetry axis of a column cross-section.](image2)

References

Dynamic buckling and dynamic progressive buckling of open-section columns

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Abstract

The paper is focused on the investigation into criteria, which indicate a transition from local buckling failure mode (progressive dynamic buckling) to the global bending, similar to Euler buckling, which takes place at certain critical conditions (buckling length and critical velocity). A theoretical study is carried out using the analytical-numerical method based on the asymptotic approach, which allows to determine buckling modes of a column subjected to impulse of compressive load (dynamic buckling), in order to predict failure modes at impact, crushing load. The results of theoretical analysis are compared with FE simulations (explicit dynamic analysis) taking into account both geometrical and physical non-linearity, and the results of impact tests performed on steel top hat section columns.

Keywords: thin-walled columns, open sections, dynamic buckling, axial impact

1. Introduction

Dynamic load acting on a thin-walled structure can be divided into two categories. The first one is a dynamic, impact (crushing) load, with accompanying perturbation propagation (a phenomenon that occurs with the sound wave propagation speed in the structure), leading to progressive, dynamic buckling (dynamic crushing). The second category is a dynamic pulse load, which results in the phenomenon of dynamic buckling.

The response of a thin-walled structural members of closed-sections subjected to impact, axial compressive load has been investigated extensively by many researchers [1]. However, some problems concerning crushing behavior of open section members have not been solved yet. Relatively long thin-walled columns (particularly of open sections) exhibit additional buckling modes, similar to the Euler buckling. Usually, a transition takes place at a certain “critical” instant of the crushing process [2]. Since buckling modes, similar to the Euler buckling, lead to poor energy absorption, the problem proves to be important for structural members acting as energy absorbers. Thus, threshold conditions between progressive local buckling and global bending still form an open and significant question.

Although the phenomenon of progressive dynamic buckling (crushing) and dynamic buckling due to the pulse load differ, however, a comparison of structural response in those two cases may bring a clearer understanding into the transition from one failure mode to another in the case of impact crushing load.

Thus, the aim of the presented research was to perform an analysis of the structural response of columns subjected to pulse load in order to predict failure modes under axial impact. The structural response to impact (crushing) was analysed on the basis of FE simulations (explicit dynamic analysis – Abaqus code) and the results of impact tests performed on steel top hat section columns [2].

In recent years research was carried out into the transition from progressive local buckling to global bending failure mode in thin-walled columns (bars) under axial impact.

In the case of circular tubes subjected to axial impact compressive load the problem was investigated e.g., by Alves and Karagiosova [3]. Teramoto and Alves [4] conducted research into the same problem for open, channel-section columns. They did not find clear condition of the transition from progressive local buckling to global bending.

Kotelko et al. in [2] performed tests on steel channel and top hat section columns. They also observed a transition mentioned above at channel section columns subjected to axial impact and relatively good agreement of limit lengths of columns with theoretical model, originally developed by Alves and Karagiosova [3]. However, they did not observe such a transition, even for long columns, in the case of top hat sections.

![Figure 1: Column under investigation](image)

<table>
<thead>
<tr>
<th>Symbol</th>
<th>a</th>
<th>b</th>
<th>w</th>
<th>L&lt;sub&gt;cr&lt;/sub&gt;E</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>60</td>
<td>60</td>
<td>15</td>
<td>351</td>
</tr>
<tr>
<td>B</td>
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<td>12</td>
<td>272</td>
</tr>
<tr>
<td>C</td>
<td>145</td>
<td>45</td>
<td>12</td>
<td>262</td>
</tr>
<tr>
<td>D</td>
<td>60</td>
<td>45</td>
<td>15</td>
<td>278</td>
</tr>
</tbody>
</table>

Figure 1: Column under investigation

2. Problem formulation

The subject of investigation was a thin-walled open top hat section column (Fig. 1) subjected to axial compressive load. Two cases: the dynamic pulse load resulting in dynamic buckling and impact (crushing) load, leading to progressive dynamic buckling were under investigation. The dimensions of sections allowed for a local buckling at static load.

The analysis was carried out for the section shown in Fig. 1 of a different length, starting from l = 200 to l = 500 [mm], with the interval 50 [mm]. Dimensions of sections tested and critical buckling lengths determined analytically for axial impact [2] are given in Fig. 1.

3. Dynamic buckling analysis

A dynamic buckling phenomenon occurs at dynamic load of a mean amplitude and a pulse duration comparable to the fundamental natural flexural vibration period or at quasi-static load of a low amplitude and a load pulse duration approximately twice as long as the period of fundamental natural vibrations. For the dynamic load, the effects of damping can be neglected in practice.
Long thin-walled prismatic beam-columns of the length \( \ell \), composed of plane, rectangular plate segments interconnected along longitudinal edges, simply supported at both ends, are considered. A plate model is adopted for the structures. Wave propagation effects were neglected, as it is done in the majority of works of dynamic stability. For the plate component, the precise geometrical non-linear relationships are assumed in order to consider both out-of-plane and in-plane bending of the plate [5].

The nonlinear problem of dynamic stability is solved with the asymptotic perturbation method. Let \( \lambda \) be a load factor. The displacement fields \( \hat{U} \) and the sectional force fields \( N \) (Koiter’s type expansion for the static buckling problem) are expanded into power series with respect to the dimensionless amplitude of the \( r \)-mode deflection \( \zeta_r \) (see [5]). If the structure contains geometric imperfections \( \hat{U} \) (only the linear initial imperfections determined by the shape of \( r \)-th buckling modes), where \( \hat{U} = \zeta_r \hat{U}_r \), then, the total potential energy can be written in the form [5]:

\[
\Pi = \frac{1}{2} \sigma(t) \pi_r \left[ \frac{1}{2} \sum_{s=1}^{s} \pi_s \zeta_s(t) \left( 1 - \frac{\sigma(t)}{\sigma_s} \right) + \frac{1}{3} \sum_{s=1}^{s} \sum_{r=1}^{r} \pi_{sr} \zeta_s(t) \zeta_r(t) + \sum_{r=1}^{r} \frac{1}{4} \sum_{s=1}^{s} \sum_{t=1}^{t} \pi_{sr} \zeta_s(t) \zeta_t(t) \right] + \sum_{r=1}^{r} \frac{\sigma(t)}{\sigma_r} \sum_{s=1}^{s} \pi_{sr} \zeta_s(t) \zeta_r(t) \right] + \frac{1}{2} \sum_{r=1}^{r} m_r \zeta_r(t) \left( t \right)
\]  

(1)

The nonlinear static stability (i.e., for \( \zeta_{s,lt} = 0 \) in Eqn (1)) of thin-walled structures in the first order approximation of Koiter’s theory (Lagrange equations) is solved with the modified analytical-numerical method (ANM) presented in [5]. The second order post-buckling coefficients are estimated with the semi-analytical method [6].

4. Dynamic progressive buckling – axial impact

Impact tests were carried out on steel specimens of different \( a/b \) ratio and different length from 250 to 500 [mm]. The wall thickness of all columns was \( t = 1 \text{ mm} \). The tests were carried out on the drop hammer rig with the initial impact velocity up to 10 m/s and impact energy up to 5 kJ. Selected results of the experiments were published in [2].

5. Final remarks

The comparative analysis of two different phenomena, namely, dynamic buckling under pulse load and dynamic crushing (impact) allowed the authors to formulate a hypothesis concerning the structural behavior of open section columns with edge stiffeners (top hat section). The columns, where the analysis of dynamic buckling under pulse load indicated a local, symmetric second buckling mode, displayed local progressive buckling at impact (Fig. 2), even if their length was greater than a critical one, determined from the theoretical rod model [1]. The longest top hat section columns showed the buckling modes in dynamic buckling under pulse load in agreement with the failure mode at impact (Fig. 3). It was a torsional – flexural mode. This hypothesis is however of a preliminary character and should be confirmed by a more detailed theoretical analysis and experimental tests.

References


Figure 2: Buckling and failure modes for B column: \( l = 350 \text{ mm} \), a) - impact test \( v_0 = 7.75 \text{ m/s} \), (local progressive buckling), b) – dynamic buckling modes for pulse load

Figure 3: Buckling and failure modes for D column: \( l = 400 \text{ mm} \), a) - impact test \( v_0 = 7.75 \text{ m/s} \), (local progressive/flexural – torsional), b) – dynamic buckling modes for pulse load (flexural-torsional)
Dynamic buckling simulation of single-layer graphene sheets by the molecular mechanics method

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Abstract
Dynamic deformation and buckling of single-layer graphene sheets (SLGSs) is simulated using the molecular mechanics method. Bonded interactions between carbon atoms are described by the DREIDING force field. Computer simulation was performed of dynamic deformation and buckling of compressed SLGSs. A dynamic buckling criterion for SLGSs was used, i.e., quasi-bifurcation points were found of the integral curves, to determine the critical times, loads, and buckling modes for a SLGS compressed at a constant rate by prescribed displacements on the opposite sides of the sheet and the critical times and buckling modes for a SLGS compressed by step forces of constant magnitude. At low compression rates of the sheet by prescribed atom displacements and under the action of step compression forces greater than a certain threshold value but, at the same time, smaller or slightly greater than the linear force $F_{cr}$ (the critical buckling force of the sheet under quasi-static deformation), a buckling mode occurs similar to the Euler buckling mode of an elastic plate with one half-wave along the free edges of the sheet. At high compression rates of the sheet and under the action of step compression forces much greater than $F_{cr}$, buckling modes occur with different numbers of half-waves along the free edges of the sheet.

Keywords: single-layer graphene sheet, molecular mechanics method, dynamic buckling, computer simulation

1. Introduction
Korobeynikov et al. [4] presented the use of molecular mechanics (MM) method to determine the critical forces, times, and buckling modes of nanostructures based on the stability theory of discrete elastic systems. They demonstrated effectiveness of the developed approach to the solution of problems of quasi-static deformation and buckling of compressed single-layer graphene sheets (SLGSs) (cf., [5]). However, the determination of critical times, buckling modes, and critical forces in dynamic buckling of SLGSs requires approaches different from the buckling simulation of SLGSs under quasi-static deformation. In [1] and others, dynamic buckling problems for compressed SLGSs were solved by direct integration of the molecular dynamics (MD) and MM equations. The SLGS buckling is initiated by perturbations introduced in the integration process randomly (due to the thermal motion of the atoms, computation errors, etc.) or by intentional onset of perturbations (atomic positions in the lattice, external forces, properties of potential interactions, etc.) in the problem solution. However, such approaches to the simulation of dynamic buckling of discrete elastic systems are not reliable enough (cf., [2]). Due to [2], it is preferred to use buckling criteria that the critical points of the integral curves and the buckling modes of these systems can be determined directly by the fundamental solution (without perturbations of parameters of the system). In the present study, the dynamic buckling criterion of discrete elastic systems based on the quasi-bifurcation points of the integral curves is used to determine the critical times, external forces, and buckling modes of SLGSs in the problems of buckling by the MM method over a wide range of deformation rates. Dynamic buckling of SLGS is analyzed for compression in two ways: by prescribed displacement of its edges and applying a step force to compress its edges.

2. Molecular mechanics equations for SLGS deformation and buckling
The vector equation of motion for a nanostructure gives (see, e.g., [3])

$M \ddot{U} + F(U) = R$ \hspace{1cm} $U(0) = U_0, \hspace{0.5cm} \dot{U}(0) = V_0.$ \hspace{1cm} (1)

Hereinafter, F and R are internal and external force vectors of the ensemble of nanostructure atoms, respectively; $U$, $U_0$, and $V_0$ are the vectors of displacements, initial displacements, and velocities of the nanostructure atoms, respectively; $M > 0$ is the diagonal mass matrix masses of the nanostructure atoms including; the dot above denotes time derivative.

In the MM method, equations (1) are solved numerically using Newmark’s implicit integration scheme. In the step-by-step integration of equations (1) by Newmark’s method, it is required to determine the symmetric tangential stiffness matrix of the nanostructure

$K \equiv \frac{\partial F(U)}{\partial U}. \hspace{1cm} (2)$

At each time step, we can solve the generalized eigenproblem

$(K - \mu M)\Phi = 0, \hspace{1cm} (3)$

and determine the eigenpairs $(\mu_i, \Phi_i)$ $(i = 1, \ldots, \text{NEQ})$ where $\text{NEQ}$ is the number of scalar equations in (1). According to [6, 2] (see also [4]), the time $t(= T_{cr})$ at which the equality $\mu_i = 0$ $(i = 1, \ldots, \text{NEQ})$ in (3) is first satisfied is

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critical, the points of the integral curves at which the matrix $K$
degenerates are called quasi-bifurcation points, and the eigenvectors $\Phi_i$
corresponding to $\mu_i = 0$ determine the buckling modes.

In the work we consider graphene elements characterized by N-body potentials (an element consists of N atoms), where N runs over 2, 3, and 4 in accordance with the DREIDING potential force field [7] for the bonded interatomic forces described using four types of potential energy: bond stretching (N = 2), bond angle bending (N = 3), bond torsion (N = 4), and bond inversion (N = 4). In addition, we take into account the nonbonded van der Waals interatomic forces.

3. Results of numerical simulation of dynamic deformation and buckling of compressed SLGS

![Snapshots of the deformed configurations of the sheet compressed by a linear step force](image)

Figure 1: Compressed SLGS and perturbing forces fitted to static buckling modes: a) #1, b) #2 c) #3, d) #11.

The problems of deformation and buckling of SLGSs were solved with the help of our PIONER code. A modified parameter set of the DREIDING force field is used to describe mechanical parameters of graphene more adequately than the standard parameter set of this force field presented in [7]. We consider a quadratic SLGS with two opposite constrained (clamped) edges loaded by prescribed displacements or forces and two opposite free edges (Fig. 1).

![Snapshots of the post-buckling equilibrium configurations of a compressed SLGS at a constant rate $U = 0.05$ nm/ps](image)

Figure 2: Snapshots of the post-buckling equilibrium configurations of a compressed SLGS at a constant rate $U = 0.05$ nm/ps of prescribed displacements of the edge atoms.

The SLGS consists of 1530 carbon atoms, whose sizes are close to 6 nm. The SLGS is loaded by prescribed displacements of the compressed atoms at the opposite edges of the sheet at different rates: 0.000005, 0.000005, 0.0005, 0.005, and 0.05 nm/ps or by step forces applied to the atoms at the sheet edges. It is found that at low rates of atomic displacements (0.000005, 0.000005, 0.0005, and 0.005 nm/ps), the sheet buckling mode is close to the Eulerian buckling mode of an elastic plate with one half-wave along the free edge of the sheet, but when the rates of displacement of the atoms at the edges are equal to 0.05 nm/ps, a buckling mode occurs which coincides with one of the higher buckling modes (buckling mode #11) of the same sheet in a quasi-static deformation. Snapshots of the deformed configurations of the sheet compressed by prescribed atomic displacements at rates of 0.05 nm/ps are given in Fig. 2. It can be seen that during the continuing compression of the sheet the higher buckling mode is transformed into the Eulerian buckling mode of an elastic plate with one half-wave along the free edge of the sheet.

In the case of step forces applied to the atoms of the sheet edge, two types of buckling modes occur, depending on the values of compression forces. The Eulerian buckling mode of an elastic plate with one half-wave along the free edge of the sheet is observed at a linear force $F < 2.5F_c$, where $F_c = 0.2421$ nN/nm is the critical linear compressive force in quasi-static deformation of the sheet, buckling modes similar to some buckling modes of the same sheet in quasi-static deformation are observed at a linear force $F > 2.5F_c$. Snapshots of the deformed configurations of the sheet compressed by a linear step force $F = 3.0F_c = 0.7263$ nN/nm are given in Fig. 3. It can be seen that due to this value of $F$, the higher Eulerian buckling mode of an elastic plate occurs, with two half-waves along the free edge of the sheet.

References


Analysis of sloping brace stiffness influence on stability and load bearing capacity of a truss

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Abstract

The paper is focused on the numerical study of stability and load bearing capacity of a truss with side elastic braces. The structure is made in reality. The rotational and sliding brace stiffnesses were taken into account. Linear buckling analysis and non-linear static analysis with geometric and material nonlinearity were performed for the beam and shell model of the truss with respect to the angle of sloping braces. As a result the buckling load and limit load in relation to the truss bracing stiffness was obtained. The threshold bracing condition necessary to provide maximum buckling resistance of the structure was proposed.

Keywords: truss buckling load, truss bearing capacity, brace stiffness

1. Introduction

Steel trusses have a much greater strength and stiffness in plane than out-of-plane therefore should be braced against lateral deflection and twisting. The problem of bracing requirements necessary to provide lateral stability of compressed members is present in a design code [7]. The out-of-plane buckling of trusses was studied in numerical research [2,4] or experimental and numerical analysis [5,6].

The research is focused on lateral buckling of truss with sloping linear elastic side supports. The sliding and rotational stiffness of braces were taken into account. A full bracing condition was investigated, defined as the bracing stiffness necessary to obtain the maximum buckling load (linear buckling analysis - LBA) or limit load (non-linear static analysis - GMNIA) of the truss.

2. Description of the truss

In the present research the model of a steel truss (Fig. 1) braced by four elastic side supports was taken into consideration. The truss length was \(L = 28.0\) m and the height was \(h = 1.8\) m. The distance between the braces was 5.6 m. The top chord consist of 2 x L160 x 15 rolled profiles, and the bottom chord of 2 x L150 x 15. The diagonals and battens (at the top chord) were made of C140 profile except three most compressed members near the supports (at each side) which were made of 2 x L90 x 10 rolled profiles. The yield strength of the truss steel was 235 MPa.

In numerical analysis it was assumed that the load is applied in the form of point forces at four top chord braced joints. The truss was modelled by beam and shell elements using software [1] or [3]. The beam model of the truss was made by means of standard 1D elements with 6 degrees of freedom at node. Each member of top and bottom chord (each angle bar) was modelled separately. In the shell model (3D), 4-node shell elements QUAD4 (with 6 degrees of freedom in node) were used (Fig. 1b). The truss members were connected by means of RIGID links. The lateral sloping supports (sloped at angles \(\alpha = 0^\circ, 15^\circ, 30^\circ, 45^\circ\) measured from the horizontal line) were modelled by means of ROD elements of axial stiffness only. The DOF-spring elements were used as the elastic supports with rotational stiffness.

3. Numerical analysis results

Linear buckling analysis confirmed that buckling load depended on brace stiffness and an angle \(\alpha\) (Fig. 2). There was a threshold (minimum) stiffness \(k_s\ [kN/m]\) – sliding stiffness and \(k_{rot}\ [kNm/deg]\) – rotational stiffness of braces which ensures that buckling load does not increase (or the load increase is less than 5%) (Table 1).

![Figure 1: Truss binder with sloping braces: a) roof construction, b) FEM model – bottom chord joint detail, c) static schema](image)

![Figure 2: Comparison between the buckling load of the truss with respect to the stiffness of braces for beam (1D) and shell (3D) model](image)
The threshold rotational stiffness \( k_{rot-lim} \) was found for the models with sliding stiffness equal to \( k_{sl-lim} \). The buckled shape of the truss was presented in Fig. 3. For the beam models with rigid braces (sliding stiffness \( k_s = 10^6 \) kN/m) the buckled form of the structure was the same as presented in Fig. 3a for \( \alpha = 0^\circ \) and Fig. 3b for \( \alpha = 45^\circ \). The reason for large differences between magnitudes of buckling load for 1D and 3D models (up to 30%) might be the built-up cross section layout of the truss members (influence of battens).

Variants of non-linear static analysis (shell model) with material and geometric non-linearity for the imperfect truss model were carried out. The initial imperfection was implemented in the form of arc curvature at the top chord with an amplitude \( L/500 \) according to the code requirements [7]. The results for the models with braces of sliding stiffness equal to \( 10^6 \) kN/m are presented in Fig. 4. For the truss with horizontal side supports the rotational stiffness had no influence on the magnitude of limit load. In the case of truss with braces situated at the angle equal to \( 45^\circ \) the magnitude of limit load increased about 5% due to the increase of brace rotational stiffness.

### Table 1: Threshold stiffness of the braces \( k_{sl-lim} \) [kN/m], \( k_{rot-lim} \) [kN/m/deg], \( P_{cr} \) [kN]

<table>
<thead>
<tr>
<th>Beam Model</th>
<th>Shell Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha = 0^\circ )</td>
<td>( \alpha = 15^\circ )</td>
</tr>
<tr>
<td>( k_{sl-lim} )</td>
<td>( k_{rot-lim} )</td>
</tr>
<tr>
<td>12000</td>
<td>12000</td>
</tr>
<tr>
<td>( P_{cr} = 4704 )</td>
<td>( P_{cr} = 4756 )</td>
</tr>
</tbody>
</table>

The threshold rotational stiffness \( k_{rot-lim} \) was overestimated for the models with sliding stiffness equal to \( k_{sl-lim} \). The buckled shape of the truss was presented in Fig. 3. For the beam models with rigid braces (sliding stiffness \( k_s = 10^6 \) kN/m) the buckled form of the structure was the same as presented in Fig. 3a for \( \alpha = 0^\circ \) and Fig. 3b for \( \alpha = 45^\circ \). The reason for large differences between magnitudes of buckling load for 1D and 3D models (up to 30%) might be the built-up cross section layout of the truss members (influence of battens).

## 4. Conclusions

The linear buckling analysis results confirmed that buckling load depends on sliding and rotational brace stiffness and angle. The threshold sliding stiffness of braces rises with the increase of brace angle (except \( \alpha = 0^\circ - 3D \) model).

The threshold rotational stiffness of braces decreases with the increase of brace angle. In all cases (LBA 1D and 3D model) implementation of the rotational brace stiffness resulted in the increase of buckling load.

For the shell model of the structure with rigid braces, the top chord of the truss buckled locally in the truss plane (bending and torsion of the top chord cross section). In this case the influence of battens (neglected in 1D beam model) on stiffness of the built-up cross section was very important.

The results obtained from non-linear static analysis for the shell model of the truss showed that rotational stiffness of braces may cause an increase in bearing capacity (up to 5%).

## References

Cylindrical shells with one longitudinal cut under different conditions of axial compression

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Abstract

The numerical buckling analysis of axially compressed elastic thin-walled cylinders with one longitudinal cut is realized in ANSYS software. The research studies two buckling solutions: 1) geometrically linear buckling solution for computation of eigenvalues; 2) geometrically nonlinear stress-strain state solution for evaluation of limit loads. Simulations are performed for five loading schemes which represent the loading nature (force loading – schemes 1, 3, 5 or kinematical loading – schemes 2, 4) and three different conditions of applying axial compression (with out-of-plane edge rotation – schemes 1, 2; with parallel edge displacement – schemes 3, 4; with out-of-plane edge displacements – scheme 5).

Keywords: cylindrical shell, longitudinal cut, axial compression, loading schemes, numerical simulation

1. Introduction

The paper [1] was focused on the numerical analysis of deformation and buckling of axially compressed elastic circular cylindrical shells under local quasi-static actions in ANSYS software. The research dealt with five different schemes of longitudinal compression (Fig. 1). These schemes reflected the nature of an axial loading: 1) force loading performed by application of axial forces (schemes 1, 3 and 5), 2) kinematic loading represented as axial displacements (schemes 2 and 4). Besides, there were realized three different conditions of applying an axial compression: 1) with out-of-plane edge rotations (schemes 1, 2); 2) with full restriction of edge rotations (schemes 3, 4); 3) with free out-of-plane edge rotations and displacements (scheme 5).

Results of linear buckling analysis of axially compressed cylindrical shells without external local actions showed that critical loads found for five considered loading schemes were almost identical (a difference was noted in the third-fourth significant digit). Similar results were obtained for geometrically nonlinear buckling analysis. On the other hand, local quasi-static actions changed buckling behaviour of axially compressed cylinders and influenced the values of buckling loads for different loading conditions. The differences consisted not only in the values of loads, but also in qualitative features of shell behaviour. Note that local quasi-static actions generated a strong non-uniform in circumferential and longitudinal directions stress-strain state. Such a type of non-uniform pre-stress-strain state obviously caused a high sensitivity of buckling loads to the loading conditions. This fact was confirmed by the results of numerical buckling analyses of cylindrical shells with one longitudinal or one transversal cut axially compressed according to the schemes 4 and 5 [2,3].

The aim of the work is to perform linear and geometrically nonlinear buckling analyses of elastic circular cylindrical shell with one longitudinal cut of different lengths for possible conditions of applying an axial compression (according to five considered loading schemes).

2. Numerical simulations

Numerical analysis is performed on 3-D FE models of half a shell due to the plane of symmetry in the middle of the shell length. The material is assumed to be linear and isotropic with the following mechanical and geometrical parameters: modulus of elasticity $E = 191$ MPa, Poisson’s ratio $\nu = 0.3$; diameter $2R = 183$ mm, thickness $h = 0.305$ mm ($R/h = 300$), $L = 192$ mm ($L/R = 2.1$); cut length $l = (0\div0.8)L$. FE models also consider elastic isotropic models of rigid disks ($E=2\times10^{11}$ Pa, $\nu=0.3$; $2R = 183$ mm, $H = 4$ mm) for the loading schemes 1, 2, 3. Shells and disks mesh is generated by standard ANSYS elements SHELL181 and SOLID185, respectively. A total number of elements varies between 15,500 and 16,900, including shell FE – between 6,200 and 7,600 depending on the cut length.

The boundary conditions correspond to the hinged support with tangential and radial displacements restrained on the top of the shell (schemes 3, 4, 5) and also on the external surfaces of rigid disks (schemes 1, 2, 3). At the plane of symmetry, there are symmetric boundary conditions.

An axial compression is applied as longitudinal concentrated forces (schemes 1, 3) or displacements (scheme 2) to the external surfaces of rigid disks and as uniform longitudinal forces or displacements (schemes 4 and 5, respectively) distributed on the edges.

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3. Discussion of results

Figure 2 presents dependences \( N' - l' \) for five loading schemes demonstrating numerical results of linear and geometrically nonlinear buckling analyses. Here, \( N' = N/\sqrt{\pi} \), and \( N = 2\pi G t^2 / \sqrt{3(1-\nu^2)} \) corresponds to the classical value of the critical axial compressive force.

![Dependences N' - l'] for different loading schemes](image)

3.1. Linear solutions

Linear buckling analysis is represented by smooth curves \( N' = N/\sqrt{\pi} \). Critical loads \( N' \) drastically descend in the region of small cuts \( l<0.2 \). Then \( N' \) go down slowly with subsequent stabilization at the level less than 0.2\( N' \) near \( l=0.8 \).

The results of linear solutions found for fives considered loading schemes are almost the same. Differences can be observed in the zones of \( l=0 \) and \( l=L \). When \( l \leq \frac{L}{2} \), critical loads \( N' \) decrease to the level of \( (0.36-0.44) N' \). However, this decrease of loads \( N' \) is deeper for the schemes 1, 2, 5 which allow out-of-plane rotations of the shell edge.

When the cuts are large enough \( l>0.2 \), the dependence \( N' \) is essentially determined by loading conditions. The scheme 5 is the most dangerous because buckling loads turn out to be the lowest for any cut length. For the largest cut lengths \( l \) limit loads are higher than the ones in the cases of possible out-of-plane rotations (schemes 1, 2).

3.2. Nonlinear solutions

The comparison of the results of geometrically linear and nonlinear analyses shows that critical loads are higher than limit loads \( (N'') \) for the small cuts. In this region the bearing capacity of the shells depends on initial imperfections magnitudes. For the rest of cut lengths linear solutions become lower than nonlinear solutions.

In the region of small cuts \( l<0.2 \), limit loads \( N'' \) decrease to the level of \( (0.36-0.44) N'' \). However, this decrease of loads \( N'' \) is deeper for the schemes 1, 2, 5 which allow out-of-plane rotations of the shell edge.

In the region of \( l>0.2 \), limit loads are almost constant about 0.41\( N' \) (schemes 1, 2) and 0.43\( N' \) (schemes 3, 4). If a cut length \( l>0.4 \) cutouts grow to the level of 0.46\( N' \) (schemes 1, 2) and 0.50\( N' \) (schemes 3, 4). Notice that limit loads coincide for loading schemes 3 and 4, and for loading schemes 1 and 2 they are similar. Thus, the nature of loading (force or kinematic loading) has no influence on the limit loads, but restrictions of out-of-plane edge rotations lead to their increase up to 10 %.

4. Conclusions

The buckling problem of axially compressed elastic cylindrical shells with a single longitudinal cut is discussed for five different conditions (schemes) of load application. In all cases for the cuts \( l>0.2 \), critical loads of linear buckling solutions are less than limit loads of geometrically nonlinear analyses. Critical loads are close for considered loading schemes, and they can be described with smooth curves that drop for small cuts and stabilize at 0.19\( N' \) for large cuts \( l=L \). A similar behaviour of nonlinear buckling analysis is observed for the scheme 5. A cut increase first leads to a decrease of limit loads and then to their slight augmentation for the loading schemes 1-4. Though, the nature of loading (either force or kinematic compression) hardly influences the values of limit loads. In the case of parallel edge displacements (schemes 3, 4) limit loads are higher than the ones in the cases of possible out-of-plane rotations (schemes 1, 2).

References


Dynamics of a rotating thin-walled composite beam mounted on in-plane moving hub

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Abstract

The paper discusses dynamics of a cantilever, thin walled composite beam built into a rigid hub performing rotation and in-plane translation. Mathematical model of flexible structure considers numerous non-classical effects, e.g. material anisotropy, cross-sectional rotatory inertia, transverse shear deformation and both primary and secondary cross-section warping. Governing equations of the system are derived applying the Hamilton’s principle. It is shown that they are mutually coupled, forming a nonlinear system of PDEs.

A detailed study on the system dynamics is performed for a specific case of a thin-walled box-beam of a closed cross-section made of circumferentially asymmetric stiffness laminate. This material configuration results in the system of governing equations of the eighth order exhibiting twist/in-plane bending/in-plane transverse shear motion coupling. The impact of modes coupling and its magnitude resulting from filament orientation on the system performance is discussed. The effects of variation of selected parameters of the system such as the hub rotational speed, the base radius, the hub/beam inertia ratio are investigated too.

Keywords: rotating structure, vibrations of a composite beam, Timoshenko beam model, base movement

1. Introduction

Rotating beams are often models to study dynamic properties and behaviour of rotating blades in turbomachinery, helicopter rotor blades, flexible links of robotic manipulators or lightweight satellite structure appendages. The primary interest is usually aimed at planar bending of rotating links, often coupled to torsion of shafted systems. Studies performed within aircraft dynamics domain are also focused on the behaviour of tapered, twisted and prestretched rotating blades. In all mentioned areas of possible engineering applications a frequent case is the in-plane base excitation in the form of a translational acceleration affecting the overall performance of the rotating structure, – see numerous examples in literature e.g. [1, 4, 5, 8, 9].

Vast majority of the research within the discussed field deals with structures made of isotropic materials. General formulations to analyse the dynamics of flexible, isotropic bodies subjected to coupled rigid-body translation and rotation were formulated by e.g. Haug et al. [3], Banerjee and Dickens [1] and Zhang et al. [10], both in linear and nonlinear regime. Later review articles summarizing current state-of-the-art achievements were published – e.g. Mayo et al. [7].

The papers dealing with dynamic properties of rotating, slender members fabricated of fiber reinforced laminates and subjected to in-plane base translation are not common; some aspects were reported by Madenci and Barut [6].

In order to address the situation the paper discusses dynamics of thin-walled composite beam built into the rigid hub performing rotation and in-plane translation. The mathematical model of a flexible system takes into account non-classical effects like material anisotropy, cross-sectional rotatory inertia, transverse shear deformation and both primary and secondary cross-section warping. The paper extents former studies of the authors, related to dynamics of rotating, thin walled composite beams [2].

2. Problem formulation

Let us consider a slender, straight and elastic composite thin-walled beam clamped at the rigid hub of radius $R_0$ experiencing rotational motion as well as translational one; see Figure 1. In order to describe the motion of the structure a hybrid co-ordinate formulation of flexible system is introduced i.e. displacements of the body are partitioned into summands due to large overall motions and displacements at elastic deformations. Multiple reference frames are established here. A general one is a fixed, inertial Cartesian frame of reference $(X_S, Y_S, Z_S)$, the next is a Cartesian coordinate system $(X_0, Y_0, Z_0)$ attached at the center of the hub $O$, experiencing translation along the inertial axis $(SX_S)$; its position at any time instant is given by a variable $h(t)$. Moreover, a floating frame $(X_1, Y_1, Z_1)$ rotating with an arbitrary angular velocity $\psi(t) = d\psi(t)/dt$ about $Z_1 = Z_0$ axis is given. In order to describe the deformation of a cantilever beam a local frame $(x, y, z)$ of reference is defined. The origin $o$ of the $(x, y, z)$ coordinates is set on the clamping point and coincides with the center of the beam cross-section. A presetting (pitch angle) of the beam is given by $\theta$ constant.

Figure 1: A rotating thin-walled beam with in-plane base translation

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2.1. Assumptions

As given in the introductory part of ‘Problem formulation’ section the hub shaft coincides with \((OZ_b)\) axis and is allowed to move along \((SX_b)\) axis only. Besides, while translating it remains parallel to the inertial frame axis \((SZ_o)\) all the time.

Regarding possible deformation of the beam the original shape of the cross-section is maintained in its plane, but is allowed to warp out of plane. Moreover, since the specimen is made of composite material, transverse beam shear deformations \(\psi\) are taken into account. Further assumptions concerning system kinematics are given in detail in the former paper [2] of the authors.

For the convenience of the presented analysis the composite material is assumed linearly elastic and uniform spanwise. It is also assumed that the perimeter arrangement of reinforcing fibers follows the circumferentially asymmetric stiffness (CAS) configuration scheme. This setting allows to decouple the full set of equations of motion (6 d.o.f.) into two sub-systems: one exhibiting twisting/bending/shear coupling and the second, where axial and second bending/shear modes are coupled.

The equations of motion and boundary conditions of a rotating beam are derived according to the extended Hamilton’s principle of the least action:

\[
\delta J = \int_{t_1}^{t_2} (\delta T - \delta U) dt = 0
\]

where \(J\) is the action, \(T\) is the kinetic energy, \(U\) is the potential energy. Full derivation of the equations of motion for a complete model including both transversal/lateral bending directions, shear deformations and primary and secondary warping effect, as well as arbitrary presetting angle and non-constant rotational speed can be found in [2]. A simplified system of equations is presented here, the presetting angle \(\theta\) is fixed at \(\pi/2\). Moreover, warping restraint is neglected in further analysis.

3. Governing equations

Accounting for the simplifications given above we arrive at the following equations expressed in terms of unknown kinematic variables of the cross-section reference point located at \(\alpha\) \((\alpha)\) axis:

- \(w_o\) in plane displacement

\[
b_1 \ddot{w}_o - 2b_2 \dot{w}_o \psi(t) - b_1 w_o \dot{\psi}^2(t) - b_1 (R_0 + x + u_o) \dot{\psi}(t) + \nonumber\]

\[
+ b_1 (R_0 + x + u_o) \dot{\psi}(t) + b_1 L \sin \psi(t) = a_{55} \ddot{\varphi}_y - a_{55} \dot{w}_o'' - (T_w \dot{w}_o)' = 0
\]

with boundary conditions,

\[
w_o|_{x=0} = 0, \quad (\dot{\varphi}_y + \dot{w}_o')|_{x=1} = 0
\]

- \(\varphi_y\) in plane transverse shear angle

\[
B_4 \ddot{\varphi}_y - B_4 \dot{\psi}^2(t) \ddot{\varphi}_y + B_4 \dot{\psi}(t) \ddot{\varphi}_y + \nonumber\]

\[
+ a_{55} \dot{\varphi}_y + a_{55} \ddot{w}_o'' - a_{37} \ddot{\varphi}_y = 0
\]

with boundary conditions,

\[
\ddot{\varphi}_y|_{x=0} = 0, \quad \left( a_{33} \ddot{\varphi}_y + a_{37} \varphi'' \right)|_{x=1} = 0
\]

- \(\varphi\) twist angle

\[
(B_4 + B_5) \ddot{\varphi} + (B_4 - B_5) \dot{\psi}^2(t) \ddot{\varphi} + \nonumber\]

\[
- a_{37} \ddot{\varphi}_y - a_{37} \varphi'' - (T_r \varphi')' = 0
\]

with boundary conditions,

\[
\varphi'|_{x=0} = 0, \quad \left( a_{37} \ddot{\varphi}_y + a_{37} \varphi'' \right)|_{x=1} = 0
\]

In foregoing relations \(B_r\) and \(b_i\) factors depict the inertia terms and \(a_{ij}\) ones correspond to beam stiffnesses. Their detailed definitions are given in [2]. Term \(T_r(x)\) is defined as

\[
T_r(x) = b_1 (L - x) \left\{ \dot{\psi}^2(t) \left[ R_0 + \frac{1}{2} (L + x) - \bar{h} \cos \psi(t) \right] \right\}
\]

and it corresponds to systems stiffening/softening resulting from transportation motions, while \(T_w(x)\) is

\[
T_w(x) = \frac{B_4 + B_5}{m_0 \beta} - T_r(x)
\]

where \(m_0\) is mass of the beam per its unit length and \(\beta\) is a perimeter of the cross-section.

4. Results

Equations of motion of a thin-walled composite beam subjected to translational and rotational transportation motions are derived. The resulting PDEs are coupled and incorporate the mutual influence of elastic and rigid body motions. Assuming the constant translational acceleration a strong nonlinearity of system emerges. Thus, in order to get the solution of the problem nonlinear methods together with bifurcation and stability analysis have to be applied.

References


Free vibrations of multi-layered beams with fractional-derivative viscoelastic layers

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Abstract

The paper concerns dynamic analysis of composite beams containing elastic and viscoelastic (VE) layers. A method to determine of the dynamic characteristics of multi-layered beams with VE layers (i.e. natural frequencies, non-dimensional damping ratios and modes of vibration) is presented. The Euler beam theory and the Timoshenko theory are used to describe the elastic and VE layers, respectively. In order to determine mechanical properties of a VE layer, the four-parameter rheological model with a fractional derivative is applied. The virtual work principle and the finite element method together with the Laplace transform are used to derive the equation of motion in the frequency domain. The dynamic characteristics of a beam with VE layers are obtained as a solution to a properly defined nonlinear eigenvalue problem. The continuation method is adopted for solving the nonlinear eigenproblem. Several conclusions concerning the validity of results are presented on the basis of numerical studies.

Keywords: beams with viscoelastic layers, fractional derivatives, nonlinear eigenvalue problem, dynamic characteristics

1. Introduction

Viscoelastic (VE) layered composites are structures which consist of VE layers sandwiched between elastic ones. The composite structures are widely used for dissipating energy in structures, to reduce vibrations, noise and fatigue failure. The effectiveness of viscoelastic damping materials in the form of constrained layers has been recognized for many years. Different approaches to analyzing the dynamic behavior of beams with VE layers were applied. Early investigations were presented in [1,2] where analytical techniques were used. Numerical methods are presented in [3,4].

Viscoelastic materials have complex characteristics which depend on many parameters, excitation frequencies and temperature are the most important. Several models were proposed. The complex modulus model is used in [5] while the generalized Maxwell model and the Biot model are used in [3,6]. In [7,8] the Anelastic Displacement Field model and/or the Golla-Hughes-McTavish model is adopted. The fractional derivative rheological model is used to describe VE layers in [4]. This model is able to correctly describe the properties of VE materials using a small number of parameters, while classical VE models require a greater number.

In the paper the solution in frequency domain is presented. The formulation utilizes the fractional Zener model for the description of VE layers because the Zener model is a simple one which takes into account all rheological properties of VE materials. The objective of the paper is to develop a method for determination of dynamic characteristics of multiple-layered beams containing VE layers. The virtual work principle and the Laplace transform are used in the formulation. The dynamic characteristics are obtained solving the nonlinear eigenvalue problem. The results of calculations are presented and discussed.

2. Finite element formulation in frequency domain

The multilayered finite element of a beam with elastic and VE layers is considered. The number of layers is optional but each VE layer must be constrained by two elastic ones. The material of each layer is isotropic and homogeneous. All inertia forces are taken into account. The Euler theory is applied for a description of the elastic layers. The VE layers are described using the Timoshenko theory. Damping in elastic layers is omitted. Physical properties of the VE layer are represented by a four-parameter fractional derivative Zener model.

2.1. Description of elastic layer

Each elastic layer is modeled as a two node element shown in Fig. 1. Horizontal displacements are approximated by linear functions while the Hermite’s polynomials are used to approximate vertical ones. The Hooke’s law is used to describe the elastic properties of layer.

Figure 1: Elastic layer as a beam finite element

The main relations for this layer are written in the form:

\[ u_i(x,t) = H_i(x) q_i(t), \quad e_i(x,t) = B_i(x) q_i(t), \]

\[ \sigma_i(x,t) = E_i e_i(x,t), \]

where \( u_i(x,t) = \text{col}(u_i, w_i, \phi_1, \phi_2) \) is the vector of displacements, \( q_i(t) = \text{col}(u_{i1}, u_{i2}, w_1, w_2, \phi_1, \phi_2) \) is the vector of nodal parameters, \( H_i(x) \) is the matrix of shape functions, \( e_i(x,t) = \text{col}(e_1, e_2, e_3(x,t)) \) is the vector of generalized strains, \( \sigma_i(x,t) = \text{col}(N_i(x,t), M_i(x,t)) \) is the vector of generalized stresses, \( E_i \) is the strain matrix and \( E_i = \text{diag}(E_{A1}, E_{L1}) \) is the matrix of elastic constants.

2.2. Description of viscoelastic layer

The Timoshenko theory is used to describe VE layers. Each VE layer is a sandwich between two elastic layers what means that the horizontal displacement of the top fibre of i-th VE layer is equal to that of the bottom fibre of the upper elastic layer. Similarly, the horizontal displacement of the bottom fibre of the VE layer is equal to the top fibre of the bottom elastic layer.

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Taking these assumptions into account we describe displacements of VE layers using nodal parameters of elastic layers. For VE layer relations (1) and (2) are also valid but with different definitions of appearing quantities, namely:

\[ \mathbf{q}_i(t) = [u_{1i}(t), u_{2i}(t), u_{3i}(t), \phi_i(t)] \]

\[ \mathbf{e}_i(t) = [\gamma_i(t), \gamma_i(t)] \]  (3)

The explicit forms of \( \mathbf{B}_i(x) \) and \( \mathbf{H}_i(x) \) matrices are omitted.

In terms of internal forces of layer (i.e. \( N_i(x,t), M_i(x,t), T_i(x,t) \) and generalized strains of layer (i.e. \( \varepsilon_i(x,t), \kappa_i(x,t), \gamma_i(x,t) \)), the following constitutive equations are:

\[ N_i(x,t) + \tau_{ii} \alpha_i M_i(x,t) = E_0 \varepsilon_i(x,t) + E_{ii} \varepsilon_i(x,t), \]

\[ T_i(x,t) + \tau_{ii} \alpha_i M_i(x,t) = G_{0i} \kappa_i(x,t) + G_{ii} \kappa_i(x,t), \]

\[ M_i(x,t) + \tau_{ii} \alpha_i M_i(x,t) = E_{0i} \gamma_i(x,t) + E_{ii} \gamma_i(x,t), \]  (4)

where \( \alpha_i \) is the order of fractional derivative and \( \tau \) is the model parameter.

After applying the Laplace transform the constitutive equations could be rewritten in the following matrix form:

\[ \mathbf{q}_i(s) = \left[ E_{0i} + \frac{s^\alpha \tau_{ii} \alpha_i}{1 + \tau_{ii} \alpha_i} (E_{ai} - E_{0i}) \right] \mathbf{e}_i(s), \quad \text{(5)} \]

where \( s \) is the Laplace variable, \( \mathbf{q}_i(s), \mathbf{e}_i(s) \) are Laplace transforms of \( \mathbf{q}_i(t), \mathbf{e}_i(t) \). Moreover, \( E_{0i} = \text{diag}(E_{0i}, A_i, E_{0i} I, G_{0i} A_i) \), \( E_{ai} = \text{diag}(E_{ai}, A_i, E_{ai} I, G_{ai} A_i) \).  (6)

2.3. Virtual work equation

The virtual work equation for the finite element of multilayered beam performing free vibration is in the form:

\[ \sum_i \int_0^L \left( \mathbf{K}_{ei}(x) \mathbf{e}_i(x, t) dx + \sum_i \int_0^L \mathbf{K}_{ei}(x) \mathbf{e}_i(x, t) dx = 0, \quad (7) \]

where \( \mathbf{K}_{ei}(x) \) and \( \mathbf{K}_{ei}(x) \) are the vectors of virtual displacements and virtual strains of the \( i \)-th layer.

Applying the Laplace transform and using the element finite procedure Eqn (7) could be rewritten in the form:

\[ \mathbf{K}_{ei}(s) \mathbf{e}_i(s) = 0, \quad \text{(8)} \]

where \( \mathbf{M}_i \) and \( \mathbf{K}_i \) are mass and elastic stiffness matrices. \( \mathbf{K}_{ei}(s) \) is the VE stiffness matrix obtained from

\[ \mathbf{K}_{ei}(s) = \sum_i \frac{\tau_{ii} \alpha_i}{1 + \tau_{ii} \alpha_i} \mathbf{K}_{ei}, \quad \mathbf{K}_{ei} = \mathbf{B}_i^T (E_{ai} - E_{0i}) \mathbf{B}_i \]  (9)

and the summation is over all VE layers of the finite element. The elastic stiffness matrix is assembled as by adding the stiffness matrices of all the layers and using the matrix \( \mathbf{K}_i \) for the elastic layer or the matrix \( \mathbf{K}_{ei} \) for VE layer, i.e.:

\[ \mathbf{K}_i = \mathbf{B}_i^T (x) \mathbf{E}_i \mathbf{B}_i (x) dx, \quad \mathbf{K}_{ei} = \mathbf{B}_i^T (x) \mathbf{E}_{ai} \mathbf{B}_i (x) dx \]  (10)

Finally, after performing the finite element procedure for a whole beam, we obtain the following nonlinear eigenvalue problem:

\[ (s^\alpha \mathbf{M} + \mathbf{K}_i(s)) \mathbf{q}_i(s) = 0, \quad (11) \]

where \( \mathbf{q}_i(s) \) is the global vector of the Laplace transform of displacements. The matrices \( \mathbf{M}, \mathbf{K}, \mathbf{K}_i \) are built as usual.

The eigenvalue problem (11) is solved with the help of continuation method in order to obtain the eigenvalues \( s = \mu + i\eta \) and eigenvectors \( \mathbf{q}_i \) which all are complex and conjugate numbers and vectors. The natural frequency \( \omega \) and nondimensional damping ratio \( \gamma \) are defined by \( \omega^2 = \mu^2 + \eta^2 \), \( \gamma = -\mu/\omega \).

3. Results of calculation

The influence of fractional order derivative \( \alpha \) on nondimensional damping ratios is shown in Fig. 2 for the fixed-pinned sandwich beam. The non-dimensional damping ratios for higher modes of vibration are smaller than those for the lower ones for values of \( \alpha \) up to approximately 0.8. The opposite conclusion can be formulated for \( \alpha \approx 1 \).

Figure 2: Non-dimensional damping ratios vs. parameter \( \alpha \)

References

Soil-structure interaction of cylindrical water tanks with linearly varying wall thickness

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Abstract

An effective method of the analysis of soil-structure interaction including behaviour of cylindrical storage tank with a varying wall thickness is presented. An elastic half space and a Winkler foundation model were used for the description of subsoil. The soil-structure interaction is considered using the power series. A computational example of reinforced concrete (RC) tank is given. The analysis performed for the model incorporating an elastic half space shows the decrease of radial displacements as well as substantial changes in the distribution of bending moments compared to Winkler foundation. Additionally, local increase of subsoil reaction around the slab circumference is observed for the case of elastic half space, in contrast to Winkler model.

Keywords: cylindrical shell, linearly varying wall thickness, ground slab, elastic half space, Winkler model

1. Introduction

In the paper effective method of the analysis of soil-structure interaction is presented including the behaviour of cylindrical storage tank with varying wall thickness. Elastic half space and Winkler foundation are used for the description of subsoil. The well-known Gorbunov-Posadov method, based on Borowicka model [1], employed herein for the analysis of the ground slab of the cylindrical tank structure, enables the description of subsoil as a perfectly elastic half space. This approach gives the advantage of being relatively simple and exact when compared to analysis methods based on different representation of soil-structure interaction. However, in the available literature, this method has not been widely used. Alternative approaches require using Hankel transforms and Bessel function series (Hemsley [2]) or numerical methods (Melerski [3], Horvath & Colasanti [3]), leading to complicated algorithms or approximate results. Kukreti & Siddiqi [4] use quadrature methods involving the use of polynomial functions with weights at selected points.

2. Elastic half space loaded by the pressure of ground slab

Displacement of the elastic and isotropic half space loaded with a circular slab is described by means of the Green function method. Boussinesq solution to the problem of the half space loaded with a concentrated load is used to construct the discussed Green function.

The pressure exerted by circular slab on a half space is expressed in terms of non-dimensional coordinates \( r/R \). The Green function is integrated over the surface of the circular slab. Finally, the elastic subsidence of isotropic half space, \( v(p) \), due to the loading \( p(r) \) transmitted by the circular slab, takes the form:

\[
v(r) = \frac{4(1-v^2)}{\pi E_0} \left[ \frac{1}{r^2} \int_0 \frac{dp}{p} \int_0 \frac{d\chi}{(p/\rho)^2} \sin^2 \chi \right] + \int_0 \frac{dp}{p} \int_0 \frac{d\chi}{(p/\rho)^2} \sin^2 \chi \right]
\]

A differential equation describing bending of a circular plate resting on the elastic half space and undergoing loading \( q \), uniformly distributed over the surface of a circle of radius \( a \), is used as a second equation of the problem. The function \( p(r) \), describing the interaction between the tank foundation and the elastic half space, as well as the slab deflection, is assumed in the form of power series expansion.

3. Analysis of soil-structure interaction of cylindrical reservoir of varying wall thickness with a ground slab

The theory of boundary perturbations was used for the analysis of the soil-structure interaction. A cylindrical reservoir is considered with the wall of a varying thickness, fixed at its base in the ground slab, subjected to axisymmetric loading. The conditions of applicability of the theory of boundary perturbations are fulfilled in the considered case. Thickness of the shell changes linearly with height, \( z \) (\( h(z) = \mu z \)). The free body diagram of the shell and the ground slab is shown in the Figure 1.
4. Computational examples

A computational example is given of a cylindrical RC tank with walls of linearly varying thickness, rigidly connected to the ground slab. Due to series describing the interaction of the ground slab with soil and providing its sufficient accuracy the first 100 terms of expansion were taken into account. Assumed were: modulus of elasticity of concrete: 20000 MPa, Poisson’s ratio of concrete: 0.2, modulus of elasticity of the subsoil – an elastic half-space: 30 MPa, Poisson’s ratio of the subsoil: 0.3, the coefficient of vertical Winkler springs 30000 kN/m$^2$, the coefficient of horizontal springs 5000 kN/m$^2$. Cylindrical wall thickness varied from 20 cm at the top to 30 cm at the bottom, the thickness of the circular ground slab: 20 cm, the geometric parameters of the shell: radius 7 m, height 5 m, the loading parameters; water pressure: gravity 10 kN/m$^3$. Analytical results are presented in Figures 2 to 5.

5. Results and conclusions

A significant difference between results obtained for both subsoil models can be observed. Due to the elastic half-space model the moments in the vicinity of the slab centre are much greater (Fig. 3). The slab supported on the elastic half-space can be bent due to an uniformly distributed load, while the same structure assumed to be supported on Winkler springs is bent only due to the edge forces, which results in opposite values of the moments at the joint of the slab with the cylindrical shell (Fig. 2 and 3). In case of the model of elastic half-space a local unlimited growth of subsoil reaction around the slab perimeter is observed, in contrast to the Winkler model (Fig. 4). In fact, the local plasticization of the soil may occur in this area.

References

Theoretical and experimental study of thin-walled growing laminated structures

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Abstract

The technology of additive manufacturing is under intensive development now. The advantages of this technology are well known. However, the implementation of new technologies is associated with a number of challenges. In particular, these include the problem for minimizing the distortion of geometrical shape that is most significant with additive manufacturing of thin-walled structures. Present communication deals with the issues of mathematical modeling for the distortion of thin-walled solids caused by additive manufacturing and experimental identification of proposed models. The growing bodies are represented as bodies whose inhomogeneity is caused by junction of incompatible deformed parts. Pivotal role in the theoretical modelling is given to implant field \( K_\alpha \) that determines the inhomogeneity. An experimental procedure is based on holographic interferometry of distortion of thin-walled structure during the stereolithography process. It allows to identify the distribution of \( K_\alpha \) with respect to various regimes of an additive process.

Keywords: growing thin-walled structures, incompatible deformations, residual stresses, theoretical models, experimental identification

1. Introduction

Conventional methods for complex shapes manufacturing imply technological processes associated with the removal of the material, for example with cutting, milling, drilling, etc. An alternative class of processes is based on the synthesis of shapes by a successive deposition of the material on a plain substrate or on a surface by local polymerization, electrochemical reaction, laminating, fusing, sintering, etc. Similar processes belong to a class of additive technology.

At present, the technology of additive manufacturing is under intensive development. It allows to create part of arbitrarily complex shape and, theoretically, of any material. The implementation of new technologies is associated with a number of challenges. In particular, these include a problem of minimizing the distortion of a geometrical shape. Actually, despite the potential for high accuracy positioning of the process unit, deviations of the geometric shape of the product remains high, primarily due to residual deformation (thermal, shrinkage, etc.). These deformations induce residual (internal) stresses inevitably arising in additive technology. Residual stress can cause buckling of the thin-walled parts and fracture of volumes, buckling and fracture may occur during the manufacturing process. The distortion minimizing problem is most significant with additive manufacturing of thin-walled structures. The communication deals with the issues of mathematical modeling for the distortion of thin-walled solids caused by additive manufacturing and experimental identification of the proposed models.

2. Mathematical model for growing shell

As shown in [1] the growing bodies can be viewed as a special class of inhomogeneous bodies whose inhomogeneity is caused by junction of incompatible deformed parts. In the following, we use the concept of a body as a smooth manifold equipped with a material connection [2, 3]. The notion of material connection formalizes the idea of a local uniform reference configuration that brings an infinitesimal neighbourhood of a material point into some uniform, typically natural strain state. In the simplest cases, the entire body may be brought into a uniform state by some global configuration. In this case, the connection proves to be Euclidean, and the theory becomes trivial. In general, we cannot simultaneously bring infinitesimal neighbourhoods of all material points into a uniform state by a smooth mapping. A materially uniform body, i.e. a body of all material points of the same kind, exhibits intrinsic (structural) inhomogeneity. From the mechanical viewpoint, such bodies have no stress-free shapes. The only way to return the neighbourhood of all material points to a natural state and thus relax the residual stresses is to split the body into infinitely many parts and allow them to deform independently. This fictitious process may be considered the inverse of the growth process [1].

Consider growing bodies representable as continuous families of nested bodies. A layerwise growing body is a continuous family of bodies monotone with respect to inclusion, i.e.

\[
E = \{B_\alpha\}_{\alpha \in I}, \quad B_\alpha \subset B_\beta \quad \text{for } \alpha < \beta,
\]

where \( I = (a, b) \subset \mathbb{R} \) is an open interval.

The bodies represent themselves in the physical space \( E \) as shapes. Every shape is the image of a configuration \( \alpha : B_\alpha \rightarrow B_\alpha \). To each element of the family (1), we assign two shapes, a reference shape \( B_\alpha^0 = \alpha_0^0 \) and an actual shape \( B_\alpha = \alpha_0 \). Thus, the family (1) induces the corresponding families of reference and actual configurations and shapes.

We assume that the bodies \( B_\alpha \) are materially uniform, simple, and elastic, so that their response can be described by the constitutive equation \( \sigma_\alpha = \mathcal{E}(H_\alpha) \), where \( \mathcal{E}(\ldots) \) is the response functional, which, in general, is nonlinear. We assume that \( \mathcal{E}(\ldots) \) does not explicitly depend on the evolution parameter
the smooth tensor field \( H_\alpha \) represents the local distortion and admits the multiplicative decomposition \( H_\alpha = F_\alpha \circ K_\alpha \), where \( F_\alpha \) is the conventional strain gradient, i.e., the linearization of the mapping \( \gamma_\alpha : B^n_\alpha \to B_\alpha \). The tensor field \( F_\alpha \) is compatible: there exists a vector field with gradient \( F_\alpha \). This property is not true in general for the smooth tensor field \( K_\alpha \), which is called an implant field [3] and is a field of linear transformations that combines the incompatible infinitesimal parts without gaps.

It was noted above that the response functional \( \mathcal{E} \) does not depend on \( \alpha \) explicitly. From the physical point of view, this means that the properties of the already accepted material do not change in the subsequent course of the growth process. Thus, the “implantation” parameters corresponding to a material point are completely determined at the moment of its accretion and do not change further. We refer to growing bodies under such condition as nonrearranging if

\[ \forall \alpha < \beta \in I \quad \forall \tau \in \mathcal{B}_\alpha \quad K_\alpha(\mathbf{x}) = K_\beta(\mathbf{x}). \]  

(2)

Let \( G_\alpha \) be the family of vector fields on \( \mathcal{B}_\alpha \) specifying the external bulk forces. Although the form of momentum equations in the bulk is identical to the classical one

\[ \nabla \cdot \sigma_\alpha + G_\alpha - \rho_\alpha \partial_t v_\alpha = 0, \]

(3)

the equations on the boundary \( \partial \mathcal{B}_\alpha \) are distinct:

\[ n_\alpha \cdot \sigma_\alpha \bigg|_{\partial \mathcal{B}_\alpha} = p_\alpha + V [\rho_\alpha v_\alpha], \quad A_\alpha \cdot \sigma_\alpha A_\alpha \bigg|_{\partial \mathcal{B}_\alpha} = T_\alpha. \]

(4)

Here \( v_\alpha \) is the velocity field, \( \rho_\alpha \) is the mass density, \( V \) is the growth velocity, and \( p_\alpha \) and \( T_\alpha \) are predefined families of fields. The former defines the traction on the boundary \( \partial \mathcal{B}_\alpha \), and the latter defines the tension of the boundary treated as an elastic material surface in contact with a three-dimensional elastic solid [4]. The vector \( n_\alpha \) is the unit outward normal to \( \partial \mathcal{B}_\alpha \), \( A_\alpha \) is the projection onto the corresponding tangent plane, and \([\ldots]\) is the jump of a field on the surface \( \partial \mathcal{B}_\alpha \).

In the present communication we assume that all bodies \( B^n_\alpha \) belong to the class of thin-walled structures, i.e., bodies for which the ratio of minimum diameter of a ball inscribed in all admissible shapes of \( B^n_\alpha \) to the maximum diameter of a circumscribed ball is a small parameter. In this case, the family (1) represents a growing shell. This definition is rather general, and we restrict ourselves to the following subclass of growing shells [5, 6]: A transversely growing shell is a growing shell such that each reference shape \( B^n_\alpha \) is bounded by an overall ruled surface \( S^n_\alpha \) and pairs of face surfaces \( S^+_\alpha \) and \( S^-_\alpha \) whose shape and position depend on the evolution parameter \( \alpha \).

In the framework of any shell theory, a certain representation of the displacements is sought by an ordered set of \( n \) kinematic fields \( (\xi^\alpha_k)_{k=1}^n \) defined on a 2D manifold \( S \) (the reduction surface) and introduce integration over thickness, i.e., along the normal coordinate \( z \) from \( h_\alpha^- \) to \( h_\alpha^+ \). This procedure gives the stress resultant tensor \( \mathbf{T}_\alpha \) and the stress couple tensor \( \mathbf{M}_\alpha :\)

\[ \mathbf{T}_\alpha = \int_{h_\alpha^-}^{h_\alpha^+} \sigma_\alpha \mathbf{dz}, \quad \mathbf{M}_\alpha = \int_{h_\alpha^-}^{h_\alpha^+} \sigma_\alpha \times z n \mathbf{dz}. \]

(5)

Here the \( \sigma_\alpha \) are some “reduced” stresses related to the stress tensor components by a reduction procedure that depends on the chosen theory [7]:

\[ \sigma_\alpha = \hat{\mathbf{E}} \begin{pmatrix} \xi_1^\alpha & \cdots & \xi_n^\alpha & K_\alpha; z \end{pmatrix}. \]

(6)

In this regard, one has the following mappings that do not depend on the evolution parameter \( \alpha \) explicitly:

\[ \mathbf{T}_\alpha = \hat{\mathbf{T}} \begin{pmatrix} \xi_1^\alpha & \cdots & \xi_n^\alpha & K_\alpha; h_\alpha^- & h_\alpha^+ \end{pmatrix}, \]

(7)

\[ \mathbf{M}_\alpha = \hat{\mathbf{M}} \begin{pmatrix} \xi_1^\alpha & \cdots & \xi_n^\alpha & K_\alpha; h_\alpha^- & h_\alpha^+ \end{pmatrix}. \]

(8)

According to common considerations typical of shell theories, the tensor fields (5) should satisfy the equations of motion

\[ \nabla \cdot \mathbf{T}_\alpha + g_\alpha - \xi_\alpha \cdot \partial_t \xi_\alpha - \mathbf{O}; \partial_t^2 \xi_\alpha = 0, \]

(9)

\[ \nabla \cdot \mathbf{M}_\alpha + (\mathbf{T}_\alpha)_S + m_\alpha - \xi_\alpha \cdot \partial_t \xi_\alpha - \mathbf{O}; \partial_t^2 \xi_\alpha = 0. \]

(10)

Here \( \mathbf{O} \) is an appropriate surface Hamilton operator, \( (\ldots)_S \) is a vector invariant (Gibbsian cross, [7, p. 249]) of a tensor, and \( g_\alpha \) and \( m_\alpha \) are external force and moment fields distributed over the reduction surface. The matrices \( \mathbf{O}; \) and \( \mathbf{O}; \) represent “dissipative” terms due to the last expression in Eq. (4).

In the simplest cases, the implant field \( K_\alpha \) is predefined for all values of the evolution parameter \( \alpha \). In most cases we have to determine \( K_\alpha \) from some conditions corresponding to the evolution process, particularly from predefined displacements of the face boundaries or their predefined membrane tension:

\[ A_\alpha \cdot \sigma_\alpha A_\alpha \bigg|_{\Gamma_\alpha^+} = \tau_\alpha^+, \quad A_\alpha \cdot \sigma_\alpha A_\alpha \bigg|_{\Gamma_\alpha^-} = \tau_\alpha^- \].

(11)

Note that \( \tau_\alpha^+ \) and \( \tau_\alpha^- \) are the surface stresses that act on the face boundaries \( S^n_\alpha \) and \( S^n_\alpha \). Thus, the equations (11) turn the evolution problem for a growing solid into a family of problems for shells with surface tension.

3. Experimental identification of the model

As shown, the distortion of thin-walled structure depends on implant field \( K_\alpha \). In turn, \( K_\alpha \) is implemented by the additive process. In this regard the problem may be stated with experimental identification of the distortion in a various conditions of an additive process in the framework of proposed model.

The experimental procedure is based on holographic interferometry of distortion of thin-walled structure during the stereolithography process. The experimental setup includes an open stereolithography system implemented on a vibration-isolated table together with the installation of holographic interferometry. The form of the object a created is designed as a thin-walled cylindrical tube with base. The interferometrical system records the quasistatic evolution of displacements field for the base in time. The measured data and the corresponding inverse problem solution allow to identify the field \( K_\alpha \) with respect to various regimes of an additive process.

References


Buckling and vibrations of seven-layer beams with lengthwise corrugated main core

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Abstract

The paper is focused on one orthotropic thin-walled sandwich beam with a trapezoidal core and three-layer facings. The inner layers of facings are trapezoidal corrugated – in perpendicular direction to the corrugation of the main core, but the outer layers are flat. The beam is attached a lengthwise corrugated main core and crosswise corrugated inner layers of facings. The mathematical and physical model of this beam is formulated, the field of displacements is obtained. Based on Hamilton’s principle the equations of motion are derived for the considered beam and their analytical solutions are given. The main goal of the study is to solve the problem of buckling. The critical load, and the natural frequency of the beam are computed.

Keywords: sandwich structures, orthotropic thin-walled beam, buckling, vibrations, mathematical modelling

1. Introduction

The basic facts on layered structures and their applications can be found in the following monographs, for example: Libove and Hubka [6], Allen [1], Ventsel and Krauthammer [11]. In many papers, various layered structures were introduced and examined recently. We refer the reader to Biancolini [2], Magnucki et al. [9], Biancolini [2] considered numerical approach to evaluate the stiffness parameters for corrugated board. The method is based on a detailed micromechanical representation of a region of corrugated board modelled by means of finite elements. The strength problem of five-layer sandwich beam with corrugated core under three-point bending was presented in [4,7,8,12]. The analytical and numerical (FEM) solutions and experimental investigations were presented. Magnucki et al. [9] presented analytical and numerical (FEM) calculations as well as experimental verification of the obtained results devoted to a sandwich beam with a crosswise or lengthwise corrugated core. The considered beam was made of an aluminum alloy. The corrugated core and the plane faces (outer layers) were glued together. Strength and buckling problems of sandwich beams-structures with corrugated core are examined in [10]. In particular based on a bending moment the analytical solutions were presented. The zig-zag hypotheses for multilayered plates can be found in [3].

In the paper buckling and vibrations problems are studied for seven-layer beams. The one orthotropic thin-walled sandwich beam with trapezoidal core and three-layer facings is considered. The inner layers of facings are trapezoidal corrugated – in perpendicular direction to the corrugation of the main core, but outer layers are flat. The beam shows a lengthwise corrugated main core and crosswise corrugated inner layers of facings (Fig. 1).

2. Analytical studies

The analytical study includes formulation of mathematical model, the crucial parts of which are: the hypotheses related to deformation of the cross section of the beam, the description of displacement and stress fields for particular layers, formulation of equations of motion, and their solution.

In this section the following field of displacements and strains are introduced:

- the outer sheets : \(-\frac{1}{2}t_1 + 2t_1 + t_2 \leq z \leq -\frac{1}{2}t_1 + t_1 + t_2\),
- the crosswise corrugated cores of faces : \(-\frac{1}{2}t_1 + t_1 + t_2 \leq z \leq -\frac{1}{2}t_1 + t_2 + t_1\),
- the inner sheets : \(-\frac{1}{2}t_1 + t_1 \leq z \leq -\frac{1}{2}t_1 + t_1 + t_2\) and the inner sheets

\[ u(x,z) = -t_0 \left( \frac{d}{ds} \varphi(x) \right), \quad v_s = -t_1 \left( \frac{d^2 w}{ds^2} + \frac{dw}{ds} \right), \quad \gamma_s = 0, \]

- the lengthwise corrugated main core : \(-\frac{1}{2}t_1 \leq z \leq \frac{1}{2}t_1\).

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\[ u(x, z) = -t_{xz} \left( \frac{dw}{dx} - 2 \psi(x) \right), \quad \varepsilon_z = -t_{xz} \left( \frac{d^2 w}{dx^2} - 2 \frac{d \psi}{dx} \right), \]
\[ \gamma_z = 2 \psi(x), \]
where \( \psi(x) = u(t)/t_{xz} \) – dimensionless function, \( \zeta = z/t_{xz} \) – dimensionless coordinate.

Stresses in all layers of the plate, with respect to Hooke’s law, are as follows:

- the outer sheets, the crosswise corrugated cores of faces and the inner sheets \( \sigma_z = E_\varepsilon_z, \)
- the lengthwise corrugated main core \( \sigma_z = E^{(i)}_{\varepsilon_z}, \)
- \( \tau_{xz} = 2G^{(i)}_{\psi z}(\psi(x)). \)

The equations of motion are analytically derived from Hamilton’s principle

\[ b t_{xz} c_p \frac{\partial}{\partial t} \frac{\partial W}{\partial x} + t_{xz} c_p \left( \frac{\partial^2 W}{\partial x^2} - c_p \frac{\partial \psi}{\partial x} \right) = -F_0 \frac{\partial^2 W}{\partial x^2}, \]
\[ c_{pr} \frac{\partial^2 \psi}{\partial x^2} - 2 c_{pr} \frac{\partial^2 \psi}{\partial x^2} + 4 G^{(i)}_{\psi z}(\psi(x)) = 0, \]
where \( t \) – time, \( \rho \) – mass density, \( E \) – Young’s modulus of the outer sheet of the beam, \( F_0 \) – axial compressive force,
\[ c_{aw} = c^{(w)}_{aw} + \frac{x_{xz}}{x_{zz}} c^{(z)}_{aw} + \frac{1}{2} E^{(w)}_{aw}, \]
\[ c_{aw} = c^{(w)}_{aw} + \frac{x_{xz}}{x_{zz}} c^{(z)}_{aw} + \frac{1}{2} E^{(w)}_{aw}, \]
\[ c_{aw} = 2 x_i + \frac{x_{xz}}{x_{zz}} c^{(z)}_{aw} + \frac{1}{2} E^{(w)}_{aw}, \]
\[ c_{aw} = 2 \left( 1 + 2 x_i + x_i \right) \frac{x_{xz}}{x_{zz}} c^{(z)}_{aw} + \frac{1}{2} E^{(w)}_{aw}, \]
\[ c_{aw} = 2 \left( 1 + 2 x_i + x_i \right) \frac{x_{xz}}{x_{zz}} c^{(z)}_{aw} + \frac{1}{2} E^{(w)}_{aw}, \]
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\[ c_{aw} = 2 \left( 1 + 2 x_i + x_i \right) \frac{x_{xz}}{x_{zz}} c^{(z)}_{aw} + \frac{1}{2} E^{(w)}_{aw}, \]
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\[ c_{aw} = 2 \left( 1 + 2 x_i + x_i \right) \frac{x_{xz}}{x_{zz}} c^{(z)}_{aw} + \frac{1}{2} E^{(w)}_{aw}, \]
\[ c_{aw} = 2 \left( 1 + 2 x_i + x_i \right) \frac{x_{xz}}{x_{zz}} c^{(z)}_{aw} + \frac{1}{2} E^{(w)}_{aw}, \]
\[ c_{aw} = 2 \left( 1 + 2 x_i + x_i \right) \frac{x_{xz}}{x_{zz}} c^{(z)}_{aw} + \frac{1}{2} E^{(w)}_{aw}, \]

The natural frequency of the beam vibrations is as follows

\[ \omega = \left( \frac{\pi^2}{L^2} \right) \sqrt{\frac{E_{aw}}{\rho c_{aw}}}, \]

Exemplary calculations of the critical load and the natural frequency of the beam vibrations have been carried.

3. Conclusions

The analytical model and the obtained results were verified numerically (FEM). Solutions are similar, with the maximum difference not exceeding 5 percent. Results for the beams with the crosswise and lengthwise corrugated main core are different.

References

Buckling and vibrations of sandwich rectangular plates with trapezoidal core and three-layer faces

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Abstract

The subject of the paper is a simply supported rectangular sandwich plate. The plate is compressed in plane. An orthotropic thin-walled sandwich plate with trapezoidal core and three-layer facings is considered. The inner layers of facings are trapezoidal corrugated – in perpendicular direction to the corrugation of the main core, but outer layers are flat. The mathematical and physical model of the plate is formulated, the field of displacements too. Buckling and vibrations problems of the plate are investigated. Based on Hamilton’s principle the equations of motion are derived for the considered plate and their analytical solutions are given. Critical loads for a family of sandwich plates are numerically determined. Results of the calculation are shown in figures.

Keywords: sandwich structures, orthotropic thin-walled plate, buckling, vibrations, mathematical modelling

1. Introduction

In recent years sandwich beams, plates and shells have been applied in mechanical engineering, particularly in vehicles and building engineering.

There exist many works on the theory and analysis of sandwich plates. Several of them are presented only. The monographs concerning theoretical investigations of such structures are [1,6,11,12]. We also refer the reader to [5,7]. Kazemahvazi et al. [5] developed an analytical model and experimental tests for the compressive and shear response of monolithic and hierarchical corrugated composite cores. Magnucki et al. [7] presented analytical and numerical (FEM) calculations and experimental verification of the obtained results of a sandwich beam with a crosswise or lengthwise corrugated core. Similar analyses were conducted in [3,9,10,14]. Significant amount of research was also dedicated to modelling composite laminated plates due to their wide use. The review of such problems and a rich bibliography may be found in a review work [2]. Isaksson et al. [4] analyzed elastic properties of corrugated board panels. Vinson [13] introduced the field of sandwich structures discussed into the structural mechanics. Similar problems of sandwich plates with corrugated cores were presented by Magnucka-Blandzi et al. [8]. Bending and buckling analysis was carried out to determine deflections, critical load and a buckling shape. Moreover, a finite element model (FEM – ABAQUS system) for the plate was formulated in order to verify the results.

In the paper we study similar problems for seven-layer beams. An orthotropic thin-walled sandwich plate with trapezoidal core and three-layer facings is studied (Fig. 1). The inner layers of facings are trapezoidal corrugated – in perpendicular direction to the corrugation of the main core, but outer layers are flat. The metal of the flat and corrugated sheets is isotropic.

The mathematical and physical model of the plate is formulated, the field of displacements is stated too.
\[\begin{align*}
&\left(-\left(\frac{1}{2} + x_1 + x_2\right) \leq \zeta \leq \left(\frac{1}{2} + x_1 + x_2\right), \\
&\frac{1}{2} + x_1 + x_2 \leq \zeta \leq \frac{1}{2} + 2x_1 + x_2:
\end{align*}\]

\[\begin{align*}
\varepsilon_x & = -t_{\alpha} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial \psi}{\partial x}\right), \\
\varepsilon_y & = -t_{\alpha} \left(\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial \psi}{\partial y}\right), \\
\gamma_{xy} & = \gamma_{yx} = 0, \\
\gamma_y & = t_{\alpha} \left(2 \frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial \psi}{\partial y}\right),
\end{align*}\]

where \(\psi(x,y)\) is the corrugated cores of faces, \(\gamma_{xy}, \gamma_{yx}\) are the corrugated main core, and \(\gamma_y\) is the corrugated main core.

\[\begin{align*}
\varepsilon_x & = -t_{\alpha} \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial \psi}{\partial x}\right), \\
\varepsilon_y & = -t_{\alpha} \left(\frac{\partial^2 \psi}{\partial y^2} + \frac{\partial \psi}{\partial y}\right), \\
\gamma_{xy} & = \gamma_{yx} = 0, \\
\gamma_y & = -2t_{\alpha} \left(\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial \psi}{\partial y}\right),
\end{align*}\]

Figure 3: Scheme of the sandwich plate under compression

Stresses in all layers of the plate, with respect to Hooke’s law, are as follows:

- the sheets
  \[\sigma_x = \frac{E}{1 - \nu^2} (\varepsilon_x + \nu \varepsilon_y), \quad \sigma_y = \frac{E}{1 - \nu^2} (\varepsilon_y + \nu \varepsilon_y), \quad \tau_{xy} = G \gamma_{xy},\]

- the corrugated cores of faces
  \[\sigma_x = E_x \varepsilon_x, \quad \sigma_y = E_y \varepsilon_y, \quad \tau_{xy} = G_{xy} \gamma_{xy}, \quad \tau_{xy} = G_{xy} \gamma_{xy},\]

- the corrugated main core
  \[\sigma_x = E_{\alpha} \varepsilon_x, \quad \sigma_y = E_{\alpha} \varepsilon_y, \quad \tau_{xy} = G_{\alpha} \gamma_{xy}, \quad \tau_{xy} = G_{\alpha} \gamma_{xy} .\]

Based on Hamilton’s principle the equations of motion are derived for the considered plate and their analytical solutions are given. The plate with sizes \(a, b\) carries a uniform compressive forces \(N_{a}^0, N_{b}^0\) (Fig. 3).

The critical load will be presented, and the natural frequency of the plate.

References


Post-buckling analysis of orthotropic circular cylindrical shell with inner corrugated layer

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Abstract

The subject of the paper is an orthotropic shell of revolution subjected to an external pressure. The circular cylindrical shell is a two-layered structure composed of an outer and an inner corrugated layer. The finite elements analyses were performed and the investigation was limited to the elastic range. Non-linear post-buckling analysis was conducted for two selected orthotropic shells. The dependency between a corrugation pitch of the inner layer on the post-buckling behaviour of the orthotropic shell is pointed out. The calculations were performed with the use of the ANSYS code for imperfect orthotropic shells. Different equilibrium paths for orthotropic shells and an equivalent single-layer shell of the same weight are presented and discussed.

Keywords: cylindrical shell, stability, external pressure, post-buckling, FEM

1. Introduction

Stability problems of thin-walled structures were extensively studied first of all for purposes of development of the aerospace engineering. For these structures a post buckling behaviour can be very different depending on their geometry. The basis of the post-critical buckling analysis of structures can be found in papers [1,2,3]. The stability problems of cylindrical orthotropic shells and porous-cellular shells were presented in papers [4,5,6].

From the results of the linear analysis [7] it is seen that the buckling pressure of the presented orthotropic shell under the conditions of external pressure is always higher than for equivalent single layer cylindrical shell of the same weight.

In this paper the post-buckling behaviour of the orthotropic cylindrical shell with an inner corrugated layer is analysed. The parametric analysis based on the non-linear procedure allowed to plot equilibrium paths showing the behaviour of all layers of the two-layered shell in the post-buckling range.

2. Cylindrical shell geometry

The corrugated layer geometry is defined by a repeating arrangement of the corrugation pitch b (Fig. 1). In this study, the corrugation pitch is based on a cosine curve and corrugated specimens consist of several repetitions of an identical unit in axial direction. Between the layers bonding conditions were imposed. The orthotropic shell is described by the radius R, the length L and the two thicknesses of the outer layer marked, t1 and the corrugated shell, t2. The corrugated inner shell is controlled by two dimensions: corrugation pitch b and amplitude of corrugation a. The external pressure is applied to the outer shell. The following dimensionless parameters and dimensionless coordinate $\xi$ are assumed: $x_1 = a/b$, $x_2 = a/t_1$, $x_3 = t_2/t_1$, $A = L/R$, $k = L/b$, $\xi = x/b$. The corrugation of a middle surface of the inner layer is assumed as in the paper [7] and is in the form as below

$$f(\xi) = -0.5a \cos(2\pi \xi).$$

(1)

The geometry of the orthotropic shell is illustrated in Fig. 1.

3. FE model of orthotropic shell

The shell is loaded with the uniform external pressure and simply supported along both edges. Only the normal and circumferential displacements are blocked. The procedures available in the ANSYS code will be used. A thin shell element SHELL181 with four nodes is used. A total number of finite elements was established on the basis of a mesh convergence analysis. Two numerical examples are presented in Fig. 2. The corrugation of the inner layer by splines (with 11 points along the length of the corrugation pitch) was modelled with the use of the function expressed by Eqn (1).

Figure 1: Geometry and boundary conditions of two-layered shell with corrugated inner layer

Figure 2: Mesh convergence of numerical calculations, R=1.3m
4. Numerical results

Both kinds of numerical analysis of the orthotropic shell were made – linear and non-linear. Two cases of orthotropic shell with the different corrugation pitch were analysed: \( b=0.25; 1.8 \text{ m} \). The other dimensions constants are: \( R=1.3 \text{ m}, L=18 \text{ m}, a=0.045 \text{ m}, t_0=0.003 \text{ m} \). Mechanical properties (linear elastic): Young modulus \( E=205 \text{ GPa} \), Poisson’s ratio \( \nu=0.3 \). The total weight of the orthotropic shell: \( m_w=2\pi \rho RL \left( t_1+s_0 \right) \), where \( \rho \)-mass density, \( s_0 \)-the dimensionless length of the inner layer: \( s_0=s_0/b=\int_0^1 \sqrt{1+(\pi t_1)^2 \sin^2(2\pi \xi)} d\xi \). An equivalent single-layer shell of the same material, the radius and the length has the weight: \( m_w=10\pi \rho RL \). It was assumed that the thickness of outer layer \( t_1 \) is constant and a thickness of equivalent single-layer shell is also being constant \( t_1=5 \text{ mm} \) \( (t_0=0) \), hence the thickness of outer layer of orthotropic shell was calculated from the expression \( t_2=(5-t_1)/s_0 \). The calculated critical pressures of orthotropic shells with the corrugation pitch 0.25 m and 1.8 m are \( p_{cr}=0.2466; 0.0261 \text{ MPa} \), respectively. The critical pressure of the equivalent single-layered shell equals 0.01191 MPa. In the first case of the orthotropic shell \( (b=0.25 \text{ m}) \) the critical pressure is 20.7 times greater then the single-layered shell. In the second case \( (b=1.8 \text{ m}) \) this value equals 2.2. The non-linear procedure based on the arc-length method and with an initial geometrical imperfection has been used. A shape of imperfection has a form of the first eigenmode and has the maximum deviation from the perfect structure equals 1 mm. Dimensionless displacements for each layer were assumed: \( \tilde{w}_A=w_A/t_1 \), \( \tilde{w}_B=w_B/t_2 \), \( \tilde{w}_C=w_C/(t_1+t_2) \). As can be seen from Fig. 3 typical equilibrium paths for two-layered shell \( (t_1=3 \text{ mm}, t_2=1.86 \text{ mm}) \) is unstable one. One of the presented paths relates to the single-layered shell of this same weight. The negative slope of the paths for three selected points \( (\text{A,B,C}) \) is significant smaller then for the single-layered shell.

Figure 3: Equilibrium paths of orthotropic shell, \( b=0.25 \text{ m} \)

In Fig. 4 post-buckling paths for the orthotropic shell \( (t_1=3 \text{ mm}, t_2=1.99 \text{ mm}) \) with the corrugation pitch equals 1.8m are shown. In the first part of the equilibrium paths of the orthotropic shell the smooth transition seen near limit point. In the second part of paths displacements grow with a decrease in pressure and in the last part displacements grow with an increase in pressure (a snap-through phenomena).

Figure 4: Equilibrium paths of orthotropic shell, \( b=1.8 \text{ m} \)

5. Conclusions

This paper presents examples of numerical investigations of elastic stability and post-buckling for two selected orthotropic shells with corrugated inner layer. The buckling load for cylindrical orthotropic shells with the corrugated inner layer is from a few to tens of times larger than for the single-layered shell of the same weight. The character of equilibrium paths and values of the critical pressure of the orthotropic shell depend strongly on the corrugation pitch and the corrugation amplitude. Generally the equilibrium paths of presented orthotropic shells are unstable. It can also occur the snap-through phenomena in the orthotropic shell for greater values of the corrugation pitch. However, such an orthotropic shell has many additional advantages, e.g. good acoustic and thermal properties. A free space between the inner and the outer layer can be used for a cooling medium.

References

Numerical simulations and experimental of work riveted joints occurring in semi monocoque structures

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Abstract

It is generally accepted that the rivets in a riveted joint are loaded in the rivet section plane perpendicular to its length. Therefore, the calculation of the strength of the rivet comes to checking the conditions of shear and surface thrust. In contrast, the case exists of skin becoming buckled - particularly if the frame is asymmetrical and the skin is mounted on one side. In this case, after crossing the critical force, the nature of the work of the rivet may dramatically change. The rivet in addition to working on shear and surface thrust, undergoes bending and tension. The paper presents the results of research and analysis, which was performed for the idealized case of a complex load status of a riveted joint. Such a situation may arise when the load exceeds the critical load and the skin becomes buckled.

Keywords: rivet joints, thin walled, semi monocoque, FEM

1. Introduction

Riveted joints are classified as persistent connections, in which parts do not change their relative position. Accordingly, design of riveted joints usually verifies the basic conditions, for shear and thrust. This simplification does not take into account internal stresses resulting from the manufacturing process, like rivet driving Ref. [2,9]. Moreover, taking into account the fact that the riveted joints are at least partially friction connections Ref. [4,6] - it turns out that checking the basic conditions of the strength, in some cases it may not be sufficient. It can greatly reduce development of the structure, in the necessity of using high safety factors. Thus, for many years more and more effort is aimed at a detailed analysis of riveted joints, expected to show the factors to cause the formation of local stress concentration, which reduces the durability of construction due to a crack initiation. The work Ref. [3] presents the results of research on the mechanism of failure of a riveted joint, depending on the type of load. Due to the fact that in semi monocoque structures elastic buckling of skins for service loads is allowed, what makes definite change in the nature of work connection Ref. [7,8], the author decided to develop and test a method for the analysis of a riveted joint work in complex load conditions corresponding to diagonal tension.

2. The object of research

Due to the fact that diagonal tension is a complex stress of a riveted joint (tension, torsion and bending), the test object idealization of the structure fragment where this type of buckling occurs (Fig. 1).

A sample (Fig. 2) was made of 2mm thick sheet metal. Flat head rivets with a diameter of 5mm where the connectors. Used materials: metal - D16TN, rivets - PA25 anodized.

Figure 1: Scheme of selection the shape of the sample and the type of load

Figure 2: Test sample - size, arrangement of measurement points A and B

3. Methods and scope of research and analysis

In order to fulfil the task analysis was done due to tension (breaking load) to determine the maximum load; then the tests for load within the elastic range covering: tension; torsion; tensile with torsion, whose aim was to investigate the effect of torsion on the resistance of connections.

The study was done on IPBM PW MTS testing machine 809 ARIAL. In order to measure strain in the immediate
vicinity of heads of rivets rosette strain gauges 13-031RB EA-120 were used from Vishay-based measurement of 0.6 mm, stuck at points A and B (Fig. 2).

The FEM analysis was performed using the program Abaqus. All cases were done due to elastic - plastic models of material with isotropic and kinematic hardening, using approximated, multi-line characteristics. Material models were based on the results of compression tests. Due to a nonlinear nature of the problem, analysis was performed using the Newton method with regard to large deformations.

The analysis takes into account the necessity of implementing initial stresses arising in the rivet driving process Ref. [1,5]. Simulation of the process included non-linear character of pressure raising in the contact task. Analysis was performed using the shell and solid models and the following models:

• "Model I" - the rivet connection modelled using rigid elements, without modelling the holes;
• "Model II" - the rivet connection modelled making holes and using beam elements as rivets and introducing initial stresses obtained in the closing rivet simulation (Model III);
• "Model III", a solid model of a single rivet environment was used to simulate closing of the rivet; then was used as a submodel, whose load were displacements obtained in the Model II analysis.

4. Summary and conclusions

Rivet driving simulation allowed to determine the state of stress occurring in the audited joint. The results were basic to determine the value of the preload of Model II (Fig. 3). The obtained results confirm the presence of negative radial stress in both in sheet metal on the side of driven head and on the side of the head.

Based on the results of the "Model I" and "Model II" it can be said that the zone of plastic deformation in both models is limited to the rivet and its immediate surroundings. Not confirmed by experimental results, it means that such modelling methods of riveted joints can be used only for global analysis. Comparison of the experimental results with the results of the analysis, turned out the best for "Model III" Fig. 3. In other cases, the differences were much greater. As we can see in the example of normal stress shown in Fig. 3, both results of the analysis and studies confirm a significant effect of twisting on the stress state formed around the rivet head.

Figure 3: Comparison of the results "Model III" with the experimental results for the measuring of point "A" - the course of stress [MPa]: normal to the direction perpendicular to the stretching direction

The analysis results confirm usefulness of the proposed method of the analysis of riveted joints in the structure, not excluding buckling.

References


Numerical simulation of micro-slip occurring in riveted joints of semi monocoque structures

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Abstract

It is generally accepted that the rivets in a riveted joint are loaded in the rivet plane of a section perpendicular to its length. Therefore, the calculation of the strength of the rivet, comes down to verify the conditions of shear and surface pressures. It is completely different in case, due to buckling occurrence for skin – especially, if the skeleton of the structure is asymmetrical and the skin is mounted just on one side. In such a case, after exceeding the critical force, the nature of the work of the rivet may dramatically change. The rivet, in addition to working on shear and pressure, begins its work at bending and stretch. This may cause the local contact stress concentration and micro-slips. The work is focused on determining of the impact of tension field on the state of loads and deformations of riveted joints occurring in thin-walled structures - especially in the semi-monocoque constructions. The conditions of the tension field formation in thin-walled structures were examined. Analysis was also performed to investigate the micro-slips between the rivet and the hole under conditions of normal and torsional loads sheets to be joined.

Keywords: rivet joints, thin walled, semi monocoque, micro-slips, stress, FEM

1. Introduction

It is known that micro-slips occurring in the riveted joints may cause fretting (fretting corrosion). That’s why they may cause the decrease of joints durability. The papers Ref. [4,5,6] examined the occurrence of peripheral micro-slips and pointed out how great influence they may put on the durability of the structure.

Assumption is made that in the semi-monocoque structures, after crossing the critical force and buckling the skin, a significant change in the nature of joints work occurs (ceases to be flat). Thus, the effort was examined of riveted joint working in such conditions mostly to determine how the micro-slips in the axial direction. An example, was used of a thin-walled panel, whose load condition caused the tension field.

2. The object of research

The object of research was a rectangular panel with a frame elements located on the one side of a skin. The following data were taken: dimensions: 250x200 [mm], vertical and horizontal stiffeners - aluminium angle bar 25x25x3 [mm], thickness of sheet metal = 1mm, rivets diameter 2.5 mm, pitch of a riveted joint 12 mm, load: P = 9000 N. The fixing and the load are shown on Fig. 1. Angle bars and sheet are made of D16TN aluminium alloy while rivets are made of PA25.

Figure 1: The object of research - the way of fixing and load

3. Models and types of analyses

In order to fulfil the task three models were analysed (Fig. 2).

Model A - shell model of a whole structure with a simplified rivet model.

Model B - local nature model made as a shell model of a selected portion of the structure, exhibiting the greatest stresses and deformations in the area of a riveted joint. Rivets were modelled as beam elements cooperating with the holes of a suitable diameter.

Model C - micro-local nature model, made-up as a solid, covering an area of influence of a single rivet.

The following analysis types were performed:
- analysis of the critical forces and the buckling modes (Model A);
- preliminary analysis of a panel - calculation of deformation and overall effort after crossing the critical force (Model A);
- driving rivet simulation (Model C);
• analysis of the most loaded portion of the panel, including the initial stresses resulting from the driving process of rivet (Model B);
• effort analysis of a single rivet taking into account the initial stresses - resulting from driving rivet simulation and from the tension field (Model C).

The FEM analyses was performed using the Abaqus program. In all analyses elastic-plastic material models were used with an isotropic, kinematic model of material strengthening, applying the approximate, multiline characteristics. Material models are based on the results of compression tests. Due to the nonlinear nature of the task analyses were performed using a Newton method Ref. [1] taking large strains into account.

Due to the necessity of initial stress resulting from the driving process of rivet Ref. [2,3] simulation of this process was performed. The simulation takes into account multipair contact and non-linear nature of pressure rise in the contact task. Tangential behavior was defined as Coulomb friction (friction coefficient = 0.1). Normal behaviour (pressure-overclosure relationship) was described with an exponential law Ref. [1]. The same contact conditions were defined in the Model A and B between the sheet and the framework elements. In the C model was not included clearance between the rivet and the hole.

4. A summary of the analyses and the conclusions

Conclusion of the results of Model A is drawn that the elastic loss of stability of a covering occurred with approx. 47% of maximum capacity. Excess of the critical force caused significant deformations of skin in the area of contact with the frame elements. Taking into account shear forces in the rivets, angles of twist and deflections of a sheet metal, the course of the relative sheet metal deflection against the inner edge of angle bar (in this case the highest values, reaching 0.15 mm, was obtained for a bottom angle iron), a fragment of a riveted joint, bottom angle iron, located in the right bottom corner of the structure (Fig. 3 a) was chosen for a further analysis.

Figure 3: a) The dimensions and the position of the area extracted for further analysis (Model B); b) Rivet joint - the zone of maximum pressure and micro-slips.

The next step was a simulation of a process of rivet driving (Model C). The results were the initial stresses in the Model B. Model B was loaded by displacements obtained from the analysis of Model A. The results of Model B analysis allowed to extract the most loaded rivet. The nodal displacements served as a load in the next stage of Model C computations.

In the last step a detailed analysis was made of the most loaded portion of a riveted joint in the tested panel. Model C with initial stresses was used after a rivet closure simulation. Model C is a sub-model, its boundary conditions - displacements, were obtained during the analysis of Model B. Displacements were introduced to the second step of the analysis (the first step in this model was rivet-driving) in the form of time varying boundary conditions, defined on the proper surfaces of the solid model.

Analysis the results obtained from the Model C directs us to the stresses in a rivet. Among the results of the second step of the analysis the most Mises stress, virtually unchanged (490 MPa), but definitely changed the character (Fig. 4). Investigating the stress distribution it may certainly be concluded that the rivet is stretched (increase in stress value from 80 MPa to 250 MPa in the rivet shank) and bent, with an uneven stress distribution.

Figure 4: Mises stress [MPa], a) the rivet after driving; b) the rivet after driving and after the introduction of loads arising from the tension field

Analyzing the condition of a relative displacement of the mating surfaces, and a condition of contact pressure, an area indicated by the thick line in Fig. 3.b deserves special attention. At this point, relative displacements of both radial and axial directions reach 0.01 mm. However, in all three cases, the zones of greatest relative displacements overlap the largest pressure zones (Fig. 5). It should be noted that the relative circumferential displacements are the least, reaching 0.003 mm.

Figure 5: The maximum pressures and relative displacements.

References

The role of imperfections in nonlinear buckling analysis of a spherical shell roof

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Abstract

The buckling resistance against external pressure or/and internal vacuum is usually the most decisive criterion in the design of isotropic roofs of steel tanks. The existing codes and recommendations provide an opportunity to assess the buckling resistance in different ways. Discrepancies between results obtained by different methods raise a question about the actual buckling resistance measure of the analysed roof. Different aspects of the existing codes and recommendations are considered and compared in the paper. A new imperfection mode inspired by experiments is proposed and the whole range of imperfection amplitudes corresponding to existing fabrication tolerance quality classes is considered. Different boundary conditions of the analysed roof were considered. The case of the whole tank (roof plus cylindrical wall) was taken into account as well. Calculations based on relatively simple formulae taken from different codes and recommendations and advanced numerical analyses (called GMNIA) are presented in the paper. Final conclusions resulting from the performed analyses are formulated.

Keywords: buckling resistance, spherical shells, steel roofs, tanks, numerical analyses

1. Introduction

The buckling resistance assessment of the pressurized, steel shells is always very important from the engineering point of view.

The determination of the actual value of the buckling resistance, which can be affected by initial imperfections and elastic-plastic deformations of some portions of the structure, is very difficult and laborious (cf. Ref. [1,2,3].) Reference [1] presents procedures of the buckling resistance assessment in the most typical cases. Any other cases should be considered individually by very laborious procedures requiring a software based on finite element method, appropriate for geometrically and materially nonlinear analysis called GMNIA (Ref. [1]). In these analyses the mode of imperfection is required. Due to the common opinion the first or the second buckling mode obtained in the linear buckling analysis is the worst imperfection mode. This rule is not confirmed in all cases of shells encountered in the engineering practice. Imperfections associated with manufacturing process are sometimes more important.

Besides the imperfection mode in GMNIA analysis the imperfection amplitude is required, too. It should be chosen in accordance with the assumed fabrication tolerance quality class of a considered shell structure (cf. Refs. [1,2]). In the paper the detailed procedure leading to the buckling resistance assessment of a steel spherical roof is presented. Different imperfection modes were considered. The results were applied. Different meshes were adopted and the most appropriate one was eventually chosen. The model was not validated by any experimental test.

In GMNIA analysis three different imperfection modes were taken into account (cf. Fig. 2). First two nodes were adopted as the first and the second buckling modes obtained in LBA analysis of a considered shell. These two shapes will be referred further as Mode 1 and Mode 2 and are shown in Figs. 2a and 2b.

The third imperfection mode was generated as follows. The shell was loaded on the oval area of the surface by a uniformly distributed pressure. The displacement field obtained as a result of a linearly static analysis, after relevant reduction to the required amplitude, was used as a disturbance mode of the
original geometry. The imperfection mode obtained in such a way was shown in Fig. 2c to be referred further as the oval dimple.

The range of amplitudes was chosen in accordance with fabrication tolerance quality classes defined in the Ref. [1], section 8.4. In a particular case of the considered shell this range was extended to <0.05\(t\), 1.75\(t\). Classes A, B and C correspond to particular values: 0.697\(t\), 1.116\(t\) and 1.743\(t\) respectively. These limits were depicted in Fig. 3.

The definition of the buckling resistance is defined in Fig. 8.6 of the Ref. [1]. Almost all performed GMNIA analyses the case C1 detected the, i.e. the buckling resistance was defined as the maximum on geometric and material nonlinear equilibrium path. Different situation took place in a case of the imperfection mode shown in Fig. 2c. All equilibrium paths manifested sharp maximum at relatively small lateral displacements. Next a sudden drop was observed and the course of the path was nearly horizontal until the emergence of very large values of lateral displacements. In these cases two resistance measures, upper and lower, were distinguished. Both were shown in Fig. 3, in the author’s opinion the latter is the adequate measure of buckling resistance.

3. Conclusions and recapitulation

Results of performed analysis were collected in Fig. 3 just to observe that buckling resistance for Mode 1 and Mode 2 is nearly identical for all values of imperfection amplitudes. It is worth mentioning that numerical analyses revealed that both kinds of boundary conditions (BC2 and BC3, cf. Fig. 3) gave nearly the same results, while predictions by Ref. [2] led to different results. The most reliable boundary conditions are these correspond to the whole tank (cf. Fig. 1b).

The whole tank was considered with oval dimples on the roof and three cases of amplitudes (0.75\(t\), 1.25\(t\) and 1.75\(t\)). For all these three imperfection amplitudes the following results were obtained: \(p_{\text{max}} = 92\) kPa and \(p_{\text{min}} = 41\) kPa. The last value is probably the most adequate assessment of the buckling resistance of the considered roof.

Analysing all obtained results one can conclude that the most reliable buckling resistance assessment of the spherical roof refers to the case of a whole tank modelling, with the oval dimple was taken into account. Due to all three fabrication quality classes the obtained buckling resistances are even lower than those predicted by very conservative recommendations of Ref. [2].

References


Analytical solution to buckling problems of plates with different boundary conditions under combination of patch load and bending

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Abstract

The past studies on the stability of rectangular plates under arbitrary loads were based on assumptions of simplified stress distributions, which put the question of the accuracy of the results thus obtained. The procedure of applying the exact stress functions on the problem of elastic stability of plates with different boundary conditions due to effects of patch loading and bending is presented in the paper. Mathieu (1890) obtained the exact solution for the plane-strain state for a rectangular element for certain types of variable stresses on the boundaries while Baker et al. (1993), following Mathieu’s results, extended solution to general problem of a rectangular plate loaded by completely arbitrary distributions of (normal and/or shear) stresses along the edges of the plate. The problem of the elastic stability of rectangular plates is investigated using the Ritz energy technique with introduction of exact stress distributions of Mathieu’s theory of elasticity through the potential energy of the plate associated with the work done by external loads. By adopting the exact stresses within a plate and using the double Fourier series to represent any possible buckled profile, the buckling loads can be obtained in a very accurate way. All results for the critical load are reaffirmed by numerical FE runs.

Keywords: elastic stability of plates, exact stress function, patch load and bending combination, Ritz energy method

1. Introduction

The past studies on the stability of rectangular plates under arbitrary loads were based on assumptions of simplified stress distributions. In the series of papers based on Mathieu’s method from 1890 Ref. [3], Baker et al. Ref. [1] and later Liu Ref. [2] and Mijušković Ref. [4] developed a precise approach to exact stress function determination for main case of rectangular plate under arbitrary external load. The method was based on splitting the solution into eight fundamental problems. Superposition of these basic cases is a background for the definition of internal stress distributions for any type of external load (Fig. 2). Such solutions are a basis for the analysis of complex stability problems in real steel structures.

2. Analytical approach to plate buckling

The problem of the elastic stability of rectangular plates with different boundary conditions is investigated using the Ritz energy technique. The strain energy due to bending of the plate is defined in the traditional way. On the other hand, the exact stress distribution of Mathieu’s theory of elasticity is introduced through the potential energy of the plate associated with the work done by external loads. Analytical approach to plate buckling under bending and patch loading is presented in the examples of the rectangular simply supported plates (SSSS) as well as in plates with two edges simply supported and other two clamped (CSCS). In order to assure the accuracy, the double Fourier series (Eqs. (1) and (2)) are used to represent buckled profiles of the two chosen types of plates.

\[ w = \sum \sum w_i \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} \left( x + \frac{a}{2} \right) \left( y + \frac{b}{2} \right) \]  

(1)

\[ w = \sum \sum w_i \cos \frac{\left( m - 1 \right) \pi x}{a} \cos \frac{\left( n + 1 \right) \pi y}{b} \left( x + \frac{a}{2} \right) \left( y + \frac{b}{2} \right) \]  

(2)

These series satisfy all boundary conditions, term by term, and, as it has been proved, are capable of representing any possible buckled profile for very wide range of aspect ratios and load cases.

After the definition of the strain energy of the plate bending \( U \) and of the value which responds to the work done by external forces \( V \), the total potential energy of the system can be written in a form \( E = U + V \). From the minimum potential energy principle, the existence of nontrivial solution, expressed through condition of a zero determinant of the system, leads to the solution of the classical eigenvalue problem. In its scope, the lowest value has the only practical importance, presenting the requested critical load.

Surely, the usage of the corresponding software (symbolic programming in the Mathematica) was necessary in the solving process because of the complexity of the analytical procedure. Its complexity directly depends on the adopted number of terms of the stress functions, as well as on number of terms of the deflection functions.

3. Examples and results

An example selected for the analytical procedure presentation is the analysis of long plates (5 m x 1 m, \( \phi = a/b = 5 \)) with different boundary conditions under patch-load (\( l = 20 \) cm as shown in Figure 1).

Figure 1: Analyzed case of patch-load and bending combination
3.1. The process of analytical model creation

Long plates with different boundary conditions were analyzed analytically and numerically (Ansys) in order to define the active web zone involved in the process of elastic buckling due to patch-loading (Fig. 3). At the same time both methods were also used to obtain the values of buckling loads necessary for later control and comparison.

3.2. Results

Table 1 contains the buckling coefficients for long plate models (Fig. 3), as well as the corresponding values for analytical models with task to simulate only active (buckling) web zone subjected to complex load combinations (Fig. 4). Also, numerical results as well as the existing discrepancy between the obtained solutions are listed.

Table 1: Buckling coefficients in form \( \sigma_{tb}^2/\pi^2 D \)

<table>
<thead>
<tr>
<th>Plate</th>
<th>Analytical</th>
<th>Numerical</th>
<th>Discr. (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSSS Long plate</td>
<td>9.8551</td>
<td>9.8314</td>
<td>-0.25</td>
</tr>
<tr>
<td>Active web zone</td>
<td>9.9173</td>
<td>9.7310</td>
<td>-1.78</td>
</tr>
<tr>
<td>CSCS Long plate</td>
<td>23.0308</td>
<td>22.9178</td>
<td>-0.49</td>
</tr>
<tr>
<td>Active web zone</td>
<td>23.2509</td>
<td>22.4963</td>
<td>-3.20</td>
</tr>
</tbody>
</table>

4. Conclusions

A chosen example illustrates the main goal of this paper is to present an analytical approach for stability problems of plates with different boundary conditions under very complex external loads. The accuracy of the defined method is, tested in a wide range of load types and plate aspect ratios. Particular attention is paid to the appropriate choice of a basic model representing buckling web zone subjected to real edge loads. The results of such analysis will be presented in future works.

References

Tolerance modelling of stability of thin plates with a dense system of ribs

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Abstract

The contribution deals with thin skeletal plates with a dense system of ribs. The aim of the analysis is to derive and apply a macroscopic model of stability of the plate with a non-uniformly oscillating microstructure. The main feature of the proposed mathematical model is the similarity of the microstructure length parameter \( l \) compared to thickness \( h \) of the plate \((l \ll h)\). The formulation of approximate mathematical model of these plates is based on a tolerance averaging approximation \([7]\). The general results of the contribution will be illustrated by the analysis of a specific problem. Validation of the obtained mathematical model is done by comparison of results obtained from model equations and from finite element method (Abaqus program).

Keywords: stability of thin plates, tolerance averaging, composite, FGM

1. Introduction

The considered skeletal plate is made of two families of thin beams whose axes are perpendicular. The regions between the beams are filled with a homogenous matrix material (Fig. 1). It is assumed that the width of beams can vary slowly in the midplane of the plate. Thus, we deal with the composite plate of a space-varying microstructure. The generalized period \( l = \sqrt{\epsilon_3^2} \) of heterogeneity is assumed to be sufficiently small comparing to thickness \( h \) of the plate. At the same time it is assumed that the macrostructure length parameter \( l \) is similar to the measure of the midplane of the plate. From a formal point of view, the structure under consideration can be described in the framework of the well-known theories for thin elastic plates. However, due to the inhomogeneous microstructure of the plate, this direct description of the structure leads to plate equations with discontinuous and highly oscillating coefficients.

The aim of the paper is to formulate and apply the macroscopic mathematical model describing stability of the composite plate under consideration. The formulation of the macroscopic mathematical model for the analysis of stability of these plates is based on the tolerance averaging approximation. The general modeling procedures of this technique are given by Woźniak at el. in books \([7, 8]\). The applications of this technique for the stability modelling of various periodic composites is given in a series papers. Baron analyzed dynamic stability of an uniperiodic medium thickness plate \([1]\). In the paper \([3]\) of Michalak the stability of elastic slightly wrinkled plates is analyzed. The stability of thin periodically stiffened cylindrical shells was analyzed by Tomczyk \([5]\). In the paper of Wierzbicki et al. \([6]\) stability of micro-periodic materials under finite deformations is discussed. The approach based on the tolerance averaging technique, formulating macroscopic stability model of functionally graded plates was presented by Jędrysiak and Michalak \([2]\). In the paper \([4]\) of Perliński et al. stability of annular plate with functionally graded structure is considered. The analyzed plate is interacting with the elastic micro-heterogeneous subsoil.

2. Direct description

The starting point is the direct description of the composite structure in the framework of the well-known theory of thin plates. The displacement field of the arbitrary point of the plate is presented below

\[
w_{\alpha}(\xi^*, z) = w_{\alpha}(\xi^*) - \partial_\alpha w_{3}(\xi^*) z.
\]

Denoting the external forces by \( p(\xi^*) \) and setting \( \partial_\alpha = \partial / \partial \xi^\alpha \) we also introduce gradient operator \( \nabla \equiv (\partial_1, \partial_2) \), in the framework of the linearly approximated theory for thin plates, we obtain the following system of equations

(i) strain-displacement relations

\[
\begin{align*}
\varepsilon_{\alpha\beta}(\xi^*, z) &= \varepsilon_{\alpha\beta}(\xi^*) + \kappa_{\alpha\beta}(\xi^*) z, \\
\varepsilon_{\alpha\beta} &= \nabla_\beta u_\alpha + \frac{1}{2} \partial_\beta w_3 \partial_\alpha w_3, \\
\kappa_{\alpha\beta} &= -\nabla_\alpha w_3;
\end{align*}
\]

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Lagrangian have the form

\[\mathcal{E}(\xi^\prime) = \frac{1}{2}(\mathbf{B}^{\alpha\beta\gamma\delta} \kappa_{\alpha\beta} K_{\gamma\delta} + \mathbf{D}^{\alpha\beta\gamma\delta} \varepsilon_{\alpha\beta} \varepsilon_{\gamma\delta}),\]  

(3)

where normal forces \(n^{\alpha\beta}\) = \(\mathbf{D}^{\alpha\beta\gamma\delta} \varepsilon_{\gamma\delta}\); work of external forces

\[\mathcal{F} = p^\alpha w^0_\alpha + p^3 w_3.\]  

(4)

3. Averaged description

The modeling technique will be based on the tolerance averaging approximation and on the restriction of the displacement field under consideration given by

\[w_3(\xi^\prime, z) = V_3(\xi^\prime),\]
\[w_\alpha(\xi^\prime, z) = (-\partial_\alpha V_3(\xi^\prime) + h^A(\xi^\prime) u^A_\alpha(\xi^\prime)) z.\]  

(5)

In order to derive the equations of motion we shall introduce tolerance averaged Lagrangian \((\mathcal{L}) = \langle \mathcal{L} \rangle + \langle F \rangle\). The governing equations derived from stationary action principle of the averaged Lagrangian have the form

\[\nabla_{\alpha\beta}(\mathbf{\tilde{B}}^{\alpha\beta\gamma\delta} \nabla_{\gamma\delta} V_3 - \mathbf{\tilde{B}}^{\alpha\gamma\delta\beta} u^A_\beta) +\]
\[\quad + \nabla_\alpha (A^{\alpha\beta\gamma\delta} \nabla_\beta V_3) = (p^3),\]  

(6)

where \(N^{\alpha\beta} = \langle n^{\alpha\beta} \rangle\), \(\mathbf{\tilde{B}}^{\alpha\beta\gamma\delta} = \langle \mathbf{B}^{\alpha\beta\gamma\delta} \rangle\), \(\mathbf{\tilde{B}}^{\alpha\gamma\delta\beta} = \langle \mathbf{B}^{\alpha\gamma\delta\beta} \rangle\).

Note that \(u^A_\beta\) can be eliminated from above equations

\[u^B_\mu = K^{\mu\nu\alpha\beta} \mathbf{\tilde{B}}^{\alpha\gamma\delta\beta} \nabla_{\gamma\delta} V_3,\]  

(7)

where \(K^{\mu\nu\alpha\beta}\) determines the linear transformation, inverse to that given by \(\mathbf{\tilde{B}}^{\alpha\beta\gamma\delta} A^{\gamma\delta}\). Denoting

\[F^{\alpha\beta\gamma\delta} = \mathbf{\tilde{B}}^{\alpha\beta\gamma\delta} - \mathbf{\tilde{B}}^{\alpha\beta\gamma\delta} K^{\mu\nu\alpha\beta} \mathbf{\tilde{B}}^{\gamma\delta\beta},\]  

(8)

after simple transformations we finally obtain the following equation for the averaged displacements \(V_3(\xi^\prime)\)

\[\nabla_{\alpha\beta}(F^{\alpha\beta\gamma\delta} \nabla_{\gamma\delta} V_3) + \nabla_\alpha (A^{\alpha\beta\gamma\delta} \nabla_\beta V_3) = (p^3).\]  

(9)

Coefficients in the above equation are smooth and functional in contrast to equations in direct description with the discontinuous and highly oscillating coefficients. The form of the equation is identical to the equation of motion for thin plates with functional coefficients.

References


Dynamic response of an annular plate with a variable three-layered structure

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Abstract

The paper presents the dynamic response of an annular plate with a three-layered structure. This work is a review-character paper. Plate is subjected to load variable in time. The main goal of analysis is evaluation of critical state of plates with a variable composition. A plate is built of elastic, viscoelastic, composite and electrorheological layers. Both symmetric and asymmetric modes of the loss of dynamic stability are considered. The problem is solved by means of approximation methods: finite difference and finite element methods. The results critical dynamic loads and deflection time histories show the dynamic behaviour of analysed plates with variable structure. They indicate sensitivity of the plate structure.

Keywords: dynamic stability, three-layered plate, viscoelastic, composite, electrorheological material

1. Introduction

The dynamic stability investigations of non-homogeneous plates concern the analysis of both buckling and the formulation of structured relations. The following works [1], [3] present the instability problem and the way of solution of a multi-layered structure.

In the paper both the dynamic behaviour problem of annular, sandwich plate and the influence of structure composition on plate stability are presented.

2. Problem formulation

The three-layered plate is the subject of the analysis. The cross-section of a structure is symmetric built of facings and the core. Four variants of a plate structure of a variable material of layers are considered. The structures are composed of: thin elastic facings and thicker elastic core, thin elastic facings and thicker viscoelastic core, thin composite facings and thicker elastic core, thin elastic facings and thin electrorheological core. The core thickness the geometrical plate parameter is analysed too. Structures with thin, elastic facings and elastic core with medium thickness and the core considered thick create two distinct groups. The analysed structure composed of three elastic layers and medium core thickness is the basic one, a reference for the comparison with the results of other examined structures.

A plate is loaded on its facings with the stress uniformly distributed on its perimeter, increasing in time according to the formula:

\[ p = st \] (1)

where: \( p \) – compressive stress, \( s \) – rate of loading growth, \( t \) – time.

The plate edges are slidably clamped. Both axisymmetric and circumferential waved forms of the loss of plate stability are analysed. The sandwich plate theory with the broken line hypothesis was accepted in formulation of the geometrical relations. According to the accepted criterion (Ref. [5]) the loss of plate stability occurs when the speed of the plate point of maximum deflection reaches the first maximum value. Figure 1 shows of the cross-section of an analysed plate.

![Figure 1: Scheme of three-layered annular plates composed of facings – layers 1, 3 and core – layer 2](image)

A detailed information about problem formulation and solution is presented in work [4].

3. Basic equations

In the problem solution orthogonalization and finite difference methods were used to formulate the basic system of differential equations. A proper form of a system of equations for an analysed plate structure reflects the dynamic deflections of examined plate. General, universal form of the system of equations for analysed plates structures of a various layer material is as follows:

\[ PU + Q + P_1 \dot{U} + Q_1 - W \cdot \dot{U} - K_E \ddot{U} = W \cdot \ddot{U} \] (2)

\[ M_{Y(V,Z)} Y(V,Z) = Q_{Y(V,Z)} \] (3)

\[ M_{Y(V,Z)} \dot{Y}(V,Z) = \dot{Q}_{Y(V,Z)} \] (4)

\[ M_{DG} \ddot{D} = M_{DG} D + M_{DG} \dot{U} + M_{DG} \ddot{U} + M_{DG} G + M_{DG} \dot{G} + ER \] (5)

\[ M_{GGL} G = M_{GGL} G + M_{GGL} U + M_{GGL} \dot{U} + M_{GGL} \ddot{U} + M_{GGL} D + M_{GGL} \dot{D} + E \Theta \] (6)

where: \( WI, W2, K_E \) – expressions,

\[ U, Y, Z, U, \dot{U}, Y, V, Z - \] vectors of plate additional deflections and components of the stress function,

\[ Q, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, D, G, D, G, \dot{E}, \dot{E}, \Theta - \] vectors of expressions composed of plate model parameters,

\[ M_{DG}, M_{GGL}, M_{GGL}, M_{GGL}, M_{GGL}, M_{GGL}, M_{GGL}, M_{GGL}, M_{GGL} \]
\textbf{M}_{GUL} - \text{matrices of elements composed of plate parameters,}
\textbf{M}_1, \textbf{M}_2, \textbf{M}_3 - \text{matrices of elements composed of the plate radius,
value of the length of the interval in the finite difference method
and number of buckling mode } m.\

Formulation of the system of equations is based on relations
for non-linear Kármán plate, applying the Hooke’s law for
facings; linear viscoelastic relations expressed by a standard
model for core and physical relations of Bingham body for
electrorheological plate core. Composite plate facings
are expressed by quasi-isotropic relations for a laminate.

4. Finite element method solution

In the course of a finite element method two models of a
plate were built: a circularly symmetrical annular form and a
simplified form built of axisymmetric elements. Facings
are built of shell elements but the core mesh is built of solid ones.
The calculations were carried out at the ACC CYFRONET-
CRACOW (KBN/SGI_ORIGIN_2000/Plódzka/030/1999) using
the ABAQUS system.

5. Example results

The results are presented for a plate compressed on the
outer edge. The rate of loading growth \( s \) is equal to \( s=931\) MPa/s. The form of buckling is axisymmetric \( m=0 \)
and circumferentially waved \( m=7 \). Thin facings \( h_1=h_2=0.001 \) m
are steel or made of laminated composite with code \( [0\text{-}45\text{-}90\text{-}90\text{-}45\text{-}0] \) and parameters: \( E_1=53.781\) GPa, \( E_2=17.927\) GPa,
\( G_{12}=8.964\) GPa, \( v_{12}=0.25\), \( \mu=2900 \text{ kg/m}^3 \) \cite{2}. Composite facing
consists of \( n=8 \) layers each of thickness equal to \( h'=0.000125\) m.
Results presented in Fig. 2 are obtained for quasi-isotropic
composite as facing material. The core thickness is equal to:
\( h_2=0.002 \) m for electrorheological core, \( h_2=0.005 \) m and \( h_2=0.06 \)
m for plates with thick core. Kirchhoff modulus of core material
is equal to \( G_2=5\) MPa or \( G_2=0.5\) MPa for electrorheological core
with value of mass density adopted as \( \mu=64 \text{ kg/m}^3 \). Viscosity
constants of electrorheological fluid core are equal to:
\( \eta=1700\) Pa-s and \( \eta=17\) Pa-s. The standard constants of viscoelastic core
model are the following: \( G_2=5\) MPa, \( G_2'=3.13\) MPa, \( \eta'=2129.2\)
GPa-s. The marked point denotes the loss of plate dynamic
stability.

Figure 3 shows the comparison of responses of plates with
thin elastic core \( h_2=0.002 \) m and electrorheological one.\! It
shows the possibility to control the plate work and to limit
supercritical vibration field.

Figure 4 indicates a possible response of plate with thick
core and very thin facings. The observed buckling form differs
from the investigated one, the former not being global. The
values of critical dynamic loads could be smaller than the
calculated ones for plate model with global buckling.

6. Conclusion

The presented analysis generalizes the undertaken research
of the described plates. Sandwich structures of plates with
various layers materials and thickness create the multi-
parameter, complex dynamic stability problem. An effective
solution to the problem enables a quick evaluation of plate
responses and can be useful in structure design.

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Shear stresses in hybrid thin-walled section: development of detail numerical algorithm based on the graph theory

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Abstract

The paper deals with the problem of shear stress calculation at the longitudinal edges of a hybrid thin-walled section for a general loading case. Hybrid section of a thin-walled bar may consist of several closed (connected and/or disconnected) contours as well as open parts. A detailed solution algorithm for the formulated problem using the graph theory and the set theory was developed. A proposed algorithm is intended for a software implementation in a computer-aided design system for thin-walled bar structures.

Keywords: shear stresses, shear flow, thin-walled section, thin-walled bar, closed contour, graph theory, software, algorithm

1. Introduction

Let us consider the problem of shear stresses calculation on longitudinal edges of a hybrid thin-walled section which consists of several closed (connected and/or disconnected) contours and/or also open parts. This problem has been studied in [3] for general loading case of thin-walled bar.

Further investigation in this area requires development of a detailed algorithm intended for a software implementation in a computer-aided design system for thin-walled bar structures. It is reasonable to construct this algorithm using mathematical background of the graph theory and the set theory.

The graph theory has been applied in [2,4] for the calculation of geometrical sectional properties of thin-walled bars with combined (open-closed) types of cross-sections. The graph theory has also been used to analyse thin-walled bars with multi-contour cross-sections by G. Alfano in [1]. Herewith, the problem of the contoured distribution of shear stresses flows for hybrid (including multi-contour, open-closed) thin-walled sections was out of consideration.

2. Construction of connected graph \( \Gamma_2 \) associated with a thin-walled section

In a general case there are four internal forces (shear forces \( Q_y \) and \( Q_z \), pure torsional moment \( H \) and flexural-torsional moment \( M_\omega \)) apply at the shear centre of the thin-walled section.

A thin-walled section with an arbitrary configuration can be determined with the set of sectional points \( P = \{ \bar{p}_p = (y_p, z_p) \} \), \( p = 1, \ldots , n_p \), and the set of sectional segments \( S = \{ \bar{s}_s = (p^+_s, p^-_s) \} , s = 1, \ldots , n_s \) (see Fig. 1).

It is also convenient to associate a hybrid thin-walled section with planar connected non-oriented graph \( \Gamma_2 \) determined on the sets of \( \Gamma_2 = \{ V_2, R_2 \} \), where \( V_2 \) – finite set of graph vertices, \( R_2 \) – the set of graph edges or the set of unordered pairs on \( V_2 \) (see Fig. 2). Hereinafter, for each graph edge \( r_2 = [u_2, v_2] \in R_2 \) we assume that \( u_2 \neq v_2 \).

3. Distribution of the shear forces flows taken along closed contours for a hybrid thin-walled section

More generally the following resolving system of equations for calculation the distribution factors \( \bar{a}_k , k = 1, \ldots , n_k \), of shear forces flows taken along closed contours of the section was formulated as presented:
here diagonal element of the matrix is weight of \( k \)th closed contour, \( p_{ab} = p_k \), \( k = 1, \ldots, n_s \); element \( \Omega_k \) – double area embraced by \( k \)th closed contour \( \Gamma_k \); \( \Omega_0 \) – double area for all closed contours of the section \( \Phi^c \), \( \Omega_0 = \sum_{k=1}^{n_c} \Omega_k \). Other elements of matrix \( p_{ab} \) were presented in the paper.

4. Composition and solution the system of linear equations

Let us consider an additional square matrix of dimensions \((n_r + n_s)^2 \times (n_r + n_s)\):

\[
M = \begin{bmatrix}
\frac{1}{G} P \xi_{\text{sum}} & 2 \mathbf{G}^T \Lambda \delta^c
\end{bmatrix}
\]

where \( n_r \) and \( n_s \) – number of edges and vertices of graph \( G_s \), respectively; \( \mathbf{G} \) – incidence matrix of graph \( G_s \) with dimensions \((n_r - 1) \times n_s \), truncated by the last row; \( P \xi_{\text{sum}} \) – weighting matrix of nonrammable sectional parts (edges of graph \( G_s \)), square matrix with dimensions \( n_r \times n_s \), and diagonal elements \( p_{rj} = 1, \ldots, n_s \), presented by paper; \( \Lambda \delta^c \) – column vector with modulus \( n_s \), of sectorial coordinate increments \( \Lambda \delta^c = (\Lambda \delta^c)^T \), \( j = 1, \ldots, n_r \), with components presented in the paper.

Let us formulate the system of \( n_r + n_s \) algebraic equations relating to the unknown column vector \( \tilde{b}_s \) with modulus \( n_s \):

\[
M \times \begin{bmatrix}
\tilde{b}_s
\Lambda
\end{bmatrix} = \begin{bmatrix}
0
\lambda
\end{bmatrix}
\]

where \( \Lambda = (\Lambda_{i,j})^T \), \( f = 1, \ldots, n_s - 1 \) – unknown column vector of Lagrange multipliers with modulus \( n_s - 1 \); \( \Lambda_{i,j} \) – additional unknown Lagrange multiplier; \( \lambda \) and \( \lambda_{n_s - 1} \) – zero column vectors with modulus \( n_s \) and \( n_s - 1 \) accordingly.

Let us formulate the system of \( n_r + n_s \) algebraic equations related to the unknown column vector \( \tilde{b}_r \) with modulus \( n_s \):

\[
M \times \begin{bmatrix}
\tilde{b}_r
\Lambda
\end{bmatrix} = \begin{bmatrix}
\frac{\Delta \omega}{\delta^c} / G
\frac{\Delta \omega}{\delta^c} / S_{\text{sum}}
\end{bmatrix}
\]

here \( \Lambda = (\Lambda_{i,j})^T \), \( f = 1, \ldots, n_s - 1 \) – unknown column vector of Lagrange multipliers with modulus \( n_s - 1 \); \( \Lambda_{i,j} \) – additional unknown Lagrange multiplier; \( \Delta \omega \) – a matrix whose components those are absolute values of matrix \( \mathbf{G} \) components; \( S_{r}, \ S_{s} \), \( S_{\text{sum}} \) – section properties presented by paper.

Formulation system of \( n_r + n_s \) algebraic equations relating to the unknown column vectors \( \tilde{b}_s \) and \( \tilde{b}_r \), with modulus \( n_r \), can be performed by analogy and was presented in the paper.

Then the following transformations should be performed:

\[
\forall k = 1, \ldots, n_s - 1: \ S_{s,0}^r \leftarrow \{S_{s,0}^r - b_{s,0}\}; \ S_{s,k}^r \leftarrow \{S_{s,k}^r - b_{s,k}\}; \ S_{s,k-1}^r \leftarrow \{S_{s,k-1}^r - a_k T_k / \Omega_k\}.
\]

5. Resulting shear flows calculation

Let us determine the set with length \( n_r - 1 \) (by the number of sectional segments) of shear forces values at the start, middle and end points of the segment midline \( T^c = \{T^c_k\} \), \( k = 1, \ldots, n_r - 1 \) as presented below:

\[
T^c_k = \frac{\phi H}{\Omega_k} a_k \frac{\Phi_k}{T_k} \frac{S_{s,0}^r + S_{s,k}^r - S_{s,k-1}^r - M_k}{I_k} S_{s,k-1}^r.
\]

Finally, shear stresses for each \( k \)th sectional segment \( \tau^c = \{\tau_{s,0}^c(k), \tau_{s,k}^c(k), \tau_{s,k-1}^c(k)\} \), \( k = 1, \ldots, n_r - 1 \) can be calculated as follows:

\[
\tau_{s,k}^c = T^c_k \left\{ \frac{\delta^c}{H} \left[ (1 - \psi) H \delta^c / I_k \right] \right. \}
\]

where component \( T^c_k \left\{ \delta^c / H \right\} \) determines the value of the shear stresses referenced to the segment middle line. Transition from the shear stresses in the middle line to the shear stresses at the longitudinal edge of the segment can be performed by addition or subtraction the last term \( (1 - \psi) H \delta^c / I_k \) in equation (5).

6. Conclusions

The problem of shear stress calculation at the longitudinal edges for a hybrid thin-walled section was considered for general loading case. The hybrid section of a thin-walled bar may consist of several closed (connected and/or disconnected) contours as well as open parts.

The detail solution algorithm for the formulated problem using the graph theory and the set theory was developed. The proposed algorithm is intended for a software implementation in a computer-aided design system for thin-walled bar structures.

References


On the resultant six-field linear theory of elastic shells

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Abstract

The extended static-geometric analogy is established for the resultant six-field linear theory of elastic shells. This allows to formulate the shell relations in the complex domain, which reduce the PDEs of the linear six-field shell model from 12th order in real domain to only of 6th order in complex domain. For an arbitrary deformation of the shell space, the 2D resultant stress power is known to contain additional terms not expressible through 2D shell stress and strain measures alone. When these terms are consistently linearised, they still depend on gradients of the 2D strain and stress measures. The nature of this consistent approximation within the resultant six-field linear theory of elastic shells is discussed.

Keywords: linearly elastic shell, six-field theory, static–geometric analogy, complex shell relations, constitutive equations

1. Introduction

The resultant non-linear theory of elastic shells was proposed by Reissner [1], developed in a number of papers and summarised in monograph for example by Libai and Simmonds [2] and chrócielewski et al. [3]. In this formulation the 2D non-linear shell equilibrium equations are derived by the exact through-the-thickness integration of equilibrium conditions of non-linear elasticity. Then the 2D virtual work identity allows one to construct uniquely the 2D shell kinematics consisting of the translation vector \( u \) and rotation tensor \( Q \) fields (six independent scalar variables) defined on the shell base surface. The 2D surface stretch and bending measures follow then again uniquely as direct consequence of the exact resultant equilibrium equations. When such a resultant shell model is linearised for infinitesimal translations, rotations, stretches and bending measures, the linearised drilling rotation remains as the independent kinematic variable, as well as two linearised drilling couples and two work-conjugate drilling bending measures remain in the description of 2D stress and strain state. The latter features contradict all classical shell formulations of the Kirchhoff-Love and Timoshenko-Reissner type following from linear elasticity by any 3D-to-2D reduction technique.

2. Extended static-geometric analogy

Exact resultant 2D relations of the six-field non-linear theory of shells are given in the books [2,3]. For the general system of notation let me refer to our recent paper [5]. When translations and rotations of the shell base surface \( M \) are assumed small, linearisation of the component form of exact equilibrium equations gives (see [5], Eqs. (34))

\[
\begin{align*}
N^\alpha_{\alpha\beta} - b_{\alpha\beta}Q^\alpha + f^\alpha &= 0, \quad Q^\alpha_{\gamma\beta} + b_{\gamma\beta}N^\gamma - f = 0 \quad (1) \\
M^\alpha_{\beta\gamma} - Q^{\alpha\beta}b_{\gamma}^\beta + m^\alpha &= 0, \quad M^\alpha_{\alpha\beta} + c^\beta(N^\alpha_{\gamma\beta} - b_{\gamma}^\beta M^\gamma) + m^\alpha = 0
\end{align*}
\]

These twelve PDEs involve the drilling stress couples \( M^\alpha \) as well, which are not present in analogous PDEs of the classical linear shell models of the K-L and T-R type.

The corresponding exact 2D compatibility conditions of the six-field non-linear theory of shells follow from integrability conditions \( \varepsilon^\alpha_{\alpha\beta} u_{\alpha\beta} = 0 \) for the surface translation vector and \( \varepsilon^\alpha_{\alpha\beta} Q_{\alpha\beta} = 0 \) for the surface rotation tensor fields. For small stretch and bending surface measures the component form of non-linear compatibility conditions can be linearised to

\[
\begin{align*}
\varepsilon_{\alpha\beta}(E_{\alpha\beta\gamma} - E_{\beta\gamma\delta} + e_{\alpha\beta}K_{\gamma\delta}) &= 0, \quad \varepsilon_{\alpha\beta}(E_{\alpha\beta\gamma} + E_{\beta\gamma\delta} + K_{\alpha\beta}) = 0 \\
\varepsilon^\alpha_{\alpha\beta}(e_{\alpha\beta}\theta_{\alpha\beta}^\delta + b_{\beta}^\delta K_{\alpha\beta}) &= 0, \quad \varepsilon^\alpha_{\alpha\beta}(K_{\alpha\beta} + \varepsilon_{\alpha\beta}\theta_{\alpha\beta}^\delta) = 0 \quad (2)
\end{align*}
\]

These twelve PDEs involve the 2D surface drilling bending \( K_{\alpha\beta} \) measures. Again, the fields \( K_{\alpha\beta} \) are not present in compatibility conditions of any classical linear shell models.

Between the homogeneous equilibrium equations (1) and the compatibility conditions (2) there exists the following correspondence:

\[
\begin{align*}
N^\alpha_{\alpha\beta} &\leftrightarrow \varepsilon_{\alpha\beta} e_{\alpha\beta} K_{\alpha\beta}, \quad Q^\alpha &\leftrightarrow -\varepsilon_{\alpha\beta} K_{\alpha\beta} \\
M^\alpha_{\beta\gamma} &\leftrightarrow -\varepsilon_{\alpha\beta} e_{\alpha\beta} E_{\beta\gamma}, \quad M^\alpha &\leftrightarrow -\varepsilon_{\alpha\beta} E_{\alpha\beta} \quad (3)
\end{align*}
\]

When the resultant 2D stress measures in (1) are replaced by the 2D strain measures according to (3), the homogeneous equilibrium equations (1) are converted exactly to the compatibility conditions (2). The correspondence (3) can be called the extended static-geometric analogy in the resultant six-field linear theory of shells. Within the classical K-L type shell model the analogy was noted by Gol’denveiser (1939), while for the shell model of T-R type it was noted by Pelekh and Lun’ (1970).

The property (3) allows one to introduce the 2D resultant complex stress and strain measures in analogy to those used by Novozhilov et al. [6,7] in the K-L type linear shell theory,

\[
\begin{align*}
\hat{N}^\alpha_{\alpha\beta} &= N^\alpha_{\alpha\beta} - i E h c \varepsilon_{\alpha\beta} e_{\alpha\beta} \hat{K}_{\alpha\beta}^\delta, \quad \hat{Q}^\alpha = Q^\alpha + i E h c \varepsilon_{\alpha\beta} \hat{K}_{\alpha\beta}^\delta \\
\hat{M}^\alpha_{\beta\gamma} &= M^\alpha_{\beta\gamma} + i E h c \varepsilon_{\alpha\beta} e_{\alpha\beta} \hat{E}_{\beta\gamma}^\delta, \quad \hat{M}^\alpha = M^\alpha + i E h c \varepsilon_{\alpha\beta} \hat{E}_{\alpha\beta}^\delta
\end{align*}
\]

where \( i = \sqrt{-1}, \quad c = h \sqrt{12(1 - v^2)} \), the star means a particular solution of inhomogeneous PDEs (1), and \( \hat{K}_{\alpha\beta}, \hat{E}_{\beta\gamma} \) are the 2D complex strain measures constructed from the complex translations and rotations,

\[
\hat{u}_\alpha = u_\alpha + i \hat{v}_\alpha, \quad \hat{w}_\beta = w + i \hat{\Psi}_\alpha, \quad \hat{\Phi}_\alpha = \Phi_\alpha + i \hat{\Psi}_\alpha, \quad \hat{\Psi}_\alpha = \Psi_\alpha + i \hat{\Xi}_\alpha \quad (5)
\]
where the overbar denotes six stress functions. This leads to the following complex linear equilibrium equations:

\[ \tilde{N}^{\text{th}}_{\text{th}} - h^{\theta}_{\text{th}} \tilde{Q}^{\theta} + f^{\theta} = 0, \]
\[ \tilde{M}^{\text{th}}_{\text{th}} - \tilde{Q}^{\theta} + e^{\theta}h_{\theta}^{\theta}M^{\theta} = m^{\theta} = 0, \]
\[ \tilde{M}^{\text{th}}_{\text{th}} + e^{\theta}h_{\theta}^{\theta}M^{\theta} = m^{\theta} = 0. \]

(6)

In terms of (5) the above system of PDEs is of 6th order in complex domain as compared with PDEs (1) of 12th order in real domain. The above complex shell equations may be applied with the hope to obtain more accurate analytical results than those based on the linear K-L type [6,7] and T-R type shell models.

3. Gradients of 2D shell measures in the resultant stress power

Distribution of translations through the shell thickness is non-linear, in general. For an arbitrary deformation of the shell space, we have introduced in [3,4] the intrinsic deviation vector \( \epsilon(\theta', \xi) \) defined by

\[ \epsilon = Q^{\theta} \xi - \xi n = e^{\theta}(\xi)g_{\text{th}}(\xi) + e(\xi)n \]

(7)

where \( Q^{\theta} \) is a measure of deviation of the deformed curved material fibre from its approximately linear rotated shape \( \xi O_{\text{th}}n \), see Fig. 1.

Figure 1: Finite deformation of the shell cross section

The 3D stress power density is defined by \( \Sigma = (FS):F \), where \( F \) is the 3D deformation gradient, \( S = Q^{\theta} \otimes g_{\text{th}}n \) is the 2nd Piola-Kirchhoff stress tensor, and \( \Lambda \neq \Lambda^{\text{th}} \) denotes the double-dot (scalar) product in the tensor space. Using the modified polar decomposition \( F = QA \), where \( \Lambda \neq \Lambda^{\text{th}} \) is the modified stretch tensor, we can calculate exactly the resultant 2D stress power density in the form, see [4], section 6,

\[ \Sigma = \int_{\xi} \Sigma_{\text{th}} d\xi = N^{\text{th}}_{\text{th}} E_{\text{th}} + Q^{\theta} E_{\text{th}} + M^{\text{th}} K_{\text{th}} + M^{\theta} K_{\text{th}} + 
\]
\[ + \int (\epsilon \times S^{\theta}) \mu d\xi \cdot \kappa_{\text{th}} + \int (\Lambda^{\theta} : V) \mu d\xi \]

(8)

where \( \kappa_{\text{th}} = e_{\text{th}} K_{\text{th}} a^{\theta} + K_{\text{th}} n \), and \( V \) means the 3D gradient performed in the undeformed shell space. The last two integrals in (8) represent that part of \( \Sigma \) which is not expressible through the 2D shell stress and strain measures alone.

We have thoroughly analysed the principal terms of the last two integrals in (8) within the accuracy of equilibrium problems of the linear theory of isotropic elastic shells. Let us apply some results obtained in [5,8],

\[ e_{\rho} = k(\xi)q_{\rho} + g(\xi)e_{\rho} \]
\[ e = r(\xi)q + s(\xi)d \]

(9)

\[ r(\xi) = \frac{20\xi^2 h^2 - 1}{h^2}, \quad s(\xi) = \frac{4\xi h^2}{h^2 - \xi} \]

(10)

\[ q = \frac{1}{40} h^2 D^{\theta6} K_{6(\theta)}, \quad d = \frac{h^2}{48} D^{\theta6} E_{6(\theta)6(\theta)} \]

(11)

while \( k(\xi), g(\xi), q_{\rho}, e_{\rho}, D^{\theta6} \) are defined in [5]. Then for the principal terms of (8), after appropriate transformations we are able to obtain the following consistently estimated relations:

\[ \int (\epsilon \times S^{\theta}) \mu d\xi \cdot \kappa_{\text{th}} = \frac{2}{3} Q^{\theta} q_{\text{th}} - Eh^{2} \left( \frac{3}{160} \eta \right), \]

(12)

\[ \int (\Lambda^{\theta} : V) \mu d\xi = -9M^{\theta} q_{\text{th}} - \frac{25}{3} h^{2} D^{\theta6} q_{\text{th}} - \text{Enh}^{2} \left( \frac{3}{160} \eta \right). \]

Within the first-approximation linear theory of elastic shells, all terms in (12) can be neglected as compared with the main terms \( -\text{Enh}^{2} \) in (8). However, in the Timoshenko-Reissner type and the resultant six-field linear shell models the terms (12) are of the same order (aside from small numerical factors) as those following from \( Q^{\theta} \). Their role should still be discussed if these linear models are to be considered energetically consistent ones.

References


Multi-objective crashworthiness optimization of thin-walled conical groove tubes filled with polyurethane foam

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Abstract

Energy absorbers are used in various – especially in the automotive – industries as a solution to reduce the damages stroke on the passengers, and to enhance automobile safety. Nowadays, thin-walled tubes as one of the most efficient energy absorption systems have found increasing applications. In the study, optimization of crashworthiness parameters is performed to investigate the impacts of designing variables related to thin-walled conical groove tubes using a finite element analysis. However, since the output of these studies depends to design methods of computational experiments, the design of experiments technique (DOE) is used in the study along with the finite element analysis. Thus a conical aluminium tube filled with polyurethane foam is simulated that the grooves are designed with a given distance on the inside and outside surfaces under quasi-static load. Objectives of the design include crush force efficiency (CFE), the total absorbed energy (E), and design variables including conical angle, groove distance, foam density, groove depth, and tube thickness. Multi-objective optimization using response surface methodology (RSM) showed that the foam density, conical angle, and the tube thickness, respectively and linearly relate to the amount of energy absorbed. Also the foam density, groove depth, tube thickness, grooves distance and conical angle respectively and non-linearly impact on CFE. In addition to the interaction between the tube thickness and foam density, the interaction between the groove depth and conical angle also was found to affect on the CFE. These results can be helpful in the design of thin-walled tubes with different conical geometries.

Keywords: crashworthiness, crush force efficiency, energy absorption, Multi-objective optimization, finite element analysis, Design of experiments (DOE).

1. Introduction

Thin-wall pipes are considered by manufacturers of automotive industry as energy absorption systems due long crushing length, lightness and sustainability in the collapse.

The crushing behaviour of thin-walled tubes was examined in many studies. For example, crashworthiness of thin-walled conical groove tubes was examined in 2014 changing the number and distance of grooves using simulation method [1]. In 2011, Multi-objective crashworthiness optimization of thin-walled conical tubes was performed changing parameters of a conical angle , number and radius of grooves and thickness of tube using numerical simulation [2]. Due to a number of variables and goals of designing in absorber design, finding the optimal conditions and studying multiple variables with the lowest number of tests is possible only with the help of the design of experiments technique and multi-criteria decision-making techniques [3,4]. Therefore in this paper, optimization of grooved thin-walled conical tubes filled with polyurethane foam was done using the technique of multi-criterion decision-making. Geometric parameters affecting crashworthiness were determined too.

2. Methodology

In order to improve the design of thin-walled conical tubes, first the designing variable range were determined. In the next step tubes crashworthiness evaluation criteria were determined, objectives of the design include total absorbed energy (E) and crush forces efficiency (CFE) as shown in Table 1.

Next, using response surface methodology (RSM) design of experiment is created and numerical simulation of samples is done using ABAQUS.

Table 1: Variables and objectives of the design

<table>
<thead>
<tr>
<th>f₁</th>
<th>E (t', d, ρ, λ, α)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f₂</td>
<td>CFE (t', d, ρ, λ, α)</td>
</tr>
</tbody>
</table>

3. Multi-objective optimization

In order to find the effect of design variables, the importance and influence of each parameter and their interaction effects, analysis of variance (ANOVA) was used. The maximum values were estimated by maximizing the objective functions (Eqn (1,2)).

\[ E = -3.67286 - 0.47036 \times \alpha - 0.20090 \times \lambda + 0.081362 \times \rho + 1.06673 \times d + 4.84219 \times t' \]   

\[ \text{CFE} = -3.67286 - 0.47036 \times \alpha - 0.20090 \times \lambda + 0.081362 \times \rho + 1.06673 \times d + 4.84219 \times t' \]   

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\[ CFE = -34.81593 + 6.91765 \times \alpha + 81.57145 \times \lambda + 0.59644 \times \varphi \]
\[-246.67339 \times \varphi - 157.76668 \times \lambda - 1.18794 \times \lambda \times \varphi\]
\[+0.051560 \times \lambda \times \varphi - 48.71180 \times \lambda \times \varphi \times \alpha \]
\[+0.29463 \times \varphi \times \alpha - 1.20690 \times \alpha \times \lambda \times \varphi\]
\[+98.13785 \times \lambda \times \varphi \times \alpha + 0.75186 \times \alpha^2 - 3.40190 \times \lambda^2 + 1.94044e-3 \times \alpha^2 \]
\[+448.35746 \times \alpha^2 + 103.94177 \times \alpha^2 \times t' \]

The obtained crashworthiness parameters are compared by simulation and optimization in Table 2.

<table>
<thead>
<tr>
<th>Crashworthiness</th>
<th>Simulation</th>
<th>Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crush force efficiency</td>
<td>-</td>
<td>398</td>
</tr>
<tr>
<td>total energy absorption</td>
<td>KJ/Kg</td>
<td>16.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16.90</td>
</tr>
</tbody>
</table>

Table 2: Comparison of the simulation and optimization

The diagram of load - displacement and the crushed shape of an optimal specimen are given in Fig. 1.

Figure 1: Load-displacement curve and crushed shape of optimal specimen

4. Conclusion

In the paper, energy absorption optimization and efficiency of crushing in grooved thin-walled conical tubes filled with polyurethane foam was done using a multi-criteria decision-making technique. In the study, the effect of design variables including the tube thickness in groove, groove distance, conical angle and density of the polyurethane foam was estimated on the responses using numerical simulation in ABAQUS.

The results showed that:

1. The foam density, conical angle and the tube thickness in groove, respectively and linearly relate to the amount of energy absorbed. While increasing the tube thickness in groove, and foam density, the absorbed energy is increased and increasing the conical angle reduces it.
2. The foam density, groove depth, tube thickness in groove, grooves distance and conical angle respectively and non-linearly impact on CFE. Increasing the density of foam and groove depth, crush force efficiency was increased. However, increasing the thickness of tube in groove, the distance between grooves and conical angle reduces crush force.
3. In addition to the interaction between the tube thickness in groove and foam density, the interaction between the groove depth and conical angle also was found to affect on the CFE. In a tube with a low groove depth, changing the conical angle, reduces the changes in crush force efficiency. However, in high a groove depth, the increased conical angle reduces the crush force efficiency.

In this study response surface methodology optimization method is used. Several optimization methods can be used, and determined the best way to achieve the target.

References

A computational formulation for liquid shells based on $C^1$-continuous finite elements

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Abstract

The paper presents a new computational model for liquid shells considering $C^1$-continuous surface discretizations based on isogeometric finite elements. The $C^1$-continuity allows for a rotation-free description of the discretized shell – making it possible to use a standard, displacement-based finite element formulation. The formulation is based on classical thin shell theory considering a suitable stored energy function in conjunction with a new in-plane stabilization scheme. The paper discusses the corresponding theory and numerical formulation, and shows several numerical examples. Those are compared with experimental and analytical results. The proposed formulation is also compared to classical solid shell formulations.

Keywords: shell theory, lipid bilayers, in-plane stabilization, rotation-free shell discretizations, isogeometric finite elements

1. Introduction

Liquid shells are mechanical structures that behave like solids in the out-of-plane direction, but behave like liquids within the surface plane. They can therefore not be fully described by classical shell theories, even under quasi-static conditions. An example for liquid shells are cell membranes, which are made of lipid bilayers. The following sections discuss the basic theory and numerical formulation suitable for liquid shells. A numerical example is given in Section 4.

2. Theory

2.1. Kinematics

Consider a surface $S_0$ defined by the mapping $X = X(\xi^1, \xi^2)$ of surface points from a parameter domain (defined by coordinates $\xi^\alpha$, $\alpha = 1, 2$) to the physical domain $S$. The surface deforms due to mechanical loading into configuration $S$, defined by the mapping $\mathbf{x} = \mathbf{x}(\xi^1, \xi^2)$. The two mappings fully characterize the surface geometry and kinematics according to the relations of differential geometry [3, 6]. In particular the components $a_{\alpha\beta}$ and $b_{\alpha\beta}$ of the metric tensor and the curvature tensor can be found.

2.2. Equilibrium

The equilibrium equation for quasi-static shells can be expressed as [8]

$$T^{\alpha\beta} + f = 0 \, ,$$

(1)

where

$$T^{\alpha} = N^{\alpha\beta} a_{\beta} + S^{\alpha} n \, ,$$

(2)

is the traction vector on the surface normal to $a^{\alpha}$. Here

$$N^{\alpha\beta} = \sigma^{\alpha\beta} + b_{\alpha\beta} M^{\gamma\beta} \, ,$$

$$S^{\alpha} = -M^{\alpha\beta} \, ,$$

(3)

where $\sigma^{\alpha\beta}$ and $M^{\alpha\beta}$ are given through constitution. The vector

$$f = f^{\alpha} a_{\alpha} + p n$$

(4)

denotes ‘body’ forces acting on the surface. Equation (1) is complemented by the boundary conditions

$$\mathbf{x} = \varphi \, \text{ on } \partial S \, ,$$

$$T = T \, \text{ on } \partial T S \, ,$$

$$M = M \, \text{ on } \partial N S \, ,$$

(5)

where $T$ and $M$ denote boundary tractions and moments.

2.3. Constitution

Considering hyperelastic material behavior, the stress and moment components follow from the stored surface energy density $W = W(a_{\alpha\beta}, b_{\alpha\beta})$ as

$$\sigma^{\alpha\beta} = \frac{2}{J} \frac{\partial W}{\partial a_{\alpha\beta}} \, ,$$

$$M^{\alpha\beta} = \frac{1}{J} \frac{\partial W}{\partial b_{\alpha\beta}} \, ,$$

(6)

where $J$ denotes the surface area change during deformation.

A suitable stored energy function for liquid shells is [7]

$$W = J (k H^2 + \tilde{K} \kappa) + \frac{K}{2} (1 - J)^2 \, ,$$

(7)

where $k$ and $\tilde{K}$ are bending moduli, and $K$ is the in-plane bulk modulus, resisting area changes. $H$ and $\kappa$ are the mean and Gaussian curvatures of surface $S$. According to (7), the liquid shell has

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no in-plane shear stiffness. In the dynamic case shear resistance is provided by viscosity. For the quasi-static case, the in-plane shear deformation has to be stabilized [4, 7].

2.4. Weak form

Considering a suitable variation of the deformation, denoted as $\delta x \in \mathcal{V}$, the weak form of equilibrium equation (1) can be derived as [5]

$$G_{\text{int}} - G_{\text{ext}} = 0 \quad \forall \delta x \in \mathcal{V}, \quad (8)$$

with

$$G_{\text{int}} = \int_{\mathcal{S}} \frac{1}{2} \delta a_{\alpha \beta} \sigma^{\alpha \beta} \, da + \int_{\mathcal{S}} \delta b_{\alpha \beta} \, M^{\alpha \beta} \, da,$$

$$G_{\text{ext}} = \int_{\mathcal{S}} \delta x \cdot f \, da + \int_{\partial \mathcal{S}} \delta x \cdot T \, ds + \int_{\partial \mathcal{S}} \delta n \cdot M \, ds. \quad (9)$$

3. Finite Element Approximation

Displacement based finite elements, with three degrees-of-freedom per node are considered. The geometry of the reference surface and the current surface are approximated by the finite element interpolations

$$X \approx X^h = \sum_{I} N_I X_I,$$

and

$$x \approx x^h = \sum_{I} N_I x_I,$$  

where $N_I = N_I(\xi^1, \xi^2)$ denotes the nodal shape function defined in parameter space. In order to ensure $C^1$-continuity of the surface, NURBS-based shape functions are used [1]. The tangent vectors of the surface are thus approximated by

$$A_{\alpha} = \frac{\partial X}{\partial \xi^\alpha} \approx \sum_{I} N_{I, \alpha} X_I,$$

and

$$a_{\alpha} = \frac{\partial x}{\partial \xi^\alpha} \approx \sum_{I} N_{I, \alpha} x_I.$$  

From these the normal vector to the surface are defined according to

$$N = \frac{A_1 \times A_2}{\|A_1 \times A_2\|},$$

and

$$n = \frac{a_1 \times a_2}{\|a_1 \times a_2\|}.\quad (15)$$

The variation of $x$ and $a_{\alpha}$ are approximated in the same manner, i.e.

$$\delta x \approx \sum_{I} N_I \delta x_I.$$

and

$$\delta a_{\alpha} \approx \sum_{I} N_{I, \alpha} \delta x_I.$$  

These approximations are inserted into weak form (8). The resulting non-linear equations are solved iteratively using the Newton-Raphson method. For this the system is fully linearized [2, 6]. Further details can be found in [2, 7].

4. A numerical example

We consider a circular disk, with radius $L$ with an applied point load in the middle. The rotation at the boundary of the disk is restricted, but in-plane motions are allowed. The point load is induced by a prescribed displacement at the center. The material parameters are $k = -0.7 k$, $K = 20000 k/L^2$ and $\sigma = 200 k/L^2$. Parameters $k$ and $L$ are used for normalization. Figure 1 shows the deformed liquid shell for a prescribed displacement of $L$. The figure shows, that a thin tube is drawn out of the disk.

![Figure 1: Drawing of a tube from a liquid shell [7]. The coloring shows the mean curvature $H/L$.](image)

5. Conclusion

The paper presents a theory and numerical formulation suitable for liquid shells. The formulation is based on $C^1$-continuous finite element discretization and is capable of analyzing challenging numerical examples.

References


Optimization of foam-filled grooved circular tubes for energy absorption using response surface method

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Abstract

The main objective of the research is improving the design and performance of the polyurethane foam-filled thin-walled aluminum grooved circular tubes using the multi-objective optimization (MOO) technique. The tubes are shaped with the inner and the outer circular grooves at different positions along the axis. In the study, several numerical simulations using ABAQUS finite element explicit code are performed to study the energy absorption characteristics of these structures. The effects of the groove distance, tube diameter, grooves depth, foam density, and tube thickness are investigated on the crashworthiness parameters of grooved circular tubes. An MOO technique is carried out using finite-element analysis (FEA) and response surface methodology (RSM), for two crashworthiness parameters termed as the specific energy absorption (SEA) and the crush force efficiency (CFE). A finite-element analysis is performed with the design of experiments technique (DOE) at the different combinations of the design parameters. Finally, the optimal density of polyurethane foam and geometric parameters of circular grooved tube are obtained based on the objectives of the design.

Keywords: grooved tubes, crush force efficiency, specific energy absorption, multi-objective optimization (MOO), finite element analysis, design of experiments (DOE)

1. Introduction

Thin-walled metal tubes as energy absorption systems were widely used in cars, trains, ships and elevators due to their ability for high energy absorption, lightness, easy to fabricate, low price, and stable during crushing. The efforts made so far by several researchers to improve the stabilization of the collapse process, to reduce the peak load magnitude at the initial stage of the collapse process and to increase the length of progressive crushing of thin-walled structures under axial load. Rezvani, Nouri and Rahmani [1] investigated the effect of geometry and number of internal and external grooves on the behaviour of the thin-walled cylindrical tubes under axial compression. The results showed that the load-displacement curves and absorbed energy can be controlled by the introduction of grooves with different distances and shapes. Niknejad et al. carried out experimental test and theoretical model to predict the mean crushing load, total absorbed energy per unit of tube length and specific absorbed energy per unit of total mass by the polyurethane foam-filled grooved tubes under axial compression. [2]. They concluded that the absorbed energy by the folding process of the polyurethane foam-filled tubes can be controlled by the introduction of grooves of different distances. Mirzaei et al. [3] performed a multi-objective optimization of cylindrical aluminum tubes under axial impact load. They considered specific absorbed energy and the maximum crushing force as objective functions and diameter, length and thickness as design variables.

In the paper optimization of thin-walled grooved cylindrical tubes filled with polyurethane foam is done using a multi-criteria decision-making technique and a response surface method. Geometric parameters affecting their crashworthiness are determined too.

2. Methodology

In the study, a multi-objective optimization technique is performed to optimize specific energy absorption (SEA) and crush force efficiency (CFE) of foam-filled grooved tubes. Design of computational experiments was created by Design-Expert software [4], response surface method, then numerical simulation were carried out using explicit finite element software ABAQUS. Equation (1) shows design constraints and objectives.

\[
\begin{align*}
\min f_1 &= \text{SEA}(t', d, \rho, \lambda, D_i) \\
\min f_2 &= \text{CFE}(t', d, \rho, \lambda, D_i)
\end{align*}
\]

subjected to:

\[
\begin{align*}
0.6 \leq t' \text{(mm)} &\leq 1.7 \\
0.4 \leq d \text{ (mm)} &\leq 1.3 \\
60 \leq \rho \text{ (kg/m}^3\text{)} &\leq 300 \\
5 \leq \lambda \text{ (mm)} &\leq 16 \\
42 \leq D_i \text{ (mm)} &\leq 92
\end{align*}
\]

where \(t'\) is the thickness of tube in groove, \(d\) groove depth, \(\rho\) foam density, \(\lambda\) grooves distance, \(D_i\) internal diameter of tube. Also the constants of designing include tube material, tube length \(L\), the groove shape and groove width \(w\).
The details of the specimen dimensions are given in Fig. 1.
3. Results

Analysis of variance (ANOVA) was used to find significant terms for objective functions. Equations (2) and (3) show the significant terms and related coefficients, for both SEA and CFE.

\[
\text{Max } f_{\text{SEA}} = 18.49 + 1.05 \times t - 0.92 \times d + 2.19 \times D
+ 1.37 \times \rho - 1.8 \times D
\]

\[
\text{Max } f_{\text{CFE}} = 60.91 - 5.16 \times t + 9.33 \times d + 15.79 \times \rho +
- 18.54 \times D - 5.40 \times d + 4.86 \times t \times \rho +
- 1.58 \times t + 2.14 \times t \times D = 0.31 \times d \times \rho +
- 9.52 \times D \times D - 2.89 \times \rho \times D + 0.78 \times t^2 +
+ 3.97 \times d^2 + 2.68 \times \rho^2 + 13.6 \times \rho^2 + 2.36 \times D^2
\]

Table 1 shows optimal design with the same importance for the design objectives.

<table>
<thead>
<tr>
<th>t' (mm)</th>
<th>d (mm)</th>
<th>(\rho) (g/cm³)</th>
<th>(\lambda) (mm)</th>
<th>(D_i) (mm)</th>
<th>SEA (kJ/kg)</th>
<th>CFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.37</td>
<td>1.03</td>
<td>230.44</td>
<td>8.19</td>
<td>56.49</td>
<td>21.2312</td>
<td>117.74</td>
</tr>
</tbody>
</table>

Also, Fig. 2 shows the load-displacement curve and the final shape of collapse for optimal sample. As can be seen, with the design of a bumper according to specified parameters, a stable collapse with increasing crashworthiness can be established on the foam-filled grooved circular tube.

4. Conclusions

The quasi-static axial collapse response of foam-filled circular grooved tubes was investigated using finite element software ABAQUUS. In this study, the effect of design variables including the tube thickness in groove, the groove distance, the groove depth, the internal diameter of tube and the density of polyurethane foam were estimated on the response of specific energy absorption and crush force efficiency by using numerical simulation. Then, an multi-objective optimization (MOO) technique was carried out using response surface methodology (RSM). The results showed that:

- The foam density, the inside diameter of tube, the distance between grooves and tube thickness in the groove were respectively and linearly associated with specific energy absorption rate. By increasing the internal diameter of the tube for energy absorption was reduced. But, by increasing the foam density, the distance between grooves and thickness of tubes in the groove the specific energy absorption increased.
- The distance between grooves, foam density, groove depth, the inner diameter of tube and tube thickness in the groove a effect the efficiency of crashworthiness by second degree model. Increasing the distance between grooves and the tube thickness in the groove, efficiency of crashworthiness was decreased. On the other hand, efficiency of crashworthiness was increased by increasing the foam density, inner diameter of tube and groove depth.
- The interaction between grooves and the groove depth were effective on the efficiency of crashworthiness.

Implementation of such an analysis for solid structures and other sections, such as square or rectangular can also be effective.

References

Nonlinear analysis of functionally graded plates resting on elastic foundation using the higher order plate theory

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Abstract

A thermal post-buckling analysis for perfect and imperfect plates subjected to uniform and non-uniform parabolic temperature loading and resting on a two-parameter elastic foundation is presented. Material properties of the plate are assumed to be graded in the thickness direction, from the ceramic top surface of the plate to the metal bottom plate. The results based on the third order shear deformation plate theory are compared with those obtained for classical plate theory (CPT) and first-order shear deformation plate theory (FGM). The influences of the side-to-thickness ratio, gradient index and elastic foundation parameters on the buckling temperature are discussed. The problem has been solved using a new 16-noded Mindlin plate element of the Lagrange family which is free from shear locking and a 32-noded zero-thickness interface element representing the response of the foundation.

Keywords: Buckling problem, FGM plates, elastic foundation, finite element method

1. Introduction

In recent years, a new class of composite materials known as functionally graded materials (FGMs) has taken considerable attention. The FGMs are heterogeneous composite materials made of a mixture of ceramic and metal constituents in which the material properties vary gradually throughout the thickness from the top to the bottom surface. Excellent characteristics of ceramics in heat and corrosive resistances combined with the toughness of metals allowed the FGMs to be considered as significant reinforcement for high-performance materials in engineering structures subjected to high temperatures. Relatively few studies have been made on the nonlinear structural response of FGM plates and shells resting on elastic foundations [1-3]. The study of Akavci [2] investigated the suitability of the higher-order hyperbolic shear deformation theory for the buckling analysis of FGM plates resting on two parameter elastic foundation in thermal environments. The refined trigonometric, hyperbolic or higher-order plate deformation theories do not require any shear correction factor to satisfy shear-stress boundary conditions on the top and bottom faces of the plate. Swaminathan et al. [3] presented a comprehensive review of analytical and numerical methods employed to study the static, dynamic and buckling behaviour of functionally graded material plates.

The effect of variation of material properties through the thickness, type of load case, boundary conditions, edge ratio, side-to-thickness ratio and the effect of nonlinearity on the buckling of FGM plates has been discussed in the paper. The problem has been solved using a 16-noded Mindlin plate element of the Lagrange family.

2. Problem formulation

2.1. Modelling properties of FGM

A FGM plate is presented in Fig. 1. Material properties are assumed to vary through the thickness:

\[ P = P_m V_m + P_c V_c \]  \hspace{1cm} (1)

where \( P \) is an effective material property, such as Young modulus, density, Poisson’s ratio or coefficient of thermal expansion, \( P_m \) and \( P_c \) represent corresponding properties of metallic and ceramic fraction, respectively and \( V_m \) and \( V_c \) - volume fractions, the ceramic fraction calculated according to the exponential law:

\[ V_c = \left( \frac{1}{2} + \frac{z}{t} \right)^n \quad (n \geq 0) \]  \hspace{1cm} (2)

where and \( n \) is the volume fraction exponent.

Figure 1: FGM plate

*The work has been performed in the scope of project Static and dynamic analysis of layered plate structures resting on elastic foundation which has been financed by Polish National Science Centre (NCN) under contract 2012/05/B/ST6/03086. The support is gratefully acknowledged.
2.2. Finite element formulation

Two shear deformation theories are employed in the present study. In the first order theory displacements are governed by the following relationships

\[ \begin{align*}
u & = z \theta_z; \quad \nu = -z \theta_z \\
\end{align*} \]  

(3)

Accounting for large deflections, initial deflections \( w_I \) and temperature increment, the equation for strain component \( E_{xx} \), is written:

\[ E_{xx} = w_{xx} + w_{x}^t w_{xx} + \frac{1}{2} w_{x}^t w_{xx} - \alpha \Delta T \]  

(4)

We also note that formulating the constitutive relationships, the first order shear deformation theory calls for a shear correction factor to represent correctly shear stresses.

The following displacement functions are used in the third-order shear deformation theory, developed by Reddy [4], on the assumption of zero shear stresses at the top and bottom of a plate

\[ u = -\frac{4z^3}{3M} w_{xx} + z \left( 1 - \frac{4z^2}{3M} \right) \theta_z; \quad v = -\frac{4z^3}{3M} w_{xx} - z \left( 1 - \frac{4z^2}{3M} \right) \theta_z \]  

(5)

The strain is therefore

\[ E_{xx} = \frac{4z^3}{3M} w_{xx} + z \left( 1 - \frac{4z^2}{3M} \right) \theta_z + w_{x}^t w_{xx} + \frac{1}{2} w_{x}^t w_{xx} - \alpha \Delta T \]  

(6)

Starting with the principle of virtual work, employing strain and stress rates and a finite element approximation, including the formulations for a two-parameter elastic foundation, we arrive at the final equation in the form

\[ \begin{align*}
\begin{bmatrix} \mathbf{K}_{ip}^{(p)} + (\mathbf{K}_{ip}^{(f)} + \mathbf{K}_{ip}^{(t)}) \end{bmatrix} \mathbf{d}_{ip}^{(0)} &= \begin{bmatrix} \mathbf{r}_{ip}^{(p)} \end{bmatrix} - \Delta t \mathbf{F}_{ip}^{(t)} \\
\end{align*} \]  

(7)

where \( \mathbf{K}_{ip}^{(p)} \) is a plate stiffness matrix, \( \mathbf{K}_{ip}^{(f)} \) is a stiffness matrix of the elastic foundation, \( \mathbf{r}_{ip}^{(p)} \) is an internal force vector, \( \mathbf{r}_{ip}^{(t)} \) is a vector of thermal loading, \( \Delta t \mathbf{d}_{ip}^{(0)} \) is a nodal displacement vector, and \( \Delta t \) is an incremental parameter.

Equation (7) was solved using the Newton-Raphson method employing the constant arc-length algorithm.

3. Numerical examples

Nonlinear behaviour of FGM plates subject to temperature rise, for the initial temperature \( T_0 = 300[K] \), is presented. The constant temperature across the plate thickness has been applied in the calculations. The data of the plates follow the data set given by Zhao et al. [5] for a combination of aluminium and alumina. In Fig. 2 comparison between first- (FSDT) and third-order (TSDT) shear deformation theories is presented for a thin plate \( b/t=1 \), for two values of exponent \( n \) in Eqn (2); \( n=0 \) and \( n=2 \). A slight difference of results of two-theories can be observed for thick plate \( b/t=10 \) (Fig. 3). The influence of the exponent on the temperature-displacement for a third-order theory is presented in Fig. 4.

4. Conclusions

A nonlinear problem of FGM plates subject to temperature rise was formulated in the framework of the finite element method using first and third-order shear deformation theory. Numerical results proved convergence of results using the two theories for thin plates and confirmed good performance of the new 16-noded finite element in nonlinear analysis.

Figure 2: FGM plate response to temperature, plate \( b/t=1 \)

Figure 3: FGM plate response to temperature, plate \( b/t=10 \)

Figure 4: FGM plate response to temperature, plate \( b/t=10 \)

References


Topological optimization of formworks meshes for free-form surfaces

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Abstract

Application of the so called free-forms in architecture became very popular recently as CAD tools like Rhino 3D made it possible to shape forms by operating on curved surfaces. However, if the designed surface structure is a reinforced concrete shell, there are serious difficulties in execution. There is lack of sufficient tools and methods to design a proper formwork for such structures. In the reference object [1] [2], the doubly curved concrete walls required construction of a complex steel structure, a developed and time-consuming adjustment of geometry and the application of sprayed concrete. An alternative approach proposed in this paper is based on structural properties of polyhedral surfaces. A certain group of polyhedra retains geometric invariance, i.e. stability, if manufactured in the form of panels (corresponding to the polyhedron facets), hinge-connected along the edges. These are the so called plate-stable polyhedra. This paper describes the implementation of optimization process (variational shape approximation) to the approximation of the continuous, smooth doubly curved surface with the use of panels equivalent to facets of a plate-stable polyhedron.

Keywords: free-form surfaces, 3-valent meshes, plate-stable polyhedra, hinge connection

1. Introduction

Transformation of free-form surfaces into meshes, called “discretisation”, is an established method of determining formwork panels for concrete walls based on these surfaces. The challenge is to distribute mesh faces over surface, so that the error between assumed surface and realised concrete wall is minimised. Also, the formwork should be easily assembled on construction site and stable during the assembly process and during concrete casting. The problem is to find optimal algorithm to adjust the mesh vertices and the application of sprayed concrete. An alternative approach proposed in this paper is based on the approximation of the continuous, smooth doubly curved surface with the use of panels equivalent to facets of a plate-stable polyhedron.

2. Format

2.1. Plate stability of polyhedral surfaces

In the work on structural stability of polyhedra Ture Wester [5], states that any general (i.e. abstract in topological meaning) polyhedron in a form of hinge connected faces (plates) is structurally stable, only if it is 3-valent (vertex valency). This means that for each of its vertices exactly three edges and exactly three plates are incident with this vertex. An example of such plate-stable polyhedron is a dodecahedron – its stability can be achieved connecting rigid plates by hinged connectors placed along edges.

The main problem with construction of such systems is to develop a proper polyhedral mesh reasonably well approximating the original smooth surface. In the case of triangular discretization of a free-form surface every vertex lays on that surface, and the corresponding triplets of vertices always make a planar face, whereas in a 3-valent system, the plates are tangent to the assumed surface, but their vertices result from intersection of triplets of neighbouring plates, the plates themselves are polygons of more than three edges and vertices.

2.2. Offsetability

The property of offsetability is important due to the thickness of a constructed and a thickness of the applied plates. The ability of offset, described in [3], means that the topology of polyhedron is preserved. A polyhedron and its offset show pairs of plates that are parallel, distant by a constant value. In triangular meshes the offset can cause additional intersections between faces that have been previously connected by a single vertex [6]. The property of offsetability appears in 3-valent polyhedra. Thanks to it, the second side of formwork can be easily generated and the wall thickness is maintained. Additionally, in order to be able to fit formwork elements, each plate has to be chamfered along each edge by an angle expressed by the formula:

\[ \alpha = \frac{\pi - \beta}{2} \] (1)

where \( \alpha \) is a chamfering angle and \( \beta \) is an angle between a chamfered plate and a neighbouring plate corresponding to each edge. By means of offsetting a polyhedron a new set of edges, parallel to corresponding edges of the previous polyhedron, is created. Surfaces connecting the corresponding edges are instantly the same as the made by chamfering, so they meet the formula (1).

2.3. Assembly process

Assuming the 3-valent system, all plates can be performed by CNC Cutting Machine, e.g. in plywood, providing the correct angles of all edges. In order to simplify the assembly process the assumed topology of connections have to be marked on each plate i.e. each plate with its number and each edge with the number of its neighbouring plate. As described in section 2.1, 3-valent topologies guarantee plate-stability. If properly assembled, formworks will recreate proper geometry without additional survey works. For each triple of connected plates, the
angle deficiency (according to Descartes equation sum of angles on faces incident with one vertex is less than \(2\pi\)) determines convexity of an apex at the intersection of these connections. Thus connections along edges do not necessarily counteract against bending between plates. Consequently, hinge connectors can be applied and one type of connector is sufficient for any angle of connection.

3. Theoretical base

3.1. Variational shape approximation

The discretisation of free-form surface refers to Variational Shape Approximation process [4]. The VSA algorithm was introduced in computer graphics for the simplification of complex triangular meshes and for architectural design purposes in [3]. Basically the algorithm distributes proxy points over the free-form surface, where faces of final polyhedra are tangent with that surface. The distribution refers to plane curvature so that proxies are denser in areas, where the curvature has greater values what minimizes the error of discretization. Precisely, the algorithm is an iterative process of assigning faces of dense triangular surface discretization into clusters. The process starts with random proxy distribution, the number of proxies refers to plane curvature so the proper selection of triangle triplets is a subject of our future study, as well as the outcome of the suggested error metric (4).

3.2. Enhancement

During the tests on VSA algorithm implemented in Rhino3D software, an issue, also mentioned in [3] occurred. In the areas of a surface with a negative Gaussian curvature, the faces of discretized polyhedron are concave polygons. In the defined conditions these polygons can be self-intersecting. Our observations, and the conclusions from [3] suggest that it is caused by exclusion of the distance between the planes consisting two neighbouring proxies in the formula (2). Hence our proposal for the enhanced error metric is:

\[
\|n_i - N_i \| T_i \sum_{t \in k} R_i
\]

where \(n_i\) is a triangle normal vector, \(T_i\) is its area and \(N_i\) is proxy normal vector. One triangle error can be calculated with many proxies if it neighbours with them or their clusters. The smallest error indicates triangles assigned to appropriate clusters. After assigning all triangles to clusters, the proxies are re-distributed within their clusters. The best fitting triangle for new proxy is the one with a normal vector closest to a vector given by a formula:

\[
\sum_{t \in k} \| T_i \| n_i
\]

where \(R_i\) is a set of triangles of one cluster. The process of distortion minimising flooding and redistributing proxies repeats until the best spots of plate tangency are found.

3.3. Approximation of polyhedral mesh

Finally, polyhedral discretization is achieved assigning intersections between triplets of planes given by proxy triangles. These intersection points around proxies create planar, convex or concave polygons which can be used as outlines of formwork plates. The proper selection of triangle triplets is a subject of our future study, as well as the outcome of the suggested error metric (4).

4. Conclusions

The proposition presented in the paper to improve the VSA algorithm brings about the possibility of implementing a new effective method for the design and construction of self-stable formwork for doubly curved surfaces.

References

Buckling of sandwich conical shells including the plasticity and unloading

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Abstract

The paper presents the analysis of stability and the stress state in a sandwich open conical shell panel. The shell under consideration is geometrically symmetric and consist two load-carrying, isotropic facings and a lightweight core layer. Lateral pressure and longitudinal force are the external loads acting the shell. It is possible that, the effective stress in the facings can exceed the yield stress and the shell deforms into the plastic range before the buckling. The stability equations are obtained by the strain energy approach. Final solution is achieved using the Ritz method and the iterative numerical algorithm. The results obtained from the analysis show that during stability loss, the effective stress may temporarily decrease. Then the unloading and hardening phenomena could occur and this may influence the equilibrium paths.

Keywords: buckling, unloading, stress, deformation, stability

1. Introduction

Thin-walled symmetrical sandwich structures detect high structural efficiency, so they are interesting for designers of lightweight, modern constructions because of a comparatively high the load-to-weight ratio. The main advantages of these structures are high bending stiffness and high compressive buckling strength on one hand and comparatively low weight on the other. Strength and stability of conical shells are considered by many authors. Works refer to single-layer shells [1, 2], as well as bi-layered [4] and sandwich ones [5 - 8]. Various analysis methods of sandwich structures are presented in [3] and a large reference list is given there. The majority of publications accept initial assumptions concerning changes in stress values. Most publications assume the active loading process (Shanley approach) [10 - 13]. If the effective stress temporarily decreases during stability loss, the unloading and hardening of the material after stability loss may influence the achieved equilibrium paths. An analysis of this case is the main objective of this paper.

2. The object of study

The analyzed object is a sandwich conical shell panel (Fig. 1). The shell under consideration consists of two thin facings and one core layer between them, which is much thicker than the facings. The facings are geometrically and physically symmetrical, so they are of equal thicknesses and exhibit the same material properties (steel sheet).

2.1. Initial assumptions

The initial assumptions, which were taken with respect to the foregoing shell model, are as follows:

– the shell is shallow, thin-layered and the general theory of thin-layered shells and the geometrically nonlinear theory are obligatory;
– the elastic-plastic properties of the faces are obligatory and the bilinear stress-strain relation for the facings is accepted;
– the pre-buckling stress state is the membrane stress state;
– the Kirchhoff-Love hypothesis is valid within the entire cross-section of the shell;
– the post-buckling stress state can be either elastic, elastic-plastic or plastic;
– the constitutive relations of the Nadai-Hencky deformation theory with the Huber-Mises-Hencky yield condition are accepted in the analysis;
– there are no imperfections in the considered shell.

Figure 1: The shell under consideration: (q, Na, Nb) external loads

2.2. Loading and stresses

The longitudinal forces \((N_a, N_b)\) applied at the edges of the shell and the surface pressure \(q\) directed perpendicularly to the shell main surface are the loads acting on the shell under consideration (Fig. 1). According to the main assumptions, there is a membrane stress state in the shell. The internal forces in that stress state vary on the \(x\)-coordinate and are described as follows:

\[
N_x = \frac{1}{2}q \cdot x \cdot \tan(\alpha) \left( x / l \right)^2 - 1 \cdot N_y(x / l),
\]

\[
N_y = -q \cdot x \cdot \tan(\alpha).
\] (1)

The effective stress in the pre-buckling state of stress can accordingly be expressed by the external forces:

\[
\sigma_t = \frac{q \cdot x \cdot \tan(\alpha)}{4b} \left( k^2 - 2k + 4 \right),
\]

\[
k = 1 - \left( x / l \right)^2 \cdot \left( 1 - 2x / \tan(\alpha) \right), \ K = N_a / (q \cdot x) = \text{const}.
\] (2)
2.3. Stability analysis

In order to derive the stability equations the variation strain energy method was accepted and the Ritz method was used to solve the stability equations. Geometrical relations for the displacement components: $u, v, w$ of the shell were described by the so-called broken line approach [5,6,8]. The strains and changes in the curvature are expressed depending on the displacements, using non-linear geometrical relations given in [8]. The effective stress in the shell facings can exceed the yield stresses for the material of these facings. When this occurs, the constitutive relations of the Nadai-Hencky deformation theory are used to obtain stability equations. The virtual work principle and the strain energy methods comprise a basis for obtaining equilibrium equations for the considered shell. Using the displacement functions that satisfy the boundary conditions concerning the free-supported shell in the non-linear geometrical equations, a full description of the shell potential energy is obtained. Next applying the Ritz method, a set of algebraic equations for the considered shell were obtained. Solving that set of equations yields the final solution in the form of a non-linear algebraic equation:

$$
q = \frac{e_1 A_1 + e_2 A_2^2 + e_3 A_3^3}{A_4 e_4 \kappa + e_5}
$$

where: $e_i$ are the coefficients of the stability equation which among others depend on external loads, $A_i$ is the free parameter of the deflection function, $\kappa = \text{Eq.}(2)$.

The solution was obtained numerically by means of iterative methods, because the coefficients $e_i$ depend on the external loads. Therefore, the iterative methods were and a special computer algorithm to perform numerical calculations and elastic-plastic analysis was developed.

3. Numerical results

The input data for the calculations are presented in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the shell –</td>
<td>$L$</td>
<td>m 1,4</td>
</tr>
<tr>
<td>Smallest radius of the shell –</td>
<td>$r_1$</td>
<td>m 2,2</td>
</tr>
<tr>
<td>Thickness of the core –</td>
<td>$2c$</td>
<td>mm 16</td>
</tr>
<tr>
<td>Thickness of the facings –</td>
<td>$b$</td>
<td>mm 0,8</td>
</tr>
<tr>
<td>Angle of inclination –</td>
<td>$\alpha$</td>
<td>deg. 30</td>
</tr>
<tr>
<td>Radial angle –</td>
<td>$\beta$</td>
<td>deg. 30</td>
</tr>
<tr>
<td>Young’s modulus (facings) –</td>
<td>$E$</td>
<td>MPa 210 000</td>
</tr>
<tr>
<td>Tangent modulus (facings) –</td>
<td>$E_t$</td>
<td>MPa 30 000</td>
</tr>
<tr>
<td>Yield stress (facings) –</td>
<td>$\sigma_p$</td>
<td>MPa 320</td>
</tr>
<tr>
<td>Shear modulus (core) –</td>
<td>$G_3$</td>
<td>MPa 16</td>
</tr>
<tr>
<td>Dimensionless deflection parameter – $\kappa$</td>
<td>–</td>
<td>400</td>
</tr>
</tbody>
</table>

In Fig. 2. there are sample diagrams of the equilibrium paths made for the shell described by parameters given in Tab. 1. Three types of the equilibrium path could be observed there. The first (dashed line) was achieved by the analysis made with active loading process. The second (solid line) was achieved by the analysis, when the unloading process and hardening of the material after stability loss were taken into account. The third (dotted line) is the approximation line for the second path.

4. Conclusions

The work presents changes in the form of equilibrium path for different assumptions about the loading process. If active loading process is assumed, the critical loads can be determined and we can observe increasing of the deflection without increasing of the load or even with decreasing of the load. If the unloading process is taken into account, increasing of the deflection occurs with continuous increase of the load.

References

A new tolerance model of dynamic problems for thin biperiodic cylindrical shells

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Abstract

The objects of consideration are thin linearly elastic Kirchhoff-Love-type circular cylindrical shells having a periodically micro-inhomogeneous structure in axial and circumferential directions (biperiodic shells). The aim of this note is to formulate a new non-asymptotic averaged model for the analysis of selected dynamic problems for these shells. The, so-called, general tolerance model is derived by means of a certain extended version of the known tolerance modelling of micro-heterogeneous media proposed by Tomczyk and Woźniak in Ref. [1]. Contrary to the starting exact shell equations with highly oscillating, non-continuous and periodic coefficients, governing equations of the tolerance model have constant coefficients depending also on the period of inhomogeneity. Hence, the model makes it possible to investigate the effect of a cell size on the global shell dynamics (the length-scale effect).

Keywords: micro-periodic thin cylindrical shells, mathematical modelling, dynamic problems, length-scale effect

1. Introduction, modelling procedure and tolerance model

Thin linearly elastic Kirchhoff-Love-type circular cylindrical shells with a periodically micro-inhomogeneous structure in circumferential and axial directions are analysed, Fig. 1. Shells of this kind are termed biperiodic.

The properties of such shells are described by highly oscillating and non-continuous periodic functions, so the exact equations of the shell theory are too complicated to apply to investigations of engineering problems. That is why a lot of different approximate modelling methods for shells of this kind have been proposed. Periodic cylindrical shells (plates) are usually described using homogenized models derived by means of asymptotic methods. Unfortunately, in the models of this kind the effect of a cell size (called the length-scale effect) on the overall shell behaviour is neglected.

In order to analyse the length-scale effect in selected dynamic or/and stability problems, the new averaged non-asymptotic models of periodic cylindrical shells with a periodic micro-heterogeneity either along two directions tangent to the shell midsurface (biperiodic structure) or along one direction (uniperiodic structure) have been proposed and discussed by Tomczyk in a series of publications and summarized as well as extended in Ref. [2]. These, so-called, tolerance models have been obtained by applying the non-asymptotic tolerance averaging technique presented and discussed by Woźniak in many monographs and papers; an extended list of these publications can be found in Ref. [1,2]. Governing equations of the tolerance models have coefficients which are constant or slowly-varying and depend on a cell size. An extended version of the known tolerance modelling technique has been proposed by Tomczyk and Woźniak in Ref. [1]. This version is based on a new notion of weakly slow-varying functions. This notion is a certain extension of the known concept of slowly-varying functions, cf. Ref. [1,2].

In contribution presented here, using the notion of weakly slow-varying functions we will formulate a new tolerance non-asymptotic model (the general tolerance model) for the analysis of dynamic problems for the biperiodic shells under consideration. The differences and similarities between this new general model and the corresponding known standard tolerance model derived by means of the concept of slowly-varying functions (cf. Ref. [2]) will be discussed.

Figure 1: Example of biperiodic cylindrical shell

We assume that $x^1$ and $x^2$ are coordinates parametrizing the shell midsurface $M$ in circumferential and axial directions, respectively, Fig. 1. Define $\Omega = (0, L_2) \times (0, L_1)$ as a set of midsurface parameters $x = (x^1, x^2)$ in $R^2$, where $L_1, L_2$ are length dimensions of $M$. Sub- and superscripts $\alpha, \beta, \ldots$ run over 1,2 and are related to $x^1, x^2$, summation convention holds.

Partial differentiation related to $x^a$ is represented by $\partial_a$. Moreover, it is denoted $\partial_{a,b} = \partial_{a} \cdot \partial_{b}$. Let $a^{\alpha\beta}$ stand for the contravariant midsurface first metric tensor.

Let $d(x)$ and $r$ be the shell thickness and the midsurface curvature radius, respectively. The time coordinate is denoted by $t$, $t \in I = [t_1, t_2]$. Let $\lambda_1$ and $\lambda_2$ be the period lengths of the shell structure respectively in $x^1$- and $x^2$-directions, cf. Fig. 1. Define the basic cell $\Delta = [-\lambda_1/2, \lambda_1/2] \times [-\lambda_2/2, \lambda_2/2] \subset R^2$.

The diameter $\lambda = \sqrt{(\lambda_1)^2 + (\lambda_2)^2}$ of $\Delta$ is assumed to satisfy conditions: $\lambda /d_{max} \gg 1$, $\lambda /r \ll 1$ and $\lambda /\min(L_1, L_2) \ll 1$. Hence, the diameter will be called the microstructure length parameter.
Let \( u_a = u_a(x,t), \quad w = w(x,t), \quad (x,t) \in \Omega \times I \) denote the shell displacements in directions tangent and normal to \( M \), respectively. Elastic properties of the shell are described by shell stiffness tensors \( D^{(o)}(x) \), \( B^{(o)}(x) \). Let \( \mu(x) \) stand for a shell mass density per midsurface unit area.

It is assumed that the behaviour of the shell under consideration is described by the action functional determined by Lagrange function \( L \) being a highly oscillating function with respect to \( x \) and having the well-known form
\[
L = \frac{1}{2}(D^{(o)} \dot{u}_a \dot{u}_a + D^{(o)} \dot{w} \dot{w} + B^{(o)} \dot{w} \dot{w} - \mu u_a u_a - \mu w^2).
\]

(1)

Applying the extended version of the known tolerance modelling technique, we obtain the averaged form of Lagrange function (1). Then, using the principle of stationary action, we arrive at the tolerance model equations.

The fundamental concepts of the modelling approach under consideration are those of two tolerance relations between points and real numbers determined by tolerance parameters, weakly slowly-varying functions and the averaging operation. A continuous, bounded and differentiable function \( F(\cdot) \) defined in \( \Omega \) (interior of \( \Omega \)) is called weakly slowly-varying of the R-th kind with respect to cell \( \Delta \) and tolerance parameters \( \delta, F \in W^{1,2}(\Omega, \Delta) \), if can be considered constant (together with its derivatives up to the R-th order) on an arbitrary cell. Nonnegative integral \( R \) is assumed to be specified in every problem under consideration. Note that the main difference between the weakly slowly-varying and slowly-varying functions is that the products of derivatives of weakly slowly-varying functions and micro-structure length parameter \( \lambda \) are not negligible small.

The averaging operator for an arbitrary function \( f(\cdot) \) being integrable and bounded in every cell is defined by
\[
< f > (x) = \frac{1}{|\Omega|} \int_{\Omega(x)} f(z) \, dz, \quad z \in \Delta(x), \quad x \in \Omega.
\]

(2)

The tolerance modelling is based on two assumptions. The first of them is called the tolerance averaging approximation (tolerance relations which are strictly related to the concept of weakly slowly-varying functions), cf. Ref. [1,2]. The second one is termed the micro-macro decomposition. In the problem under consideration, the micro-macro decomposition of displacements \( u_a, w \) is assumed in the form
\[
\begin{align*}
\hat{u}_a(x,t) & = u_a(x,t) + h(x) \hat{U}_a(x,t), \\
\hat{w}(x,t) & = w(x,t) + \hat{g}(x) \hat{W}(x,t),
\end{align*}
\]

where averaged variables \( \hat{u}_a, \hat{w} \) and fluctuation amplitudes \( \hat{U}_a, \hat{W} \) are the new weakly slowly-varying unknowns, i.e. \( \hat{u}_a, \hat{w} \in W^{1,2}(\Omega, \Delta), \hat{w} \in W^{1,2}(\Omega, \Delta) \). Functions \( h(x) \) and \( \hat{g}(x) \) are \( \lambda \)-periodic, continuous and highly-oscillating fluctuation shape functions. These functions are assumed to be known in every problem under consideration. They depend on \( \lambda \) as a parameter and have to satisfy conditions \( -\mu h > 0 = \mu g > 0 \), where \( \mu(\cdot) \) is the shell mass density being a \( \lambda \)-periodic function in \( x \).

In the first step of modelling, we substitute the right-hand sides of (3) into (1). The resulting Lagrangian is denoted by \( L_M \). Then, we average \( L_M \) over cell \( \Delta \) using averaging formula (2) and applying the tolerance averaging approximation. As a result we obtain function \( < L_M > \) called the tolerance averaging of starting lagrangian (1) in \( \Delta \) under micro-macro decomposition (3). In the second step, applying the principle of stationary action to the tolerance averaged action functional determined by means of averaged lagrangian, we obtain the system of Euler-Lagrange equations for \( \hat{u}_a, \hat{w}, \hat{U}_a, \hat{W} \), which can be written in explicit form as

\[
\begin{align*}
\text{– the constitutive equations} & \quad \text{\( N^{(\delta)} := D^{(o)} \hat{u}_a \hat{u}_a + r^2 < D^{(o)} > w^2 + < D^{(o)} > \hat{h} > U, \)} \\
& \quad \text{\( M^{(\delta)} := B^{(o)} \hat{w} \hat{w} + B^{(o)} \hat{g} \hat{g} > W + < B^{(o)} > \hat{g} \hat{g} > \hat{w}, \)} \\
& \quad \text{\( H^\delta := \frac{1}{2} < \hat{h} > D^{(o)} \hat{u}_a \hat{u}_a + r^2 < \hat{h} > D^{(o)} > w^2 + \)} \\
& \quad \text{\( < \hat{h} > D^{(o)} \hat{w} \hat{w} + U_{\delta} - < \hat{h} > J > \hat{U}_{\delta} > U, \)} \\
& \quad \text{\( G := \frac{1}{2} < \hat{g} > B^{(o)} > \hat{u}_a \hat{u}_a + < \hat{g} > B^{(o)} > \hat{g} \hat{g} > \hat{w}, \)} \\
& \quad \text{\( + < B^{(o)} > \hat{g} \hat{g} > \hat{g} \hat{g} > W + < B^{(o)} > \hat{g} \hat{g} > \hat{w} > W, \)} \\
& \quad \text{\( - 4 < B^{(o)} > \hat{g} \hat{g} > \hat{g} \hat{g} > \hat{w} > W + < B^{(o)} > \hat{g} g > \hat{U}, \)} \\
\end{align*}
\]

(4)

\[
\begin{align*}
\text{– the dynamic equilibrium equations} & \quad \text{\( \frac{\partial}{\partial t} \hat{U}_a - \mu \hat{U}_a = 0, \)} \\
& \quad \text{\( \frac{\partial}{\partial t} \hat{W} - \mu \hat{W} = 0, \)} \\
& \quad \text{\( \frac{\partial}{\partial t} \hat{U} + H = 0, \)} \\
& \quad \text{\( \frac{\partial}{\partial t} \hat{W} + G = 0. \)}
\end{align*}
\]

(5)

Equations. (4) and (5) together with the micro-macro decomposition (3) constitute the general tolerance model for analysis of selected dynamic problems for biperiodic shells under consideration. Coefficients of the derived model equations are constant and some of them depend on micro-structure length parameter \( \lambda \) (underlined terms). It can be shown that neglecting the underlined in (4), we obtain constitutive relations of the known standard tolerance model of biperiodic shells under consideration, cf. Ref. [2].

2. Remarks and conclusions

The tolerance modelling technique based on the notion of weakly slowly-varying function is proposed as a tool to derive a new mathematical non-asymptotic averaged model for analysis of dynamic problems for thin cylindrical shells with micro-periodic structure in axial and circumferential directions.

Contrary to exact shell equations with highly oscillating non-continuous periodic coefficients, the tolerance model equations have constant coefficients depending also on a cell size. Hence, the tolerance model makes it possible to describe the effect of a length scale on the global shell behaviour.

The basic unknowns of the tolerance model equations must be the weakly slowly-varying functions in periodicity directions. This requirement can be verified only a posteriori.

The basic unknowns of the tolerance model equations must be the weakly slowly-varying functions in periodicity directions. This requirement can be verified only a posteriori.

References


Stability of hybrid rotating shaft with imperfect boundary conditions

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Abstract

Dynamics of rotating shaft was considered in a weak (variational) form. Dynamics equation of shaft subjected to the axial stochastic force in the weak formulation was derived. The weak form of dynamics equations of the rotating with constant angular velocity is obtained using Hamilton’s principle. The almost sure stochastic stability of beam equilibrium, without the previous discretization, was analysed by means of direct Liapunov method. The stability analysis method is developed for distributed dynamic problems with relaxed assumptions imposing solutions. Sufficient stability conditions were established for the imperfect boundary conditions and two limit cases: the simply supported beam and the clamped beam are obtained.

Keywords: rotating shaft, weak equations, imperfect boundary conditions, Liapunov method, stability analysis

1. Introduction

A strong form of dynamics equations involves irregularities which lead to computational difficulties for estimation and control problems. In order to avoid irregular terms resulting from differentiation of the force and moment terms the dynamics equations are written in a weak form. The weak form of systems is useful for the development of identification methods and general computational methods [1]. We consider rotating shafts described by partial differential equations that include the time-dependent coefficients implying parametric vibrations [3], [5], [6]. The response of such systems lead to a new increasing mode of oscillations and the structure dynamically buckles. The classical Liapunov technique for stability analysis of continuous elements is based on choosing or generating of a functional which is positive definite in the class of functions satisfying structure boundary conditions. The time-derivative of Liapunov functional has to be negative in some defined sense. Almost sure stability of the beam equilibrium state in the strong formulation was examined in [4]. The weak form of a distributed controller in an active system with suitable space characteristics is useful for the feedback theoretical developments. For the purpose of active vibration and noise control SMA and piezoelectric devices have shown great potential as elements of passive absorbers and active control systems, as they are light-weight, small, and can be bonded to or embedded in the main structures. They cannot be modeled by point force excitations, partial differential equations should be used to describe the response of the structures driven by them. The stability of beams and plates in the weak formulation was examined. Sufficient stability conditions were established examining properties of Liapunov functional along the weak solution [9]. An important problem in modeling of structural systems is the determination of more realistic boundary conditions in comparison with the known academic boundary conditions.

In the paper, a model for imperfect conditions is derived under assumption that a slope is admissible at the beam ends. Stability conditions are obtained for the imperfect boundary conditions and the two limit cases: the simply supported shaft and the clamped shaft. The paper examines dynamic stability due to the axial force in the form of stochastic physically possible time-dependent processes with given statistical properties.

2. Weak formulation of rotating shaft dynamics equation

The shaft modelled as a thin-walled symmetrically laminated beam, contains both the conventional fibers oriented at \( +\Theta \) and \( -\Theta \) to the shaft axis and the activated shape memory alloy fibers parallel to the shaft axis. The shaft rotates with a constant angular velocity \( \omega \). Forcing the martensite transformation of the SMA layer we modify the basic mechanical properties such as Young’s modulus \( E \) and internal damping coefficient \( b \). The shaft of length \( l \) is assumed to have a constant annular cross-section of mean radius \( a \), and thickness \( h \) without initial geometric and material imperfections. Mean density is \( \rho \), area and geometrical moment of inertia are denoted by \( A = 2\pi ah \) and \( J = \pi a^3 h \), respectively. A viscous model of external damping with coefficient \( b_e \) is assumed. The beam-like approach [2] is used in order to derive the shaft bending stiffness

\[
AE = (A_{11} - A_{12}^2 / A_{22}) \pi a^3
\]

where \( A_{ij} \), \( i,j=1,2 \) denote in-plane stiffnesses of the thin-walled shaft. Displacements of the center shaft line in the movable rotating coordinates are described by \( u \) and \( v \). Introducing the dimensionless time with the time scale \( k_t \) and the dimensionless coordinate obtained dividing by \( l \) we obtain a shaft model with the unit mass density, unit bending stiffness, dimensionless angular velocity \( \Omega \) and modified damping coefficients \( \beta_1, \beta_2 \). Assuming the axial force in the form \( f_s + f(t) \) and applying the Hamilton’s principle we obtain the weak equations satisfied for all test functions \( \Phi \) and \( \Psi \)

\[
\int_0^l \left[ u_{ss} - \Omega^2 u + 2\Omega v_{ss} \right] \Phi + \beta_1 v_{ss} (u_{ss} + \Omega v) \Phi + + \beta_1 u_{tt} \Phi + u_{xx} \Phi_{xx} + (f_a + f(t)) u_{ss} \Phi \right] dx + + c(u_x (0) \Phi_x (0) + u_x (l) \Phi_x (l)) = 0
\]

\[
\int_0^l \left[ v_{ss} - \Omega^2 v - 2\Omega u_{ss} \right] \Psi + \beta_2 v_{ss} (v_{ss} - \Omega u) \Psi + + \beta_2 v_{tt} \Psi + v_{xx} \Psi_{xx} + (f_a + f(t)) v_{ss} \Psi \right] dx + + c(v_x (0) \Psi_x (0) + v_x (l) \Psi_x (l)) = 0
\]
where $\Phi$ and $\Psi$ are arbitrary sufficiently smooth test functions satisfying essential boundary conditions. There is no demands for existence of derivatives higher than the second order.

Boundary conditions in dimensionless forms are as follows

$$\begin{align*}
    u(0,t) &= v(0,t) = u(t,1) = v(t,1) = 0 \\
    u_{xx}(0,t) &= \varepsilon u_x(0,t) & v_{xx}(0,t) &= \varepsilon v_x(0,t) \\
    u_{xx}(1,t) &= -\varepsilon u_x(1,t) & v_{xx}(1,t) &= -\varepsilon v_x(1,t)
\end{align*}$$

(4)

where $\varepsilon$ is the relative stiffness of boundary angular springs.

3. Stability analysis

The dynamics equations (2) and (3) contain the terms explicitly dependent on time. The time-dependent axial force parametrically excites shafts and increasing forms of vibration can occur. Equations (2) and (3) with zero initial conditions have a trivial solution

$$\begin{align*}
    u(x,t) &= v(x,t) = 0 \\
    u_x(x,t) &= v_x(x,t) = 0
\end{align*}$$

(5)

The trivial solution of Eq. (2) and (3) is almost surely stable if the measure of distance between perturbed solution with nonzero initial conditions and the trivial one tends to zero as time tends to infinity, with probability 1. Usually the measure of distance is defined by a positive-definite functional. In order to examine the almost sure stability of the shaft equilibrium (the trivial solution) the Liapunov functional is chosen in the form

$$V = \frac{1}{2} \int_0^1 \left[ u_t + \Omega v + \beta u \right]^2 + \left[ u_x + \Omega v \right]^2 + \left[ v_t - \Omega u + \beta v \right]^2 + 2f_o \left( u_{xx} + v_{xx} \right) dx + \frac{1}{2} \varepsilon \int_0^1 \left[ u_{xx}(0,t) + u_{xx}(1,t) + v_{xx}(0,t) + v_{xx}(1,t) \right]$$

(6)

The functional is positive-definite if the constant component of axial force is smaller then the critical static force. Therefore, the measure of disturbed solutions is chosen as a square root of functional (6). As trajectories of the solution of Eqn (6) are physically realizable the classical calculus is applied to calculation of the time-derivative of functional (6). Its time-derivative is given by

$$\frac{dV}{dt} = -\beta_e V + U$$

(7)

where the auxiliary functional $U$ is known. Now we look for a function $\chi$ satisfying the following inequality

$$U \leq \chi V$$

(8)

Substituting inequality (8) into Eqn (7) yields the first order differential inequality with solution

$$\|u(x,t)\| \leq \|u(0,x,0)\| \exp \left[ -\beta_e \int_0^t \chi(\tau) d\tau \right] t$$

(9)

Ergodicity of the axial loading leads to the following almost sure stochastic stability condition

$$\beta_e \geq \bar{\chi} = \left( \max_{\chi \geq 0} \bar{\chi} \right)$$

(10)

where $\langle \rangle$ denotes mathematical expectations. It should be noted that the way to obtain Eqn (8) is purely algebraic contrary to systems described by strong equations, where the derivation of stability conditions is based on integration by parts and manipulations with higher order partial derivatives. In order to find $\chi$ effectively we use the expansions of the shaft displacements in the series of functions satisfying imperfect boundary conditions

$$\bar{\chi}(x) = \sin k_x x + \frac{\sinh k_x x - \cosh k_x x}{e^{\cosh k_x x}} (11)$$

where $k_x$ satisfy the appropriate transcendental equation. Functions $\bar{\chi}$ are orthogonal and are useful to compute the function $\chi$.

4. Results

It should be emphasized that Eqn (7)-(10) are obtained without dealing with the third and the fourth spatial derivatives. If the density function for the time-dependent component of axial force is known numerical integration can be used to evaluate Eqn (10) for different values of parameters such as for example variance $\sigma^2$. The solution is obtained in an iterative way as the critical damping is involved in both sides of inequality. The structure of stability conditions for the imperfect clamped beams and simply supported beams is similar. The constant component of axial force has a most significant influence on the critical damping coefficient $\beta_e$. In the region of axial tension, where $\beta_e$ is negative changes of damping coefficient $\beta$ are negligible. The critical damping is very sensitive in region of axial compression particularly if $\beta_e$ is approaching the critical Euler loading. The changes of critical values of damping coefficient are most significant in the region of moderate values of relative spring stiffness.

References


Bending of a rectangular thick plate with respect to its proper weight

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Abstract

A thick elastic rectangular plate is considered. It is assumed that the plate is fixed along its contour. Normal compressive load acts at the lower face, the upper face is unloaded. It is required to estimate the stress and displacements fields in the plate with respect to its proper weight. The proposed method is based on the fact that a function of volumetric expansion is harmonic. The use of integral transformation method reduces the initial problem to the two-dimensional infinite system of linear equations, solved by the reduction method. The applicability of it is proved. The stress state of the plate is investigated in terms of its geometrical sizes.

Keywords: thick rectangular plate, integral transformations method, infinite system, two-dimensional infinite system

1. Introduction

The problems of bending of an elastic rectangle or a thin elastic rectangular plate are well known in the theory of elasticity. A detailed review of different statements and solving methods of such problems are given in Refs. [1, 6]. The problem is complex while given a three-dimensional description. Mechanical behaviour of thick plates of a rectangular thick plates with fixed edges. The idea is based on the fact that a volumetric expansion follows a harmonic function. This fact, combined with the method of the integral transformation leads to a two-dimensional infinite system of linear equations, solved by the reduction method.

2. The problem’s statement

The thick elastic plate (the shear modulus \(G\), Poisson’s ratio \(\mu\)) \(0 \leq x \leq a, \ 0 \leq y \leq b, \ 0 \leq z \leq h\) is considered. It is assumed that the plate edges are fixed

\[
\begin{align*}
&u(0,y,z)=u(a,y,z)=0, \quad u(x,0,z)=u(x,b,z)=0 \\
&v(0,y,z)=v(a,y,z)=0, \quad v(x,0,z)=v(x,b,z)=0 \\
&w(0,y,z)=w(a,y,z)=0, \quad w(x,0,z)=w(x,b,z)=0
\end{align*}
\]

(1)

The face \(z=0\) is subjected to normal pressure, the tangent stresses are equal to zero

\[
\begin{align*}
&\sigma_z(x,y,0)=-p(x,y), \quad \tau_{xz}(x,y,0)=0, \quad \tau_{yz}(x,y,0)=0
\end{align*}
\]

(2)

The face \(z=h\) is stress-free.

\[
\begin{align*}
&\sigma_z(x,y,h)=0, \quad \tau_{xz}(x,y,h)=0, \quad \tau_{yz}(x,y,h)=0
\end{align*}
\]

(3)

It is required to estimate the displacements and stresses in the plate satisfy the boundary conditions (1)-(3) and the system of equilibrium equations

\[
\Delta u + \mu_0 \theta = 0, \quad \Delta v + \mu \theta = 0, \quad \Delta w + \mu \theta = -\gamma G^{-1}, \tag{4}
\]

here \(\mu_0 = (1-2\mu)^{-1}\), \(\gamma\) is the specific gravity, the operator \(\Delta\) denotes the Laplace operator and it is taken

\[
\partial \theta(x,y,z)/\partial x = \Theta(x,y,z), \quad \partial \theta(x,y,z)/\partial y = \Theta(x,y,z), \quad \partial \theta(x,y,z)/\partial z = \Theta(x,y,z)
\]

(5)

The proposed solution method of the stated problem is based on the integral transformations the function

\[
\theta(x,y,z) = u'(x,y,z) + v'(x,y,z) + w'(x,y,z)
\]

(6)

is a harmonic function \(\Delta \theta(x,y,z) = 0\)

(7)

If the displacements \(u, v, w\) are found, the stress are given

\[
\sigma_z = 2G\mu_0 \theta + w', \quad \tau_{xz} = G(w'+u'), \quad \tau_{yz} = G(w'+v')
\]

(8)

The function \(\theta(x,y,z)\) is the additional unknown function, which is connected with the main unknown functions by the formula (6).

3. Reduction to a one-dimensional problem

Finite integral Fourier’s transformation may be applied to the equations (4) with regard to the variables \(x\) and \(y\)

\[
\begin{align*}
&\left[u'_j(y,z), v'_j(y,z), w'_j(y,s)\right] = \int_0^a \left[u(x,y,z), v(x,y,z), w(x,y,z)\right] dx, \quad a = \frac{j\pi}{a} \\
&u'_j(z), v'_j(z), w'_j(z) = \int_0^b \left[u(x,y,z), v(x,y,z), w(x,y,z)\right] dy, \quad b = \frac{k\pi}{b}
\end{align*}
\]

(9)

The finite cos-integral transformation is also used

\[
\begin{align*}
&u'_j(y,z) = \int_0^\beta \cos \alpha y u(x,y,z) dx, \quad u'_j(z) = \int_0^\beta \cos \alpha y u(x,y,z) dy
\end{align*}
\]

(9)

In the transformations domain (9) the system of the one-dimensional ordinary equations is obtained instead of the initial system (4):
Analogical transformations are executed with the equation (7). As a result, the boundary value problem for the volume expansion function is constructed, and solved exactly with the Fourier method. General solutions of the equations in the system (10) are constructed as the superposition of the general solutions of the homogeneous equations and particular solutions of the inhomogeneous ones. The latter are obtained by means of Green’s function, was constructed by the method proposed in Ref. [3]. Exact solutions of the system (10) are constructed in the transformations (9) domain in the form:

\[
\begin{align*}
\eta^{10} (z) &= \int_0^\infty G_{ji} (z, \xi) [\mu_0 \theta^{10}_{ji} (\xi) - \frac{1}{\gamma_{ji}}] d\xi - \frac{p^{10}_{ji}}{2G} \psi^{10}_{ji} (z) \\
\eta^{11} (z) &= D^{10}_{ji} sh \lambda_{ji} z + D^{10}_{ji} ch \lambda_{ji} z - \alpha_{ji} \mu_0 \int_0^\infty G_{ji} (z, \xi) \theta^{10}_{ji} (\xi) d\xi \\
\eta^{12} (z) &= D^{10}_{ji} sh \lambda_{ji} z + D^{10}_{ji} ch \lambda_{ji} z - \beta_{ji} \mu_0 \int_0^\infty G_{ji} (z, \xi) \theta^{10}_{ji} (\xi) d\xi
\end{align*}
\]

where \( G_{ji} (z, \xi) \) is the constructed Green’s function, \( D^{10}_{ji} i, m = 0,1 \) - are the unknown contacts. These constants are found from the unsatisfied boundary conditions. It leads to the solution of a two-dimensional infinite system of linear algebraic equations. The system is solved with the help of the reduction method, its applicability is proved in scheme Ref. [4].

With the help of the inverse formulas of Fourier transformations the displacements are the series

\[
[u, v, w] = \frac{4}{ab} \sum_{y=1}^{n} \sum_{x=1}^{n} \sin \alpha_j x \sin \beta_i y [u^{1\alpha}_{ji}, v^{1\alpha}_{ji}, w^{1\alpha}_{ji}]
\]

Numerical analyses of the displacements and stress fields are worked out. The stress values are investigated depending on the plate geometrical sizes.
Elastic buckling of corrugated plates

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Abstract

The paper is focused on the elastic buckling of simply supported square plates corrugated in form of circular arcs. The analytical and numerical (FEM) studies were performed. The mass of the plates is assumed to be constant. The influence of the corrugation wave number on the value of critical load is considered.

Keywords: corrugated plate, elastic buckling

1. Introduction

Many components of engineering structures incorporate corrugated panels, particularly in civil, marine and aerospace engineering. The strength and buckling problems of these structures are studied in practice since the mid of the 20th century. Plantema [11], Volmir [13], Allen [1] presented primary monographs for sandwich structures. Libove and Hubka [5] derived elastic constants for corrugated cores of sandwich plates. Briassoulis [2] investigated the equivalent flexural rigidity of plates with sine-wave and rectangular corrugations. Samanta and Mukhopadhyay [12] performed the static and dynamic analyses of trapezoidal corrugated sheets by considering both extensional and flexural rigidities. Cheng et al. [3] used the finite element method to determine the equivalent stiffness of sandwich structures with various types of cores. Peng et al. [10] investigated the equivalent elastic properties of sinusoidal and trapezoidal corrugated plates by means of a mesh-free Galerkin method. Liew et al. [6] used this method for the geometrically nonlinear analysis of corrugated plates. Both the equivalent flexural and extensional properties were considered in the analyses. The analytical and numerical (FEM) investigations related to a sandwich beam with a crosswise or lengthwise corrugated core were conducted in [4,8,9,14]. Magnucki et al [7] investigated bending and buckling problems of orthotropic sandwich beams with three-layer faces.

2. Analytical model

The plate of width \( a \) and length \( b \) is considered square \((a = b = 160 \text{ mm})\). An amplitude of the corrugation is constant and equal to 5 mm. The geometrical parameters of the plate are collected in Table 1., where \( i \) is a number of the waves per the width of the plate, \( a_i \) is a wavelength of the corrugation, \( R_i \) is a radius of the circular arc, \( s_i \) is an unfolded length of one wave of the corrugation, \( t_i \) is thickness of the plate.

<table>
<thead>
<tr>
<th>( i )</th>
<th>( a_i ) [mm]</th>
<th>( R_i ) [mm]</th>
<th>( s_i ) [mm]</th>
<th>( t_i ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>20.00</td>
<td>5.00</td>
<td>251.33</td>
<td>0.6366</td>
</tr>
<tr>
<td>7</td>
<td>22.86</td>
<td>5.77</td>
<td>232.08</td>
<td>0.6894</td>
</tr>
<tr>
<td>6</td>
<td>26.67</td>
<td>6.94</td>
<td>214.50</td>
<td>0.7459</td>
</tr>
<tr>
<td>5</td>
<td>32.00</td>
<td>8.90</td>
<td>198.86</td>
<td>0.8046</td>
</tr>
<tr>
<td>4</td>
<td>40.00</td>
<td>12.50</td>
<td>185.46</td>
<td>0.8627</td>
</tr>
<tr>
<td>3</td>
<td>53.33</td>
<td>20.28</td>
<td>174.60</td>
<td>0.9164</td>
</tr>
<tr>
<td>2</td>
<td>80.00</td>
<td>42.50</td>
<td>166.59</td>
<td>0.9605</td>
</tr>
<tr>
<td>1</td>
<td>160.00</td>
<td>162.50</td>
<td>161.66</td>
<td>0.9897</td>
</tr>
</tbody>
</table>

The arrangement of the load is illustrated in Fig. 2.

![Figure 2: The arrangement of the load](image)

The calculation of buckling load of corrugated plate is based on the solution of the following differential equation:

\[
\text{The presented research results executed under the subject of No. 04/43/DSPB/0082, were funded with grants for education allocated by the Ministry of Science and Higher Education.}\
\]
\[ D_i \left( \frac{\partial^2 w}{\partial x^2} + 2H \frac{\partial^2 w}{\partial x \partial y} + D \frac{\partial^2 w}{\partial y^2} \right) = -N_i \left( \frac{\partial^2 w}{\partial x\partial y} \right) \]

where:

\[ D_i = \frac{a_i}{s_i 12(1-\nu)} \quad \text{and} \quad D = \frac{EJ_y}{a_i} \]

in which \( N_i \) is in-plane load per unit width (Fig. 2), \( J_y \) is the moment of inertia of one wave of the corrugation, \( E \) is Young’s modulus and \( \nu \) is Poisson ratio.

The general solution of the equation (1) satisfying simply supported boundary conditions is in the form:

\[ w(x,y) = w_0 \sin \left( \frac{m \pi x}{a} \right) \sin \left( \frac{n \pi y}{b} \right) \]

Substituting equation (2) into equation (1), gives:

\[ N_i = \left( \frac{m \pi}{a} \right)^2 \left( \frac{m b}{n} \right) D_i + 2H + \left( \frac{n a}{m b} \right) D \]

where \( m \) and \( n \) are the number of half-waves in the directions \( x \) and \( y \), respectively.

The critical load \( N_{cr} \) for a square plate will be obtained by taking \( m = 1 \) and \( a = b \) in equation (3). Hence

\[ N_{cr} = \min \left[ \left( \frac{\pi}{a} \right)^2 \left( \frac{n}{m} \right)^2 D_i + 2H + n^2 D \right] \]

3. Finite element analysis

The analytical results were validated by comparing with the finite element ones. The geometrically and physically linear analysis were carried out by means of ABAQUS software. The geometrically linear analysis was modelled with symmetric boundary conditions. The finite element model is symmetric and therefore only a quarter of the plate is modelled with symmetric boundary conditions.

4. The results of the calculations

Numerical analysis as performed for a steel plate with following mechanical properties \( E = 200000 \text{ MPa}, \nu = 0.3 \) and geometrical properties from Tab. 1. The upper index \( AN \) indicates results obtained from analytical model. The index \( FEM \) indicates ABAQUS results. The results from the finite element analysis are compared with analytical ones and are shown in Tab. 2.

Table 2: The critical load

<table>
<thead>
<tr>
<th>( i )</th>
<th>( N_{cr,AN} )</th>
<th>( N_{cr,FEM} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>978.07</td>
<td>973.36</td>
</tr>
<tr>
<td>7</td>
<td>996.90</td>
<td>985.81</td>
</tr>
<tr>
<td>6</td>
<td>1017.48</td>
<td>999.11</td>
</tr>
<tr>
<td>5</td>
<td>1039.03</td>
<td>1012.28</td>
</tr>
<tr>
<td>4</td>
<td>1057.97</td>
<td>1025.24</td>
</tr>
<tr>
<td>3</td>
<td>1055.82</td>
<td>1037.18</td>
</tr>
<tr>
<td>2</td>
<td>752.51</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>258.10</td>
<td>-</td>
</tr>
</tbody>
</table>

5. Conclusion

The finite element results in Tab. 2 are in good correlation with the analytical ones. The analytical results for \( i = 1 \) and \( i = 2 \) are not considered because the plates for these parameters buckle locally. The maximum value of the critical load occurs for four waves of corrugation.

References

Buckling and vibrations of seven-layer beam with lengthwise corrugated main core – numerical study

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Abstract

The paper is focused on one orthotropic thin-walled sandwich beam with a trapezoidal core and three-layer facings. The inner layers of facings are trapezoidal corrugated – in perpendicular direction to the corrugation of the main core, but outer layers are flat. The beam is with lengthwise corrugated main core and crosswise corrugated inner layers of facings. The finite element analysis (FEA) of the sandwich beam is performed with the use of the FEM system – ABAQUS. The critical load and the natural frequencies of the beam are computed.

Keywords: sandwich structures, orthotropic thin-walled beam, buckling, vibrations

1. Introduction

Many applications require high stiffness and low weight which can be achieved with the use of sandwich structures. Therefore sandwich structures with a various shape of a core are commonly used in many engineering fields. The theory of sandwich structures is presented in many monographs, for example: Allen [1], Plantema [10], Ventsel and Krauthammer [11]. Buannic et al. [2] showed homogenisation method for sandwich plate with corrugated core and verified this method with finite element analysis. Magnucki et al [6] investigated bending and buckling problems of orthotropic sandwich beams with three-layer faces. Isaksson et al. [4] analyzed elastic properties of corrugated board panel. Magnucka-Blandzi [7] presented mathematical modeling of shear effect in sandwich beams with a metal foam core. Liew et al. [6] used this method for the geometrically nonlinear analysis of corrugated plates. Both the equivalent flexural and extensional properties were considered in the analyses. The analytical and numerical (FEM) investigations focused on a sandwich beam with a crosswise or lengthwise corrugated core were conducted in [3,8,9,12]. The present work was also inspired by the results obtained in the paper [5].

2. Numerical analysis

Numerical analysis with the use of ABAQUS code was performed. Dimensions of beam: 200 mm (width), 67 mm (depth) and 2392, 2208, 2024, 1840, 1656, 1472, 1288, 1104 mm (length). The thickness of all sheets equals 0.8 mm. The dimensions of trapezoid are presented in Fig. 2.

Figure 1: Scheme of the sandwich beam

The objective of the study is buckling and vibrations analysis of an orthotropic thin-walled sandwich beam with trapezoidal core and three-layer facings. The outer layers of facings are flat, but inner layers are trapezoidal corrugated – in perpendicular direction to the corrugation of the main core. The beam is with lengthwise corrugated main core and crosswise corrugated inner layers of facings. The metal of the flat or corrugated sheets is isotropic. The finite element model is presented in Fig. 1.

Figure 2: The dimensions of trapezoid

The shell elements S4R of size 4 mm were placed at mid-surface of the flat layers and the corrugated sheet. The interaction between the layers was modeled by means of the definition of a “tie” contact method. The load and boundary conditions were applied with using “coupling” constraints. The arrangement of load and supports is illustrated in Fig. 3.

Figure 3: Scheme of the simply supported beam

The load F₀ applied to the model equals to 1 kN. The material properties correspond to those of steel and have the following

*This work was financially supported by the project funded by the National Science Centre (Poland) allocated on the basis of the decision number DEC-2013/09/B/ST8/00170.
values: Young’s modulus $E = 200000$ MPa, Poisson’s ratio $\nu = 0.3$. The critical load, the buckling shape and the natural frequencies for the family of beams with the lengthwise corrugated core are determined. The results obtained in the numerical (FEM) analysis are collected in Table 1 and shown in Figs. 4–6. The mesh density analysis were carried out until the good agreement between results was obtained (less than 0.5%). Thus, the beam was meshed with 3 mm edge length elements.

Table 1: The critical load and the lowest natural frequency

<table>
<thead>
<tr>
<th>$L$</th>
<th>$F_c$ [kN]</th>
<th>$f$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2392</td>
<td>237.94</td>
<td>30.952</td>
</tr>
<tr>
<td>2208</td>
<td>277.21</td>
<td>36.193</td>
</tr>
<tr>
<td>2024</td>
<td>326.84</td>
<td>42.875</td>
</tr>
<tr>
<td>1840</td>
<td>390.73</td>
<td>51.574</td>
</tr>
<tr>
<td>1656</td>
<td>474.71</td>
<td>63.182</td>
</tr>
<tr>
<td>1472</td>
<td>587.72</td>
<td>79.14</td>
</tr>
<tr>
<td>1288</td>
<td>743.78</td>
<td>101.89</td>
</tr>
<tr>
<td>1104</td>
<td>-</td>
<td>135.86</td>
</tr>
</tbody>
</table>

The critical load for beam with 1104 mm is not considered because the beam for this length buckles locally.

3. Conclusion

The current investigation is a preliminary numerical analysis focused on developing an appropriate finite element model. The calculated results will be verified analytically in the future. The parametric model of the beams is created and the parametric study of the trapezoid geometry will be considered in further analysis.

References


**Strength of a damaged T-joint under Low-Cycle Loading**

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Abstract

The work attempts to undertake evaluate the residual life of a T-joint with a volumetric surface defect (VSD) located at the area opposite to branch, the following cases: inner surface – at zone I, outer surface – at zone III (Fig. 1). Reliability of a stress-strain state (SSS) has assessed by FEA calculations and checked by means of a cyclic test of a full-size T-joint.

Keywords: T-joint; strength; residual life; volumetric surface defect.

1. Introduction

The stress concentration factor (SCF) in VSD may exceed the design stress concentration at the area of pipe and branch intersection. In this case stresses in VSD are under investigation one of possible solutions to estimate damaged T-joint residual life is the direct use of engineering assessment method intended for straight pipelines [3]. This method includes:

- determination of SCF on the basis of VSD and pipeline geometrical parameters;
- definition of a fatigue curve using pipe material strength and plasticity characteristics;
- determination of cycle number of a pipeline during operation;
- calculation of the allowable number of cycles and the residual life on the basis of the abovementioned data.

2. Stress-strain state of T-joint

All the computations quoted by FEA assumed loading of T-joints by inner pressure \( P = 1 \) MPa.

2.1. Undamaged zones I and III

Some stress-strain state distinctive features at zones I and III were investigated based on:

- not-flush T-joint (diameter of pipe and branch are not equal) 159x4.8-108x4 (159x4.8 – diameter x wall thickness of pipe; 108x4 – diameter x wall thickness of branch);
- flush T-joint159x4.8-159x4.8.

In the case of 159x4.8-108x4 joint unevenness of SSS at zones I, III is insignificant, stress rising at zone I amounts to \( \approx 9\% \) and its reduction at zone III – to \( \approx 6\% \). This tendency grows as a diameter of branch approaches pipe dimension. In case of flush joint, stress increases up to \( \approx 75\% \) at zone I and decreases down to \( \approx 40\% \) at zone III.

2.2. T-joint with defect

Basic regularities of influence of defect geometrical parameters on stresses at its surface are given by the example of damage at zone I of 159x6.2-108x5.5 joint. Rapid increment of stresses is observed as the defect depth grows. So in case of the same length - 60 mm and width - 28 mm, the SCF for depths 2, 2.5 and 3 mm amounts to 1.93, 2.45 and 2.97, accordingly. Defect elongation results in stress growth in a similar manner.

![Figure 1: Generatrices designation and stress distribution on the surface of T-joint geometrical model: a – outer surface; inner surface](image)

Comparison of results obtained by FEA calculations, related to straight pipe 159x4.8 and T-joints 159x4.8-108x4, 159x4.8-159x4.8, having inner surface defect of the same size (depth - 3 mm, length - 60 mm, width - 28 mm) revealed that maximum equivalent stress arises in the case of defect location at the inner surface of straight pipeline. Stress concentration in this variant is 1.5 times higher than in T-joint with the same damage at zone I. It is important that diameter of the branch does not significantly influence stresses in defect. Taking into account abovementioned, the residual life estimation of a straight pipeline [3] applicable for evaluation of cyclic strength of T-

*This work was supported by the FP7 project INNOPIPES GA- 2012-318874.
joint with damage in the area opposite to branch, since this approach gives conservative results.

3. Experiment

For the sake of experimental investigation of cyclic strength seamless T-joint 159х6.0-108х5.0 was used manufactured according to [2] from steel 20 [1] (Fig. 2).

A T-joint specimen was subjected to cyclic hydraulic loading by a water injection inside with a pump through inlet nipple. Pressure release was performed through outlet nipple. Loading was performed in an automatic mode with a frequency 1.5 - 2 cycle per minute.

A strain measurement was carried out in the first loading cycles under stepped pressure rising.

A specimen loading history is given in Table 1. After the run of a general number of cycles \( \sum N_i = 17550 \) the loss of pressure had happened due to occurrence of trough wall crack in defect, located in zone III along generatrix \( I_{m} \) (Fig. 1).

Table 1: History of loading

<table>
<thead>
<tr>
<th>( P_{max} )</th>
<th>( N_i )</th>
<th>( e_{\alpha} )</th>
<th>( d_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPa</td>
<td>cycle</td>
<td>( \times 10^4 )</td>
<td></td>
</tr>
<tr>
<td>7.5</td>
<td>9220</td>
<td>0.98</td>
<td>0.35</td>
</tr>
<tr>
<td>9.0</td>
<td>2650</td>
<td>1.24</td>
<td>0.28</td>
</tr>
<tr>
<td>11.0</td>
<td>2670</td>
<td>2.66</td>
<td>0.41</td>
</tr>
</tbody>
</table>

* Symbols used in the table: \( P_{max} \)- maximum pressure in load cycle (minimal pressure in all cycles \( P_{min} = 0 \); \( e_{\alpha} \)- strain intensity amplitude; \( N_i \)- number of cycles of a given i-type; \( d_i \)- number of failure cycles of a given i-type; \( d_i \)- damage fraction, accumulated in defect in i-given type of cycles.

4. Analysis

Table 2 presents computational and experimental data on SSS of most loaded zones of T-joint specimen under elastic loading.

Table 2: T-joint stress-strain state

<table>
<thead>
<tr>
<th>Defect</th>
<th>Coupling pipe-branch</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-joint</td>
<td></td>
</tr>
<tr>
<td>Straight pipe</td>
<td></td>
</tr>
<tr>
<td>Surface</td>
<td></td>
</tr>
<tr>
<td>( e_{\alpha} )</td>
<td>( e_{\alpha} )</td>
</tr>
<tr>
<td>( \times 10^4 )</td>
<td>( \times 10^4 )</td>
</tr>
<tr>
<td>MPa</td>
<td>MPa</td>
</tr>
<tr>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>3.7</td>
<td>3.7</td>
</tr>
<tr>
<td>3.3</td>
<td>3.3</td>
</tr>
<tr>
<td>7.3</td>
<td>7.3</td>
</tr>
<tr>
<td>4.4</td>
<td>4.4</td>
</tr>
<tr>
<td>1.2</td>
<td>1.2</td>
</tr>
</tbody>
</table>

According to Table 1, fatigue damage accumulated by metal in defect at a point of destruction came to \( d_f = 0.56 \). It should be noted that fatigue damage, accumulated under a low cycle loading, is typically within the range \( d = 0.5 - 1.5 \). A relatively small magnitude of accumulated damage could be result of stress calculation using understated mechanical characteristics of middle, against wall thickness, layers of T-joint metal. At the same time, residual life assessment performed according to [3] resulted in safe conservative estimation, \( d = 1.89 \).

5. Conclusions

Low cycle strength assessment of a seamless T-joint with volumetric surface defect can be performed on the basis of cyclic strength evaluation of a corroded straight pipeline. If the defect is of the same size and located in the area of T-joint opposite to branch such calculation gives a conservatively safe result.

In the case of damage location at outer surface of T-joint area, opposite to branch, stress in defect, versus its geometrical parameters, could approach stress in defect of the same size at the surface of the straight pipeline and exceed stresses at the zone of pipe-branch coupling. Damage of the same size at the inner surface of mentioned area under low cyclic loading is less dangerous.

References


Investigation of stability and limit load of a truss overhead opened bridge

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Abstract

The paper presents selected methods of determining stability and limit load of a truss top chord in opened bridges. These methods include linear buckling and non-linear static analysis based on the finite element method and algorithms based on design code procedures. The described methods were tested on an example of a steel footbridge situated in Straszyn. The results of stability analysis are compared. The results received from non-linear analysis – geometrically and materially-geometrically – are close, the difference is only 1.1%. The critical forces are higher than the results from non-linear analysis by about 10%. A Polish standard procedure showed some difficulties during analysis and the final condition of stability was not satisfied, whilst a Eurocode procedure confirmed correctness of a footbridge design.

Keywords: footbridge, buckling, stability, limit load, non-linear analysis, Eurocode, standards

1. Introduction

Stability is an engineering problem which occurs in light elements due to compression and flexure. Large compressive forces appear in upper chords, diagonal posts of trusses, arches or columns. Renowned examples of such structures are truss and arch bridges [1]. One of them was subjected to a test, where stability and limit loads were investigated using different numerical methods based on the finite element method [2, 3]. The most popular method is a linear buckling analysis (LBA). As a result we obtain buckling modes with corresponding buckling forces to given loads. The LBA is often used in design offices. Critical forces are required in further analysis using standard procedures to receive a design buckling resistance which has to be higher than the greatest compressive forces.

Another approach does not apply any standards. The maximum value of the compressive forces can be obtained by means of a non-linear analysis, geometrically (GNA) or geometrically and materially (GMNA). Those procedures are more time-consuming, but more precise.

The stability conditions can be checked according to standard procedures. The Polish standard [4] recommends a verification of stability of compressive truss chords not prevented from buckling out of their planes, determining a stiffness of a so-called half-frame. It is created by posts and a transverse to prevent buckling out of their planes, determining a more time-consuming, but more precise.

The determined half-frame stiffness should satisfy the following condition:

\[ H \geq c_{1,2} \cdot H_0 \]  

where:

\[ H_0 = \frac{3 \cdot P_{\max}}{\gamma_s \mu_s d_{\min}} \]  

where:

- \( c_{1,2} \) – coefficients dependent on a way of supporting compressed chords of a truss girder,
- \( P_{\max} \) – the maximum compressive force in the compressed chord,
- \( \gamma_s \) – material factor,
- \( \mu_s \) – arithmetic mean of buckling length factors of all chord bars,
- \( d_{\min} \) – the smallest length of a chord between two half-frames.

The European standard [5] recommends to determine a design buckling resistance \( N_{b,Ed} \) which should be higher or equal to the maximum compressive force in the compressed element.

In the paper above-mentioned methods of investigating stability and limit load were used for an example of a real structure. The results obtained by means of LBA, GNA and GMNA were compared. Two standards – Polish [4] and European [5] – were used to check the stability conditions.

2. Description of the structure

The above-mentioned methods of investigating stability and limit load of structures were tested on a steel truss pedestrian bridge over Radunia river located on a hydroelectric power station field in Straszyn. Its view is presented in Fig. 1.

The footbridge is a one-span truss structure of 21 m length. Width of the bridge in girders axes is 1.20 m and its overall width equals to 2.40 m. The upper truss chord consists of two isosceles angle sections L 80×80×9, while the bottom truss chord is made of two non-isosceles angle sections L 100×65×10. Angle sections L 50×50×6 are used for posts and L
90×90×11 for diagonals. Two strips of a balustrade lie between two sections of diagonals and posts in each truss girder and are welded to them, providing a cooperation between the footbridge and the balustrade. Pedestrians walk on a deck made of truss plates. It is treated as a non-cooperating element of the structure.

Figure 1: A view of an investigated footbridge in Straszyn

3. Numerical model

Numerical calculations were conducted in a commercial program ABAQUS using the finite element method [6]. A numerical model of the investigated footbridge was fully created with 4-node shell elements with reduced integration. Each node has six degrees of freedom. All truss elements with gusset plates were included in the model. Joint work of the pedestrian bridge and the balustrade was considered as well. Although fasteners in the bridge were mostly rivets, the connections between the elements of the footbridge were modelled in a form of ties between the contacting surfaces of connected members. A steel class was assumed S235 due to the lack of any technical specifications. Geometry was set on a basis of stocktaking. The footbridge supports are bearings – rollers on one side of a bridge and fixed on the second side.

Loads with safety factors, acting on a structure were determined according to a Polish standard [1] including dead load, load of equipment and live loads – wind and pedestrians. Static analysis, LBA, GNA and GMNA were executed. Static analysis was necessary to obtain the compressive forces in the upper chord. In non-linear analysis an arc-length method by Riks was used to determine equilibrium paths. Initial arc-length increment was set as 0.1, minimum and maximum increments were default – 10^{-5} and 10^{36}, respectively.

4. Results

The compressive force in the model of the footbridge was equal to 172.2 kN. The corresponding bending moments were very low, thus omitted in further analysis. The first two buckling modes were local deformations of the strip of the balustrade. They did not affect the entire structure. The third buckling mode was global buckling of the upper chord out of its plane. The visualisation of the third buckling mode was presented in Fig. 2. The corresponding buckling factor was equal to 2.936, resulting in a critical force equal to 505.6 kN. Note that the first global buckling mode consists of two half-waves. This deformation was used as an initial imperfection in a non-linear analysis. The footbridge without any imperfections was also analysed.

Figure 2: The first global buckling mode of the footbridge

The relationship between maximum compressive forces in the upper chord and a horizontal displacement of a node in the middle of the chord was presented in the form of equilibrium paths obtained by means of a non-linear analysis. The paths incorporating perfectly elastic and elastic-plastic models were drawn in Fig. 3. The maximum compressive force in GNA was equal to 458.7 kN and in GMNA – 453.3 kN. Both values were lower than in the case of a linear buckling analysis. The differences were 9.3% and 10.3%, respectively. Plasticity did not influence strongly the maximum compressive force in the upper truss chord.

Figure 3: The evolution of maximum compressive forces and the horizontal displacement of the upper chord obtained from GNA and GMNA.

The Polish standard procedure is difficult to use in the case of developed geometry of the bridges, such difficulties were evident in the footbridge considered here. The final stability condition according to the Polish standard was not satisfied. The European standard, Eurocode 3, was less restrictive and the design buckling resistance was higher than the compressive forces appearing in the model.

References


Tolerance and Non-Asymptotic Modelling of Microstructured Media
– a Session in Honor of Prof. Czesław Woźniak in connection with His 60th Anniversary
of Scientific Activities and the recognition of important scientific achievements
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Thermal stresses in elastic periodic laminates

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Abstract

The paper is concerned with the subject of periodically layered composites. It is assumed that the constituents of the composites are homogeneous and isotropic. This work presents a model of thermal stresses, in which classical equations with discontinuous and discretely varying coefficients are substituted by equations with constant coefficients. The influence of the inhomogeneity of the medium on the distribution of displacements and temperature is described with new functions which can be determined while the averaging quantities are known. A simpler model is obtained using the tolerance averaging technique.

Keywords: theory of thermal stresses, periodical composites, tolerance averaging

1. Introduction

The paper concerns media with a certain inhomogeneous microstructure. This structure will not be arbitrary, but such that the macroscopic properties of the medium will be constant. The distribution of the constituents in the medium will be periodical.

In the paper layered composites are investigated with respect to their thermo-mechanical properties, bounding the scope of analysis to modelling thermal stresses.

The problem of thermal stresses is described with the use of a certain geometrical structure of configuration baseline.

Thermal stresses in periodic laminates were analysed e.g. in [1,2]. In the second position the author used the tolerance averaging technique. In the first position the results were obtained the other way round.

2. Thermal stresses in laminates

It is assumed that field \( \Omega = (0, L_s) \times \Pi \subset R^3 \) is the configuration baseline, where \( \Pi = (0, L_s) \times (0, L_s) \subset R^2 \) composed of \( \Omega_s \subset \Omega \), \( s = 1, 2, ..., n_s \) such that \( \Omega = \bigcup_{i=1}^{n_s} \Omega_i \), and for \( s_i \neq s_j \), \( \Omega_s \cap \Omega_s = \emptyset \). \( \Omega, \Omega_i \) denote the closure of the fields \( \Omega, \Omega_i \), respectively.

The subject of consideration are such sets in which \( \Omega_s \) will be cuboids of the dimensions \( L_i, L_j, L_k \). It is assumed that \( l_i << L_i \) and \( L_i = n_i l_i \), where \( n_i \in \mathbb{N} \) and \( \frac{1}{n_i} << 1 \).

The interval \( (0, L_s) \) was thus divided into \( n_s \) segments \( \Lambda_s = \left( x'_s \frac{L_s}{2}, x'_s \frac{L_s}{2} + \frac{L_s}{2} \right) \), \( s = 1, 2, ..., n_s \), where \( x'_s \) denote the midpoints of segments \( \Lambda_s \),

\[ x'_s = s l_i - \frac{l_i}{2} \]

where \( s = 1, 2, ..., n_s \).

Hence \( \Omega_s = \bigcup_{s=1}^{n_s} (0, L_s) \times (0, L_s) \) is obtained.

Here \( \theta \) denotes the temperature, and \( u_k, k = 1, 2, 3 \) the displacements. The following denotations were applied: mass density – \( \rho \), specific heat capacity – \( c \), the components of the thermal conduction tensor – \( K_{ij} \), the components of thermal expansion tensor – \( D_{ij} \), \( k, l = 1, 2, 3 \). The introduced quantities are functions: \( \rho : \Omega \to R^1 \), \( c : \Omega \to R^1 \), \( K_{ij} : \Omega \to R^3 \), \( D_{ij} : \Omega \to R^3 \), \( u_k : \Omega \times t \to R^3 \), \( \theta : \Omega \to R^1 \).

Each element \( \Lambda_s \) will now be divided with planes which are parallel to the plane of points of the coordinates \( (0, x_i, x_j) \) and which pass the points of division of the edge of the cuboid. These points define the segments of the length \( l'_i \), \( a_i = 1, 2, ..., a_i \). The layers which have the dimensions \( l'_a \) and which are located inside the element \( \Lambda_s \) will be denoted by \( \Lambda'_a \). If the dimensions of the fields \( \Lambda'_s \) are the same in each field \( \Lambda_s \), the medium is periodical. The constituents \( \Lambda'_s \) are assumed to be homogeneous. This means that the material properties of the medium are constant at \( \Lambda'_s \). If there are at least two elements \( \Lambda'_s \), \( \Lambda'_a \), for which at least one of the conditions \( \rho_{|c_s = 0} \neq \rho_{|c_s = 0} \), \( c_{|c_s = 0} \neq c_{|c_s = 0} \), \( K_{ij} |_{c_s = 0} \neq K_{ij} |_{c_s = 0} \), \( D_{ij} |_{c_s = 0} \neq D_{ij} |_{c_s = 0} \) is satisfied, it is a multicomponent medium.

It is assumed that the layer consists of two homogeneous constituents which are joined on interfaces. Their distribution is periodical along the direction orthogonal to the surface of the layers.

The problem of thermal stresses is described with the use of the following equations:

\[ (B_{s} m + h_{s} \gamma_{s} \theta) \partial_t \theta + \rho \partial_t \theta = \partial_t \partial_t \theta \]

\[ c \partial_t \theta - (K_{ij} \partial_i \partial_j \theta) \partial_t = f \]  

(2)
where for an arbitrary function $g$, the corresponding derivatives were denoted by $g_u = \frac{\partial g}{\partial x_u}$ and \( \hat{g} = \frac{\partial \hat{g}}{\partial t} \). $B_{h\theta}$ are material functions, $D_{\theta}$ are the components of the thermal expansion tensor, $b_i$ are components of body forces, $f_u = B_{b\theta}D_{\theta}$ and $f$ is the capacity of the heat source.

Defining two functionals

\[
P_1 = \frac{1}{2}(\tau \rho \vec{n}_u \dot{u}_i + B_{b\theta}D_{\theta}u_{s,i} + b_i u_i + \{\gamma_{u\theta}\} \tau u_i)
\]

\[
P_2 = \frac{1}{2}(\tau \rho \vec{n}_\theta \dot{\theta}_i + K_{\theta} \theta_{s,i} + b_i \theta_i + \{\gamma_{\theta\theta}\} \tau \theta_i)
\]

where $\tau$ is the parameter, the equations (2) can be written as

\[
\frac{\partial P_1}{\partial u_{i,j}} - \frac{1}{\tau} \frac{\partial P_1}{\partial u_i} - \frac{\partial P_1}{\partial u_i} = 0
\]

\[
\frac{\partial P_2}{\partial \theta_{i,j}} - \frac{1}{\tau} \frac{\partial P_2}{\partial \theta_i} - \frac{\partial P_2}{\partial \theta_i} = 0
\]

(3)

The equations (2) are linear differential equations with discontinuous and discretely varying coefficients. For such a case the work presents a simpler model, in which the coefficients for periodic bodies will be constant. This model will be obtained with the use of the tolerance averaging technique.

3. Averaged model

The simpler model is obtained with the use of the tolerance averaging technique, [3,4].

According to this technique the micro-macro decomposition of the thermal and displacement field is assumed, which in case of a medium with the following elements $\Lambda$, takes the form presented below:

\[
u(x_1,x_2,x_3,t) = \nu_0(x_1,x_2,x_3,t) +\]

\[+ h^A(x_1) \nu^A(x_1,x_2,x_3,t)
\]

\[\theta(x_1,x_2,x_3) = \theta_0(x_1,x_2,x_3,t) +\]

\[+ g^\theta(x_1) \psi(x_1,x_2,x_3,t)
\]

where $A = 1,2,...,M$, $B = 1,2,...,N$, functions $\nu_0$, $\theta_0$, $\nu^A$, $\psi^x$ are unknown slowly varying functions. Functions $h^A$, $g^\theta$ which occur in connection with (5) are given oscillating shape functions. Functions $\nu_0$ should be interpreted as averaged displacements, function $\theta$ as the averaged temperature, and functions $\nu^A$, $\psi^x$, which are called fluctuations, take into account the influence of the inhomogeneity of the material on the displacements and the heat distribution.

In the case of description of the body in the form of the relations (5), instead of four unknowns $u_i$, $\theta$, $u^A$, $\psi^x$, $\psi^y$ are obtained.

According to the tolerance averaging technique the equations for the investigated functions are as follows

\[
\left\{ \frac{\partial (P_1)}{\partial u_{i,j}} - \frac{1}{\tau} \frac{\partial (P_1)}{\partial u_i} - \frac{\partial (P_1)}{\partial u_i} \right\} = 0
\]

\[
\left\{ \frac{\partial (P_2)}{\partial \theta_{i,j}} - \frac{1}{\tau} \frac{\partial (P_2)}{\partial \theta_i} - \frac{\partial (P_2)}{\partial \theta_i} \right\} = 0
\]

(6)

where $\{P_1\}$ and $\{P_2\}$ are averaged functionals (3) and $\alpha, \beta = 2,3$.

The averaging for an arbitrary slowly varying function $f$ determined in $[0,t]$ can be defined in the following manner

\[
(f(x))_t = \frac{1}{t} \int_{t/2}^{t} f(y)dy.
\]

In the case of considered two-component laminates it is assumed that $M = N = 1$ and $h(x) = g(x)$.

After averaging the functionals (3) and applying the tolerance averaging technique (6) the following equations are obtained

\[
\left\{ \rho \nu_{0,i} - \{B_{b\theta}\} \nu_{0,\theta,i} \right\} = \{b_i\} + \{\gamma_{u\theta}\} \theta_{0,i} + \{\gamma_{\theta\theta}\} \nu_{0,i} + \{\gamma_{\theta\theta}\} \psi_{0,i} + \{\gamma_{\theta\theta}\} \nu_{0,i}
\]

\[
\left\{ \mu \nu_{0,i} - \{B_{b\theta}\} \nu_{0,\theta,i} \right\} = \{b_i\} h + \{\gamma_{u\theta}\} \theta_{0,i} + \{\gamma_{\theta\theta}\} \nu_{0,i} + \{\gamma_{\theta\theta}\} \psi_{0,i} + \{\gamma_{\theta\theta}\} \nu_{0,i} + \{\gamma_{\theta\theta}\} \nu_{0,i}
\]

\[
\left\{ \rho \theta_{0,i} - \{K_{\theta}\} \theta_{0,i} \right\} = \{f\} + \{\gamma_{\theta\theta}\} \nu_{0,i} + \{\gamma_{\theta\theta}\} \nu_{0,i} + \{\gamma_{\theta\theta}\} \nu_{0,i} + \{\gamma_{\theta\theta}\} \nu_{0,i}
\]

(7)

The presented model, described with the use of the equations (7) for averaged displacements, averaged temperature and fluctuations, in its periodical case uses constant coefficients.

References

Nonlinear dynamic response of periodically inhomogeneous Rayleigh beams

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Abstract

The paper deals with large amplitude vibrations of slender periodically inhomogeneous beams with an inhomogeneity period much smaller than their span. A beam with geometrical and physical properties varying periodically along its axis is considered. The beam can interact with a periodically inhomogeneous visco-elastic subsoil. Replacement model equations with constant coefficients are obtained via application of the tolerance modelling technique. The proposed model can serve as a tool in parametric analysis of large amplitude vibrations of considered structures.

Keywords: periodic beams, nonlinear vibrations

1. Introduction

Flexural periodic structures can exhibit desirable mechanical properties, such as band-pass filtering. Their dynamical behaviour in the small amplitude regime is well recognized, especially for infinite systems.

The main objective of the project is modelling and analysis of numerous thermomechanics problems of microheterogeneous structures. The basic concepts will be now briefly introduced. A new averaged mathematical model of this type was presented in \cite{1}.

2. Fundamental equations

The dynamics of considered beams is governed by the following system of coupled nonlinear differential equations

\[
\begin{align*}
\tilde{E}(t)\tilde{u}_t + \frac{1}{2}\tilde{u}_t\tilde{w}_t &= 0, \\
\mu\tilde{w}_t - \tilde{E}(t)\tilde{u}_w + c\tilde{w}_t + kw_t + \tilde{E}(t)\tilde{w}_w &= q, \\
\end{align*}
\]

where \(E = E(x)\) stands for the Young's modulus, \(A = A(x)\) and \(J = J(x)\) are the cross-sectional area and moment of inertia, \(\mu = \mu(x)\) and \(\vartheta = \vartheta(x)\) are the mass density and the rotational inertia per unit length, \(k = k(x)\) and \(c = c(x)\) the stiffness and damping coefficients of the subsoil. The transverse deflection and longitudinal displacements are denoted by \(w = w(x,t)\), \(u_0 = u_0(x)\).

In a general case all the coefficients of the above equations are non-continuous, highly oscillating periodic functions of \(x\).

3. The averaged description

The tolerance modelling technique was applied in analysis of numerous thermomechanics problems of microheterogeneous structures. The basic concepts will be now briefly introduced. Let \(\Delta(x) = x + \Delta\) be a periodicity cell with center at \(x \in \Pi_\Delta\), \(\Pi_\Delta = \{x \in \Pi; \Delta(x) \subset \Pi\}; \Pi = [0, L]\). The averaging operator for an arbitrary integrable function \(f\) is defined by:

\[
f > (x) = \frac{1}{L} \int_{L_0}^{L} f(y)dy, \quad (2)
\]

where \(L\) is the periodicity cell length, called the microstructure parameter.

The unknown displacements are decomposed into their averaged and fluctuating parts as follows:

\[
w(x,t) = W(x,t) + h^\alpha(x,t), \quad A = 1, \ldots, N, \\
W(x,t), V^\alpha(x,t) \in SV^r_j(\Pi, \Delta), \quad \alpha = 1, \ldots, M, \\
u_0(x,t) = U(x,t) + g^\alpha(x,t)T^\alpha(x,t), \quad K = 1, \ldots, M, \\
U(x,t), T^\alpha \in SV^r_j(\Pi, \Delta), \quad g^\alpha(x) \in FS^j(x, \Delta). \quad (3)
\]

In the above relations the new slowly-varying (SV) basic kinematic unknowns \(W(x,t)\) and \(U(x,t)\) are called the macrodeflection and the in-plane macrodisplacements, respectively; \(V^\alpha(x,t)\) and \(T^\alpha(x,t)\) are additional kinematic unknowns called the fluctuation amplitudes. The fluctuation shape (FS) functions \(h^\alpha(x)\) and \(g^\alpha(x)\) describe the disturbance of the displacements in a periodicity cell. The tolerance parameter is denoted by \(d\).

Let us introduce the following basic denotations:

\[
\begin{align*}
\xi & = \chi / L, \quad \tau = \Omega t, \\
w & = W/f_\alpha, \quad v^\alpha = V^\alpha/f_\alpha, \quad u = U/f_0, \\
\Omega & = \pi A\sqrt{E/\rho_0^2}, \quad \lambda = l / L, \quad \eta = r / L, \\
\delta_0 & = \eta \int_0^1 u_0 \, dx_0 = \eta^{-1}[u(t) - u(0)].
\end{align*}
\]

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where \( r_0, A_0, E_0 \) are the radius of gyration, area, Young's modulus and mass density per unit length of a reference cross section, \( L \) is the beam length.

Making use of the decomposition (3) and the averaging operation (2) we obtain a system of dimensionless differential equations with constant averaged coefficients (denoted by capital letters):

\[
\begin{align*}
Dw_{\text{av}} + Dv_{\text{av}} + Kw + \lambda^2 K'v' - Nw_{\text{av}} - \lambda N'v' + \\
+ Cw_{\text{av}} + Mw_{\text{av}} - \eta^2 Jw_{\text{av}} + \lambda^2 C'v' + \lambda^2 M'v' + \\
- \eta^2 \lambda^2 Jv_{\text{av}} - Q = 0,
\end{align*}
\]

(5)

\[
\begin{align*}
Dv_{\text{av}} + \lambda^2 K_{\text{av}} v_{\text{av}} + \lambda\lambda C_{\text{av}} v_{\text{av}} + \lambda\lambda M_{\text{av}} v_{\text{av}} + \lambda^4 C'v' + \\
+ \lambda^2 N_{\text{av}} v_{\text{av}} + \lambda^2 N'v' + Dv_{\text{av}} + \\
+ \lambda^2 K_{\text{av}} + \lambda^2 M_{\text{av}} + \lambda^2 J_{\text{av}} - \lambda^2 Q = 0,
\end{align*}
\]

(6)

\[
\begin{bmatrix}
\bar{N} \\
\bar{N}_{\text{av}}
\end{bmatrix}
= \begin{bmatrix}
\bar{B}_0^C \\
\bar{B}_0^{\text{BC}} \\
\bar{B}_{\text{av}}^C \\
\bar{B}_{\text{av}}^{\text{BC}}
\end{bmatrix}
\begin{bmatrix}
\bar{N}_{\text{av}} \\
\bar{N}_{\text{av}}^{\text{CD}}
\end{bmatrix}
\times
\begin{bmatrix}
\int_0^1 w_{\text{av}}v_{\text{av}} \, \dd \xi + 0 \\
\int_0^1 v_{\text{av}}^D \, \dd \xi + 0
\end{bmatrix}.
\]

(7)

4. Solution procedure

The obtained partial differential equations are transformed into a finite system of ordinary differential equations by application of the Bubnov-Galerkin method. The solutions to Eqns (5)-(7) are sought for in the form of truncated series of orthogonal functions:

\[
w(\xi, \tau) = \sum_{i=1}^M w_i(\tau) N_i(\xi), \quad v(\xi, \tau) = \sum_{i=1}^M v_i(\tau) N_i(\xi)
\]

(8)

The resulting equations can be written in convenient matrix form as follows:

\[
Kw + Cy + My = q; \quad K = K_0 + K_{\text{av}}(y)
\]

(9)

The linearized equations are solved for linear eigenvalues. Independently, Eqns (9) are rewritten as a system of first order equations

\[
Kw + Cy + My = q, \quad \dot{y} = v,
\]

(10)

and numerically integrated by means of the Runge-Kutta-Fehlberg method.

5. Applications

The obtained model is applied in the analysis of a pinned-pinned uniform cross-section beam carrying a system of periodically distributed masses \( M_1 = 10 \mu_\text{p} \) and \( M_2 = \alpha M_1 \) with mass moments of inertia \( I_1 \) and \( I_2 = \alpha I_1 \), \( \lambda = 1/10 \). The beam and its fragment are depicted in Fig. 2.

The fluctuation shape functions \( h'(y), h'(y), h''(y) \) and \( g'(y) \) can be found by means of finite element discretization of a periodicity cell as the \( l \)-periodic eigenforms of the cell vibrations, e.g. as depicted in Fig. 3.

Figure 4 depicts linear eigenfrequencies versus the \( \alpha \) parameter for a 20-element 40-degree-of-freedom finite element (FE) and a 4-degree-of-freedom tolerance averaging (TA) models of considered beam. It can be seen that the proposed model is able to determine the stop- and pass-bands' boundaries.

6. Conclusions

A novel geometrically nonlinear model of locally resonant one-dimensional flexural periodic structures. It takes into account all types of heterogeneity and initial axial tension or shortening, including vibrations of pre-buckled members.

References


Dynamic response of sandwich beam with periodic core
due to fuzzy stochastic moving load

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Abstract

In the paper the problem of dynamic response of a finite, simply supported sandwich beam with periodic structure of the core due to the fuzzy stochastic moving load is considered. The solution of the problem was found thanks to the random dynamic moving influence function which allows to apply the perturbation method and average tolerance approach allows to pass from differential equations with periodic variable coefficients to differential equations with constant coefficients.

Keywords: sandwich beam, stochastic load, periodic structure

1. Introduction

Over the past decades the sandwich beams had widespread applications in the fields of aerospace, civil and mechanical engineering as structural members with high strength to weight ratios. Dynamics response of the sandwich beams has been studied by many authors in the recent decades. Very often the cores used in load carrying sandwich beam have cellular, periodic nature like honeycomb or corrugated sheet. The problem of a dynamic response of a structure subjected to moving loads is interesting and important. This problem occurs in dynamics of bridges, roadways, railways and runways as well as missiles and aircrafts. The structural engineering is significantly influenced by uncertainty which originates from human mistakes and errors in the manufacture, from the use and maintenance of the construction and from a lack of information. Therefore, in recent years uncertainty modelling in computational mechanics received particular attention [2, 3]. In the paper the problem of the dynamic response of a finite, simply supported sandwich beam with periodic structure of the core with uncertain parameters under a moving fuzzy random load is considered. The solution of the problem was found thanks to the random dynamic moving influence function which allows to apply the perturbation method and average tolerance approach allows to pass from differential equations with periodic variable coefficients to differential equations with constant coefficients.
Figure 1: Displacements of the beam

\[ w_\alpha(x,t), \Phi_\alpha(x,t) \text{ and } V_\alpha^\mu(x,t), \phi_\alpha^\mu(x,t) \] will be referred to as the fluctuating parts of the functions 
\[ w_\alpha(x,t), \Phi_\alpha(x,t) \] (the summation convention over \( \alpha=1,2,... \) holds). The functions \( g^\mu(x) \) are a priori known oscillating \( l \)-periodic-like functions and the amplitudes \( V_\alpha^\mu(x,t), \phi_\alpha^\mu(x,t) \) are sufficiently regular and slowly varying functions.

The functions \( g^\mu(x) \) should satisfy conditions

\[
\int_{a}^{a + l} g^\mu(x) dx = 0, \quad (5)
\]

and

\[
\int_{b}^{b + l} m(b,x) g^\mu(x) dx = 0. \quad (6)
\]

After averaging equation (1) we obtain finally partial differential equations with a constants coefficients [7]

\[
\frac{\partial^2 \psi_\alpha(x,t)}{\partial x^2} - \frac{a_{\beta \mu} g^\mu(x)}{\xi_d} \frac{\partial \psi_\alpha(x,t)}{\partial x} + \frac{a_{\beta \mu}}{\xi_d} \frac{\partial W_\beta(x,t)}{\partial x} > \psi_\alpha(x,t) + +
\]

\[
- \frac{a_{\beta \mu}}{\xi_d} \frac{\partial g^\mu(x)}{\partial x} \frac{\partial \psi_\alpha(x,t)}{\partial x} + \frac{W_\beta(x,t)}{\partial x} > \frac{\partial W_\beta(x,t)}{\partial x} \frac{\partial \psi_\alpha(x,t)}{\partial x} + \frac{a_{\beta \mu}}{\xi_d} \frac{\partial \psi_\alpha(x,t)}{\partial x} \frac{\partial W_\beta(x,t)}{\partial x} > +
\]

\[
+ \frac{\partial^2 W_\beta(x,t)}{\partial x^2} + \frac{\partial^2 W_\beta(x,t)}{\partial x^2} > \psi_\alpha(x,t) > +
\]

and

\[
\frac{\partial^2 \phi_\alpha^\mu(x,t)}{\partial x^2} - \frac{a_{\beta \mu} g^\mu(x)}{\xi_d} \frac{\partial \phi_\alpha^\mu(x,t)}{\partial x} + \frac{a_{\beta \mu}}{\xi_d} \frac{\partial W_\beta(x,t)}{\partial x} > \phi_\alpha^\mu(x,t) + +
\]

\[
- \frac{a_{\beta \mu}}{\xi_d} \frac{\partial g^\mu(x)}{\partial x} \frac{\partial \phi_\alpha^\mu(x,t)}{\partial x} + \frac{W_\beta(x,t)}{\partial x} \phi_\alpha^\mu(x,t) > +
\]

\[
+ \frac{\partial^2 W_\beta(x,t)}{\partial x^2} + \frac{\partial^2 W_\beta(x,t)}{\partial x^2} > \phi_\alpha^\mu(x,t) = 0. \quad (8)
\]

After averaging the Eq. (2) we obtain finally partial differential equations with a constants coefficients

\[
\frac{\partial \psi_\alpha(x,t)}{\partial x^2} - \frac{a_{\beta \mu} g^\mu(x)}{\xi_d} \frac{\partial \psi_\alpha(x,t)}{\partial x} + \frac{a_{\beta \mu}}{\xi_d} \frac{\partial W_\beta(x,t)}{\partial x} > \psi_\alpha(x,t) + +
\]

\[
- \frac{a_{\beta \mu}}{\xi_d} \frac{\partial g^\mu(x)}{\partial x} \frac{\partial \psi_\alpha(x,t)}{\partial x} + \frac{W_\beta(x,t)}{\partial x} > \frac{\partial W_\beta(x,t)}{\partial x} \frac{\partial \psi_\alpha(x,t)}{\partial x} + \frac{a_{\beta \mu}}{\xi_d} \frac{\partial \psi_\alpha(x,t)}{\partial x} \frac{\partial W_\beta(x,t)}{\partial x} > +
\]

\[
+ \frac{\partial^2 W_\beta(x,t)}{\partial x^2} + \frac{\partial^2 W_\beta(x,t)}{\partial x^2} > \psi_\alpha(x,t) = 0, \quad (9)
\]

\[
\frac{\partial^2 \phi_\alpha^\mu(x,t)}{\partial x^2} - \frac{a_{\beta \mu} g^\mu(x)}{\xi_d} \frac{\partial \phi_\alpha^\mu(x,t)}{\partial x} + \frac{a_{\beta \mu}}{\xi_d} \frac{\partial W_\beta(x,t)}{\partial x} > \phi_\alpha^\mu(x,t) + +
\]

\[
- \frac{a_{\beta \mu}}{\xi_d} \frac{\partial g^\mu(x)}{\partial x} \frac{\partial \phi_\alpha^\mu(x,t)}{\partial x} + \frac{W_\beta(x,t)}{\partial x} \phi_\alpha^\mu(x,t) > +
\]

\[
+ \frac{\partial^2 W_\beta(x,t)}{\partial x^2} + \frac{\partial^2 W_\beta(x,t)}{\partial x^2} > \phi_\alpha^\mu(x,t) = 0. \quad (10)
\]

The boundary conditions for a simply supported beam have forms

\[
w(0,t) = w(L,t) = 0, \quad (11)
\]

\[
\frac{\partial \phi_\alpha^\mu(x,t)}{\partial x} \bigg|_{x=0} = \frac{\partial \phi_\alpha^\mu(x,t)}{\partial x} \bigg|_{x=L} = 0, \quad (12)
\]

where \( L \) is the beam length.

The set of the equations with constant coefficients (7-10) with the boundary conditions for simply supported beam (11) (12) allows to analyse the dynamic response of the beam due to fuzzy stochastic moving load.

References

Tolerance modelling of vibrations of visco-elastic thin periodic plates with moderately large deflections

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Abstract

Visco-elastic periodic plates with moderately large deflections are considered. Geometrically non-linear vibrations of these plates are investigated. In order to take into account the effect of the microstructure on dynamics of these plates the tolerance modelling is applied. This method makes it possible to derive model equations with constant coefficients with terms dependent of the microstructure size.

Keywords: thin periodic plates, microstructure size, tolerance modelling, visco-elastic, moderately large deflections

1. Introduction

There are considered thin visco-elastic plates with a periodic structure in planes parallel to the plate midplane. These plates consist of many identical small elements, called periodic cells. It is assumed that the plates have deflections of the order of their thickness and rest on a periodic visco-elastic foundation. Hence, plates of this kind can be called thin visco-elastic plates with moderately large deflections. Dynamic behaviour of these plates can be described by non-linear partial differential equations with coefficients being highly oscillating, periodic and non-continuous functions of cells parameters. Hence, plates of this kind can be called visco-elastic plates with moderately large deflections resting on an elastic foundation with damping, with constant coefficients.

2. Fundamental relations

Let us denote the orthogonal Cartesian co-ordinate system in the physical space by \( \Omega = \{(x_1, x_2) : 0 \leq x_1 \leq L_1, 0 \leq x_2 \leq L_2\} \), and the region of undeformed plate by \( \Omega = \{(x_1, x_2) : -d(x) \leq z \leq d(x)/2, x \in \Omega\} \), with midplane \( \Pi \) and the plate thickness \( d(x) \). These plates are assumed to have a periodic structure along the \( x_1 \) - and \( x_2 \)-axis directions with periods \( l_1, l_2 \) respectively, in planes parallel to the plate midplane. The periodicity basic cell on \( \Omega_{12} \) plane is denoted by \( \Delta = [-l_2/2, l_2/2] \times [-l_1/2, l_1/2] \). The cell size is described by parameter \( l \equiv (l_1), l_2 \) satisfies the condition \( \max (d) \ll l \ll \min (l_1, l_2) \), and is called the microstructure parameter. Subscripts \( i, j, \ldots, i, j, \ldots \) run over 1, 2, 3 and indices \( A, B, \ldots \) \((a,b,\ldots)\) run over \( 1, \ldots, 3 \). The partial derivatives with respect to space coordinates are denoted by \( \partial \). Assumptions of the non-linear theory of thin plates \([6, 11, 13]\) are used. Let \( w(x) \) be the plate midplane deflection, \( \nu(x) \) be the in-plane displacements along the \( x_\alpha \)-axes, \( p(x) \) be the total loadings in the \( z \)-axis; \( x \in \Omega \). Thickness \( d \) can be a periodic function in \( x \), elastic moduli \( E_{ij} \) and mass density \( \rho \) can be also periodic functions in \( z \). The non-zero components of the elastic moduli tensor are \( a_{ij}, b_{ij}, c_{ij} \), but proper visco-elastic moduli tensor by \( \tilde{c}_{ij} \).

It is also assumed that these periodic plates interact with a visco-elastic foundation, periodic in planes parallel to the plate midplane, resting on a deformable base. Hence, foundation properties: a mass density per an unit area \( \tilde{\mu} \), a Winkler coefficient \( k \), a damping parameter \( c \), can be periodic functions in \( x \). Moreover, the plate cannot be torn off from the foundation.

The plate properties, i.e. stiffness tensors: \( h_{\alpha \beta \gamma}, d_{\alpha \beta \gamma} \), visco-elastic tensors: \( h_{\alpha \beta \gamma}, d_{\alpha \beta \gamma} \), inertia properties: \( \mu, j \), are defined as:

\[
\begin{align*}
\h_{\alpha \beta \gamma}(x) & = \int_{-d/2}^{d/2} \epsilon_{\alpha \beta \gamma}(x,z)dz, & \quad d_{\alpha \beta \gamma}(x) & = \int_{-d/2}^{d/2} \gamma_{\alpha \beta \gamma}(x,z)dz, \\
\h_{\alpha \beta \gamma}(x) & = \int_{-d/2}^{d/2} \tilde{\epsilon}_{\alpha \beta \gamma}(x,z)dz, & \quad d_{\alpha \beta \gamma}(x) & = \int_{-d/2}^{d/2} \tilde{\gamma}_{\alpha \beta \gamma}(x,z)dz, \\
\mu(x) & = \int_{-d/2}^{d/2} \rho(x,z)dz, & \quad j(x) & = \int_{-d/2}^{d/2} \rho(x,z)dz, \\
\end{align*}
\]

and they are periodic functions in \( x \).

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3. Tolerance modelling

The tolerance modelling is based on some concepts and assumptions shown in [14], e.g.: an averaging operator, a slowly-varying function, a tolerance-periodic function, a highly-oscillating function, a fluctuation shape function. These concepts are used in the fundamental assumptions.

The first assumption is the micro-macro decomposition, where it is assumed that deflections \( w \) and in-plane displacements \( \bar{u}_a \), \( a=1,2 \), can be decomposed as \((A=1, \ldots, N; \alpha=1, \ldots, m)\):

\[
\hat{u}(x,t) = W(x,t) + \hat{g}^0(x,t), \quad \hat{u}_a(x,t) = U_a(x) + \hat{f}_a(x)T_a(x),
\]

with new unknowns: macrodeflection \( W \), in-plane macrodisplacements \( U_a \), fluctuation amplitudes of the deflection \( \hat{g}^0 \) and the in-plane displacements \( \hat{f}_a \), being slowly-varying functions in \( x \) and the known fluctuation shape functions \( g^0, f^0 \), assumed usually in the form of trigonometric functions.

The second assumption is the tolerance averaging approximation in which it is assumed that in the modelling terms of an order of tolerance parameter \( \delta \) are negligibly small.

4. Tolerance model equations

From the tolerance modelling procedure, cf. [14], the governing equations with constant coefficients are derived:

- the constitutive equations:

\[
M_{\alpha} = D_{\alpha\beta}W_{\beta,\gamma,\gamma} + D_{\alpha\beta\gamma\gamma}V_{\beta,\gamma,\gamma} + \bar{D}_{\alpha\beta}W_{\beta,\gamma} + \bar{D}_{\alpha\beta\gamma\gamma}V_{\beta,\gamma},
\]

\[
M^\ast_{\alpha} = D^\ast_{\alpha\beta}W_{\beta,\gamma,\gamma} + D^\ast_{\alpha\beta\gamma\gamma}V_{\beta,\gamma,\gamma} + \bar{D}^\ast_{\alpha\beta}W_{\beta,\gamma} + \bar{D}^\ast_{\alpha\beta\gamma\gamma}V_{\beta,\gamma} + \hat{S}_{\alpha\beta},
\]

\[
N_{\alpha} = B_{\alpha\beta}(U_{\beta,\gamma,\gamma} + \hat{g}^\ast_{\beta}\gamma,\gamma) + B_{\alpha\beta\gamma\gamma}(W_{\beta,\gamma,\gamma} + \hat{f}^\ast_{\beta}\gamma,\gamma) + \hat{S}^\ast_{\alpha\beta},
\]

\[
N^\ast_{\alpha} = B^\ast_{\alpha\beta}(U_{\beta,\gamma,\gamma} + \hat{g}^\ast_{\beta}\gamma,\gamma) + B^\ast_{\alpha\beta\gamma\gamma}(W_{\beta,\gamma,\gamma} + \hat{f}^\ast_{\beta}\gamma,\gamma) + \hat{S}^\ast^\ast_{\alpha\beta},
\]

\[
Q_{\alpha} = F_{\alpha}^{W}(U_{\beta,\gamma,\gamma} + \hat{g}^\ast_{\beta}\gamma,\gamma) + F_{\alpha}^{V}(V_{\beta,\gamma,\gamma} + \hat{f}^\ast_{\beta}\gamma,\gamma) + \hat{F}_{\alpha},
\]

\[
Q^\ast_{\alpha} = F^\ast_{\alpha}^{W}(U_{\beta,\gamma,\gamma} + \hat{g}^\ast_{\beta}\gamma,\gamma) + F^\ast_{\alpha}^{V}(V_{\beta,\gamma,\gamma} + \hat{f}^\ast_{\beta}\gamma,\gamma) + \hat{F}^\ast_{\alpha},
\]

\[
R_{\alpha} = F_{\alpha}^{W}(U_{\beta,\gamma,\gamma} + \hat{g}^\ast_{\beta}\gamma,\gamma) + F_{\alpha}^{V}(V_{\beta,\gamma,\gamma} + \hat{f}^\ast_{\beta}\gamma,\gamma) + \hat{F}_{\alpha},
\]

- the equilibrium equations:

\[
M_{\alpha\beta\gamma\gamma} - \left\{ \begin{array}{c}
\tilde{N}_{\alpha\beta\gamma\gamma} + \tilde{Q}_{\alpha\beta}\gamma\gamma + \tilde{M}_{\alpha\beta\gamma\gamma}\gamma \gamma + \tilde{P}_{\alpha\beta}\gamma\gamma\gamma
\end{array} \right\} + \left\{ \begin{array}{c}
\tilde{N}_{\alpha\beta\gamma}\gamma\gamma + \tilde{Q}_{\alpha\beta}\gamma\gamma + \tilde{M}_{\alpha\beta}\gamma\gamma\gamma \gamma + \tilde{P}_{\alpha\beta}\gamma\gamma\gamma\gamma
\end{array} \right\} = 0,
\]

Equations (4)-(5) (and the non-linear tolerance model of thin visco-elastic periodic plates resting on an elastic foundation with damping) describe the effect of the microstructure size on the plate behaviour by terms with the microstructure parameter \( \lambda \). There are to be formulated boundary conditions only for macrodeflection \( W \) and in-plane macrodisplacements \( \hat{u}_a \). Moreover, the basic unknowns of equations (4)-(5) have to be slowly-varying functions in \( x \).

5. Remarks

The tolerance modelling used to describe thin visco-elastic periodic plates with moderately large deflections resting on an elastic foundation leads to equations with constant coefficients, describing the effect of the microstructure size.

References

Tolerance modelling of thermoelastic phenomena in functionally graded laminates

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Abstract

The paper presents thermoelasticity problems in laminates of two components non-periodically distributed as micro-laminas along one direction. In order to obtain the governing equations of the model describing these problems, the tolerance averaging technique is applied. In the note, three models are proposed: the tolerance and the asymptotic-tolerance models, taking into account the effect of the microstructure size on the overall behaviour of these laminates, and the asymptotic model, neglecting this effect.

Keywords: thermoelasticity, functionally graded materials, tolerance modelling, asymptotic modelling

1. Introduction

In the note we deal with the problems of thermoelasticity in two-phase laminates. The lamina cells are composed of two sublayers of different material constituents. The thickness of the cells \( l \) is constant, cf. Fig. 1, and is assumed to be small compared to the minimum length of the laminate and is called the microstructure parameter \( l \).

![Figure 1: The cross-section of considered laminates: a) microstructure, b) macrostructure](image)

Figure 1: The cross-section of considered laminates: a) microstructure, b) macrostructure

It is also assumed that the composite macroscopic properties change continuously along one direction (perpendicular to the lamina). These objects can be analysed as made of functionally graded materials, cf. [8]. In the analysis of various cases concerning these composites modelling approaches are used for laminates with a periodic structure and their overall behaviour can be described by adopted methods, which are used for macroscopically homogeneous composites (e.g. for periodic composites), cf. [8]. Unfortunately, the effect of the microstructure size on the overall behaviour of the laminates is usually neglected in the equations of the proposed models.

In order to obtain the averaged equations, taking into account this effect, the tolerance averaging technique, cf. [11], was applied to describe various thermomechanical phenomena in periodic media. This way of modelling is used in many studies to derive equations that describe the various cases of periodic structures, cf. [10,11], e.g. for thermoelastic problems, cf. [1,2,9]. The tolerance averaging technique is also adopted to model problems of composite structures made of materials with functional gradation of properties. The examples of applications of this method can be found in [4,5,6,7,11]. The tolerance modelling replaces equations with functional, highly-oscillating, tolerance-periodic (or periodic) and non-continuous coefficients by a system of differential equations with slowly-varying (or constant) coefficients. For this purpose, the concepts of a slowly-varying function, a tolerance-periodic function and an averaging operation are used, cf. [3,10,11].

In the contribution equations of three models are derived. The first is the tolerance model, which equations involve terms describing the effect of the microstructure size on the overall behaviour of the laminates under consideration. The second is the asymptotic-tolerance model, which equations involve also terms with the microstructure parameter. And the third - the asymptotic model, whose equations neglect this effect.

2. The modelling procedures

The starting point of the modelling is a set of governing equations describing thermoelasticity problems of composites:

\[
\begin{align*}
\partial_t (\rho_1 \varepsilon_{ijkl} \mu_{ij}) - \rho_1 \ddot{u}_i &= \partial_t \partial \varepsilon \varepsilon + b_\varepsilon \varepsilon \partial \varepsilon + b_{ijkl} \partial \varepsilon (k_{ijkl} \partial \varepsilon) = c \theta + T_{ijkl} \partial \varepsilon \mu_{ij},
\end{align*}
\]

with highly-oscillating, non-continuous functional coefficients of \( x_1 \cdot \varepsilon_{ijkl}, \rho_1, b_{ijkl}, k_{ijkl}, c \) for displacements \( u_i \) and a temperature \( \theta \). In order to obtain equations with slowly-varying coefficients, describing the effect of the microstructure size, the tolerance or the asymptotic-tolerance modelling can be used.

In the tolerance modelling two fundamental assumptions are introduced. The first is the micro-macro decomposition assumed in the form:

\[
\begin{align*}
u_i(x,k_\xi,t) &= w_i(x,k_\xi) + \delta(x)\varepsilon_i(x,k_\xi),
\end{align*}
\]

where: \( x = x_\xi, \; \xi = (x_1,k_\xi); \; w_i, \; \delta, \; \varepsilon \) are slowly-varying functions in \( x \). Functions \( w_i \) and \( \delta \) are the basic unknowns, called the macrodisplacements and the macrotemperature, respectively, \( \varepsilon \) are additional basic unknowns, called the fluctuation amplitudes of displacements and of temperature, respectively; \( \delta(x) \) and \( g(x) \) are the fluctuation shape functions, assumed saw-like functions.

The second assumption is the tolerance averaging approximation in which terms of an order of \( O(\delta) \) are assumed negligibly small.

Substituting micro-macro decomposition (2) to governing equations (1), after averaging and manipulations, the final equations of the tolerance model can be obtained.
The asymptotic-tolerance modelling procedure can be divided into two steps, described and explained in [4,6,10]. In the first step, the asymptotic solution to the problem, by Eqn (1), is derived in the form:

$$u_d(x,t) = u(x,t) + T(x,t),$$

$$\theta_d(x,t) = \theta(x,t) + g(x)\psi(x,t),$$

As a result, the system of equations only for the macrodisplacements and the macrotemperature is obtained. The second step uses the additional micro-macro decomposition to equations, with known functions $w_i$, $\chi$, $\psi$:

$$u(x,t) = U(x,t) + f(x)t \chi(x,t),$$

$$\theta(x,t) = \Theta(x,t) + d(x)t \psi(x,t),$$

with $w_i$, $\chi$ as new unknowns being slowly-varying functions and $f$, $d$ as new additional fluctuation shape functions similar to $h$, $g$.

3. The model equations

From the tolerance modelling procedure, cf. [3,10], the equations of the tolerance model of functionally graded laminates are derived:

$$\begin{align*}
\partial_t (c_{\omega d}\partial_t w_1 + c_{\omega d}\partial_t v_1) - \partial_\theta w_1 &= \partial_\theta b_1 + \partial_\theta \beta, \\
- c_{\varepsilon d} \partial_t h_1 \partial_t w_1 - c_{\varepsilon d} \partial_t h_1 \partial_t v_1 + c_{\varepsilon d} \partial_t h_1 \partial_t w_1 + \\
+ c_{\phi d} \theta_1 &= - \partial_\theta b_1 + \partial_\theta \beta. \\
\partial_t (c_{\varepsilon d} \partial_t \beta + k_{\varepsilon d} \partial_t \psi &= \\
&= c_\beta + T_{\beta d} \partial_t \beta + T_{\beta d} \partial_t \psi, \\
k_{\varepsilon d} \partial_t \psi &= - k_{\varepsilon d} \partial_t \psi.
\end{align*}$$

The above equations describe the thermoelasticity in laminates on the macro-level.

4. Remarks

While using the tolerance averaging the differential equations of thermoelasticity for functionally graded laminates with highly-oscillating, non-continuous coefficients are replaced by the differential equations with slowly-varying coefficients.

In the proposed modelling procedures there are obtained the governing equations of two models – the tolerance and the asymptotic-tolerance, describing the effect of the microstructure size in thermoelasticity problems of the considered laminates, and of one model – the asymptotic, which neglect this effect.

References

Free vibrations of plates reinforced by rods – the homogenization with micro – local parameters

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Abstract

The object of the paper are elastic plates, periodically reinforced in one or in two directions. For this kind of bodies an averaging model was created with the use of homogenization with micro-local parameters. In the paper we analyzed free vibrations and described the effect of basic cell dimension on the displacement of these plates.

Keywords: periodic plate, non-asymptotic homogenization, free vibrations of reinforced plates

1. Introduction

The object of consideration in the paper are elastic, heterogeneous plates. Such plates are three-dimensional objects, for their description and research classic theory of elasticity may be used. However, resolving initial-boundary issues based on this theory is usually complicated, so we try to find a simpler theory – in that case a two-dimensional approach.

In the paper an averaging, two-dimensional model was made with the use of homogenization with micro-local parameters. Next, analysis of free vibrations of the plate was conducted, concluded with description of the basic-cell effect on dimension on the displacement and free vibrations form.

2. Modelling procedure

Let the configuration of the plate will be a region \( \Omega = \Pi \times \left( \frac{-h}{2}, \frac{h}{2} \right) \) where \( \Pi = (0, L_1) \times (0, L_2) \subset \mathbb{R}^2 \) and \( h > 0 \).

We shall assume that \( h \ll \min(L_1, L_2) \).

We assume that the plate is periodically heterogeneous and a repeated element (basic cell), denoted by \( \Delta \), is a rectangle of whose dimensions are \( L_1 \) and \( L_2 \). We suppose that every basic cell \( \Delta \) consists of four elements \( \Delta_{ab} \), \( a, b = 1, 2 \), Fig. 1.

In elements \( \Delta_{ab} \) there occurs a reinforcement of rods, which is described by saturation functions. These elements are averaged by us by means of this functions [1]. Averaging values of mass density \( \rho(x_1, x_2) \) and stiffness moduli \( B_{a\beta\gamma\delta} = B_{a\beta\gamma\delta}(x_1, x_2) \), \( \alpha, \beta, \gamma, \delta = 1,2 \) in the element \( \Delta_{ab} \) we designate with \( \rho^a \), \( B_{a\beta\gamma\delta}^a \). These values are constant.

Let \( w = w(x_1, x_2, t) \), \( (x_1, x_2) \in \Omega \), \( t \in (t_0, t_1) \) be a plate deflection which, according to the method, we are taken in the form

\[
w(x_1, x_2, t) = u(x_1, x_2, t) + h^v(x_1, x_2) v^v(x_1, x_2, t) \]

where \( A = 1, 2, 3, \ldots, N \), \( u \), \( v^v \) are slowly-varying functions relative to the basic cell, which are the sought after functions, and \( h^v \) are known fluctuation shape functions.

Figure 1: The basic cell of the bi-periodic plate

Let us define the functional

\[
P = \frac{t^2}{2} \rho (\dot{w})^2 - \frac{1}{2} B_{a\beta\gamma\delta}^{ab} w_{a\beta\gamma\delta} - pw \]

where \( p \) is an external load of the plate, and \( t \) is a parameter.

Model equations, in homogenizations with micro-local parameters, are obtained after an averaged functional \( P \) in the form

\[
\frac{\partial (P)}{\partial \dot{u}} = \frac{\partial (P)}{\partial \dot{v}^v} = \frac{\partial (P)}{\partial v^v} = 0 \; ; \; \frac{\partial (P)}{\partial \dot{v}^v} = 0
\]

The averaging functional \( P \) is defined

\[
\langle P \rangle = \frac{1}{m} \int_\Omega P(x_1, x_2) d\Omega
\]

Equations (3) will take the form

\[
\rho_\alpha \ddot{u} + E_{a\beta\gamma\delta}^{ab} u_{a\beta\gamma\delta} + E_{a\beta}^{ab} v^v_{a\beta} + p = 0
\]

\[
E_{a\beta\gamma\delta}^{ab} v^v_{a\beta\gamma\delta} + E_{a\alpha}^{ab} v^v_{a\alpha} = 0
\]

where

\[
\rho_\alpha = \eta^a \rho^a_0, \quad E_{a\beta\gamma\delta}^{ab} = \eta^a B_{a\beta\gamma\delta}^a,
\]

\[
E_{a\beta}^{ab} = \eta^a B_{a\beta}^a
\]

and
Let us consider a special case. Assuming that the plate is a pivot bearing, where initial conditions are as follows
\[ u(x_1, x_2, 0) = u_0(x_1, x_2) \]
where \( c_0 \) is the initial deflections value in the centre of plate whereas \( v_0 \) is movement velocity of the central surface of the plate at \( t = 0 \).

Dimensions of the plate are equal \( l_x = l_y = 5 \text{ m} \), a thickness is \( h = 0,2 \text{ m} \). Material constants are assumed as for concrete and steel.

\[ (E_{11}) \alpha \beta = (E_{22}) \alpha \beta = (E_{12}) \alpha \beta = (G_{12}) \alpha \beta = (E_{11}) \text{ for concrete, } (E_{22}) \text{ for steel} \]

Let us consider a special case. Assuming that the plate is a pivot bearing, where initial conditions are as follows
\[ u(x_1, x_2, 0) = u_0(x_1, x_2) \]
where \( c_0 \) is the initial deflections value in the centre of plate whereas \( v_0 \) is movement velocity of the central surface of the plate at \( t = 0 \).

Dimensions of the plate are equal \( l_x = l_y = 5 \text{ m} \), a thickness is \( h = 0,2 \text{ m} \). Material constants are assumed as for concrete and steel.

![Figure 2: The averaging displacement of the plate at time a) \( t = 1 \text{ s} \), b) \( t = 2 \text{ s} \)](image)

![Figure 3: The plate’s deflection in cross-section \( x_2 = L_2/2 \) at \( t = 2 \text{ s} \) and different basic cell dimensions](image)

References


Impact of tolerance averaging of heat transfer equation into exact description of a boundary effect phenomenon

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Abstract

The aim of the study is to propose a partially-averaged model of heat conduction in simple micro-periodic composite conductors. The starting point for the implementation of the proposed method of modeling is the modeling method known as “the tolerance averaging technique”, as proposed by Professor Czesław Woźniak. There are some of basic monographs referred to here on this subject, cf. [3,4,5]. In this work, it is proposed to instead apply “micro-macro hypothesis” (used in tolerance modeling), a more general technique”, as proposed by Professor Czesław Woźniak. There are some of basic monographs referred to here on this subject, cf. [1,2]. Unlike other known averaged models of this type, the resulting model allows for the formulation exact solutions to initial-boundary value problems formulated for the hyperbolic heat conduction equation. If tolerance approximations will be applied to averaged temperature field this model becomes an asymptotically exact model. The term “asymptotically exact model” refers to models in the framework of whose solutions coincide with exact solutions to the mentioned problems for the hyperbolic heat transfer equation when the microstructure parameter tends to zero. The aforementioned property of the proposed model allows us for situations to compare the behavior of the composite conductors made of the same finite number of components but different in their geometric arrangement. This comparison is made, while the reactions of these composites to the same external thermal conditions (represented by the imposed initial-boundary conditions) are be examined. Other known averaged models of this type do not allow to carry out such comparisons, because the solutions obtained in the framework of these models are only approximate and the accuracy of these solutions can be verified in the present state of knowledge only empirically.

Keywords: tolerance averaging, heat conduction, periodic conductors, boundary effect, Fourier series

1. Introduction

The aim of the study is to propose a partially-averaged model of heat conduction in simple micro-periodic composite conductors. In this model, as in many known models of this type, the type of microstructure is represented by a single scalar parameter, which is referred to as a microstructure parameter, cf. [1,2]. Unlike other known averaged models of this type, the resulting model allows for the formulation exact solutions to initial-boundary value problems formulated for the hyperbolic heat conduction equation. If tolerance approximations will be applied to averaged temperature field this model becomes an asymptotically exact model. The term “asymptotically exact model” refers to models in the framework of whose solutions coincide with exact solutions to the mentioned problems for the hyperbolic heat transfer equation when the microstructure parameter tends to zero. The aforementioned property of the proposed model allows us for situations to compare the behavior of the composite conductors made of the same finite number of components but different in their geometric arrangement. This comparison is made, while the reactions of these composites to the same external thermal conditions (represented by the imposed initial-boundary conditions) are be examined. Other known averaged models of this type do not allow to carry out such comparisons, because the solutions obtained in the framework of these models are only approximate and the accuracy of these solutions can be verified in the present state of knowledge only empirically.

2. Fundamental concepts

The starting point of consideration is the known hyperbolic heat transfer equation

\[
- \nabla (K \nabla \theta) + c \nabla \theta + \tau \nabla \theta = b + \tau \theta
\]

(1)

in which \( \theta = \theta(x,t) \), \( z \in R^3 \), \( t \geq 0 \), denotes the temperature field, \( c \) is a specific heat field, \( K = (k_{ij}) \) is the heat conductivity matrix, \( \tau \) is the relaxation time and \( b \) is the density of heat sources.

In the subsequent investigations the averaging \( \langle f \rangle(x) \) of an arbitrary integrable field \( f \) defined on \( R^n \) plays an important role, and is defined as

\[
\langle f \rangle(x) = \frac{1}{|\Delta|} \int_{x+\Delta} f(\xi) d\xi
\]

(2)

and which is a constant field provided that \( f \) is a \( \Delta \)-periodic field. Investigations of the mentioned above physical problem will be focused on the analysis of initial-boundary condition.

Examples of two-constituent conductors together with related basic cells \( \Delta \) are illustrated in Fig. 1.

Figure 1: Examples of periodic rigid conductor
The tolerance averaging technique introduce the micro-macro hypothesis, cf. [3,4,5]. In accordance with that hypothesis, the temperature field \( \theta \) can be approximated with an acceptable accuracy with

\[
\theta_{\text{av}} = \tilde{\theta}(z,t) + h^A(x)\phi_x(z,t),
\]

where fields \( \tilde{\theta} \) and \( \phi_x \), slowly varying in periodicity directions, are the tolerance averaging of temperature field and amplitude fluctuations fields, respectively. Here and in the sequel the summation convention holds with respect to indices \( A, B, \ldots = 1, \ldots, N \). Symbols \( h^A(x) \), \( A = 1, \ldots, N \), denote in (3) tolerance shape functions.

The tolerance micro-macro hypothesis can be formulated in the following form:

Tolerance micro-macro hypothesis. The residual part of the temperature field \( \theta_{\text{res}} \) being the difference between the temperature field \( \theta \) and its tolerance parts \( \theta_{av} \) given by Eqn (3),

\[
\theta_{\text{res}} = \theta - \theta_{\text{av}},
\]

can be treated as zero.

In the study, instead of the above micro-macro hypothesis, we are to introduce its extended version which allow as to obtain the exact description of the boundary effect phenomenon.

3. Modelling procedure

The course of modelling is based on the two fundamental assumptions, cf. [2].

The first modeling assumption is a certain extension of the micro-macro hypothesis, given by Eqn (3). According to this new hypothesis we are to impose on the choice of shape functions \( h^A(x) \), \( A = 1, \ldots, N \), an additional requirements under which the solution to the heat transfer equation can be investigated in the form of a certain orthogonal Fourier expansion. The orthogonality is determined here by scalar product defined by formula

\[
\langle f_1, f_2 \rangle = \int f_1 \cdot f_2.
\]

Assumption 1. (Extended micro-macro hypothesis) The component of heat flux vector \( q_{av} = KV\theta_{av} \) generated by residual part \( \theta_{av} \) of the temperature field, normal to the surface \( I \) separating constituents, vanish in almost everywhere on \( I \). Moreover, \( \theta_{av} \) should be regular enough to be represented by the certain Fourier expansion. Shape functions as well as elements of Fourier basis should depend exclusively on the geometrical structure of the composite and to be independent of its material structure.

The second modeling assumption allows the use of tolerance modeling technique as a tool to derive model equations.

Assumption 2. (The validity of the tolerance modeling approach) Coefficients of the Fourier expansion mentioned in Assumption 1. should satisfy equations obtained as the result of the tolerance variant of the orthogonalization method.

The possibility of the obtaining the expected model equations strongly depends on the existence of shape functions leading to the fulfillment of Assumptions. In general, it is not known whether such a shape functions exist for every periodic composite. It must be emphasized, that if the proposed method of modeling was applied it is possible to obtain an open formula for the effective conductivity tensor without using limit passage to the asymptotic case i.e. while the diameter of the basic cell not necessary tends to zero. Moreover, it was proved, that short-wave external disturbances temperature field are transmitted by two-phased periodic laminates regardless of the behavior of the average temperature inside the conductor.

4. Final remarks

We shall emphasize that the character of that short-wave transmission of external disturbances temperature field is still unknown. Since the near boundary regions of periodic composites usually fast damage it suggest that the proposed model includes the description of the boundary effect behaviour in which such damaging effects will be illustrated. Indeed, the obtained model equations allow for solutions in which suppressions are described by matrices with an unnecessary real spectrum.

References

Tolerance modelling of vibrations in three-layered periodic structures

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Abstract

The paper deals with a vibration analysis of three-layered composite structures. The considered structures consist of outer layers, assumed Kirchhoff type thin plates, connected by elastic, Winkler type material, described with certain elasticity modulus $k$. Both outer layers and the core may detect a certain periodic microstructure. Since the equations of motion of such structures are partial differential equations with highly-oscillating, periodic and non-continuous coefficients, the tolerance averaging technique is used to obtain approximate equations with constant coefficients. Based on the aforementioned technique, equations of the tolerance model and the asymptotic-tolerance model are derived and used to evaluate free vibration frequencies.

Keywords: periodic sandwich plates, free vibration analysis, tolerance averaging technique

1. Introduction

Sandwich plates under consideration are three-layered composite structures, their upper and lower layers are assumed to be thin plates. The third, inner layer can be made of an elastic, Winkler type material, described with an elasticity modulus $k$. The analysis of dynamic behaviour of such plate was performed e.g. by Szcześniak [2], where the motion of the structure was described with a system of well-known Kirchhoff equations of motion. The proposed method can be useful, unless the material properties and geometry (e.g. a bending stiffness) are constant for the whole structure. In the note it is assumed that both outer layers and the core can detect a certain periodic microstructure, determined by the diameter of small repetitive element, called the periodicity cell. In such a case, coefficients in Kirchhoff equations of motion are highly-oscillating, periodic and non-continuous, which cause many difficulties in solving the problem. In order to obtain a convenient tool for vibration analysis of such plates, the tolerance averaging technique, presented by Woźniak (ed.) [3], can be used.

2. Modelling foundations

Let $O_{1,2,3}$ be an orthogonal Cartesian coordinate system and let us denote $x=(x_1,x_2)$. The three-layered plate under consideration is assumed to have spans $L_1$ and $L_2$ in $x_1$ and $x_2$-axis directions, respectively, and total thickness $H(x)$. Hence, it is defined in a region $\Delta = [0,L_1] \times [0,L_2] \times [-\frac{1}{2}H(x),\frac{1}{2}H(x)]$. According to the plate microstructure, it is possible to distinguish a small, repeatable element called a periodicity cell, with its diameter being microstructure parameter $l$. Both the characteristic dimensions of the plate and the microstructure parameter must satisfy the following normalizing conditions: $H(x) < c < \min(L_1,L_2)$, cf. Fig. 1.

Let us introduce outer layer mass densities per unit area as $\mu_i(x)$ and bending stiffness as $B_i(x)$, $i=1,2$. Moreover, let us denote $t$ a time coordinate and an overdot a time derivative.

The basic concepts of the tolerance averaging technique was described e.g. by Woźniak (ed.) [3]. It is based on some introductory concepts, like: an averaging operator, a tolerance-periodic function, a slowly-varying function and two fundamental assumptions. The micro-macro decomposition of the outer plates deflections $u_i(x,t)$, $\omega_i(x,t)$ assumes, that plate deflections can be written as sums of macrodeflections $W_i(x,t)$, $\gamma_i(x,t)$ and fluctuation functions $\nu_i(x,t)$, $\gamma_i(x,t)$, $A, B=1,\ldots,N$, while the tolerance averaging approximation assumes, that certain terms can be treated negligibly small thus they can be omitted.

3. Modelling procedure and model equations

The starting point of the modelling procedure is formulating an action functional $A$. For the problem under consideration it can be written as:

$$A\left( w(x) \right) = \int_\Delta \int_0^\infty A_i\left( y, \partial_{x_1}w(y,t) \right) dt dy,$$

where the Lagrangean $A_i$ is given by:

$$A_i = \frac{1}{2} \left( \mu_i \left( \partial_{x_1}w_1 + B_i w_1 \right) \partial_{x_1}w_2 - k w_1 \right) + p_i w_2.$$

From the principle of stationary action, applied to functional $A$, we arrive at a system of equations in the form:

$$\partial_{x_1}\left( B_i \partial_{x_1}w_i + \mu_i w_i + k (w_1 - w_i) \right) = p_i,$$

where $p_i(x,t)$, $p_3(x,t)$ are loadings along the $x_3$-axis and $k(x)$ is an elasticity modulus of Winkler type core.

3.1. The tolerance model

The tolerance modelling procedure is based on the system of equations (3). After applying the averaging operator, it must be transformed using both the micro-macro decomposition and the tolerance averaging approximations. As a result, the averaged form of a system of equations (3) can be presented as:
averaged functional $A$ as a microstructure parameter differential equations with constant coefficients, where underlined terms are dependent on parameter $l$ shape functions, having the same geometry. Hence, it can be written:

$$\partial_{\alpha l}(B_\alpha \partial_{\alpha l}W_i + <B_\alpha \partial_{\alpha l}g^i >_v >_\mu >_\mu W_i + + k > (W_i - W_2) = <p_\mu >,$$

$$<B_\alpha \partial_{\alpha l}g^i >_\alpha >_\mu >_\mu W_i + <B_\alpha \partial_{\alpha l}g^i >_v >_\mu >_\mu W_i + + k > (W_i - W_2) = <p_\mu >,$$

$$<B_\alpha \partial_{\alpha l}g^i >_\alpha >_\mu >_\mu W_i + + k > (W_i - W_2) = <p_\mu >.$$  

(4)

The above system of equations is a system of $2N+2$ partial differential equations with constant coefficients, where $W_i(x,t)$ and $v_i(x,t)$ are basic unknowns and $g^i(x)$ are the known mode-shape functions, $A=1, \ldots, N$. It can be also observed, that only the underlined terms are dependent on the microstructure parameter $l$.

Equations (4) constitute the tolerance model of the periodic composite structures under consideration.

3.2. The asymptotic-tolerance model

The asymptotic-tolerance model can be obtained in two steps, which are described and explained by Woźniak (eds.) [3], Kazmierczak and Jędrysiak [1]. In the first step, the asymptotic solution to the problem is derived by computing the limit of these equations for the considered models are the fourth degree polynomial in $\omega^2$, hence it is possible to derive analytical solutions to these equations.

$$G = g_1^i = g_2^i = \hat{g}_1^i = \hat{g}_2^i = I^2 \cos (2\pi x / l).$$

(9)

Let us also assume solutions to governing equations as:

$$W_i(x,t) = A_i \sin (k_i x) e^{\omega t},$$

$$v_i(x,t) = A_i \sin (k_i x) e^{\omega t}.$$  

(10)

Keeping in mind the above assumptions, the characteristic equation for both the tolerance and asymptotic-tolerance models is obtained. These equations for the considered models are shown separately.

```math
\begin{align*}
W_i(x,t) &= A_i \sin (k_i x) e^{\omega t}, \\
v_i(x,t) &= A_i \sin (k_i x) e^{\omega t}.
\end{align*}
```

Figure 3: Dimensionless free vibration frequency parameters versus the parameter $X$ (cf. Fig. 2) (solid lines – the tolerance model, dotted lines – the A-T model)

More results and some comments and remarks will be shown separately.

References


A new combined model of dynamic problems for thin uniperiodic cylindrical shells

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Abstract

The objects of consideration are thin linearly elastic Kirchhoff-Love-type circular cylindrical shells having a periodically micro-inhomogeneous structure in circumferential direction (uniperiodic shells). The aim of this note is to formulate a new non-asymptotic averaged model for the analysis of selected dynamic problems for these shells. This, so-called, general combined model is derived by applying the combined modelling technique which includes both the asymptotic and tolerance non-asymptotic procedures. Moreover, a certain extended version of the known tolerance modelling technique, proposed by Tomczyk and Woźniak in Ref. [1], is used. Contrary to the starting exact shell equations with highly oscillating, non-continuous and periodic coefficients, governing equations of the combined model have constant coefficients depending also on the period of inhomogeneity. An important advantage of the combined model is allowing for the study micro-dynamics of periodic shells independently of their macro-dynamics.

Keywords: micro-periodic thin cylindrical shells, mathematical modelling, dynamic problems, length-scale effect

1. Introduction, modelling procedure, combined model

Thin linearly elastic Kirchhoff-Love-type cylindrical shells with a periodically inhomogeneous structure in circumferential direction (uniperiodic shells) are analysed, Fig. 1.

In various problems of these shells the effect of a cell size (called the length-scale effect) on the overall shell behaviour cannot be neglected. In order to analyse the length-scale effect in dynamic or stability problems, the new averaged non-asymptotic models of thin cylindrical shells with a periodic micro-heterogeneity either along two directions tangent to the shell midsurface (biperiodic structure) or along one direction (uniperiodic structure) have been proposed by Tomczyk in a series of publications and summarized as well as extended in Ref. [2]. These, so-called, tolerance models have been obtained by applying the non-asymptotic tolerance averaging technique presented by Woźniak in many monographs, e.g. Ref. [3]. Governing equations of the tolerance models have coefficients which are constant or slowly varying depend on a cell size.

In this note, we will formulate a new general combined model for the analysis of dynamic problems for the uniperiodic shells under consideration. This model will be derived by applying the combined modelling which includes two techniques: the consistent asymptotic modelling procedure given by Woźniak in Ref. [3] and a certain extended version of the known tolerance non-asymptotic modelling technique proposed by Tomczyk and Woźniak in Ref. [1]. This version is based on a new notion of weakly slowly-varying functions.

We assume that \( x^1 \) and \( x^2 \) are coordinates parametrizing the shell midsurface \( M \) in circumferential and axial directions, respectively, Fig. 1. Define \( \Omega = (0,L_1) \times (0,L_2) \) as a set of midsurface parameters \( x = (x^1,x^2) \) in \( \mathbb{R}^2 \), where \( L_1, L_2 \) are length dimensions of \( M \). Sub- and superscripts \( \alpha, \beta, \ldots \) run over 1,2 and are related to \( x^1, x^2 \), summation convention holds. Partial differentiation related to \( x^1 \) is represented by \( \partial_1 \).

Moreover, it is denoted \( \partial_{\alpha \beta} \equiv \partial_\alpha \partial_\beta \). Let \( a^{\alpha \beta} \) stand for the contravariant midsurface first metric tensor.

We denote \( x = x^1 \in \Omega = (0,L_1) \) and \( \xi = x^2 \in \Xi = (0,L_2) \).

Let \( d(x) \) and \( r \) be the shell thickness and the midsurface curvature radius, respectively. The time coordinate is denoted by \( t \), \( t \in [t_i,t_f] \). Let \( \lambda = \lambda_1 \) be a period length of the shell structure in \( x = x^1 \)-direction, cf. Fig. 1. Period \( \lambda \) is assumed to satisfy conditions: \( \lambda / d_{\alpha \beta} >> 1 \), \( \lambda / r << 1 \) and \( \lambda / L_1 << 1 \).

Hence, \( \lambda \) will be called the microstructure length parameter.

Define the basic cell \( \Delta = [-\lambda / 2, \lambda / 2] \).

Figure 1: Example of uniperiodic cylindrical shell

Denote by \( u_i = u_i (x, \xi, t) \), \( w = w(x, \xi, t) \), \( (x, \xi, t) \in \Omega \times \Xi \times \mathbb{I} \), the shell displacements in directions tangent and normal to \( M \), respectively. Elastic properties of the shell are described by shell stiffness tensors \( D^{\alpha \beta \gamma \delta} (x) \), \( B^{i j k} (x) \). Let \( \mu (x) \) stand for a shell mass density per midsurface unit area.

It is assumed that the behaviour of the shell under consideration is described by the action functional determined by Lagrange function \( \mathcal{L} \) being a highly oscillating function with respect to \( x \) and having a well-known form

\[
L = \frac{1}{2} \left[ D^{\alpha \beta \gamma \delta} \partial_\alpha \partial_\beta u_j \partial_\gamma \partial_\delta u_j + \frac{2}{r} D^{i j k} \partial_\alpha \partial_\beta w_k \partial_\gamma \partial_\delta w_k + \frac{1}{r} D^{i j k} \partial_\alpha \partial_\beta w_j \partial_\gamma \partial_\delta w_k + \frac{1}{r} D^{i j k} \partial_\alpha \partial_\beta w_k \partial_\gamma \partial_\delta w_j + \frac{1}{r} D^{i j k} \partial_\alpha \partial_\beta w_j \partial_\gamma \partial_\delta w_j \right].
\]

The combined modelling technique used to starting Lagrangian (1) is performed in two steps. The first step is based on the consistent asymptotic averaging of lagrangian (1) under the consistent asymptotic
decomposition, Ref. [2,3]. This decomposition states that displacement fields \( u_{\alpha} \), occurring in (1) to be replaced by families of fields depending on small parameter \( \varepsilon = 1/m, m = 1, 2, ... \) and defined in an arbitrary cell. These families of displacements are decomposed into averaged part independent of \( \varepsilon \) and highly-oscillating part depending on \( \varepsilon \).

The averaged part is described by unknown macrodisplacements \( u_{\alpha} \) being continuously bounded in periodicity direction. The strongly oscillating part is represented by the known periodic highly oscillating fluctuation shape functions \( h, g \) depending on \( \varepsilon \) and by unknown fluctuation amplitudes \( U_{\alpha} \) being continuously bounded in direction of periodicity. Introducing the asymptotic decomposition into (1), under limit passage \( \varepsilon \to 0 \) we obtained the averaged form of Lagrangian (1). Then, applying the principle of stationary action, we obtain the governing equations of consistent asymptotic model for the anisotropic shell under consideration. Coefficients of these equations are constant and independent of a cell size. Hence, the model obtained in the first step is referred to as the macroscopic model. It is assumed that within this model, solutions \( u_{\alpha} \), \( w \) to the problems under consideration are known. Hence, there are also known functions \( u_{\alpha} = u_{\alpha}^0 + hU_{\alpha} \) and \( w = w^0 + gW \).

The second step is based on the tolerance averaging of lagrangian (1) under so-called superimposed decomposition. The extended version of the known tolerance modelling technique proposed in Ref. [1] is applied.

The fundamental concepts of the tolerance approach under consideration are those of two interacting systems between points and real numbers determined by tolerance parameters, weakly slowly-varying functions and the averaging operation. A continuous, bounded and differentiable function \( F() \) defined in \( \Omega = [0, L_0] \) is called weakly slowly-varying of the \( R \)-th kind with respect to cell \( \Delta \) and tolerance parameters \( \delta, F \in WS\{^R\}(\Omega, \Delta) \), if can be treated (together with its derivatives up to the \( R \)-th order) as constant on an arbitrary cell. Nonnegative integer \( R \) is assumed to be specified in every problem under consideration.

The averaging operator for an arbitrary function \( f() \) being integrable and bounded in every cell is defined by

\[
< f > (x) = \frac{1}{\Delta} \int f(z) dz, \quad z \in \Delta(x), \quad x \in \Omega. \tag{2}
\]

The tolerance modelling is based on two assumptions. The first of them is called the tolerance averaging approximation, cf. Ref. [1,2,3]. The second one is termed the micro-macro decomposition.

In the problem under consideration, we introduce the extra micro-macro decomposition superimposed on the known solutions \( u_{\alpha}^0, w_0 \) obtained within the macroscopic model

\[
\begin{align*}
u_{\alpha}(x, \xi, t) &= u_{\alpha}^0(x, \xi, t) + c(x)Q_{\alpha}(x, \xi, t), \\
w_\alpha(x, \xi, t) &= w_0(x, \xi, t) + h(x)W(x, \xi, t),
\end{align*}
\tag{3}
\]

where fluctuation amplitudes \( Q_{\alpha}, W \) are the new weakly slowly-varying unknowns, i.e. \( Q_{\alpha} \in WS\{^R\}(\Omega, \Delta), \quad W \in WS\{^{R-1}\}(\Omega, \Delta) \). Functions \( c(x) \) and \( h(x) \) are the new \( \lambda \)-periodic, continuous and highly-oscillating fluctuation shape functions which are assumed to be known in every problem under consideration.

We substitute the right-hand sides of (3) into (1). The resulting Lagrangian is denoted by \( L_0 \). Then, we average \( L_0 \) over cell \( \Delta \) using an averaging formula (2) and applying the tolerance averaging approximation. As a result we obtain function \( < L_0 > \) called the tolerance averaging of starting lagrangian (1) in \( \Delta \) under superimposed decomposition (3). Then, applying the principle of stationary action, we obtain the system of Euler-Lagrange equations for \( Q_{\alpha}, W \), which can be written in an explicit form

\[
\begin{align*}
< D^{\alpha\alpha}\frac{\partial}{\partial t} > \tilde{\varepsilon}_{\alpha} Q_{\alpha} &- < D^{\alpha\alpha}\frac{\partial}{\partial x_{\alpha}} c_\alpha c_{\beta} > Q_{\alpha} - \mu c^{\alpha\beta} \gamma^{\alpha \beta} = 0, \\
< D^{\beta\alpha}\frac{\partial}{\partial x_{\beta}} c_\alpha > &+ < D^{\beta\alpha}\frac{\partial}{\partial x_{\alpha}} c_\beta > u_{\alpha}^0 >, \\
< B^{\alpha\beta}V_{\beta} > &+ 2 < B^{\alpha\alpha}b_{\alpha} > + \tilde{V} \tilde{V} = - < B^{\alpha\alpha}b_{\alpha} >, \\
\end{align*}
\tag{4}
\]

Equations (4) and (5) together with the micro-macro decomposition (3) constitute the superimposed microscopic model. Coefficients of the derived model equations are constant and some of them depend on a cell size \( \lambda \). Underlined terms.

The right-hand sides of (4) and (5) are known under assumption that \( u_{\alpha}^0, w_0 \) were determined in the first step of modelling.

2. Remarks and conclusions

The general combined model equations consist of macroscopic model equations formulated by means of the consistent asymptotic procedure which are combined with superimposed microscopic model equations (4), (5) derived by applying an extended version of the tolerance modelling technique (Ref. [1]) and under assumption that in the framework of the macroscopic model the solution to the problem under consideration is known.

In contrast to exact shell equations with discontinuous, highly oscillating and periodic coefficients, the combined model equations proposed here have constant coefficients. Moreover, some coefficients of the superimposed microscopic model equations depend on a cell size \( \lambda \). Thus, the combined model makes it possible to analyse the length-scale effect.

It can be shown, that under special conditions imposed on the fluctuation shape functions we can obtain superimposed microscopic model equations, which are independent of the solutions obtained in the framework of the macroscopic model. It means, that an important advantage of the combined model is that it makes it possible to describe selected problems of the shell micro-dynamics (e.g. free micro-vibrations) independently of the shell macro-dynamics.

References


Tolerance modelling of elastic-nonelastic multilayered two-component composites

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Abstract
The work concerns tolerance modelling of elastic-nonelastic periodic layered composites. A comparative analysis was performed of the tolerance model and the model with microlocal parameters. 

Keywords: multilayered periodic composite, tolerance modelling, modelling with microlocal parameters, elasticity, plasticity, viscoelasticity

1. Introduction
The work presents a tolerance model of elastic-plastic two-component periodic composites with kinematic hardening and elastic/viscoplastic composites of such type (viscous effects do not appear within the framework of elastic deformations, occurring in the yield point exceedance.

The obtained results will be compared to the results of Ref. [3], where the modelling procedure employed the method with microlocal parameters based on the non-standard analysis [3]. The equations obtained within the framework of these models and the detailed solutions will be compared.

2. Object of analysis
Let us assume that, in a reference configuration in a three-dimensional reference space, the elastic-nonelastic medium under consideration occupies an area $\Omega = (0, L_1) \times (0, L_2) \times (0, L_3)$. The points of the area $\Omega$ are denoted by $x \in \Omega$, $x = (x_1, x_2, x_3)$, where $x_1 \in (0, L_1)$, $x_2 \in (0, L_2)$, $x_3 \in (0, L_3)$.

![Figure 1: Example of a periodic layered composite](image)

3. Modelling procedure
The modelling procedure for elastic-nonelastic composites will be based on the tolerance modelling method, proposed by Cz. Woźniak and E. Wierzbicki (Ref. [5]), using the term of slowly varying function and dimensionless shape function [4]. In addition, we assume that the loading velocity changes are so small that the problem can be considered quasi-stationary.

The constitutive relations for the layered composites regarded as non-homogeneous materials will have a form presented in the subsections below.

3.1. Elastic-plastic composites (Ref. [2])

\[
\begin{align*}
\sigma^{00}(x, t) &= 2\mu(x)\kappa^{00}(x, t) + \lambda(x)\gamma^{00}(x, t) + H(f(x, t)), \\
\chi^{00}(x, t) &= H(f(x, t))H(w(x, t)) - \frac{2m(x)\mu(x)}{m(x) + \mu(x)k^2(x)}, \\
\end{align*}
\]

where:

\[
\sigma^{00}(x, t) = \sigma^{00}(x, t) - \chi^{00}(x, t), \\
\chi^{00}(x, t) = \sigma^{00}(x, t) - \frac{1}{3}G^{00}(x)\kappa^{00}(x, t), \\
\sigma^{00}(x, t) = \sigma^{00}(x, t) - \frac{1}{3}G^{00}(x)\gamma^{00}(x, t), \\
\chi^{00}(x, t) = \sigma^{00}(x, t) - \frac{1}{3}G^{00}(x)\chi^{00}(x, t),
\]

\[
\chi^{00}(x, t) = \frac{m(x)\mu(x)}{m(x) + \mu(x)k^2(x)}.
\]
3.2. Elastic/viscoplastic composites (Ref. [1])

Let $\gamma(x)$ be a viscosity coefficient and $P_{\alpha\beta\gamma}(x)$ be a tensor of stiffness moduli for an isotropic material. Let us define a convex closed set in $\mathbb{R}^6$

$$K = \left\{ \sigma^{\alpha\beta}(x,t) \in \mathbb{R}^6 \mid P_{\alpha\beta\gamma}(x) = \sigma^{\alpha\beta}(x,t), \quad F(\sigma^{\alpha\beta}(x,t)) \leq 0 \right\}$$

almost everywhere in $\Omega \times [0,t_f]$, where $F$ is some real continuous, smooth in ranges and convex function depending on arguments $\sigma^{\alpha\beta}(x,t)$.

Let

$$f_\gamma(x)\left(\sigma^{\alpha\beta}(x,t)\right) = \frac{1}{4\gamma(x)}\left[\sigma^{\alpha\beta}(x,t) - P_k \sigma^{\alpha\beta}(x,t)\right]$$

$$\left[\sigma^{\alpha\beta}_{\beta\gamma}(x,t) - P_k \sigma^{\alpha\beta}_{\beta\gamma}(x,t)\right]$$

where $(x,t) \in \Omega \times [0,t_f]$ almost everywhere, wherein an operator $P_k(\cdot)$ is the operator of orthogonal projection of the point $\sigma^{\alpha\beta}(x,t)$ in $\mathbb{R}^6$ onto the set $\tilde{K}$.

The constitutive equations will assume the form

$$\dot{\varepsilon}^{\alpha\beta}(x,t) - A_{\alpha\beta\gamma\delta}(x) \varepsilon_{\gamma\delta}(x,t) = \frac{\partial f_\gamma(x)}{\partial \sigma^{\alpha\beta}(x,t)}$$

where the functions $A_{\alpha\beta\gamma\delta}(\cdot)$ and $\gamma(\cdot)$ describe the material features of the layered composite.

In the tolerance modelling procedure we assume that the displacement field is given in the form:

$$\tilde{u}(x,t) = \tilde{u}(x,t) + h(x) \tilde{q}(x,t),$$

where the functions $\tilde{u}(x,t), \tilde{q}(x,t) \in \mathcal{S}^v_{2\delta}(0,L_4]$ - they are slowly varying functions changing their values in the direction perpendicular to the layers, $h(\cdot)$ is an a priori given shape function, for the layered composites being continuous, dimensionless and piecewise linear function.

As a result of the performed tolerance modelling procedure, equations are obtained governing the composites under consideration. These equations are fundamendal perform a comparative analysis with the equations obtained in Ref. [3].

4. Conclusions

The most important conclusion of the analysis are that:
- the behaviour for elastic-nonelastic multilayered two-component composites can be described by equations with smooth and slowly varying coefficients,
- the solutions obtained within the frames of tolerance modelling better describe the structure of elastic-nonelastic multilayered two-component composites than the theory of microlocal parameters.

References

Heat conduction in biperiodic rigid composites

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Abstract

The work concerns the tolerance modelling of heat conduction in rigid biperiodic composites. There are presented numerical solutions for some boundary problems.

Keywords: biperiodic composite, tolerance modelling, heat conduction

1. Introduction

First papers for mathematical modelling of periodic composites were on of the beginning of the twentieth century. The problem of heat conduction and thermoelasticity in these types of materials is well known and extensively described in the literature, e.g. [1,2,3,5,6,8,11]. The most important problem in describing heat conduction within the frames of Fourier equations for periodic composites is fact that equation of heat conduction has discontinuous and highly oscillating components.

There are many mathematical methods to find the approximated solution of this problem. One of the most important method of modelling periodic composites is homogenization, Ref. [5,7]. We can also mention here another methods for multilayered twocomponent periodic composites, which were proposed in Ref. [12] - microlocal parameters and in Ref. [11] - tolerance modelling.

These methods can be used for two and three direction periodic composites only for some special structures, Ref. [7]. This problem was also discussed in [4,10]. Problem of heat conduction for biperiodic rigid composites can be described within the frames of the model of tolerance modelling for example if the periodicity cell consist of three different materials. Two parts of cell are isotropic and two parts are orthotropic with perpendicular direction of orthotropy, Fig. 1. The structure of these composites must satisfy the condition of band structure additionally.

2. Aim of contribution

The aim of contribution is heat conduction in rigid biperiodic composites. Let us assume that, in a reference configuration in a three-dimensional Cartesian space, the biperiodically rigid composite under consideration occupies an area \( \Omega = (0,L_1) \times (0,L_2) \times (0,L_3) \). The points of the area \( \Omega \) are denoted by \( x \in \Omega \), \( x = (x_1,x_2,x_3) \), where \( x_1 \in [0,L_1) \), \( x_2 \in (0,L_2) \), \( x_3 \in [0,L_3) \).

It is assumed, like in Ref. [4], that the periodicity cell \( \Delta \) which is a rectangle with length of side \( h_i \), \( h_2 \) consists of four rectangles \( (\Delta^i, i,j=1,2) \) with length of sides \( h_i \), \( h_2 \), \( i,j=1,2 \), Fig. 1. Let matrixes of heat conduction \( K^{ij} \) and specific heat \( c^{ij} \), \( i,j=1,2 \) in these parts are

\[
K^{11} = \begin{bmatrix} k_R & 0 \\ 0 & k_R \end{bmatrix}, \quad K^{12} = \begin{bmatrix} k_R & 0 \\ 0 & k_M \end{bmatrix}, \quad K^{21} = \begin{bmatrix} k_M & 0 \\ 0 & k_M \end{bmatrix}, \quad K^{22} = \begin{bmatrix} k_M & 0 \\ 0 & k_M \end{bmatrix}, \quad c^{11} = c^{12} = c^{21} = c^{22}.
\]

Figure 1: Example of a biperiodically cell in composite

Fourier equation of heat conduction for considered biperiodic composites is

\[
c(x,y,z,t) = \theta(x,y,z) - \partial_{tt} [k^{ij}(x,y,z) \theta(x,y,z)] = f(x,y,z), \quad \alpha, \beta = 1,2
\]

where: \( \theta (\cdot) \) - temperature in region of \( \Omega \) for every \( t \in [0,t_0) \),

\[
\partial_{tt} (\cdot) = \frac{\partial}{\partial t^2}, \quad \partial_{x} (\cdot) = \frac{\partial}{\partial x}, \quad f(\cdot) \quad \text{- internal heat sources, } \epsilon(\cdot)
\]

specific heat, \( k^{ij}(\cdot) \) - thermal conductivities such that

\[
k^{ij}(\cdot) = 0 \quad \text{for } \alpha \neq \beta \quad \text{and}
\]

\[
k^{11}(x) = \begin{cases} k_R \quad \text{when } x \in \Delta^1 \\ k_R \quad \text{when } x \in \Delta^2 \\ k_M \quad \text{when } x \in \Delta^2 \end{cases}, \quad k^{22}(x) = \begin{cases} k_M \quad \text{when } x \in \Delta^1 \\ k_R \quad \text{when } x \in \Delta^1 \\ k_M \quad \text{when } x \in \Delta^2 \end{cases}
\]

Equation (1) is partial differential equation with discontinuous and highly oscillating coefficients \( k^{ij}(\cdot) \) and \( \epsilon(\cdot) \). The most important fact is that Eqn (1) will be replaced by equation with constant coefficients. In Section 3 tolerance model equations of stationary heat conduction for rigid biperiodic composites is presented.
3. Modelling procedure and model equations

A modelling procedure is based on a special variant of a tolerance modelling method – local homogenization method, Ref. [9]. In this method the field \( \Theta(x) \) is approximated by

\[
\Theta(x) \approx \tilde{\Theta}(x) = \Theta(x) + \gamma^1(x_1) \psi^1(x) + \gamma^2(x_2) \psi^2(x)
\]

(2)

where for every \( x_i \in (0, L_i) \) fields \( \Theta, \psi^\alpha(x_i) \in C^4((0, L_i) \times (0, L_j)) \) are unknown functions: macro-temperature and amplitude fluctuations of temperature, respectively. \( \gamma^\alpha(x_i) \), \( \alpha = 1, 2 \) are fluctuation shape functions, which are linear, continuous functions depending only on one argument \( (\gamma^1(x_1), \gamma^2(x_2)) \) and attaining values \( \pm \frac{L_i}{2} \) as maximum and minimum values, respectively.

After process of tolerance modelling for zero internal heat sources, three differential equations for unknown functions take the form

\[
\left( k^{eff}_{\alpha \alpha} \partial_{x_i} \gamma^\alpha \right) - \left( c^0 \partial_{x_i} \partial_{x_i} \gamma^\alpha \right) \psi^\alpha = 0,
\]

(3)

\[
\left( k^{eff}_{11} \partial_{y_1} \gamma^1 \right) \psi^1 + \left( k^{eff}_{12} \partial_{y_1} \gamma^2 \right) \psi^2 = 0,
\]

(4)

\[
\left( k^{eff}_{22} \partial_{y_2} \gamma^1 \right) \psi^1 + \left( k^{eff}_{12} \partial_{y_2} \gamma^2 \right) \psi^2 = 0.
\]

(5)

Equations (3) and (4) imply that

\[
\psi^1 = \frac{\left( k^{eff}_{11} \partial_{y_1} \gamma^1 \right) \psi^1}{\left( k^{eff}_{11} \gamma^1 \right)^2} \partial_{y_1} \gamma^1, \quad \psi^2 = \frac{\left( k^{eff}_{22} \partial_{y_2} \gamma^2 \right) \psi^2}{\left( k^{eff}_{22} \gamma^2 \right)^2} \partial_{y_2} \gamma^2.
\]

(6)

Substituting Eqn (6) into Eqn (3), equation for \( \tilde{\Theta}(x) \) takes the form

\[
k^{eff}_{11} \partial_{x_1}^2 \gamma^1 + k^{eff}_{22} \partial_{x_2}^2 \gamma^2 = 0
\]

(7)

where \( k^{eff}_{11}, k^{eff}_{22} \) are effective conductivities coefficients, Ref.[4].

4. Solutions

The boundary value problem for \( \tilde{\Theta}(x) \) with boundary conditions \( \tilde{\Theta}(x_1, 0) = f_1(x_1), \tilde{\Theta}(x_1, L_2) = f_2(x_1), \tilde{\Theta}(0, x_2) = g_1(x_2), \)

\( \tilde{\Theta}(L_1, x_2) = g_2(x_2) \) is Dirichlet problem.

Using the method of separation of variables, a solution to the problem is given by formula

\[
\tilde{\Theta}(x_1, x_2) = \frac{2}{L_1} \sum_{n_1=1}^{\infty} \left[ A_n \sinh \left( \frac{n \pi (L_1 - x_1)}{L_1} \right) + B_n \sin \left( \frac{n \pi x_1}{L_1} \right) \right] \cdot

\sin \left( \frac{n \pi x_2}{L_2} \right) + \left. \sin \left( \frac{n \pi L_1}{L_1} \right) \right|^{t_1} +

\frac{2}{L_2} \sum_{n_2=1}^{\infty} \left[ C_n \sinh \left( \frac{n \pi (L_2 - x_2)}{L_2} \right) + D_n \sin \left( \frac{n \pi x_2}{L_2} \right) \right] \cdot

\sin \left( \frac{n \pi x_2}{L_2} \right) + \left. \sin \left( \frac{n \pi L_2}{L_2} \right) \right|^{t_2}.
\]

(8)

where \( k = \frac{k^{eff}_{11}}{k^{eff}_{22}}, \ A_n = \int_0^{t_1} f_1(x_1) \frac{n \pi x_1}{L_1} dx_1, \ B_n = \int_0^{t_1} f_2(x_1) \frac{n \pi x_1}{L_1} dx_1, \)

\( C_n = \int_0^{t_2} g_1(x_2) \frac{n \pi x_2}{L_2} dx_2, \ D_n = \int_0^{t_2} g_2(x_2) \frac{n \pi x_2}{L_2} dx_2. \)

Substituting Eqn (8) into Eqn (6) and (2) the distribution of temperature is obtained.

5. Conclusions

Equations (2), (6) and (8) give the possibility of finding the distribution of temperature field \( \Theta(x) \) for special biperiodically composites.

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The book includes the entirety of contributions to PCM-CMM-2015 CONGRESS, SEPTEMBER 8 - 11, 2015, Gdańsk (Poland).

The PCM-CMM-2015 CONGRESS is a joint scientific event of:
• the 3rd Polish Congress of Mechanics (PCM)
• the 21st International Conference on Computer Methods in Mechanics (CMM)

The idea of a "Polish Congress of Mechanics" was firstly suggested in 2005 by the Polish Society of Theoretical and Applied Mechanics. The scope was intended to cover the whole range of problems of theoretical, experimental and computational mechanics as well as interdisciplinary issues, including industrial applications.

The 21st International Conference on Computer Methods in Mechanics continues the 44-year series of conferences dedicated to numerical methods and their applications to the mechanics-based problems. The meetings, organized biennially since 1973 provide a forum for presentation and discussion of new ideas referring to the theoretical background and practical applications of computational mechanics.

Both events - the 3rd Polish Congress of Mechanics (PCM) and the 21st Conference on Computer Methods in Mechanics (CMM) - are aimed at presenting current state-of-the-art research in the field of mechanics and providing a wide forum for discussion of new ideas on theoretical background, new technologies and computational methods in a vast domain of mechanics and related disciplines.

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